

Introduction to time-series commands in Stata

May 11th, 2023



Topics

Part I: Univariate time-series analysis

- Working with dates in Stata
- Declaring time-series data with **tsset**
- Plotting with **tsline**
- Testing parameter stability with **estat cusum**
- Smoothing with **tssmooth**
- Testing for unit roots with **dfuller**
- Difference and lag operators
- Visualizing autocorrelation with **ac** and **pac**
- Box-Jenkins models with **arima**
- Heteroskedasticity models with **arch**
- Forecasting with **predict**

Part II: Multivariate time-series analysis

- Business calendars with **bcal**
- Vector autoregression with **var** and **svar**
- Impulse-response functions with **irf**

Part I. Univariate time-series analysis

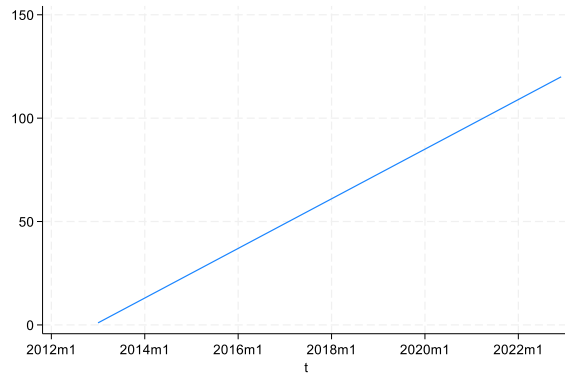
Components of a time series

$$y_t = trend_t + seasonality_t + residual_t$$

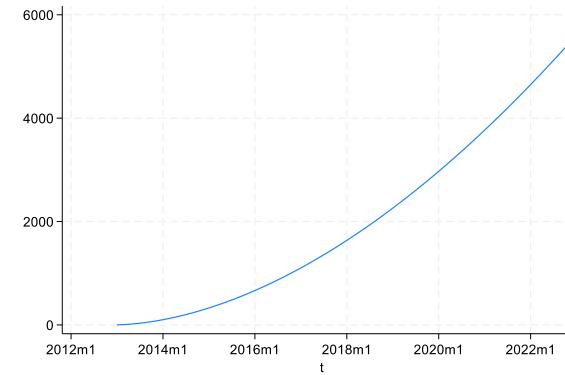
or

$$x_t = trend_t \times seasonality_t \times residual_t$$

Components of a time series

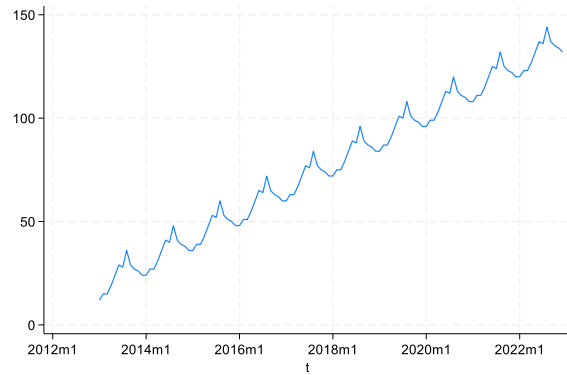


Additive trend



Multiplicative trend

Components of a time series

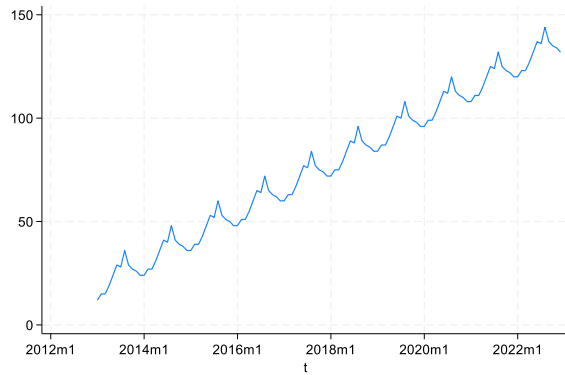


Additive trend
Additive seasonality



Multiplicative trend
Additive seasonality

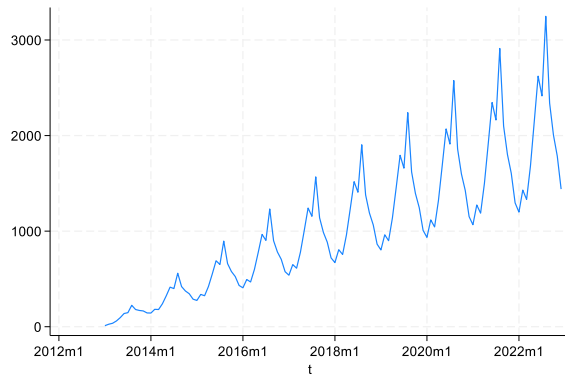
Components of a time series



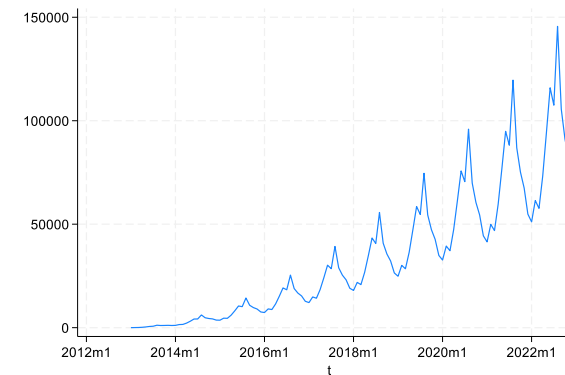
Additive trend
Additive seasonality



Multiplicative trend
Additive seasonality

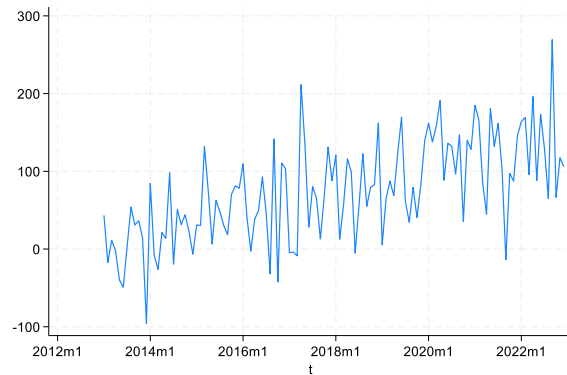


Additive trend
Multiplicative seasonality

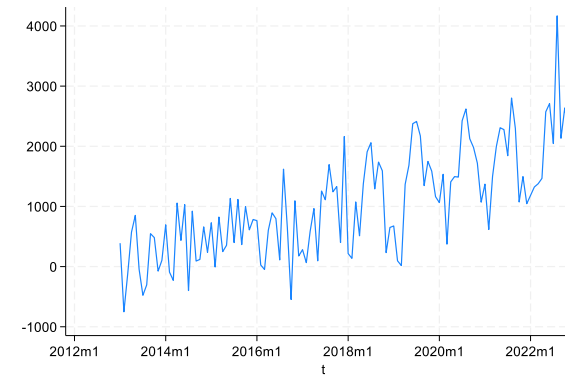


Multiplicative trend
Multiplicative seasonality

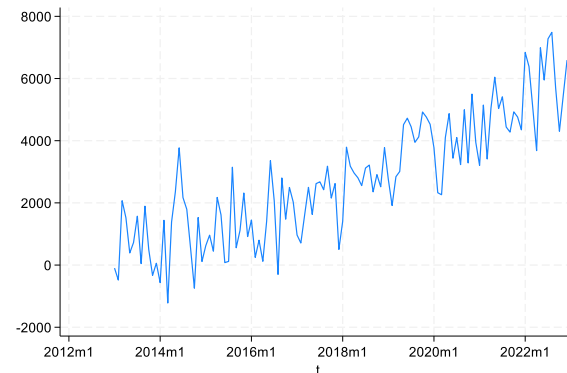
Components of a time series



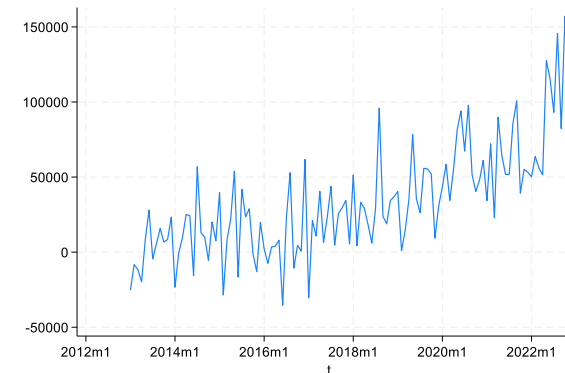
Additive trend
Additive seasonality
+ random noise



Multiplicative trend
Additive seasonality
+ random noise

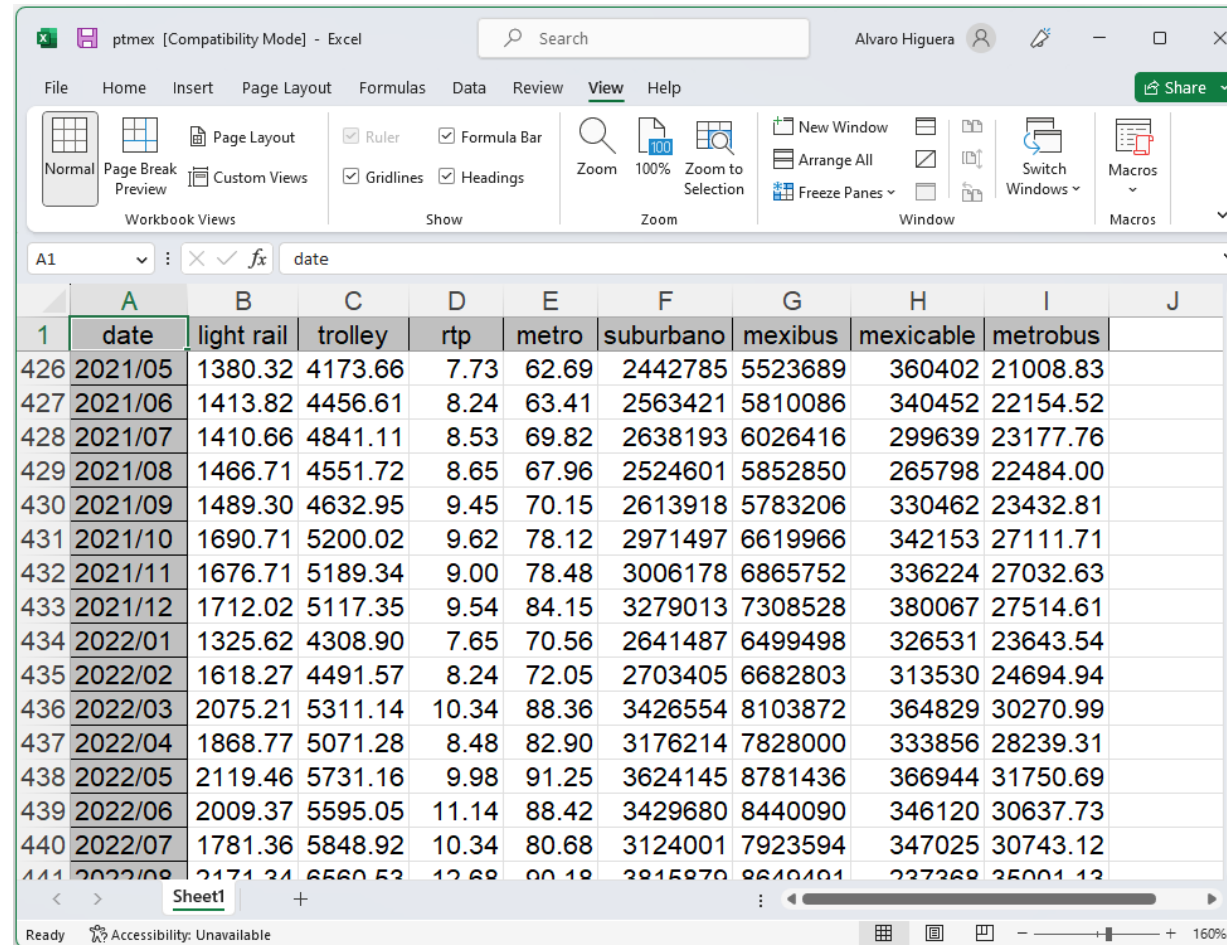


Additive trend
Multiplicative seasonality
+ random noise



Multiplicative trend
Multiplicative seasonality
+ random noise

Public transport dataset



The screenshot shows an Excel spreadsheet titled 'ptmex [Compatibility Mode] - Excel'. The 'View' tab is active in the ribbon. The spreadsheet contains a table with the following data:

	A	B	C	D	E	F	G	H	I	J
1	date	light rail	trolley	rtp	metro	suburbano	mexibus	mexicable	metrobus	
426	2021/05	1380.32	4173.66	7.73	62.69	2442785	5523689	360402	21008.83	
427	2021/06	1413.82	4456.61	8.24	63.41	2563421	5810086	340452	22154.52	
428	2021/07	1410.66	4841.11	8.53	69.82	2638193	6026416	299639	23177.76	
429	2021/08	1466.71	4551.72	8.65	67.96	2524601	5852850	265798	22484.00	
430	2021/09	1489.30	4632.95	9.45	70.15	2613918	5783206	330462	23432.81	
431	2021/10	1690.71	5200.02	9.62	78.12	2971497	6619966	342153	27111.71	
432	2021/11	1676.71	5189.34	9.00	78.48	3006178	6865752	336224	27032.63	
433	2021/12	1712.02	5117.35	9.54	84.15	3279013	7308528	380067	27514.61	
434	2022/01	1325.62	4308.90	7.65	70.56	2641487	6499498	326531	23643.54	
435	2022/02	1618.27	4491.57	8.24	72.05	2703405	6682803	313530	24694.94	
436	2022/03	2075.21	5311.14	10.34	88.36	3426554	8103872	364829	30270.99	
437	2022/04	1868.77	5071.28	8.48	82.90	3176214	7828000	333856	28239.31	
438	2022/05	2119.46	5731.16	9.98	91.25	3624145	8781436	366944	31750.69	
439	2022/06	2009.37	5595.05	11.14	88.42	3429680	8440090	346120	30637.73	
440	2022/07	1781.36	5848.92	10.34	80.68	3124001	7923594	347025	30743.12	
441	2022/08	2171.34	6560.53	12.68	90.18	3815870	8610101	337368	35001.13	

Importing data

import excel - Import Excel files

Excel file:
C:\Users\afh\Desktop\tsWebinar\ptmex.xls Browse...

Worksheet:
Sheet1 A1:I444

Cell range:
A1:I444 ...

☒ Import first row as variable names
☐ Import all data as strings

Variable case:
Preserve

Preview: (showing rows 2-51 of 444)

	date	lightrail	trolley	rtp	metro	suburbano	mexibus	mexica
2	1986/01	.	.	172.0004	110.2453	.	.	.
3	1986/02	.	.	161.2996	98.3724	.	.	.
4	1986/03	.	.	157.6009	103.9895	.	.	.
5	1986/04	.	.	171.699	112.752	.	.	.
6	1986/05	.	.	165.4997	118.0573	.	.	.
7	1986/06	.	.	176.301	114.735	.	.	.

OK Cancel

```
. import excel "C:\Users\afh\Desktop\tsWebinar\ptmex.xls",  
sheet("Sheet1") firstrow
```

Date variable

Data Editor (Browse) - [Untitled]

File Edit View Data Tools

date[1] 1986/01

	date	lightrail	trolley	rtp	metro	suburbano
1	1986/01	.	.	172.0004	110.2453	.
2	1986/02	.	.	161.2996	98.3724	.
3	1986/03	.	.	157.6009	103.9895	.
4	1986/04	.	.	171.699	112.752	.
5	1986/05	.	.	165.4997	118.0573	.
6	1986/06	.	.	176.301	114.735	.
7	1986/07	.	.	168.9004	124.2883	.
8	1986/08	.	.	167.0993	120.1064	.
9	1986/09	.	.	172.599	115.131	.
10	1986/10	.	.	176.3993	120.1281	.
11	1986/11	.	.	167.4	111.69	.
12	1986/12	.	.	206.1996	108.9278	.
13	1987/01	.	.	170.2923	115.9214	.
14	1987/02	.	.	156.8	107.3016	.
15	1987/03	.	.	177.0007	121.8114	.
16	1987/04	.	.	173.499	109.899	.

Variables

Filter variables here

Name	Label	Type	Format	Value Labels
date	date	str7	%9s	
lightrail	Light Rail passengers (t...	double	%10.0g	
trolley	Trolleybus passengers (t...	double	%10.0g	
rtp	RTP passengers (millions)	double	%10.0g	
metro	Metro passengers (milli...	double	%10.0g	
suburbano	Suburbano passengers	long	%10.0g	
mexibus	Mexibus passengers	long	%10.0g	
mexicable	Mexicable passengers	long	%10.0g	
metrobuc	Metrobuc passengers (t...	double	%10.0g	

Variables Snapshots

Properties

Variables

Name	date
Label	date
Type	str7
Format	%9s
Value label	
Notes	

Data

Frame	default
Filename	
Label	
Notes	
Variables	9
Observations	443

Ready Length: 7 Vars: 9 Order: Dataset Obs: 443 Filter: Off Mode: Browse CAP NUM

Date variable format

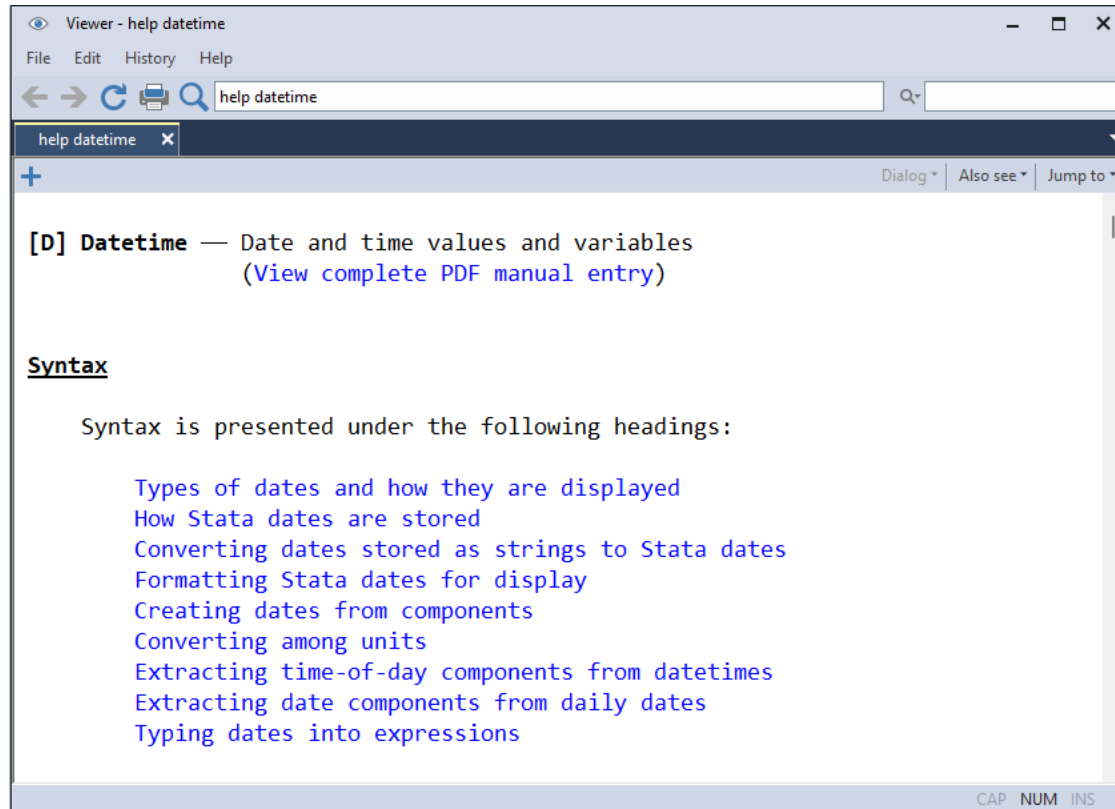
	date	t
1	1986/01	312
2	1986/02	313
3	1986/03	314
4	1986/04	315
5	1986/05	316
6	1986/06	317
7	1986/07	318
8	1986/08	319
9	1986/09	320
10	1986/10	321



	date	t
1	1986/01	1986m1
2	1986/02	1986m2
3	1986/03	1986m3
4	1986/04	1986m4
5	1986/05	1986m5
6	1986/06	1986m6
7	1986/07	1986m7
8	1986/08	1986m8
9	1986/09	1986m9
10	1986/10	1986m10

```
. generate t = monthly(date, "YM")  
. format t %tm
```

Date variables help



```
. help datetime
```

Declaring time-series data

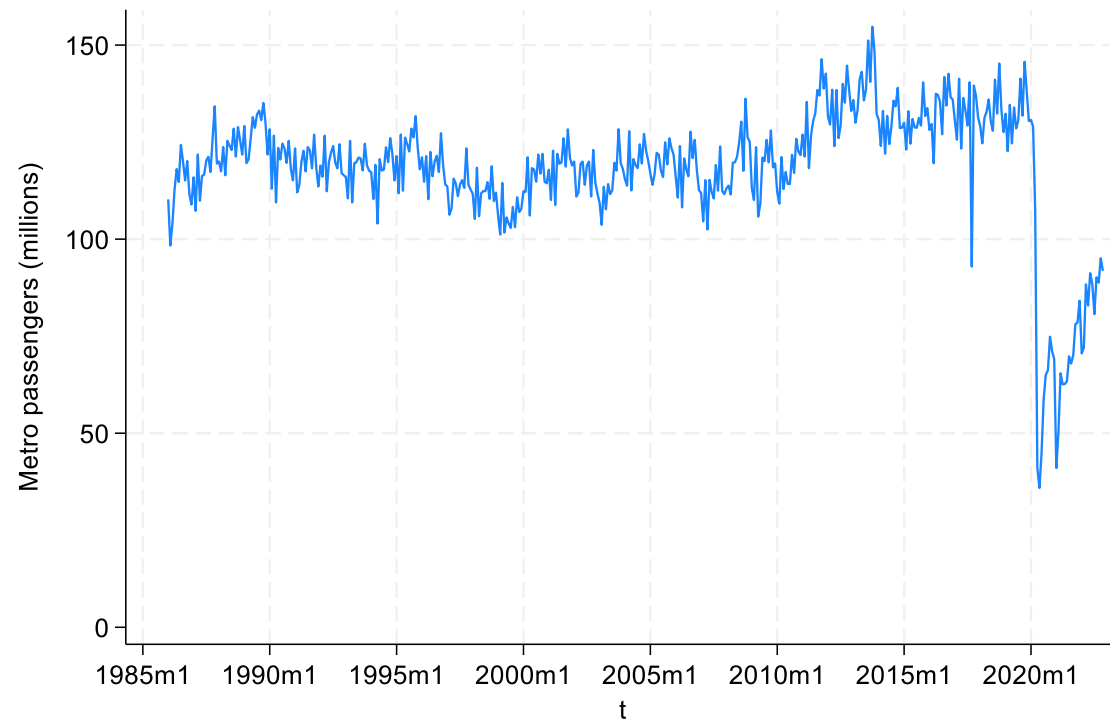
```
. tsset t
```

```
Time variable: t, 1986m1 to 2022m11  
Delta: 1 month
```


Metro

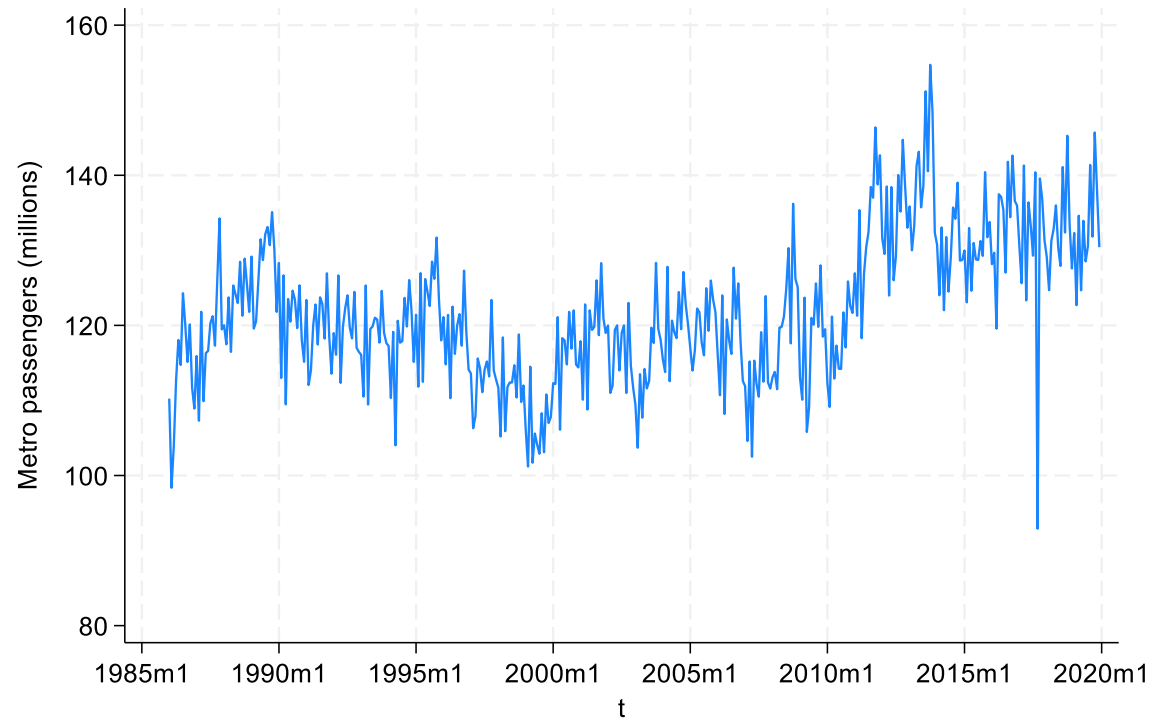


Metro: Time-series line plot



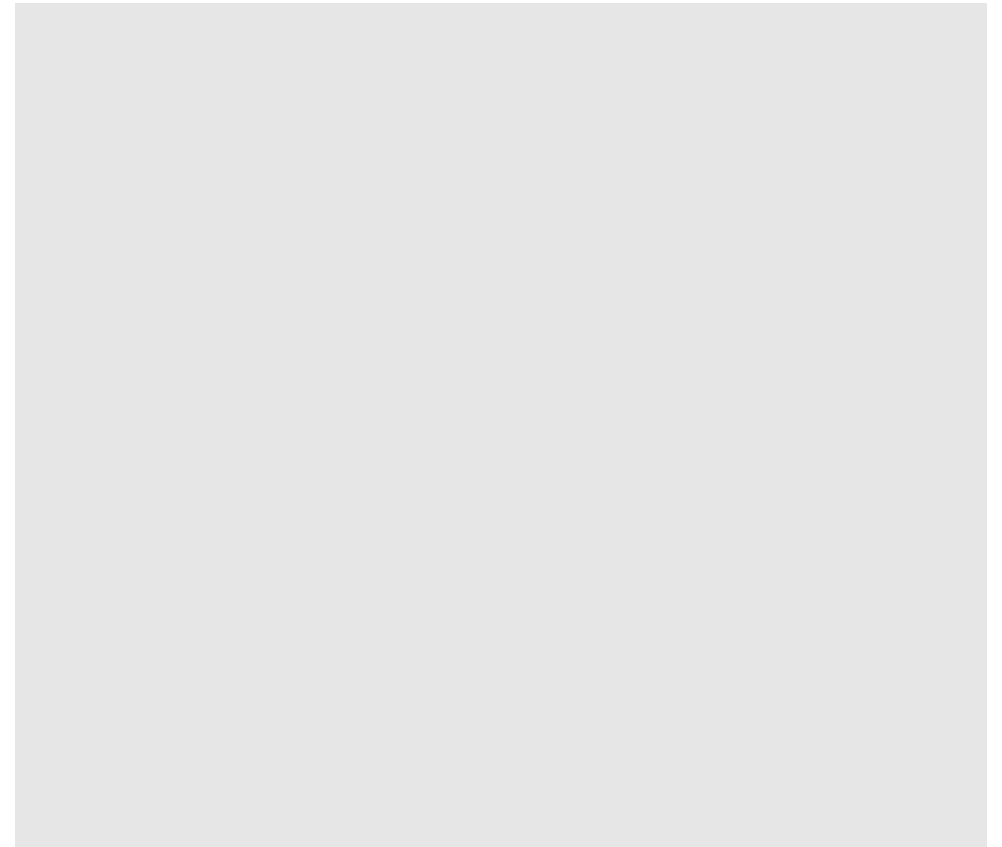
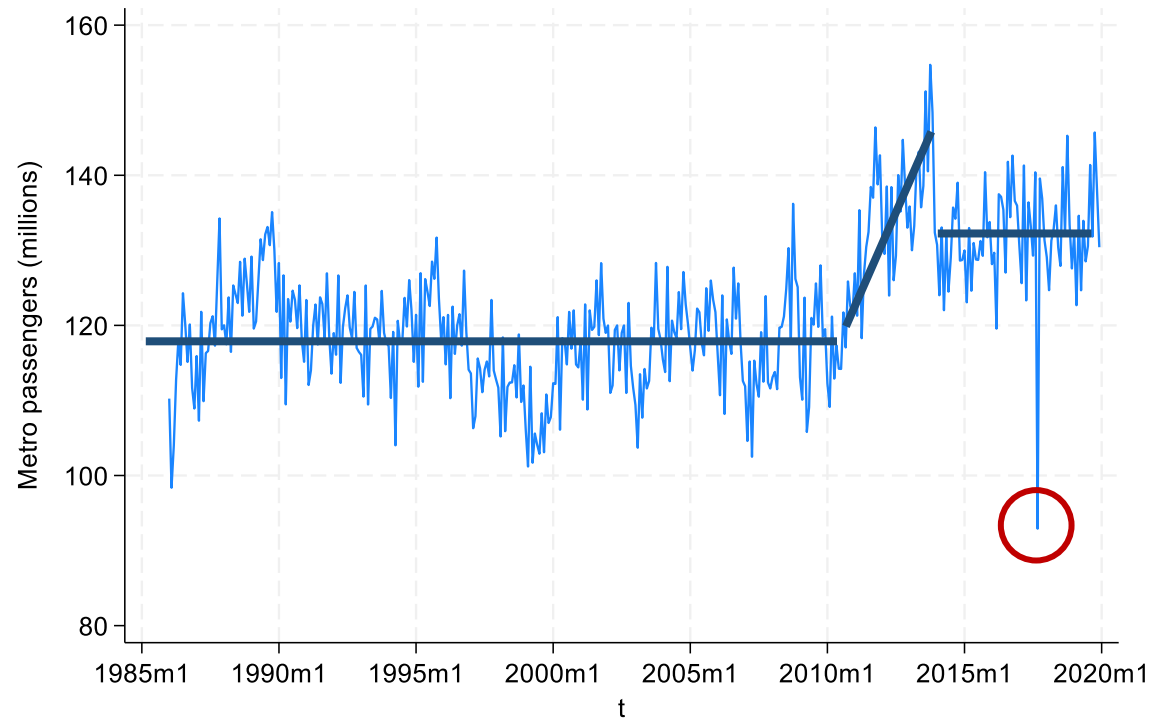
```
. tsline metro
```


Metro: Removing the pandemic



```
. keep if tin(1986m1, 2019m12)  
. tsline metro
```

Metro: First impression



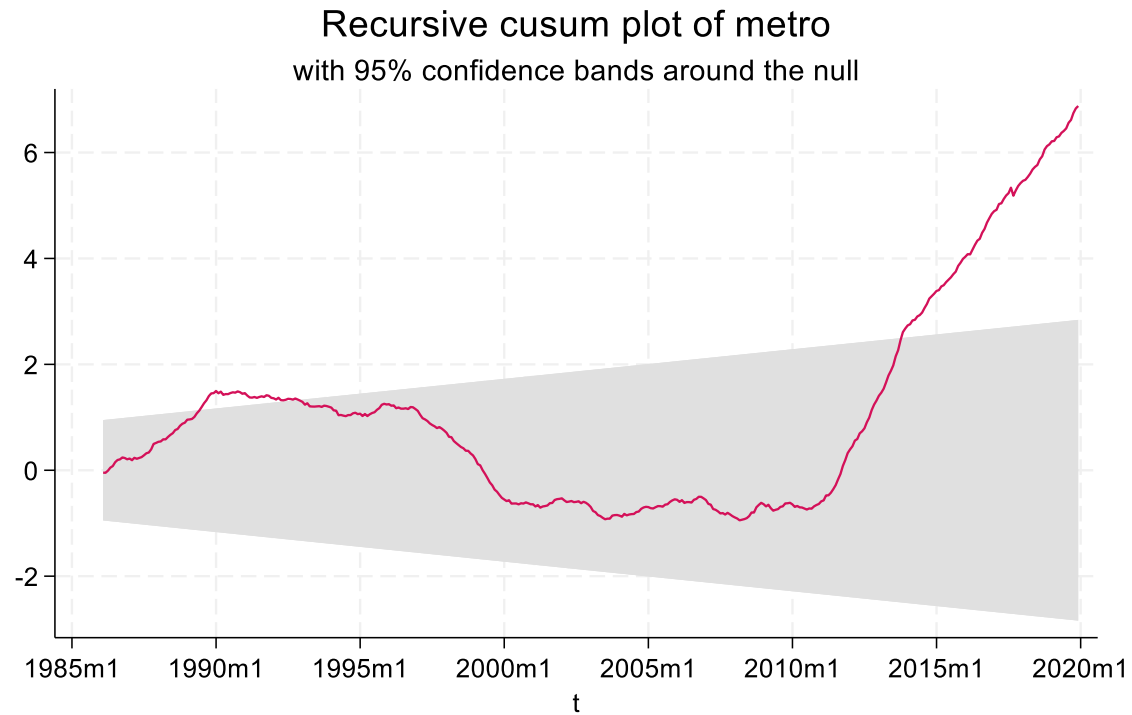
Metro: Parameter stability

Source	SS	df	MS	Number of obs	=	408
Model	0	0	.	F(0, 407)	=	0.00
Residual	38300.6572	407	94.1048088	Prob > F	=	.
				R-squared	=	0.0000
				Adj R-squared	=	0.0000
Total	38300.6572	407	94.1048088	Root MSE	=	9.7008

metro	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
_cons	121.8943	.4802593	253.81	0.000	120.9502	122.8384

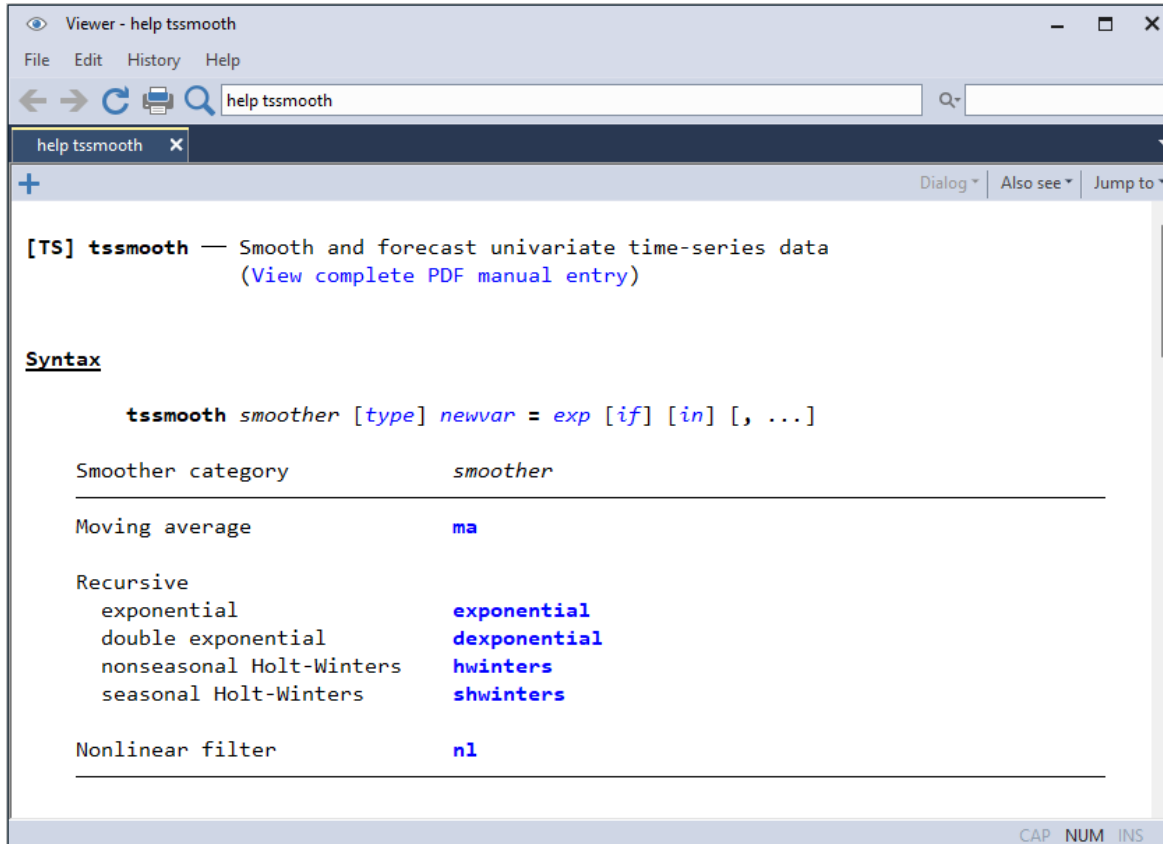
```
. regress metro
```

Metro: Parameter stability



```
. estat sbcusum
```

Smoothers in Stata



The screenshot shows the Stata help window for the `tssmooth` command. The window title is "Viewer - help tssmooth". The search bar contains "help tssmooth". The help text describes the command as "Smooth and forecast univariate time-series data" and includes a link to the "complete PDF manual entry". The "Syntax" section shows the command format: `tssmooth smoother [type] newvar = exp [if] [in] [, ...]`. Below this is a table of smoother categories and their corresponding command names.

[TS] **tssmooth** — Smooth and forecast univariate time-series data
([View complete PDF manual entry](#))

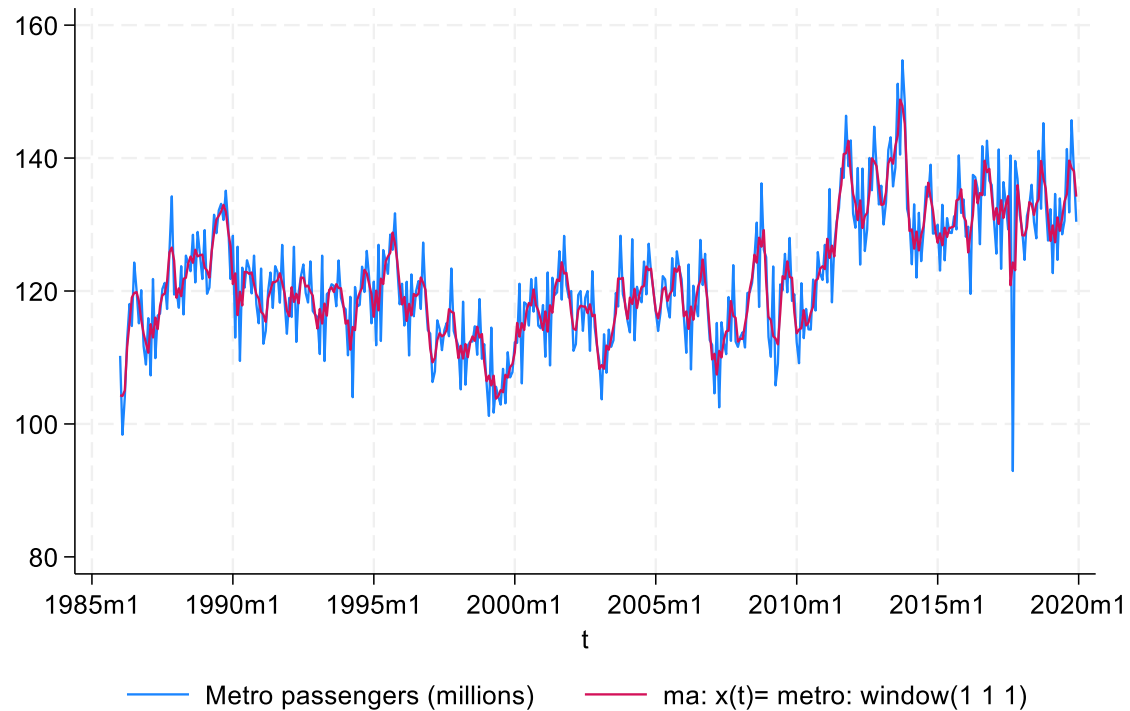
Syntax

tssmooth *smoother* [*type*] *newvar* = *exp* [*if*] [*in*] [, ...]

Smoother category	<i>smoother</i>
Moving average	ma
Recursive	
exponential	exponential
double exponential	dexponential
nonseasonal Holt-Winters	hwinters
seasonal Holt-Winters	shwinters
Nonlinear filter	nl

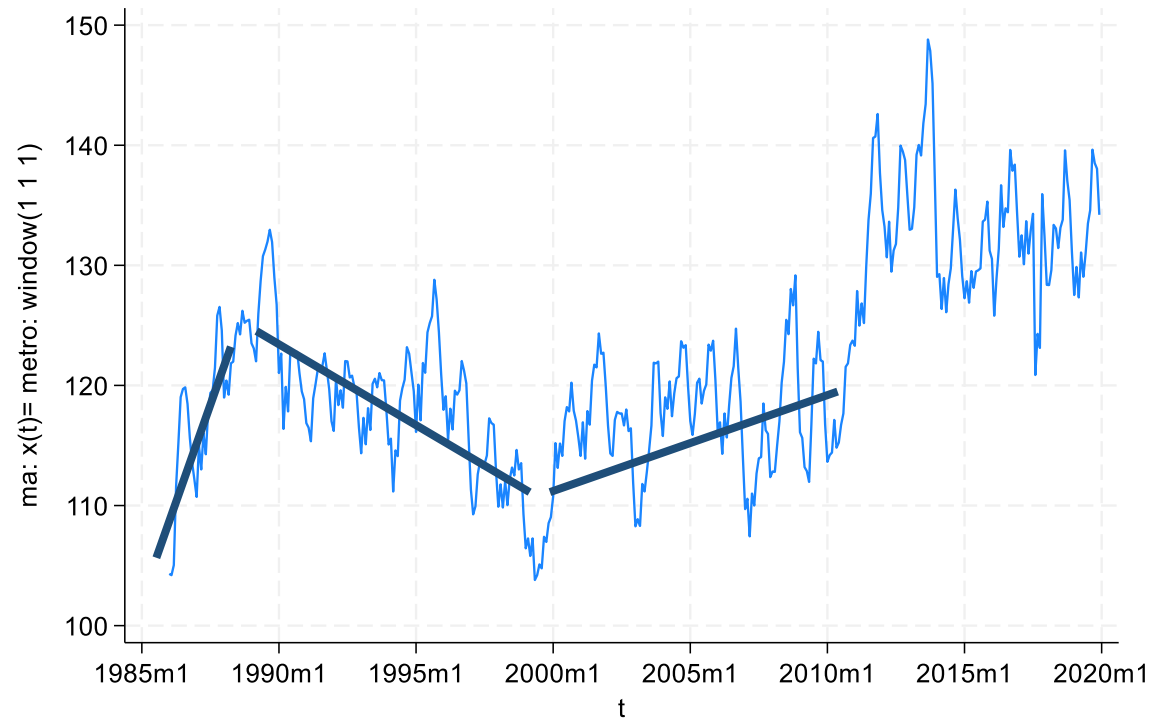
```
. help tssmooth
```

Metro: Moving average (1 1 1) smoother



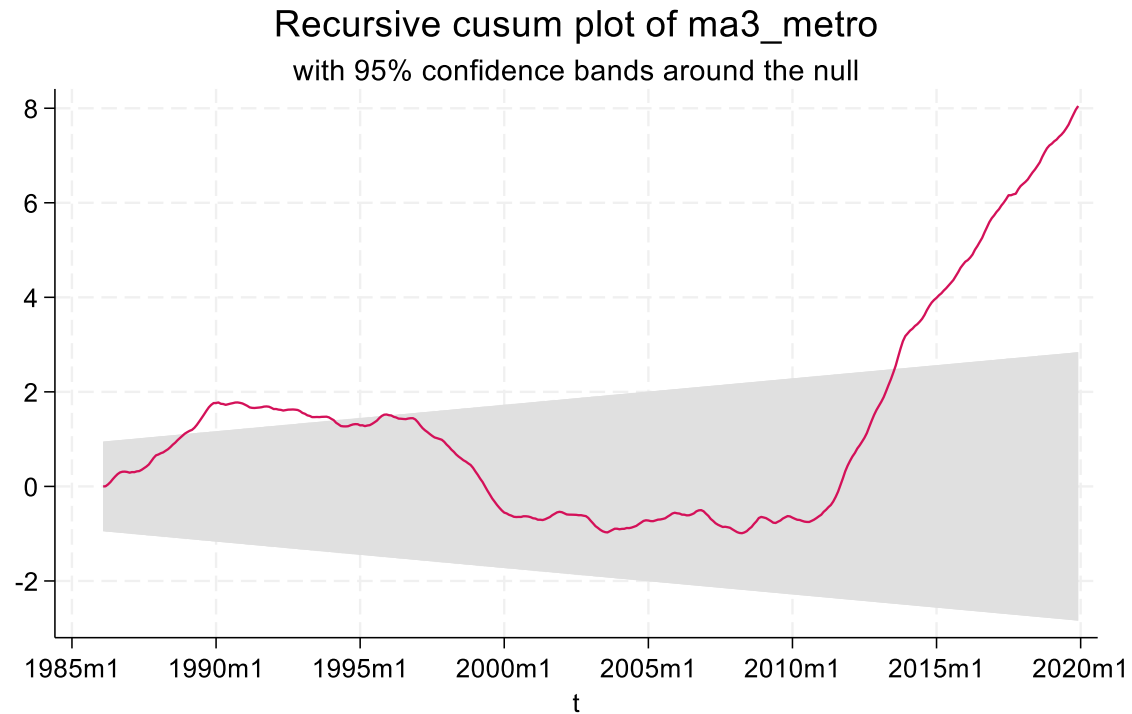
```
. tssmooth ma ma3_metro = metro,  
    window(1 1 1)  
  
. tsline metro ma3_metro
```

Metro: Moving average (1 1 1) smoother



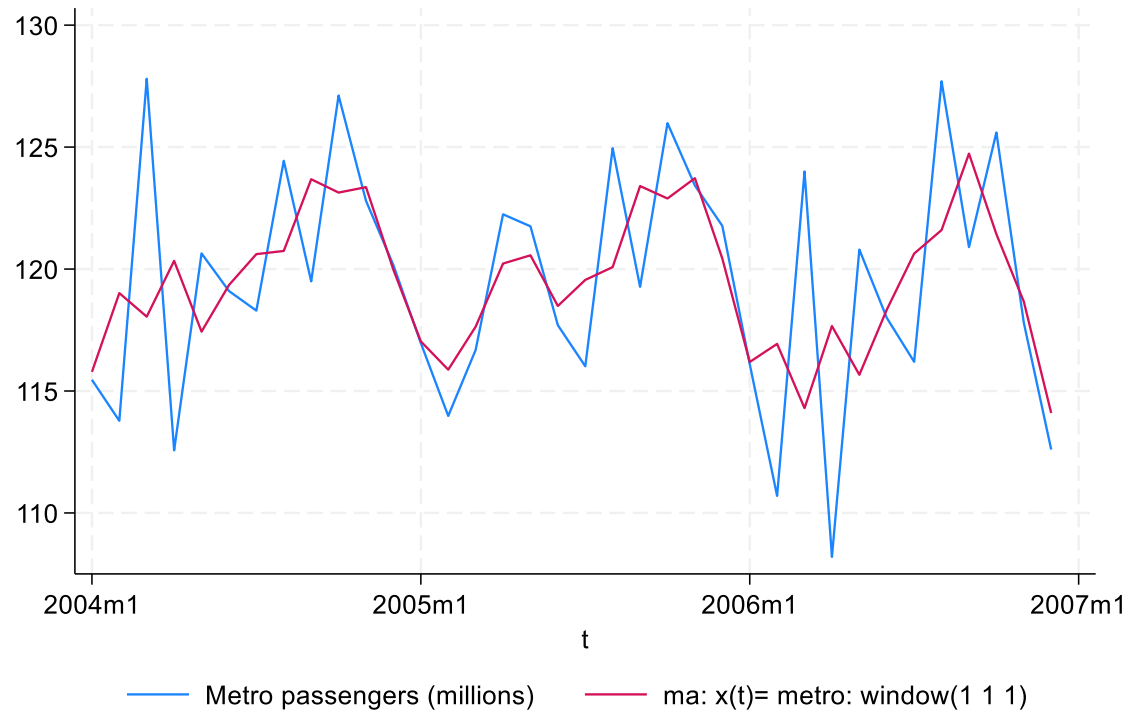
```
. tsline ma3_metro
```

Metro: Moving average (1 1 1) smoother



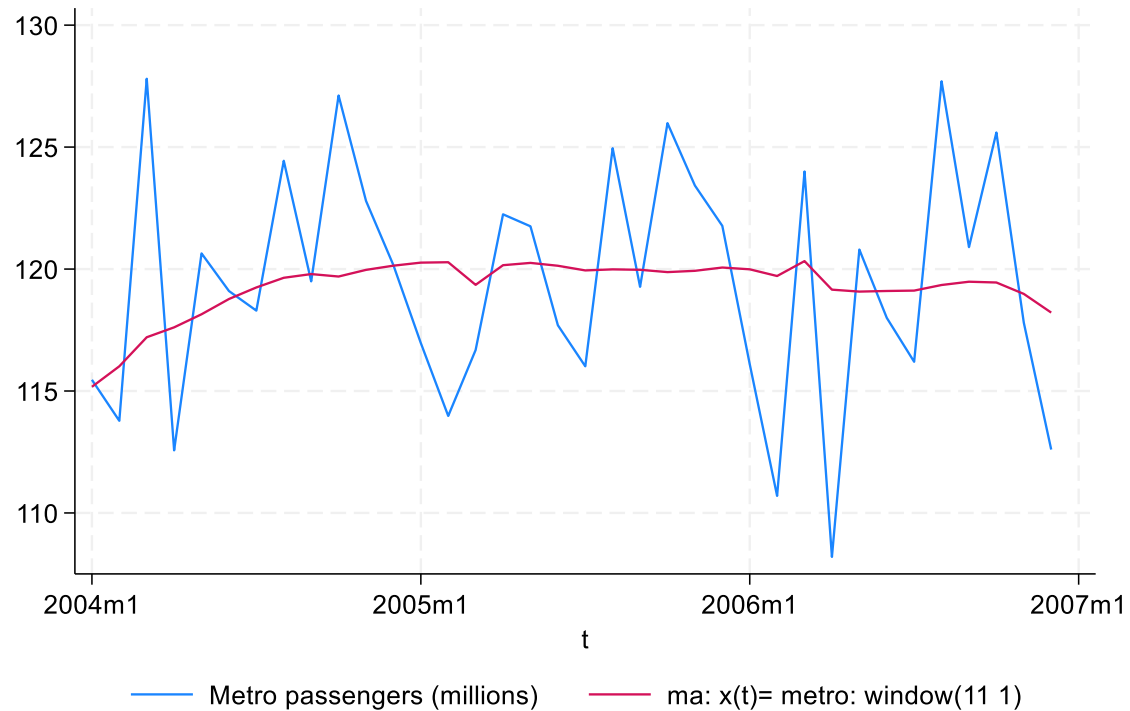
```
. regress ma3_metro  
. estat sbcusum
```


Metro: Seasonality



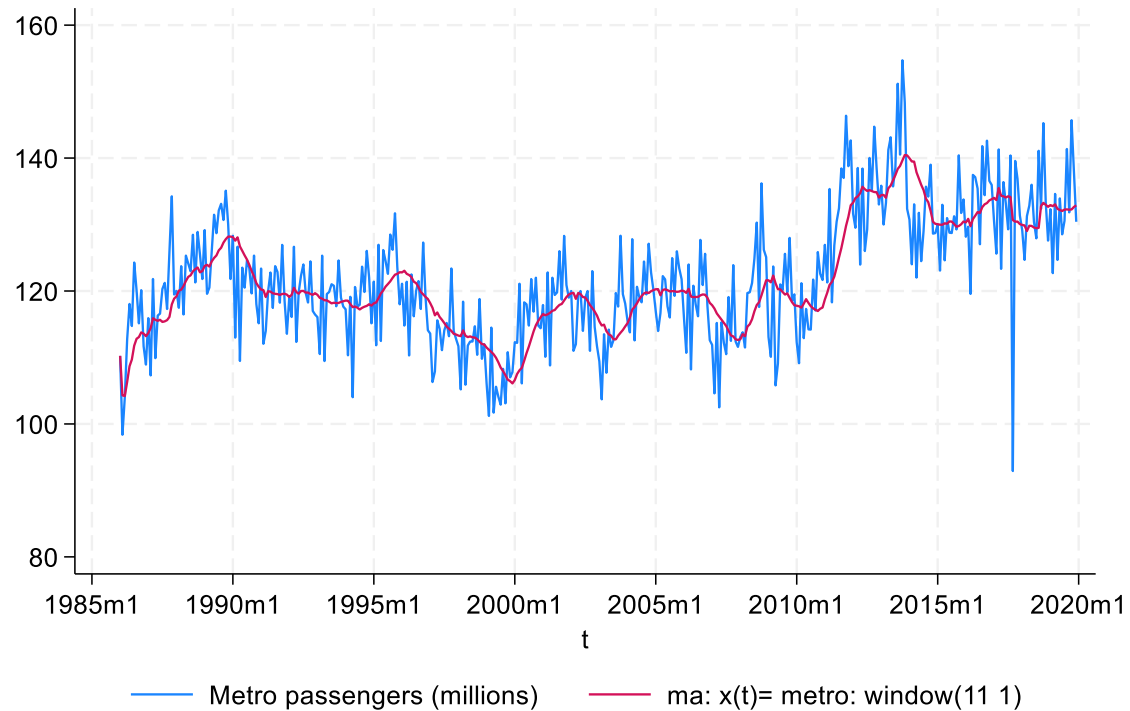
```
. tsline metro ma3_metro if tin(2004m1,  
2006m12)
```

Metro: Moving average (11 1) smoother



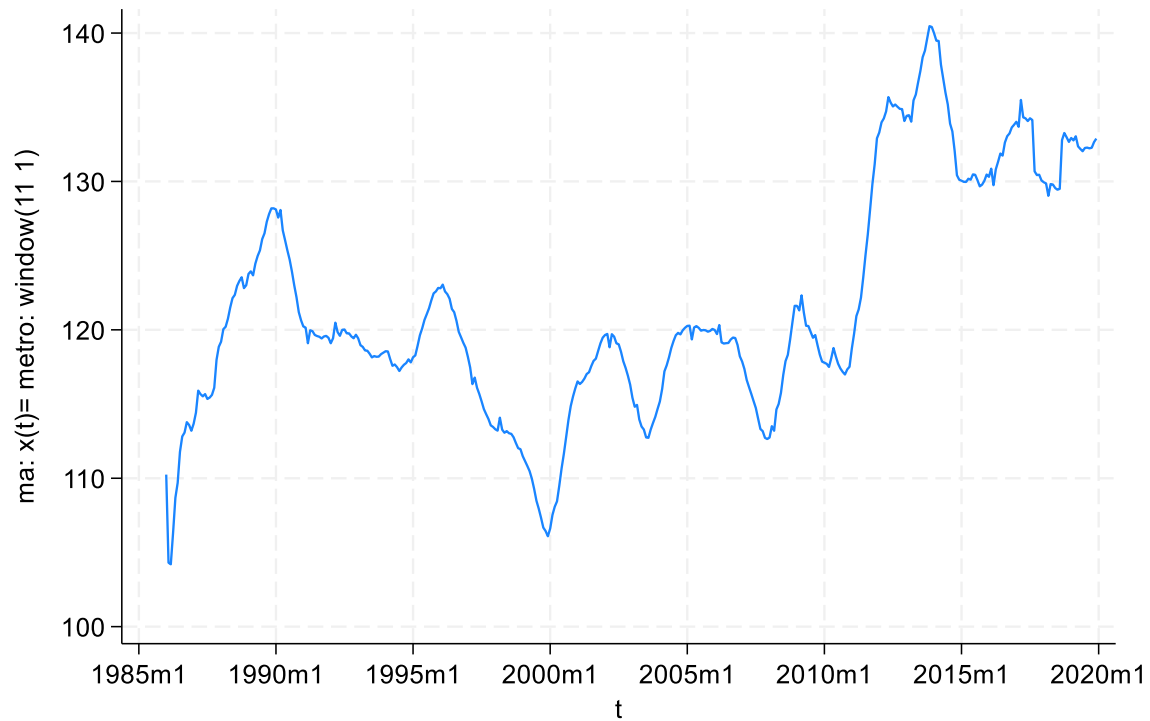
```
. tssmooth ma ma12_metro = metro,  
    window(11 1)  
  
. tsline metro ma12_metro if  
    tin(2004m1, 2006m12)
```

Metro: Moving average (11 1) smoother



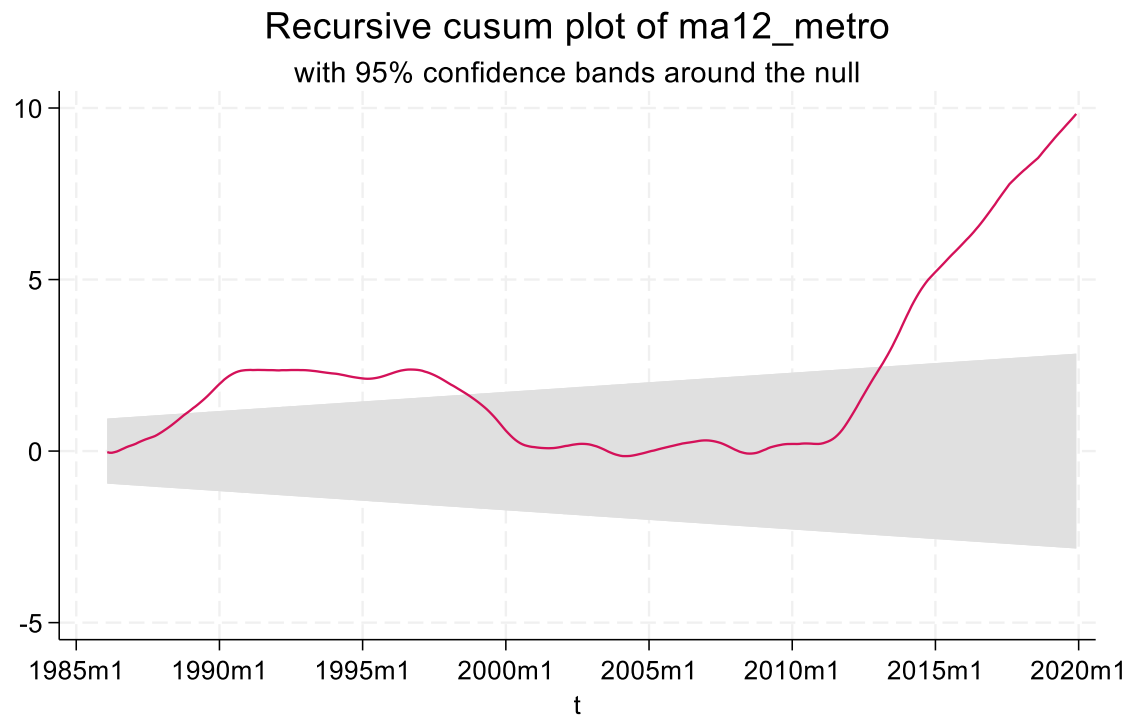
```
. tsline metro ma12_metro
```

Metro: Moving average (11 1) smoother



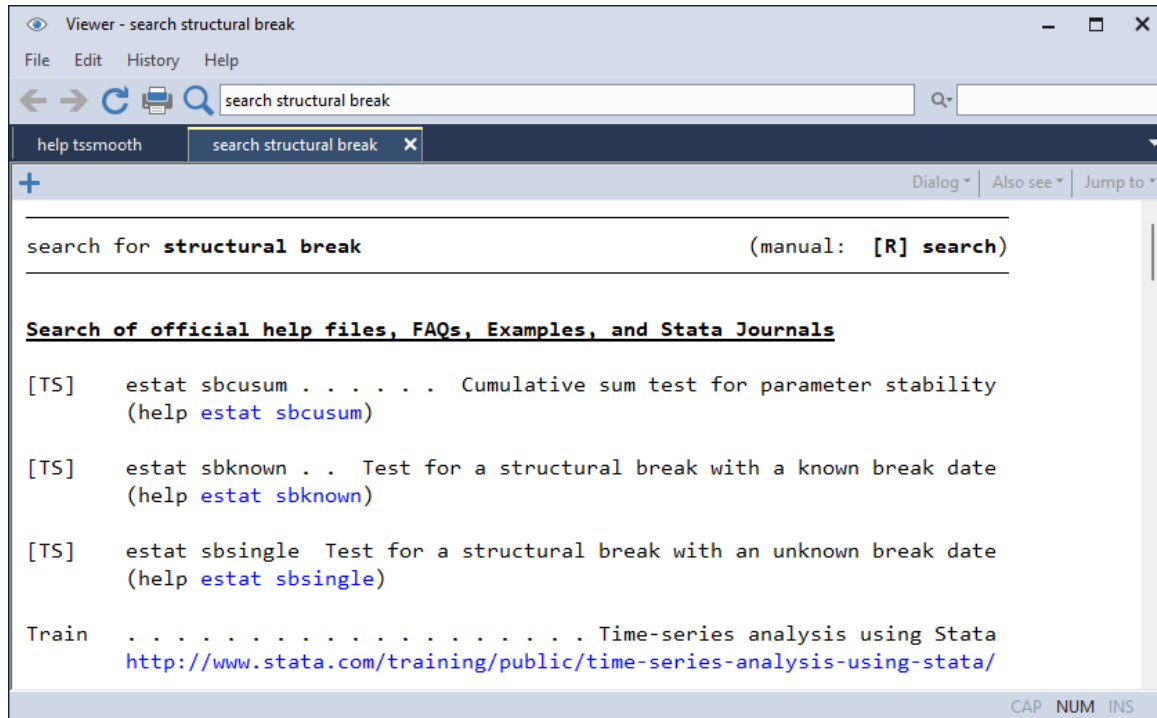
```
. tsline ma12_metro
```

Metro: Moving average (11 1) smoother



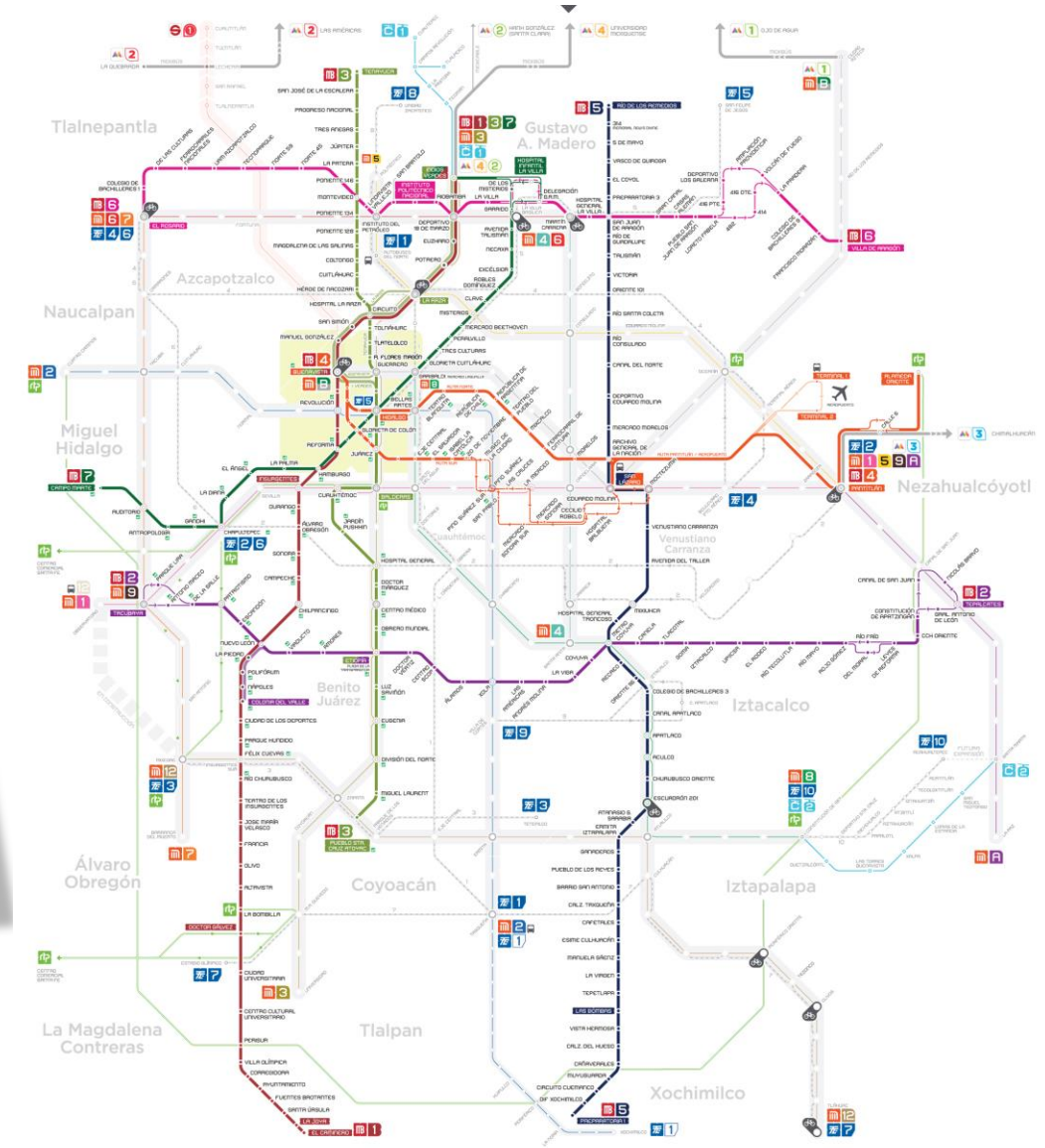
```
. regress ma12_metro  
. estat sbcusum
```

Other structural break tests

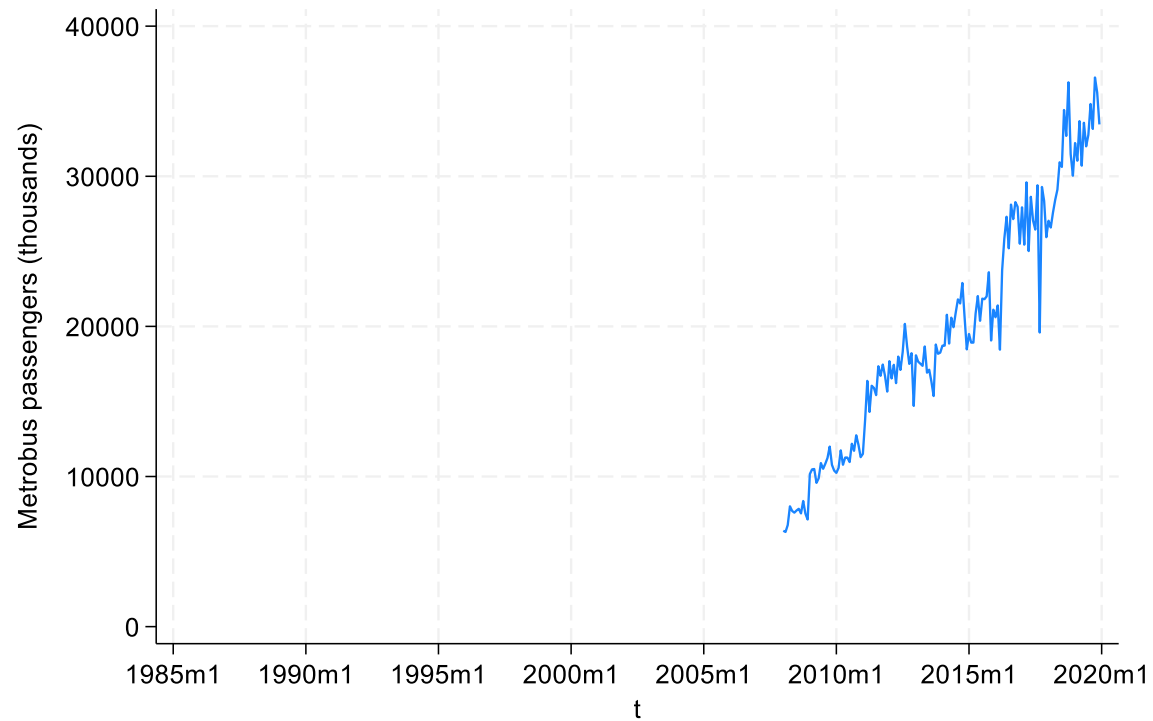


```
. search structural break
```

Metrobus

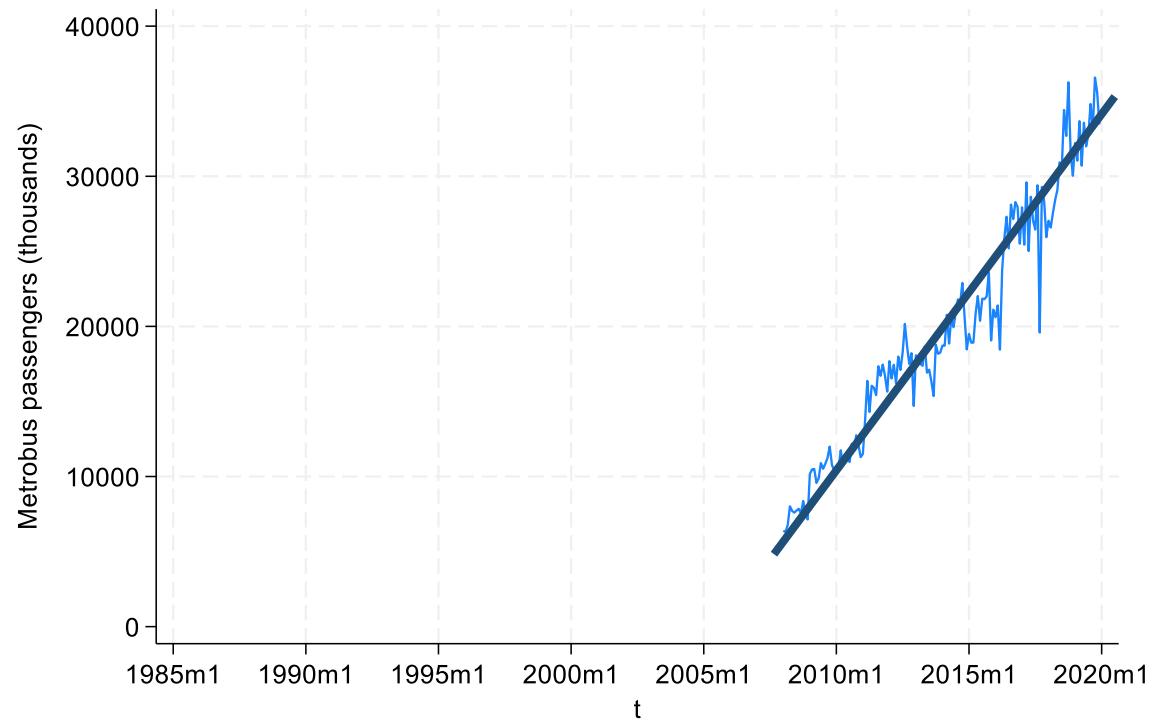


Metrobus: Time series line plot

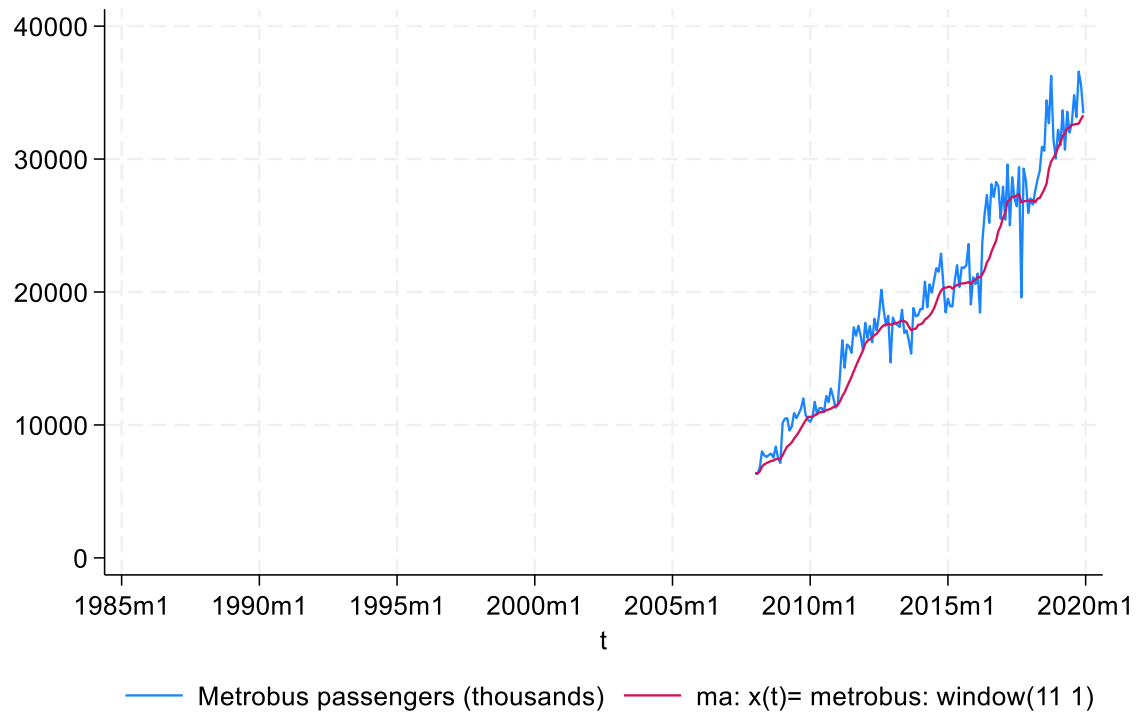


```
. tsline metrobus
```


Metrobus: Time series line plot

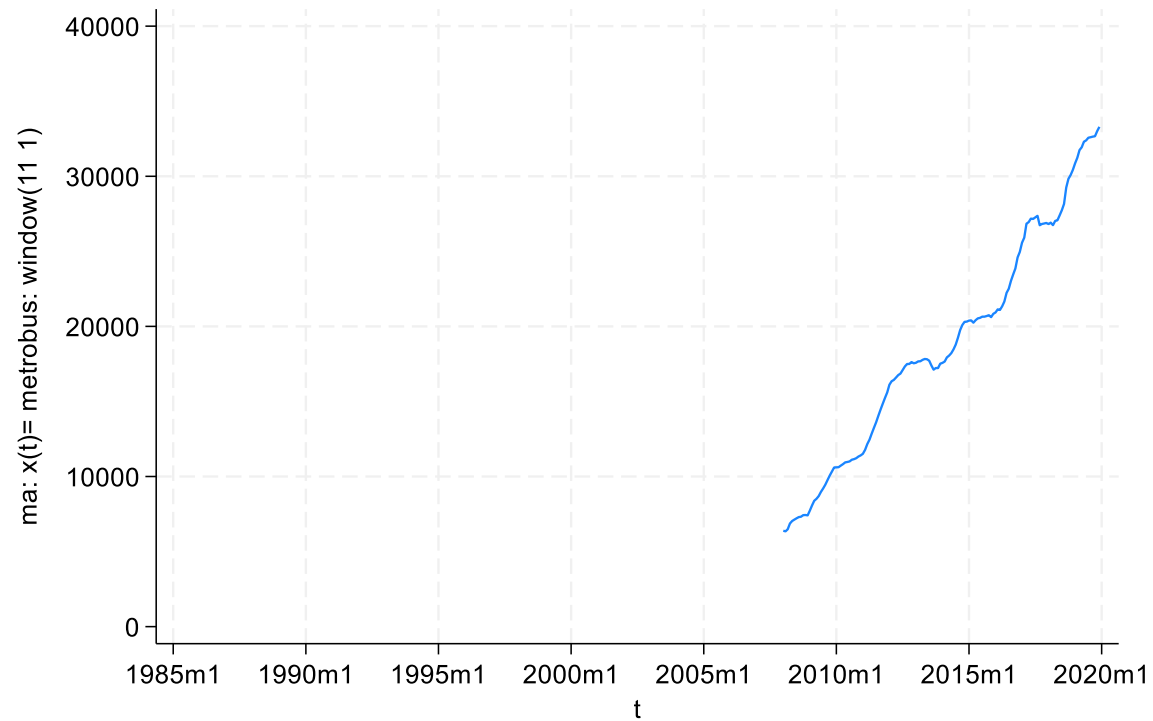


Metrobus: ma window(11 1) smoother



```
. tssmooth ma ma12_metrobus = metrobus,  
window(11 1)  
  
. tsline metrobus ma12_metrobus
```

Metrobus: ma window(11 1) smoother



```
. tsline ma12_metrobus
```

Metrobus: Unit-root test

Dickey-Fuller test for unit root Number of obs = 143
Variable: metrobus Number of lags = 0

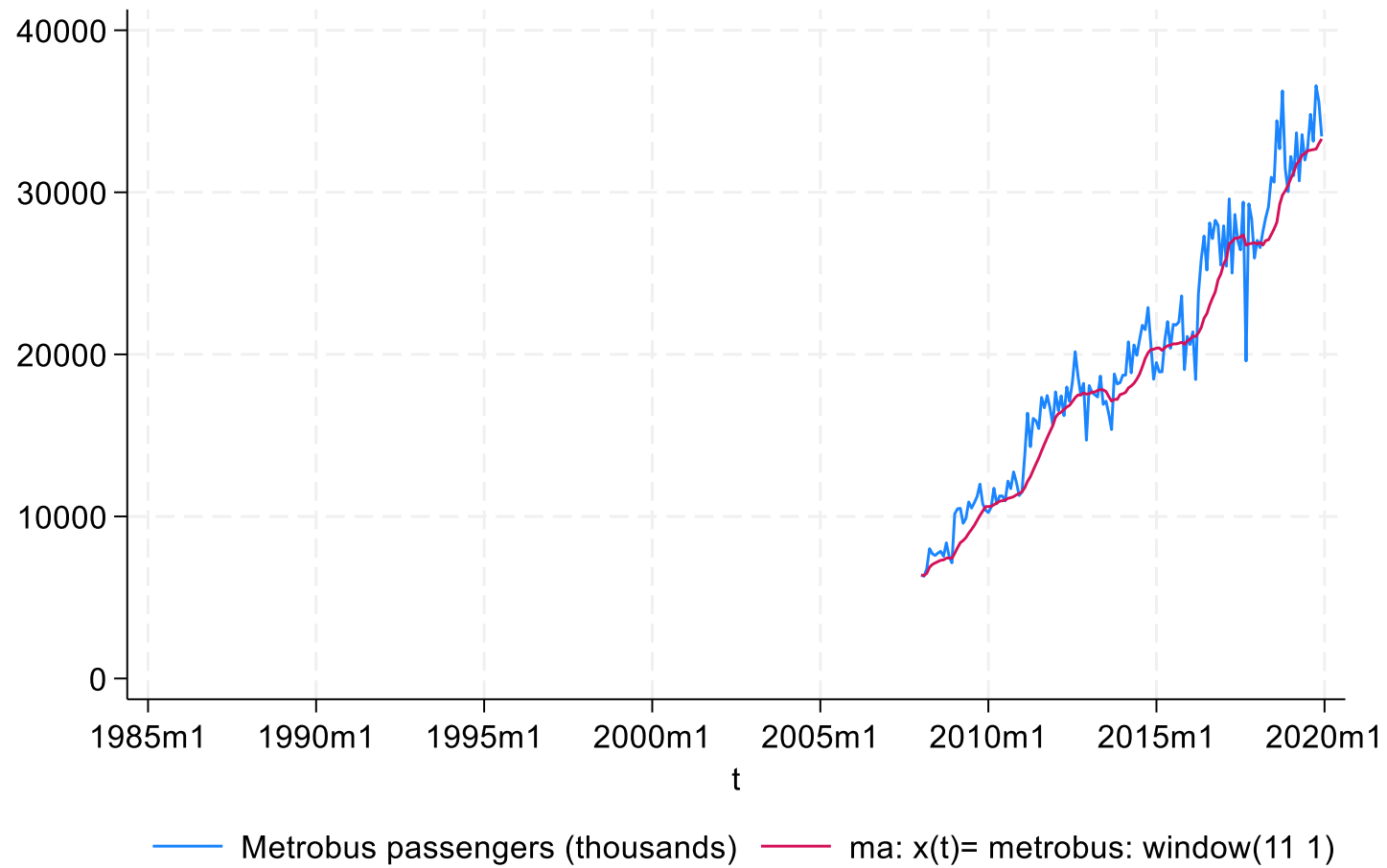
H0: Random walk without drift, $d = 0$

Test statistic	Dickey-Fuller critical value			
	1%	5%	10%	
Z(t)	-1.576	-3.496	-2.887	-2.577

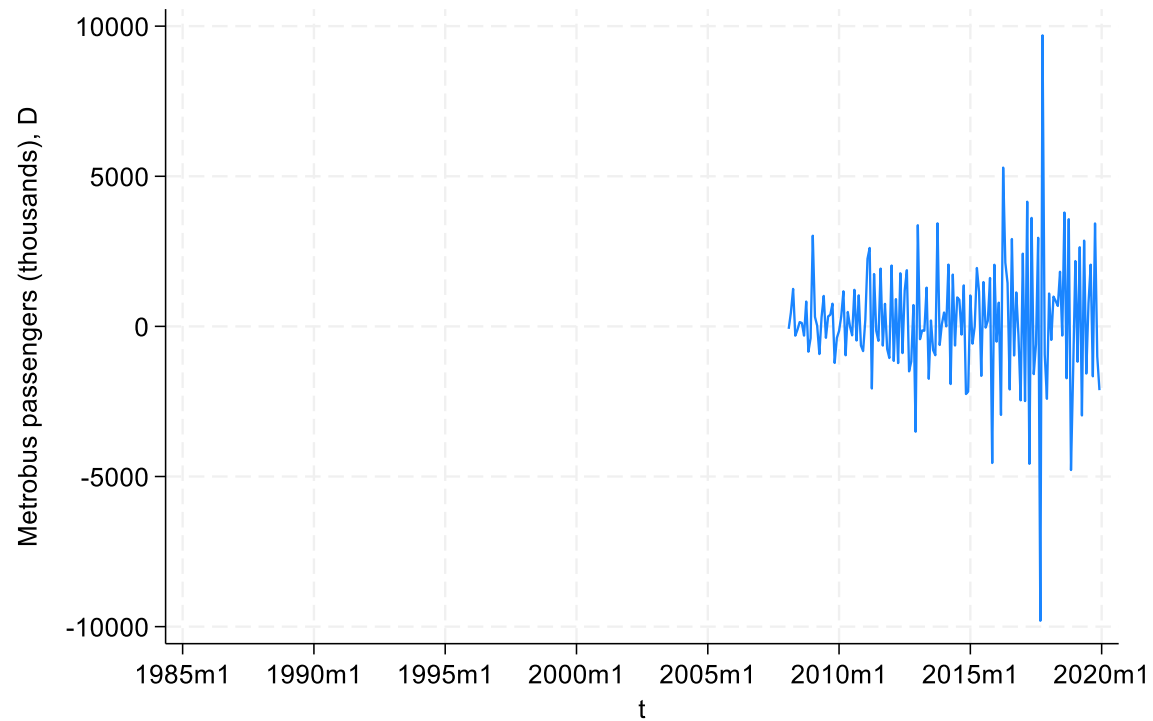
MacKinnon approximate p -value for Z(t) = 0.4956.

```
. dfuller metrobus
```

Metrobus: Unit-root test



Metrobus: First differences



```
. tsline D.metrobus
```

Aside: Difference and lag operators

Differences

D.variable
D2.variable
etc...

Lags

L.variable
L2.variable
etc...

Leads

F.variable
F2.variable
etc...

Seasonal difference

S4.variable
S12.variable

Metrobus: Unit-root test

Dickey-Fuller test for unit root Number of obs = 143
Variable: metrobus Number of lags = 0

H0: Random walk with or without drift

Test statistic	Dickey-Fuller critical value		
	1%	5%	10%
Z(t)	-7.560	-4.026	-3.444

MacKinnon approximate *p*-value for Z(t) = 0.0000.

```
. dfuller metrobus, trend
```


Metrobus: Unit-root test

Dickey-Fuller test for unit root Number of obs = 142
Variable: D.metrobus Number of lags = 0

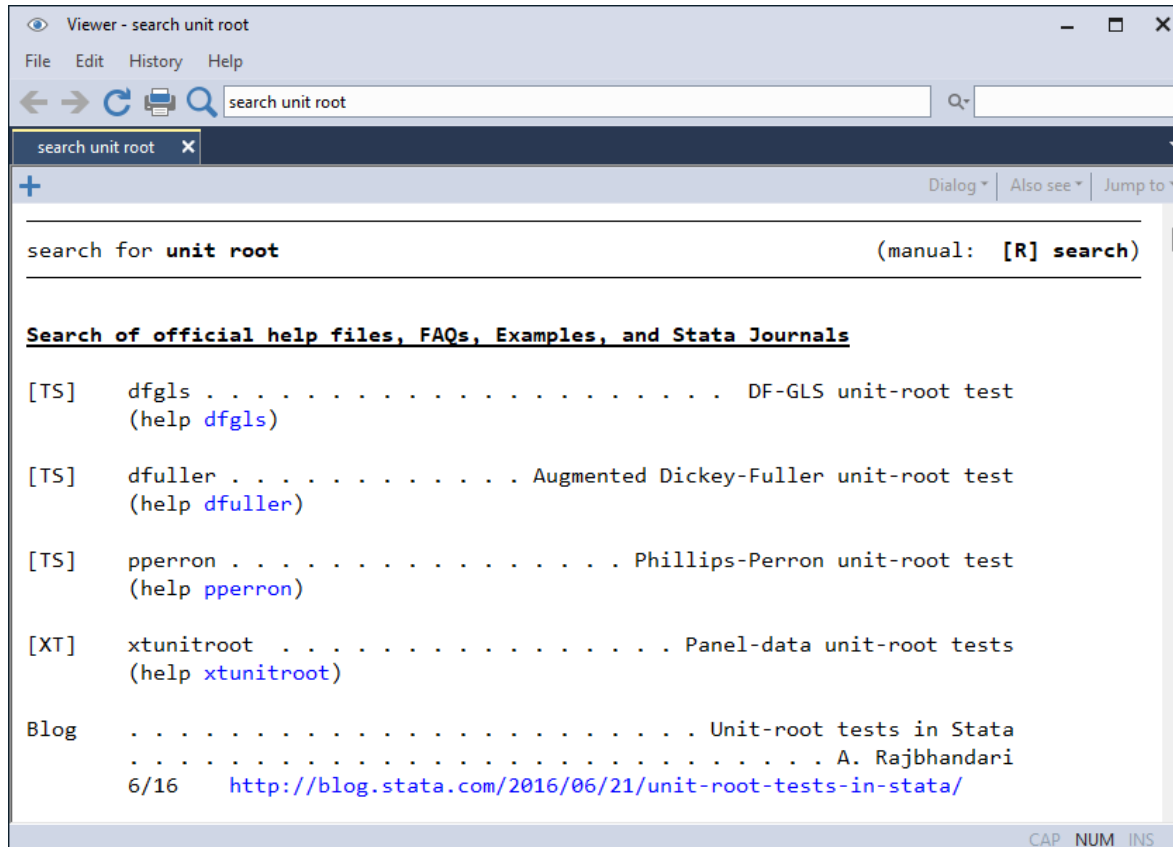
H0: Random walk without drift, d = 0

Test statistic	Dickey-Fuller critical value			
	1%	5%	10%	
Z(t)	-22.508	-3.496	-2.887	-2.577

MacKinnon approximate *p*-value for Z(t) = 0.0000.

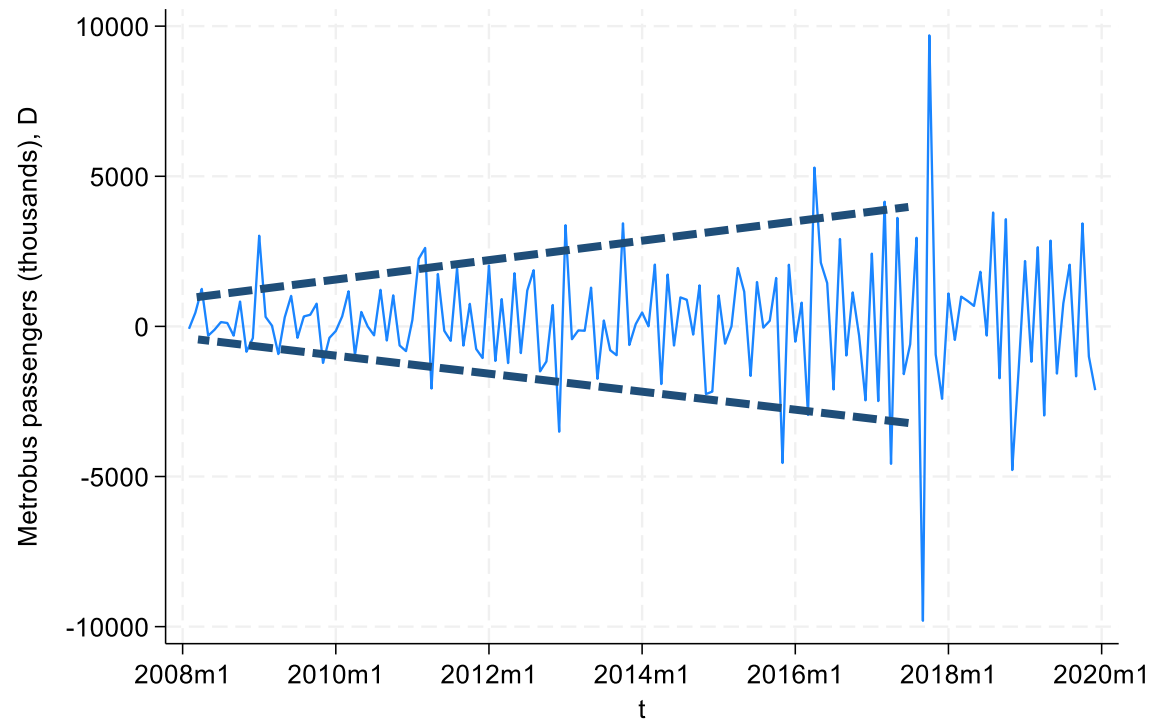
```
. dfuller D.metrobus
```

Other unit-root tests



```
. search unit root
```

Metrobus: Constant variance?

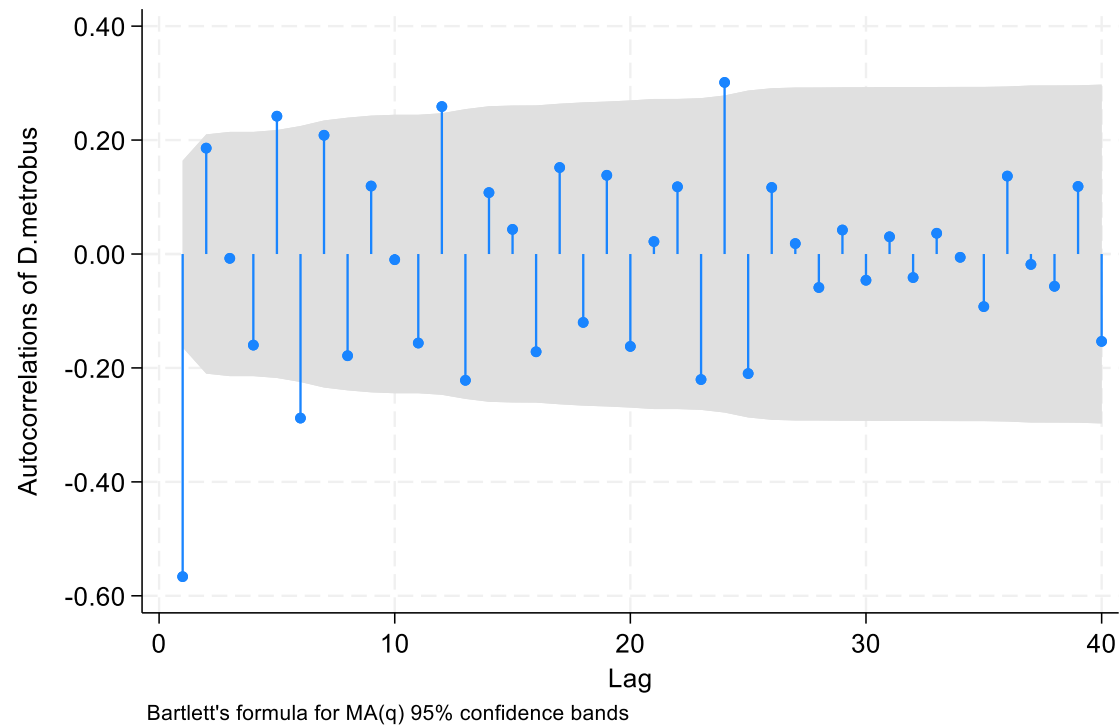


```
. tsline D.metrobus if tin(2008m1,  
2019m12)
```

Metrobus: ARIMA

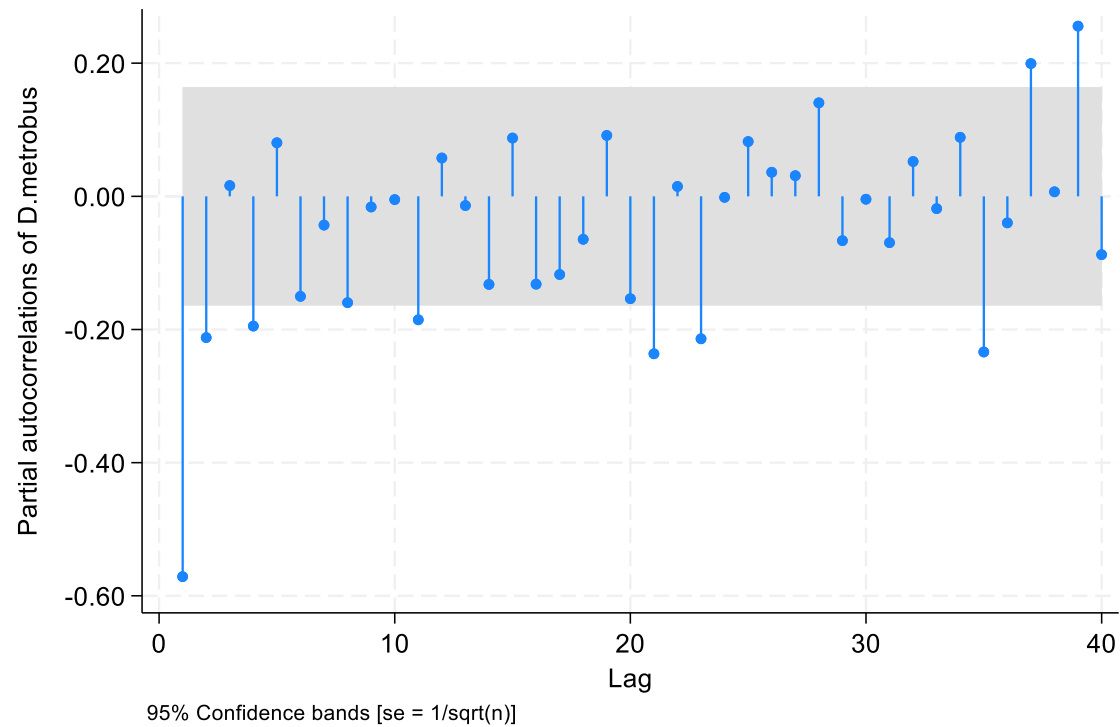
$$\begin{aligned} d.metrobus_t = & \mu + \phi_1 d.metrobus_{t-1} + \cdots + \phi_p d.metrobus_{t-p} \\ & + \epsilon_t + \theta_1 \epsilon_{t-1} + \cdots + \theta_q \epsilon_{t-q} \end{aligned}$$

Autocorrelogram



```
. ac D.metrobus
```

Partial autocorrelogram



```
. pac D.metrobus
```

Model selection with arimasoc

Lag-order selection criteria

Sample: 2008m2 thru 2019m12

Number of obs = 143

Model	LL	df	AIC	BIC	HQIC
ARMA(0,0)	-1296.576	2	2597.151	2603.077	2599.559
ARMA(0,1)	-1267.407	3	2540.814	2549.702	2544.426
ARMA(0,2)	-1266.229	4	2540.459	2552.31	2545.274
ARMA(1,0)	-1268.825	3	2543.65	2552.539	2547.262
ARMA(1,1)	-1265.545	4	2539.09	2550.941	2543.906
ARMA(1,2)	-1265.508	5	2541.017	2555.831	2547.037
ARMA(2,0)	-1265.658	4	2539.317	2551.168	2544.133
ARMA(2,1)	-1263.476	5	2536.951	2551.766	2542.971
ARMA(2,2)	-1263.171	6	2538.342	2556.12	2545.566

Selected (max) LL: ARMA(2,2)

Selected (min) AIC: ARMA(2,1)

Selected (min) BIC: ARMA(0,1)

Selected (min) HQIC: ARMA(2,1)

```
. arimasoc D.metrobus
```

Metrobus: ARIMA(2, 1, 1)

ARIMA regression

Sample: 2008m2 thru 2019m12 Number of obs = 143
Wald chi2(3) = 536.43
Log likelihood = -1263.476 Prob > chi2 = 0.0000

D.metrobus	OPG					
	Coefficient	std. err.	z	P> z	[95% conf. interval]	
metrobus _cons	197.8784	87.02933	2.27	0.023	27.30409	368.4528
ARMA						
ar						
L1.	-1.381551	.1067587	-12.94	0.000	-1.590794	-1.172307
L2.	-.5842782	.0561559	-10.40	0.000	-.6943418	-.4742146
ma						
L1.	.7644672	.1315618	5.81	0.000	.5066109	1.022324
/sigma	1659.666	72.87977	22.77	0.000	1516.824	1802.508

Note: The test of the variance against zero is one sided, and the two-sided confidence interval is truncated at zero.

```
. arima metrobus, arima(2, 1, 1)  
  
. estimates store arima21
```


Metrobus: ARCH

$$\begin{aligned} d.metrobus_t = & \mu + \phi_1 d.metrobus_{t-1} + \cdots + \phi_p d.metrobus_{t-p} \\ & + \epsilon_t + \theta_1 \epsilon_{t-1} + \cdots + \theta_p \epsilon_{t-p} \end{aligned}$$

$$\sigma_t^2 = \gamma_0 + \gamma_1 \epsilon_{t-1}^2 + \cdots + \gamma_m \epsilon_{t-m}^2$$

Metrobus: ARCH effects test

LM test for autoregressive conditional heteroskedasticity (ARCH)

lags(p)	chi2	df	Prob > chi2
1	32.956	1	0.0000

H0: no ARCH effects vs. H1: ARCH(p) disturbance

```
. quietly regress D.metrobus  
. estat archlm, lags(1)
```

Metrobus: ARMA(2,1) with ARCH(1) disturbances

ARCH family regression -- ARMA disturbances

Sample: 2008m2 thru 2019m12 Number of obs = 143
Wald chi2(3) = 182.94
Log likelihood = -1253.196 Prob > chi2 = 0.0000

D.metrobus		OPG		z	P> z	[95% conf. interval]	
		Coefficient	std. err.				
metrobus							
	_cons	146.1883	69.72694	2.10	0.036	9.525977	282.8505
ARMA							
	ar						
	L1.	-1.168511	.4180182	-2.80	0.005	-1.987812	-.3492107
	L2.	-.4133894	.2131822	-1.94	0.052	-.8312189	.0044401
	ma						
	L1.	.5519442	.4585955	1.20	0.229	-.3468865	1.450775
ARCH							
	arch						
	L1.	.6602451	.213515	3.09	0.002	.2417633	1.078727
	_cons	1395431	256825.7	5.43	0.000	892062.1	1898800

```
. arch D.metrobus, ar(1 2) ma(1)  
arch(1)
```

Metrobus: AR(2) with ARCH(1) disturbances

ARCH family regression -- AR disturbances

Sample: 2008m2 thru 2019m12
Log likelihood = -1253.454

Number of obs = 143
Wald chi2(2) = 73.36
Prob > chi2 = 0.0000

		OPG		z	P> z	[95% conf. interval]	
D.metrobus		Coefficient	std. err.				
metrobus _cons		155.7661	62.14133	2.51	0.012	33.97133	277.5609
ARMA							
ar							
L1.		-.6555086	.1004949	-6.52	0.000	-.852475	-.4585421
L2.		-.1295425	.0979355	-1.32	0.186	-.3214926	.0624076
ARCH							
arch							
L1.		.703343	.1981944	3.55	0.000	.3148891	1.091797
_cons		1362397	240384.3	5.67	0.000	891252.9	1833542

```
. arch D.metrobus, ar(1 2) arch(1)
```

Metrobus: AR(1) with ARCH(1) disturbances

ARCH family regression -- AR disturbances

Sample: 2008m2 thru 2019m12 Number of obs = 143
Wald chi2(1) = 55.66
Log likelihood = -1254.34 Prob > chi2 = 0.0000

		OPG		z	P> z	[95% conf. interval]	
D.metrobus		Coefficient	std. err.				
metrobus _cons		158.3452	70.99336	2.23	0.026	19.20079	297.4896
ARMA							
	ar						
	L1.	-.5558053	.0744961	-7.46	0.000	-.701815	-.4097956
ARCH							
	arch						
	L1.	.6144126	.1385206	4.44	0.000	.3429173	.8859079
	_cons	1454262	221030.7	6.58	0.000	1021050	1887474

```
. arch D.metrobus, ar(1) arch(1)  
  
. estimates store arch1
```

Metrobus: Model comparison

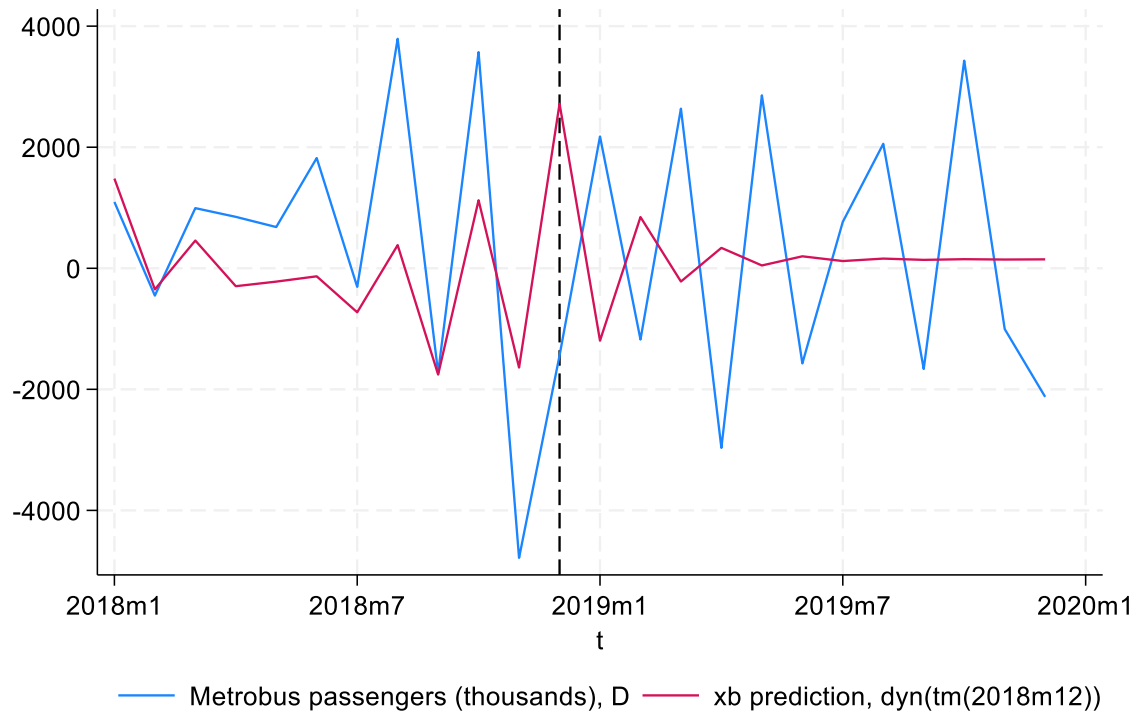
Akaike's information criterion and Bayesian information criterion

Model	N	ll(null)	ll(model)	df	AIC	BIC
arima21	143	.	-1263.476	5	2536.951	2551.766
arch1	143	.	-1254.34	4	2516.68	2528.531

Note: BIC uses N = number of observations. See [\[R\] BIC note](#).

```
. estimates stats arima21 arch1
```

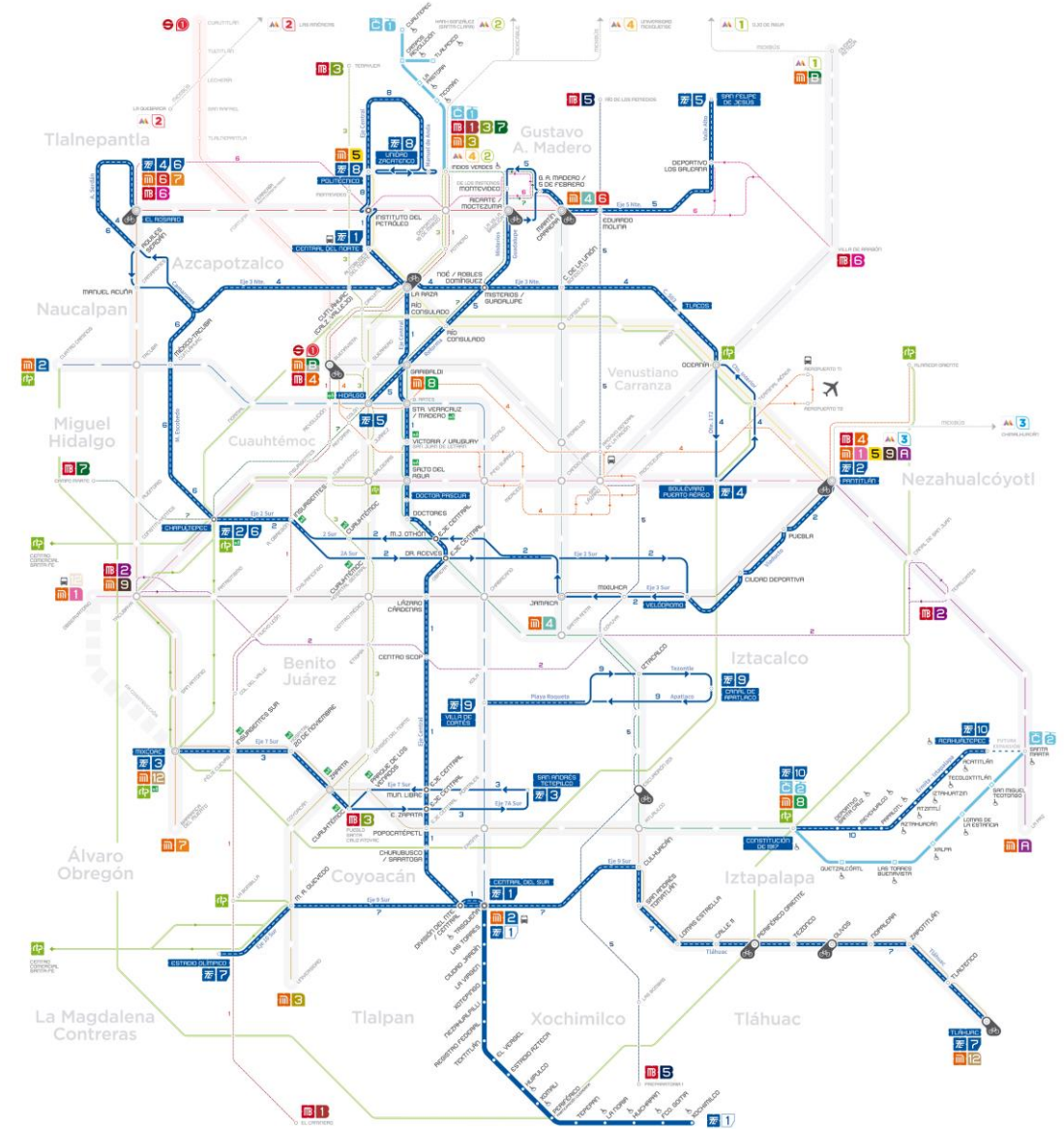
Metrobus: 2019 forecast



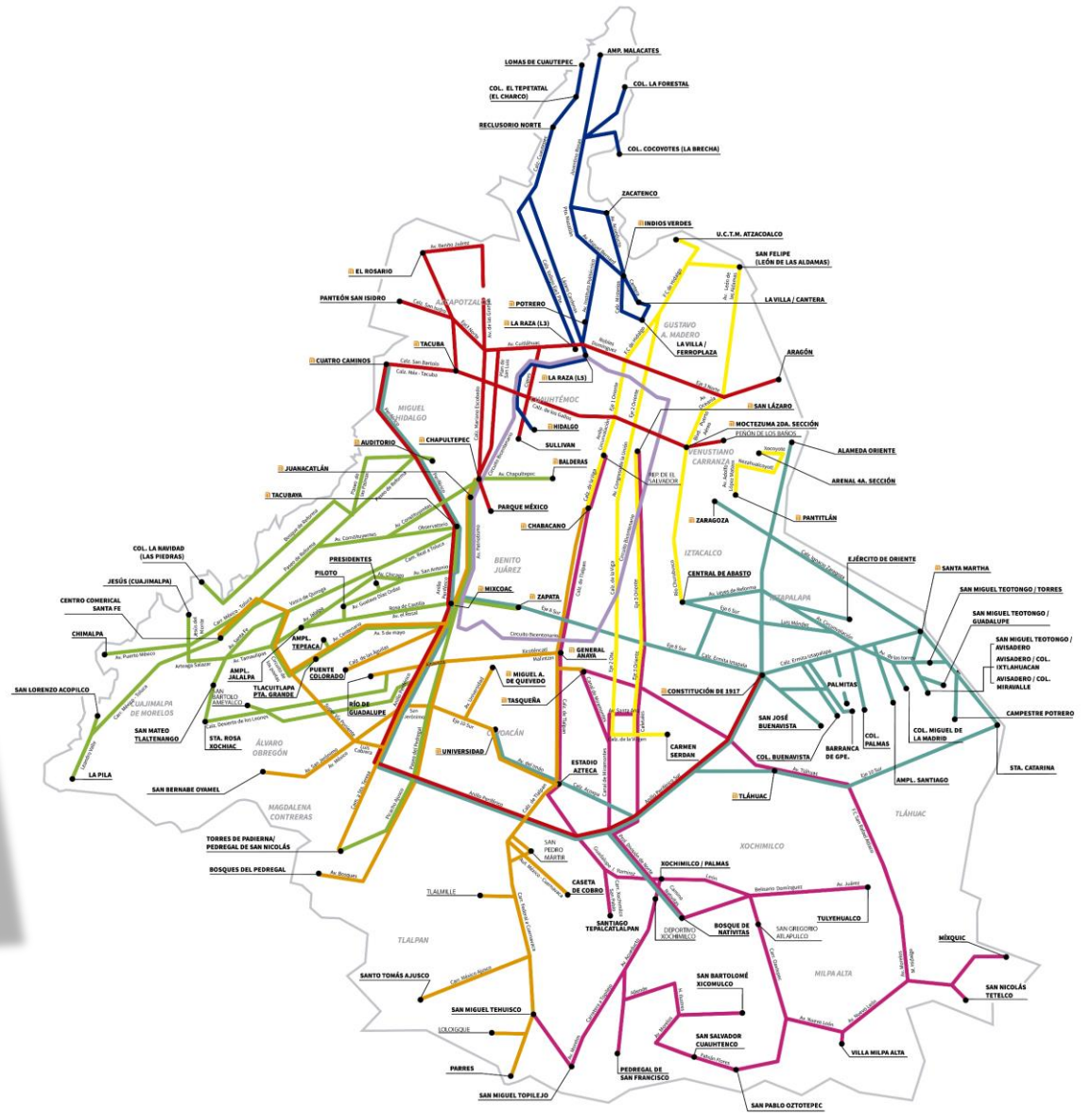
```
. arch D.metrobus if  
  tin(2008m1, 2018m12), ar(1) arch(1)  
  
. predict fcst_arch1 in tin(2008m1,  
  2019m12), dynamic(tm(2018m12))  
  
. tsline D.metrobus fcst_arch1  
  if tin(2018m1, 2019m12),  
  tline(2018m12)
```

Part II. Multivariate time-series analysis

Trolley bus



RTP



The dataset

Data Editor (Browse) - [var_ptmex]

File Edit View Data Tools

date[1] 05jan2015 DMY

	date	d_metro	d_metrobus	d_trolley	d_rtp
1	05jan2015	1.242085	3.481033	3.42668	-1.556734
2	06jan2015	.1757893	1.720243	1.196019	.2009903
3	07jan2015	.2469624	1.116559	2.394182	.1769749
4	08jan2015	-.0164806	1.023347	1.110079	.4144409
5	09jan2015	.931869	.9210421	-.9818314	.2410533
6	12jan2015	.0305322	.8562469	1.590774	.7253451
7	13jan2015	.168649	-.4410605	-.6486164	-1.70464
8	14jan2015	-.99567	-.4756604	-.1325982	.416592
9	15jan2015	.7037176	.8317384	.5290978	1.532328
10	16jan2015	.0017397	.7911649	-.6334136	-.5504839
11	19jan2015	.4245895	-1.425354	-.6473138	-1.587711
12	20jan2015	-.4003676	-.6383936	-1.295054	.1525764
13	21jan2015	-.7422836	-1.317292	-1.781026	-1.58423
14	22jan2015	-1.628885	-1.512212	.8079062	-.348318

Variables

Filter variables here

Name	Label	Type	Format	Value
<input checked="" type="checkbox"/> date		float	%td	
<input checked="" type="checkbox"/> d_metro	Metro passengers (daily change)	float	%9.0g	
<input checked="" type="checkbox"/> d_metrobus	Metrobus passengers (daily change)	float	%9.0g	
<input checked="" type="checkbox"/> d_trolley	Trolley passengers (daily change)	float	%9.0g	
<input checked="" type="checkbox"/> d_rtp	RTP passengers (daily change)	float	%9.0g	

Variables Snapshots

Properties

Variables

Name	date
Label	
Type	float
Format	%td
Value label	
Notes	

Data

Frame	default
Filename	var_ptmex.dta
Label	Mexico City transport VAR
Notes	
Variables	5

Ready Vars: 5 Order: Dataset Obs: 997 Filter: Off Mode: Browse CAP NUM

The dataset

```
. tsset date
```

```
Time variable: date, 05jan2015 to 30oct2018, but with gaps  
Delta: 1 day
```

```
. tsset date
```

The dataset

date
05jan2015
06jan2015
07jan2015
08jan2015
09jan2015
12jan2015
13jan2015
14jan2015
15jan2015
16jan2015

Step 1: Create a business calendar

Business calendar mycal (format %tbmycal):

Purpose:

Range: 05jan2015	30oct2018	
20093	21487	in %td units
0	996	in %tbmycal units

Center: 05jan2015	
20093	in %td units
0	in %tbmycal units

Omitted:	398	days
	104.2	approx. days/year

Included:	997	days
	261.0	approx. days/year

Notes:

Business calendar file mycal.stbcal saved.

```
. bcal create mycal, from(date)
```

Step 2: Load the business calendar

```
loading .\mycal.stbcal ...
```

```
1. * Business calendar "mycal" created by -bcal create-  
2. * Created/replaced on 13 Mar 2023  
3.  
4. version 17  
5. dateformat ymd  
6.  
7. range 2015jan05 2018oct30  
8. centerdate 2015jan05  
9.  
10. omit dayofweek (Sa Su)
```

```
(calendar loaded successfully)
```

```
. bcal load mycal
```


Step 3: Generate business calendar variable

	date	d_metro	d_metrobus	d_trolley	d_rtp	bcaldate
1	05jan2015	1.242085	3.481033	3.42668	-1.556734	0
2	06jan2015	.1757893	1.720243	1.196019	.2009908	1
3	07jan2015	.2469624	1.116559	2.394182	.1769749	2
4	08jan2015	-.0164806	1.023347	1.110079	.4144409	3
5	09jan2015	.931869	.9210421	-.9818314	.2410538	4
6	12jan2015	.0305322	.8562469	1.590774	.7253451	5
7	13jan2015	.168649	-.4410605	-.6486164	-1.70464	6
8	14jan2015	-.99567	-.4756604	-.1325982	.416592	7
9	15jan2015	.7037176	.8317384	.5290978	1.532328	8
10	16jan2015	.0017397	.7911649	-.6334136	-.5504839	9
11	19jan2015	.4245895	-1.425354	-.6473138	-1.587711	10
12	20jan2015	-.4003676	-.6383936	-1.295054	.1525764	11
13	21jan2015	-.7422836	-1.317292	-1.781026	-1.58428	12
14	22jan2015	-1.628885	-1.512212	.8079062	-.348318	13

```
. generate bcaldate = bofd("mycal",  
date)
```


Step 4: Format business calendar variable

	date	d_metro	d_metrobus	d_trolley	d_rtp	bcaldate
1	05jan2015	1.242085	3.481033	3.42668	-1.55673	05jan2015
2	06jan2015	.1757893	1.720243	1.196019	.200990	06jan2015
3	07jan2015	.2469624	1.116559	2.394182	.176974	07jan2015
4	08jan2015	-.0164806	1.023347	1.110079	.414440	08jan2015
5	09jan2015	.931869	.9210421	-.9818314	.241053	09jan2015
6	12jan2015	.0305322	.8562469	1.590774	.725345	12jan2015
7	13jan2015	.168649	-.4410605	-.6486164	-1.7046	13jan2015
8	14jan2015	-.99567	-.4756604	-.1325982	.416592	14jan2015
9	15jan2015	.7037176	.8317384	.5290978	1.53232	15jan2015
10	16jan2015	.0017397	.7911649	-.6334136	-.550483	16jan2015
11	19jan2015	.4245895	-1.425354	-.6473138	-1.58771	19jan2015
12	20jan2015	-.4003676	-.6383936	-1.295054	.152576	20jan2015
13	21jan2015	-.7422836	-1.317292	-1.781026	-1.5842	21jan2015
14	22jan2015	-1.628885	-1.512212	.8079062	-.34831	22jan2015

```
. format %tbmycal bcaldate
```

Business calendar variable

```
. tsset bcaldate
```

```
Time variable: bcaldate, 05jan2015 to 30oct2018  
Delta: 1 day
```

```
. tsset bcaldate
```

Vector autoregression intuition

$$\begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} = \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} \\ a_{21} & 0 & a_{23} & a_{24} \\ a_{31} & a_{32} & 0 & a_{34} \\ a_{41} & a_{42} & a_{43} & 0 \end{bmatrix} \begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} + \begin{bmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 & a_{14}^1 \\ a_{21}^1 & a_{22}^1 & a_{23}^1 & a_{24}^1 \\ a_{31}^1 & a_{32}^1 & a_{33}^1 & a_{34}^1 \\ a_{41}^1 & a_{42}^1 & a_{43}^1 & a_{44}^1 \end{bmatrix} \begin{bmatrix} metro_{t-1} \\ metrobus_{t-1} \\ trolley_{t-1} \\ rtp_{t-1} \end{bmatrix} + \dots + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \\ \epsilon_{4,t} \end{bmatrix}$$

VAR intuition: Contemporaneous effects

$$\begin{bmatrix} \text{metro}_t \\ \text{metrobus}_t \\ \text{trolley}_t \\ \text{rtp}_t \end{bmatrix} = \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} \\ a_{21} & 0 & a_{23} & a_{24} \\ a_{31} & a_{32} & 0 & a_{34} \\ a_{41} & a_{42} & a_{43} & 0 \end{bmatrix} \begin{bmatrix} \text{metro}_t \\ \text{metrobus}_t \\ \text{trolley}_t \\ \text{rtp}_t \end{bmatrix} + \begin{bmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 & a_{14}^1 \\ a_{21}^1 & a_{22}^1 & a_{23}^1 & a_{24}^1 \\ a_{31}^1 & a_{32}^1 & a_{33}^1 & a_{34}^1 \\ a_{41}^1 & a_{42}^1 & a_{43}^1 & a_{44}^1 \end{bmatrix} \begin{bmatrix} \text{metro}_{t-1} \\ \text{metrobus}_{t-1} \\ \text{trolley}_{t-1} \\ \text{rtp}_{t-1} \end{bmatrix} + \dots + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \\ \epsilon_{4,t} \end{bmatrix}$$

VAR intuition: Contemporaneous effects

$$\begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} = \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} \\ a_{21} & 0 & a_{23} & a_{24} \\ a_{31} & a_{32} & 0 & a_{34} \\ a_{41} & a_{42} & a_{43} & 0 \end{bmatrix} \begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} + \begin{bmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 & a_{14}^1 \\ a_{21}^1 & a_{22}^1 & a_{23}^1 & a_{24}^1 \\ a_{31}^1 & a_{32}^1 & a_{33}^1 & a_{34}^1 \\ a_{41}^1 & a_{42}^1 & a_{43}^1 & a_{44}^1 \end{bmatrix} \begin{bmatrix} metro_{t-1} \\ metrobus_{t-1} \\ trolley_{t-1} \\ rtp_{t-1} \end{bmatrix} + \dots + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \\ \epsilon_{4,t} \end{bmatrix}$$

VAR intuition: Contemporaneous effects

$$\begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} = \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} \\ a_{21} & 0 & a_{23} & a_{24} \\ a_{31} & a_{32} & 0 & a_{34} \\ a_{41} & a_{42} & a_{43} & 0 \end{bmatrix} \begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} + \begin{bmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 & a_{14}^1 \\ a_{21}^1 & a_{22}^1 & a_{23}^1 & a_{24}^1 \\ a_{31}^1 & a_{32}^1 & a_{33}^1 & a_{34}^1 \\ a_{41}^1 & a_{42}^1 & a_{43}^1 & a_{44}^1 \end{bmatrix} \begin{bmatrix} metro_{t-1} \\ metrobus_{t-1} \\ trolley_{t-1} \\ rtp_{t-1} \end{bmatrix} + \dots + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \\ \epsilon_{4,t} \end{bmatrix}$$

- If a commuter is not using one transportation system today, they may be using another.

VAR intuition: Lagged effects

$$\begin{bmatrix} \text{metro}_t \\ \text{metrobus}_t \\ \text{trolley}_t \\ \text{rtp}_t \end{bmatrix} = \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} \\ a_{21} & 0 & a_{23} & a_{24} \\ a_{31} & a_{32} & 0 & a_{34} \\ a_{41} & a_{42} & a_{43} & 0 \end{bmatrix} \begin{bmatrix} \text{metro}_t \\ \text{metrobus}_t \\ \text{trolley}_t \\ \text{rtp}_t \end{bmatrix} + \begin{bmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 & a_{14}^1 \\ a_{21}^1 & a_{22}^1 & a_{23}^1 & a_{24}^1 \\ a_{31}^1 & a_{32}^1 & a_{33}^1 & a_{34}^1 \\ a_{41}^1 & a_{42}^1 & a_{43}^1 & a_{44}^1 \end{bmatrix} \begin{bmatrix} \text{metro}_{t-1} \\ \text{metrobus}_{t-1} \\ \text{trolley}_{t-1} \\ \text{rtp}_{t-1} \end{bmatrix} + \dots + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \\ \epsilon_{4,t} \end{bmatrix}$$

VAR intuition: Lagged effects

$$\begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} = \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} \\ a_{21} & 0 & a_{23} & a_{24} \\ a_{31} & a_{32} & 0 & a_{34} \\ a_{41} & a_{42} & a_{43} & 0 \end{bmatrix} \begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} + \begin{bmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 & a_{14}^1 \\ a_{21}^1 & a_{22}^1 & a_{23}^1 & a_{24}^1 \\ a_{31}^1 & a_{32}^1 & a_{33}^1 & a_{34}^1 \\ a_{41}^1 & a_{42}^1 & a_{43}^1 & a_{44}^1 \end{bmatrix} \begin{bmatrix} metro_{t-1} \\ metrobus_{t-1} \\ trolley_{t-1} \\ rtp_{t-1} \end{bmatrix} + \dots + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \\ \epsilon_{4,t} \end{bmatrix}$$

- Negative autocorrelation?
Probably not. People don't go to work/school just every other day.
- Positive autocorrelation? Maybe, because days of above-average changes in passenger volumes tend to be followed by more days of above-average changes. Same for below-average days.

VAR intuition: Innovations

$$\begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} = \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} \\ a_{21} & 0 & a_{23} & a_{24} \\ a_{31} & a_{32} & 0 & a_{34} \\ a_{41} & a_{42} & a_{43} & 0 \end{bmatrix} \begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} + \begin{bmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 & a_{14}^1 \\ a_{21}^1 & a_{22}^1 & a_{23}^1 & a_{24}^1 \\ a_{31}^1 & a_{32}^1 & a_{33}^1 & a_{34}^1 \\ a_{41}^1 & a_{42}^1 & a_{43}^1 & a_{44}^1 \end{bmatrix} \begin{bmatrix} metro_{t-1} \\ metrobus_{t-1} \\ trolley_{t-1} \\ rtp_{t-1} \end{bmatrix} + \dots + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \\ \epsilon_{4,t} \end{bmatrix}$$

VAR intuition: Innovations

$$\begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} = \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} \\ a_{21} & 0 & a_{23} & a_{24} \\ a_{31} & a_{32} & 0 & a_{34} \\ a_{41} & a_{42} & a_{43} & 0 \end{bmatrix} \begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} + \begin{bmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 & a_{14}^1 \\ a_{21}^1 & a_{22}^1 & a_{23}^1 & a_{24}^1 \\ a_{31}^1 & a_{32}^1 & a_{33}^1 & a_{34}^1 \\ a_{41}^1 & a_{42}^1 & a_{43}^1 & a_{44}^1 \end{bmatrix} \begin{bmatrix} metro_{t-1} \\ metrobus_{t-1} \\ trolley_{t-1} \\ rtp_{t-1} \end{bmatrix} + \dots + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \\ \epsilon_{4,t} \end{bmatrix}$$

VAR theory review: Setup

$$\begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} = \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} \\ a_{21} & 0 & a_{23} & a_{24} \\ a_{31} & a_{32} & 0 & a_{34} \\ a_{41} & a_{42} & a_{43} & 0 \end{bmatrix} \begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} + \begin{bmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 & a_{14}^1 \\ a_{21}^1 & a_{22}^1 & a_{23}^1 & a_{24}^1 \\ a_{31}^1 & a_{32}^1 & a_{33}^1 & a_{34}^1 \\ a_{41}^1 & a_{42}^1 & a_{43}^1 & a_{44}^1 \end{bmatrix} \begin{bmatrix} metro_{t-1} \\ metrobus_{t-1} \\ trolley_{t-1} \\ rtp_{t-1} \end{bmatrix} + \dots + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \\ \epsilon_{4,t} \end{bmatrix}$$

VAR theory review: Structural to reduced form

$$\begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} = \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} \\ a_{21} & 0 & a_{23} & a_{24} \\ a_{31} & a_{32} & 0 & a_{34} \\ a_{41} & a_{42} & a_{43} & 0 \end{bmatrix} \begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} + \begin{bmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 & a_{14}^1 \\ a_{21}^1 & a_{22}^1 & a_{23}^1 & a_{24}^1 \\ a_{31}^1 & a_{32}^1 & a_{33}^1 & a_{34}^1 \\ a_{41}^1 & a_{42}^1 & a_{43}^1 & a_{44}^1 \end{bmatrix} \begin{bmatrix} metro_{t-1} \\ metrobus_{t-1} \\ trolley_{t-1} \\ rtp_{t-1} \end{bmatrix} + \dots + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \\ \epsilon_{4,t} \end{bmatrix}$$

!!!
!!!

VAR theory review: Structural to reduced form

$$\begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} = \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} \\ a_{21} & 0 & a_{23} & a_{24} \\ a_{31} & a_{32} & 0 & a_{34} \\ a_{41} & a_{42} & a_{43} & 0 \end{bmatrix} \begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} + \begin{bmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 & a_{14}^1 \\ a_{21}^1 & a_{22}^1 & a_{23}^1 & a_{24}^1 \\ a_{31}^1 & a_{32}^1 & a_{33}^1 & a_{34}^1 \\ a_{41}^1 & a_{42}^1 & a_{43}^1 & a_{44}^1 \end{bmatrix} \begin{bmatrix} metro_{t-1} \\ metrobus_{t-1} \\ trolley_{t-1} \\ rtp_{t-1} \end{bmatrix} + \dots + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \\ \epsilon_{4,t} \end{bmatrix}$$



$$\begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} - \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} \\ a_{21} & 0 & a_{23} & a_{24} \\ a_{31} & a_{32} & 0 & a_{34} \\ a_{41} & a_{42} & a_{43} & 0 \end{bmatrix} \begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} = \begin{bmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 & a_{14}^1 \\ a_{21}^1 & a_{22}^1 & a_{23}^1 & a_{24}^1 \\ a_{31}^1 & a_{32}^1 & a_{33}^1 & a_{34}^1 \\ a_{41}^1 & a_{42}^1 & a_{43}^1 & a_{44}^1 \end{bmatrix} \begin{bmatrix} metro_{t-1} \\ metrobus_{t-1} \\ trolley_{t-1} \\ rtp_{t-1} \end{bmatrix} + \dots + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \\ \epsilon_{4,t} \end{bmatrix}$$

VAR theory review: Structural to reduced form

$$\begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} = \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} \\ a_{21} & 0 & a_{23} & a_{24} \\ a_{31} & a_{32} & 0 & a_{34} \\ a_{41} & a_{42} & a_{43} & 0 \end{bmatrix} \begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} + \begin{bmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 & a_{14}^1 \\ a_{21}^1 & a_{22}^1 & a_{23}^1 & a_{24}^1 \\ a_{31}^1 & a_{32}^1 & a_{33}^1 & a_{34}^1 \\ a_{41}^1 & a_{42}^1 & a_{43}^1 & a_{44}^1 \end{bmatrix} \begin{bmatrix} metro_{t-1} \\ metrobus_{t-1} \\ trolley_{t-1} \\ rtp_{t-1} \end{bmatrix} + \dots + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \\ \epsilon_{4,t} \end{bmatrix}$$

$$\begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} - \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} \\ a_{21} & 0 & a_{23} & a_{24} \\ a_{31} & a_{32} & 0 & a_{34} \\ a_{41} & a_{42} & a_{43} & 0 \end{bmatrix} \begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} = \begin{bmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 & a_{14}^1 \\ a_{21}^1 & a_{22}^1 & a_{23}^1 & a_{24}^1 \\ a_{31}^1 & a_{32}^1 & a_{33}^1 & a_{34}^1 \\ a_{41}^1 & a_{42}^1 & a_{43}^1 & a_{44}^1 \end{bmatrix} \begin{bmatrix} metro_{t-1} \\ metrobus_{t-1} \\ trolley_{t-1} \\ rtp_{t-1} \end{bmatrix} + \dots + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \\ \epsilon_{4,t} \end{bmatrix}$$



$$\begin{bmatrix} 1 & -a_{12} & -a_{13} & -a_{14} \\ -a_{21} & 1 & -a_{23} & -a_{24} \\ -a_{31} & -a_{32} & 1 & -a_{34} \\ -a_{41} & -a_{42} & -a_{43} & 1 \end{bmatrix} \begin{bmatrix} metro_t \\ metrobus_t \\ trolley_t \\ rtp_t \end{bmatrix} = \begin{bmatrix} a_{11}^1 & a_{12}^1 & a_{13}^1 & a_{14}^1 \\ a_{21}^1 & a_{22}^1 & a_{23}^1 & a_{24}^1 \\ a_{31}^1 & a_{32}^1 & a_{33}^1 & a_{34}^1 \\ a_{41}^1 & a_{42}^1 & a_{43}^1 & a_{44}^1 \end{bmatrix} \begin{bmatrix} metro_{t-1} \\ metrobus_{t-1} \\ trolley_{t-1} \\ rtp_{t-1} \end{bmatrix} + \dots + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \\ \epsilon_{4,t} \end{bmatrix}$$

A
 y_t
 A_1
 y_{t-1}
 B
 ϵ_t

VAR theory review: Structural to reduced form

$$Ay_t = A_1y_{t-1} + \dots + A_py_{t-p} + B\epsilon_t$$

VAR theory review: Structural to reduced form

$$Ay_t = A_1y_{t-1} + \dots + A_py_{t-p} + B\epsilon_t$$

$$y_t = \underbrace{A^{-1}A_1}_{\Phi_1}y_{t-1} + \dots + \underbrace{A^{-1}A_p}_{\Phi_p}y_{t-p} + \underbrace{A^{-1}B\epsilon_t}_{u_t}$$

VAR theory review: Structural to reduced form

$$Ay_t = A_1y_{t-1} + \dots + A_py_{t-p} + B\epsilon_t$$

$$y_t = \underbrace{A^{-1}A_1}_{\Phi_1}y_{t-1} + \dots + \underbrace{A^{-1}A_p}_{\Phi_p}y_{t-p} + \underbrace{A^{-1}B\epsilon_t}_{u_t}$$

$$y_t = \Phi_1y_{t-1} + \dots + \Phi_py_{t-p} + u_t$$

VAR: Lag order

Lag-order selection criteria

Sample: 12jan2015 thru 30oct2018

Number of obs = 992

Lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	4527.17				1.3e-09	-9.1193	-9.11179	-9.09954
1	6067.36	3080.4	16	0.000	6.0e-11	-12.1923	-12.1547	-12.0935
2	6153.1	171.48	16	0.000	5.2e-11	-12.3329	-12.2653	-12.155*
3	6184.62	63.04*	16	0.000	5.0e-11*	-12.3642*	-12.2665*	-12.1073
4	6192.31	15.39	16	0.496	5.1e-11	-12.3474	-12.2197	-12.0115
5	6196.83	9.0334	16	0.912	5.2e-11	-12.3243	-12.1665	-11.9094

* optimal lag

Endogenous: d_metro d_metrobus d_trolley d_rtp

Exogenous: _cons

```
. varsoc d_metro d_metrobus d_trolley  
d_rtp
```

VAR: Output

Vector autoregression

Sample: 08jan2015 thru 30oct2018	Number of obs	=	994
Log likelihood = -2057.746	AIC	=	4.244961
FPE = .0008197	HQIC	=	4.342451
Det(Sigma_ml) = .0007383	SBIC	=	4.50139

Equation	Parms	RMSE	R-sq	chi2	P>chi2
d_metro	13	.411561	0.8329	4954.318	0.0000
d_metrobus	13	.581226	0.6617	1944.278	0.0000
d_trolley	13	.925986	0.1391	160.5786	0.0000
d_rtp	13	.981036	0.0497	51.96537	0.0000

```
. var d_metro d_metrobus d_trolley  
d_rtp, lags(1 2 3)
```

VAR: Output

	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
d_metro						
d_metro						
L1.	.0578853	.0382751	1.51	0.130	-.0171325	.1329031
L2.	.108086	.0464578	2.33	0.020	.0170304	.1991416
L3.	.1416069	.0375527	3.77	0.000	.0680049	.2152089
d_metrobus						
L1.	-.1588305	.1139093	-1.39	0.163	-.3820885	.0644276
L2.	-.2908373	.0945086	-3.08	0.002	-.4760708	-.1056038
L3.	.0977986	.0398539	2.45	0.014	.0196864	.1759108
d_trolley						
L1.	.0341848	.0200201	1.71	0.088	-.0050539	.0734234
L2.	.0284608	.0287286	0.99	0.322	-.0278462	.0847679
L3.	.0385756	.0269324	1.43	0.152	-.0142109	.091362
d_rtp						
L1.	.5980455	.074328	8.05	0.000	.4523653	.7437257
L2.	.6297207	.0803916	7.83	0.000	.4721561	.7872854
L3.	.3153713	.0453119	6.96	0.000	.2265616	.404181
_cons	-.0038782	.0129925	-0.30	0.765	-.029343	.0215867

VAR: Output

d_metrobus						
d_metro						
L1.	.3077106	.0540539	5.69	0.000	.2017669	.4136543
L2.	-.0260303	.0656099	-0.40	0.692	-.1546233	.1025628
L3.	-.0008957	.0530337	-0.02	0.987	-.10484	.1030485
d_metrobus						
L1.	.0615783	.160868	0.38	0.702	-.2537172	.3768738
L2.	-.2176465	.1334695	-1.63	0.103	-.479242	.0439489
L3.	.0721852	.0562836	1.28	0.200	-.0381286	.1824989
d_trolley						
L1.	.1889079	.0282733	6.68	0.000	.1334933	.2443225
L2.	.1050341	.0405719	2.59	0.010	.0255147	.1845536
L3.	.0957912	.0380351	2.52	0.012	.0212437	.1703387
d_rtp						
L1.	.418431	.1049695	3.99	0.000	.2126945	.6241674
L2.	.0648947	.1135328	0.57	0.568	-.1576256	.2874149
L3.	.0730707	.0639916	1.14	0.254	-.0523506	.1984919
_cons	-.0057634	.0183486	-0.31	0.753	-.041726	.0301992

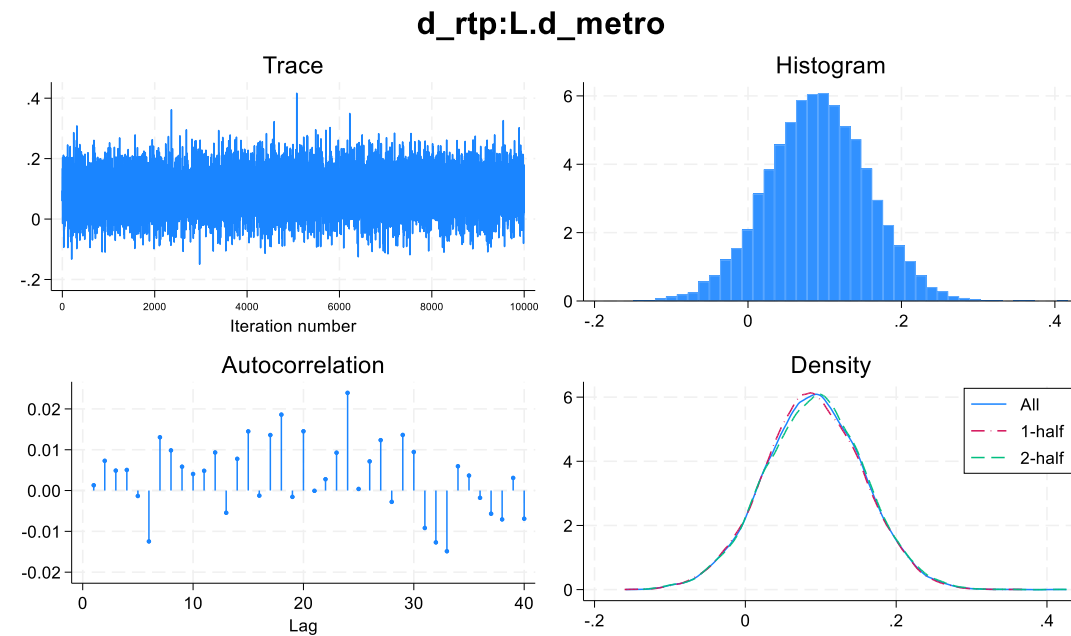
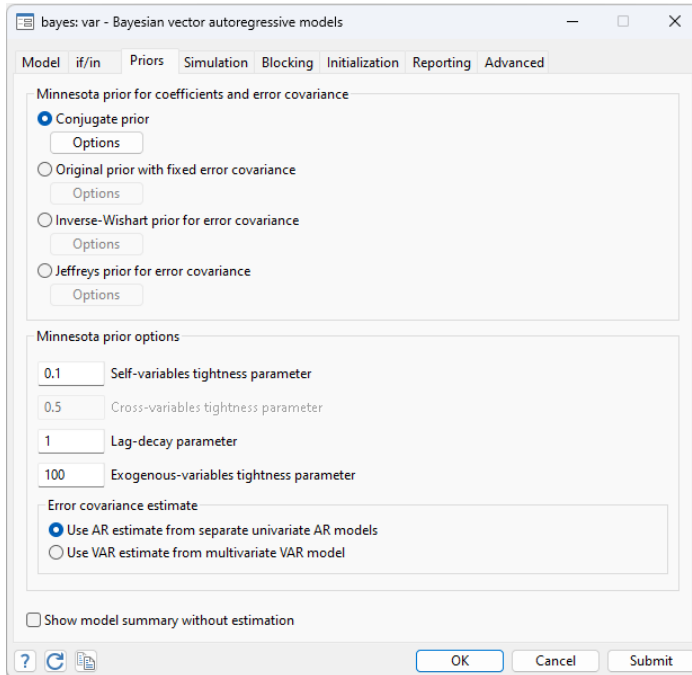
VAR: Output

d_trolley						
d_metro						
L1.	.1137616	.0861166	1.32	0.186	-.0550238	.2825469
L2.	-.0588601	.1045272	-0.56	0.573	-.2637296	.1460094
L3.	-.0256809	.0844913	-0.30	0.761	-.1912808	.1399191
d_metrobus						
L1.	.2076194	.2562887	0.81	0.418	-.2946972	.709936
L2.	.0436953	.2126385	0.21	0.837	-.3730684	.460459
L3.	.0777061	.0896688	0.87	0.386	-.0980415	.2534537
d_trolley						
L1.	-.0152065	.0450439	-0.34	0.736	-.103491	.0730779
L2.	-.0088138	.0646376	-0.14	0.892	-.1355011	.1178735
L3.	.0516183	.0605961	0.85	0.394	-.0671479	.1703845
d_rtp						
L1.	.1036687	.1672333	0.62	0.535	-.2241027	.43144
L2.	-.0593307	.1808761	-0.33	0.743	-.4138413	.2951799
L3.	-.0253078	.101949	-0.25	0.804	-.2251241	.1745085
_cons	-.005603	.0292323	-0.19	0.848	-.0628973	.0516912

VAR: Output

d_rtp						
d_metro						
L1.	.1021072	.0912362	1.12	0.263	-.0767124	.2809268
L2.	-.0348227	.1107413	-0.31	0.753	-.2518716	.1822262
L3.	.0238819	.0895143	0.27	0.790	-.1515629	.1993267
d_metrobus						
L1.	.0221895	.271525	0.08	0.935	-.5099897	.5543687
L2.	-.4005451	.2252798	-1.78	0.075	-.8420853	.0409952
L3.	.1591715	.0949996	1.68	0.094	-.0270242	.3453672
d_trolley						
L1.	.0124564	.0477217	0.26	0.794	-.0810765	.1059893
L2.	.0050517	.0684802	0.07	0.941	-.1291671	.1392705
L3.	.0785933	.0641985	1.22	0.221	-.0472335	.2044201
d_rtp						
L1.	.0610577	.1771753	0.34	0.730	-.2861995	.408315
L2.	.2549217	.1916291	1.33	0.183	-.1206644	.6305079
L3.	.1552565	.1080098	1.44	0.151	-.0564388	.3669518
_cons	-.0000959	.0309702	-0.00	0.998	-.0607963	.0606045

The bayes prefix



```
. bayes: var d_metro d_metrobus d_trolley d_rtp, lags(1 2 3)
```


Structural vector autoregression

$$Ay_t = A_1y_{t-1} + \dots + A_py_{t-p} + B\epsilon_t$$

$$y_t = \underbrace{A^{-1}A_1}_{\Phi_1}y_{t-1} + \dots + \underbrace{A^{-1}A_p}_{\Phi_p}y_{t-p} + \underbrace{A^{-1}B\epsilon_t}_{u_t}$$

$$y_t = \Phi_1y_{t-1} + \dots + \Phi_py_{t-p} + u_t$$

SVAR

$$Ay_t = A_1y_{t-1} + \dots + A_py_{t-p} + B\epsilon_t$$

$$y_t = \underbrace{A^{-1}A_1}_{\Phi_1}y_{t-1} + \dots + \underbrace{A^{-1}A_p}_{\Phi_p}y_{t-p} + \underbrace{A^{-1}B\epsilon_t}_{u_t}$$

$$y_t = \Phi_1y_{t-1} + \dots + \Phi_py_{t-p} + u_t$$

$$\Rightarrow E[u_t u_t'] = \Sigma$$

SVAR: Identification

Information we have

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} \end{bmatrix}$$

10 covariances

Information we want

$$A = \begin{bmatrix} 1 & -a_{12} & -a_{13} & -a_{14} \\ -a_{21} & 1 & -a_{23} & -a_{24} \\ -a_{31} & -a_{32} & 1 & -a_{34} \\ -a_{41} & -a_{42} & -a_{43} & 1 \end{bmatrix}$$

28 parameters

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix}$$

SVAR: Cholesky decomposition

Information we have

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} \end{bmatrix}$$

10 covariances

Information we want

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -a_{21} & 1 & 0 & 0 \\ -a_{31} & -a_{32} & 1 & 0 \\ -a_{41} & -a_{42} & -a_{43} & 1 \end{bmatrix}$$

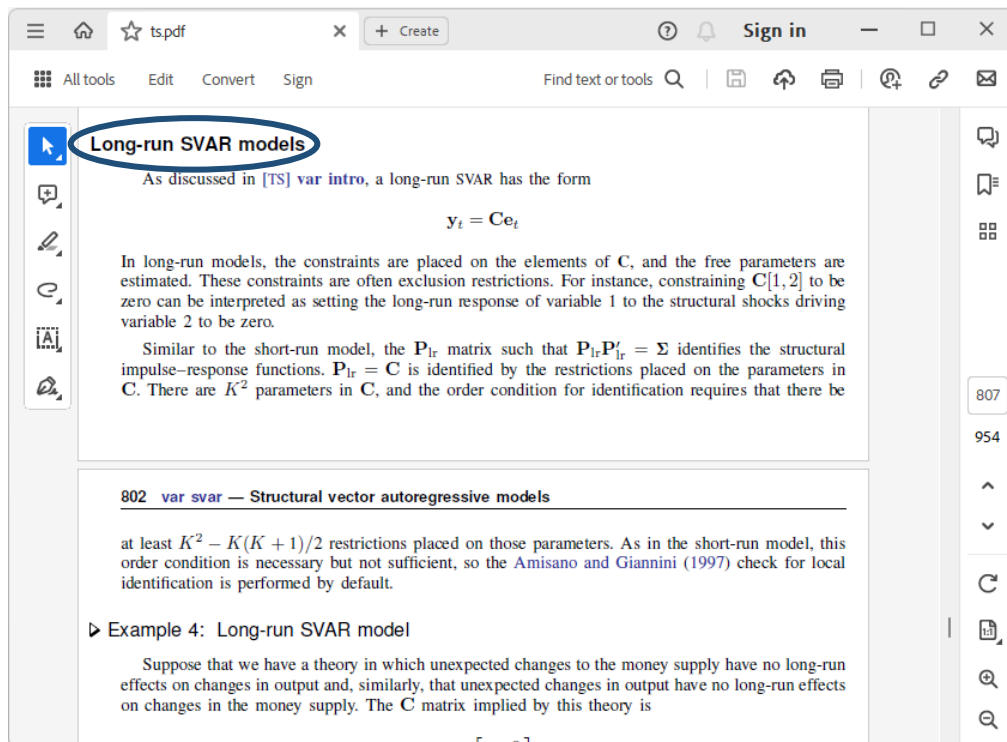
10 parameters

$$B = \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ 0 & b_{22} & 0 & 0 \\ 0 & 0 & b_{33} & 0 \\ 0 & 0 & 0 & b_{44} \end{bmatrix}$$

SVAR ordering

$$\begin{bmatrix} rtp_t \\ trolley_t \\ metrobus_t \\ metro_t \end{bmatrix}$$

SVAR: Other identification strategies



The screenshot shows a PDF viewer window with a document titled 'ts.pdf'. The left sidebar contains a list of icons, with the first icon (a blue square with a white mouse cursor) circled in blue and labeled 'Long-run SVAR models'. The main content area displays text about long-run SVAR models, including the equation $y_t = Ce_t$ and a discussion of constraints on the matrix C . The text is as follows:

As discussed in [TS] [var intro](#), a long-run SVAR has the form

$$y_t = Ce_t$$

In long-run models, the constraints are placed on the elements of C , and the free parameters are estimated. These constraints are often exclusion restrictions. For instance, constraining $C[1,2]$ to be zero can be interpreted as setting the long-run response of variable 1 to the structural shocks driving variable 2 to be zero.

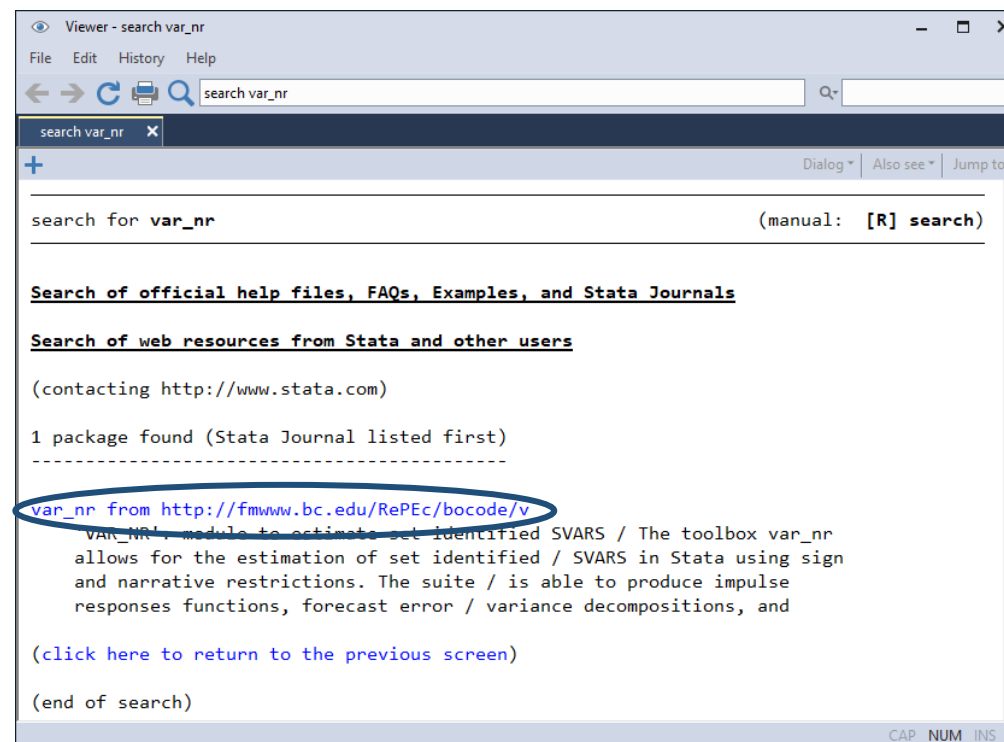
Similar to the short-run model, the P_{lr} matrix such that $P_{lr}P'_{lr} = \Sigma$ identifies the structural impulse-response functions. $P_{lr} = C$ is identified by the restrictions placed on the parameters in C . There are K^2 parameters in C , and the order condition for identification requires that there be

802 [var svar](#) — Structural vector autoregressive models

at least $K^2 - K(K + 1)/2$ restrictions placed on those parameters. As in the short-run model, this order condition is necessary but not sufficient, so the Amisano and Giannini (1997) check for local identification is performed by default.

► Example 4: Long-run SVAR model

Suppose that we have a theory in which unexpected changes to the money supply have no long-run effects on changes in output and, similarly, that unexpected changes in output have no long-run effects on changes in the money supply. The C matrix implied by this theory is



The screenshot shows a Stata search results window titled 'Viewer - search var_nr'. The search bar contains 'search var_nr'. The results are displayed in a list format. The first result is circled in blue and reads:

[var_nr from http://fmwww.bc.edu/RePEc/bocode/v](http://fmwww.bc.edu/RePEc/bocode/v)

The text below the link describes the `var_nr` module, which is used to estimate set identified SVARS in Stata. It mentions that the module allows for the estimation of set identified / SVARS in Stata using sign and narrative restrictions, and that it can produce impulse responses functions, forecast error / variance decompositions, and more.

(click here to return to the previous screen)

(end of search)

SVAR(3)

Sample: 08jan2015 thru 30oct2018
Exactly identified model

Number of obs = 994
Log likelihood = -2057.746

	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
/A						
1_1	1	(constrained)				
2_1	-.5707443	.023845	-23.94	0.000	-.6174796	-.5240089
3_1	-.5982546	.0047149	-126.89	0.000	-.6074956	-.5890136
4_1	-.8871721	.0558795	-15.88	0.000	-.9966938	-.7776504
1_2	0	(constrained)				
2_2	1	(constrained)				
3_2	.0319603	.0049952	6.40	0.000	.0221699	.0417506
4_2	.2263981	.0145668	15.54	0.000	.1978476	.2549485
1_3	0	(constrained)				
2_3	0	(constrained)				
3_3	1	(constrained)				
4_3	1.031821	.0906478	11.38	0.000	.8541541	1.209487
1_4	0	(constrained)				
2_4	0	(constrained)				
3_4	0	(constrained)				
4_4	1	(constrained)				
/B						
1_1	.9745992	.0218584	44.59	0.000	.9317576	1.017441
2_1	0	(constrained)				
3_1	0	(constrained)				
4_1	0	(constrained)				
1_2	0	(constrained)				
2_2	.7326836	.0164327	44.59	0.000	.7004761	.764891
3_2	0	(constrained)				
4_2	0	(constrained)				
1_3	0	(constrained)				
2_3	0	(constrained)				
3_3	.1153879	.0025879	44.59	0.000	.1103156	.1204601
4_3	0	(constrained)				
1_4	0	(constrained)				
2_4	0	(constrained)				
3_4	0	(constrained)				
4_4	.3297697	.0073961	44.59	0.000	.3152736	.3442658

```
matrix input A = ( 1, 0, 0, 0\ ///
                  ., 1, 0, 0\ ///
                  ., ., 1, 0\ ///
                  ., ., ., 1)
```

```
matrix input B = ( ., 0, 0, 0\ ///
                   0, ., 0, 0\ ///
                   0, 0, ., 0\ ///
                   0, 0, 0, .)
```

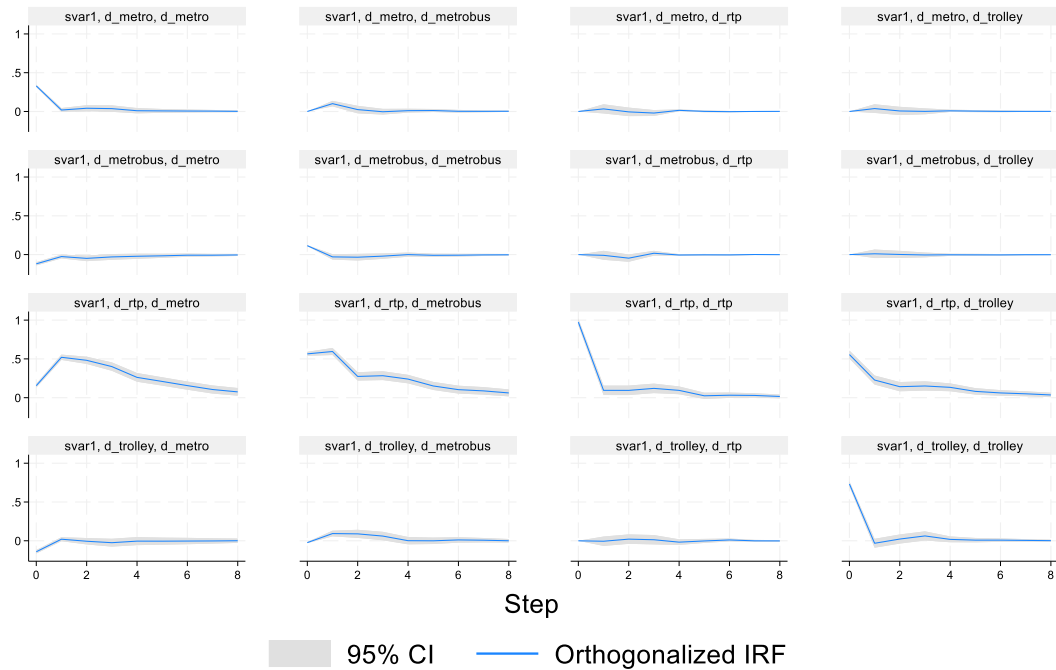
```
. svar d_rtp d_trolley d_metrobus
   d_metro, aeq(A) beq(B) lags(1 2 3)
```

Impulse-response Function

```
. irf set "myIRF"  
(file myIRF.irf created)  
(file myIRF.irf now active)  
  
. irf create svar1  
(file myIRF.irf updated)
```

```
. irf set "myIRF"  
  
. irf create svar1
```

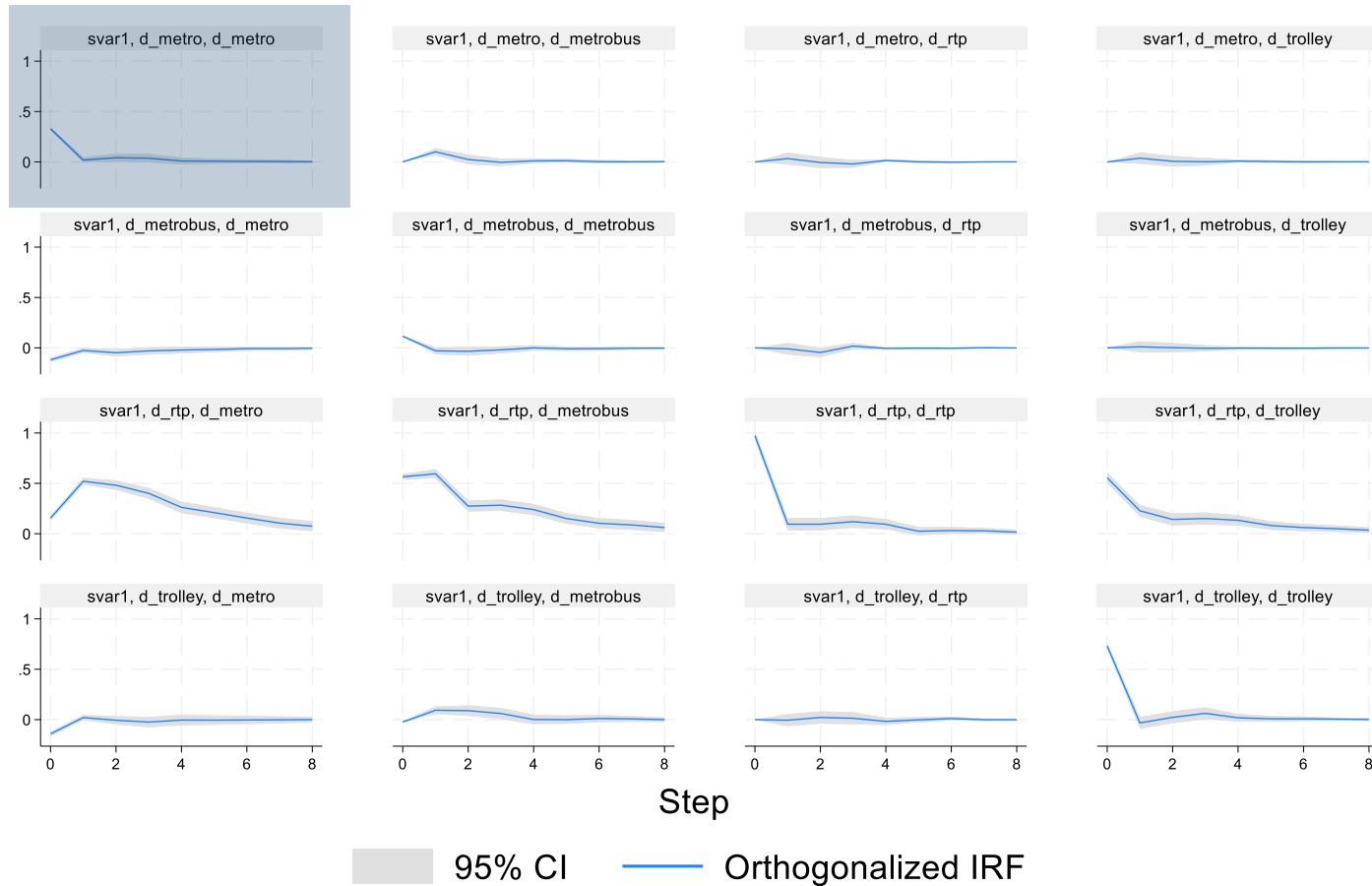

IRF graphs



Graphs by irfname, impulse variable, and response variable

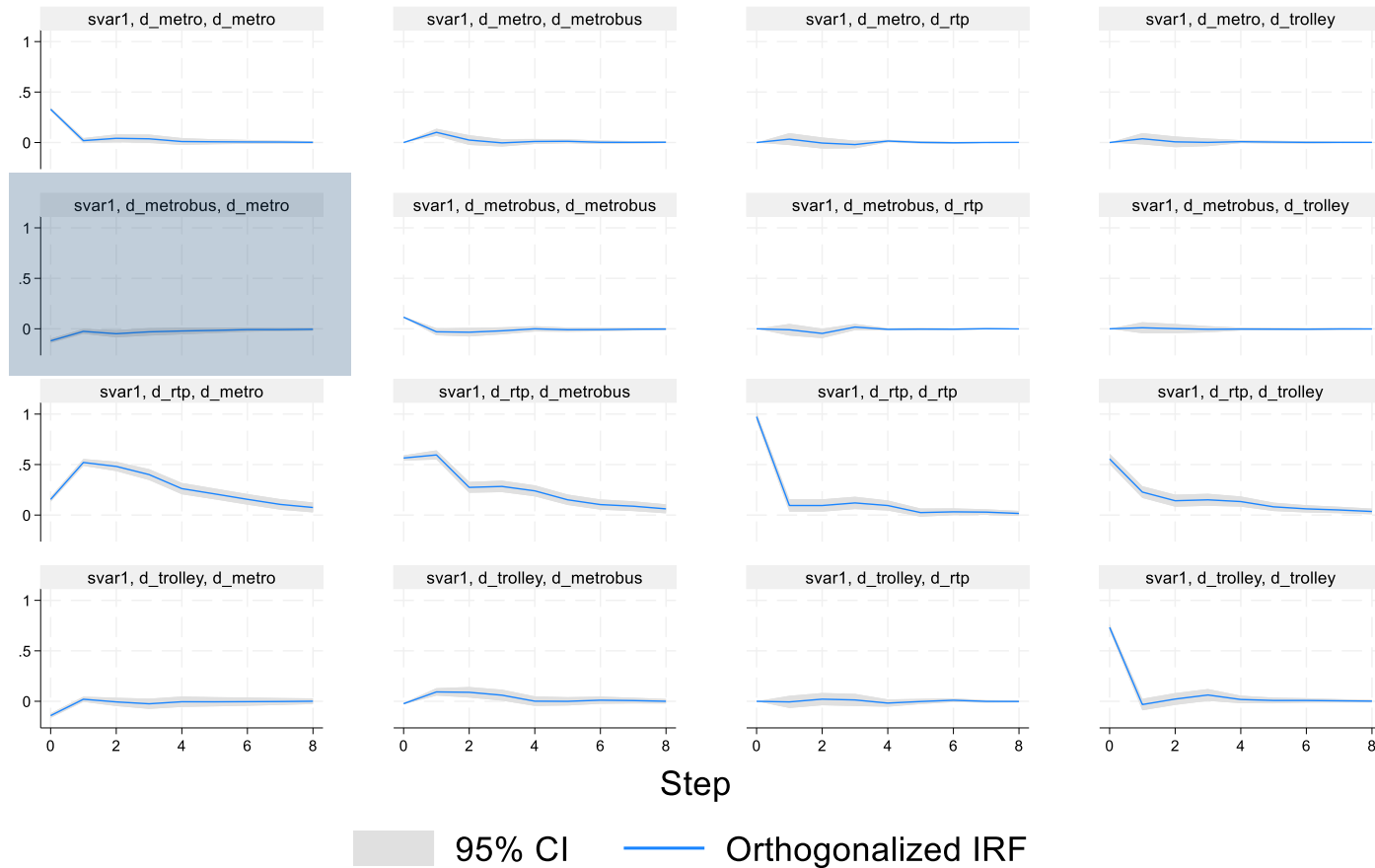
```
. irf graph oirf
```

IRF graphs



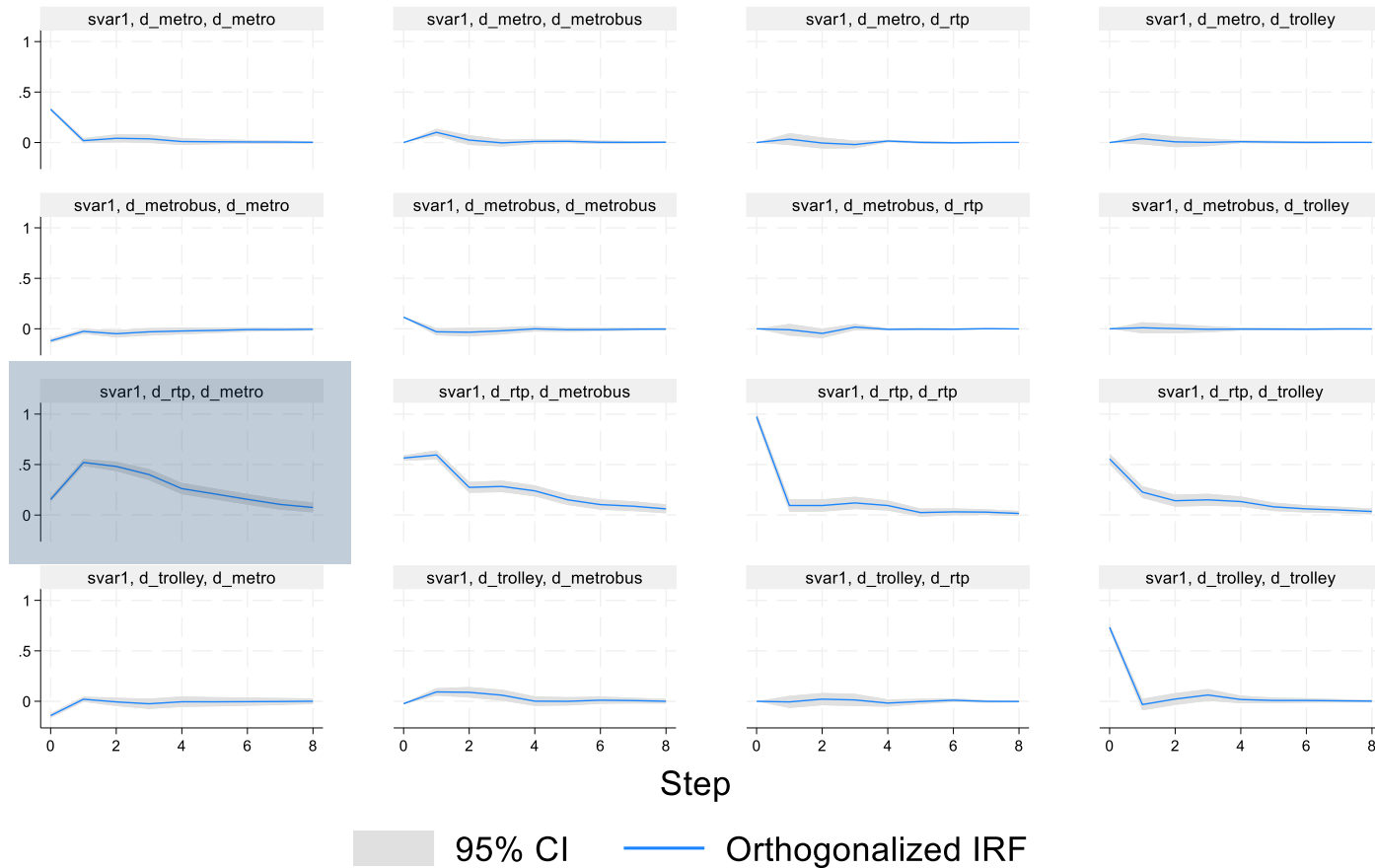
Graphs by irfname, impulse variable, and response variable

IRF graphs



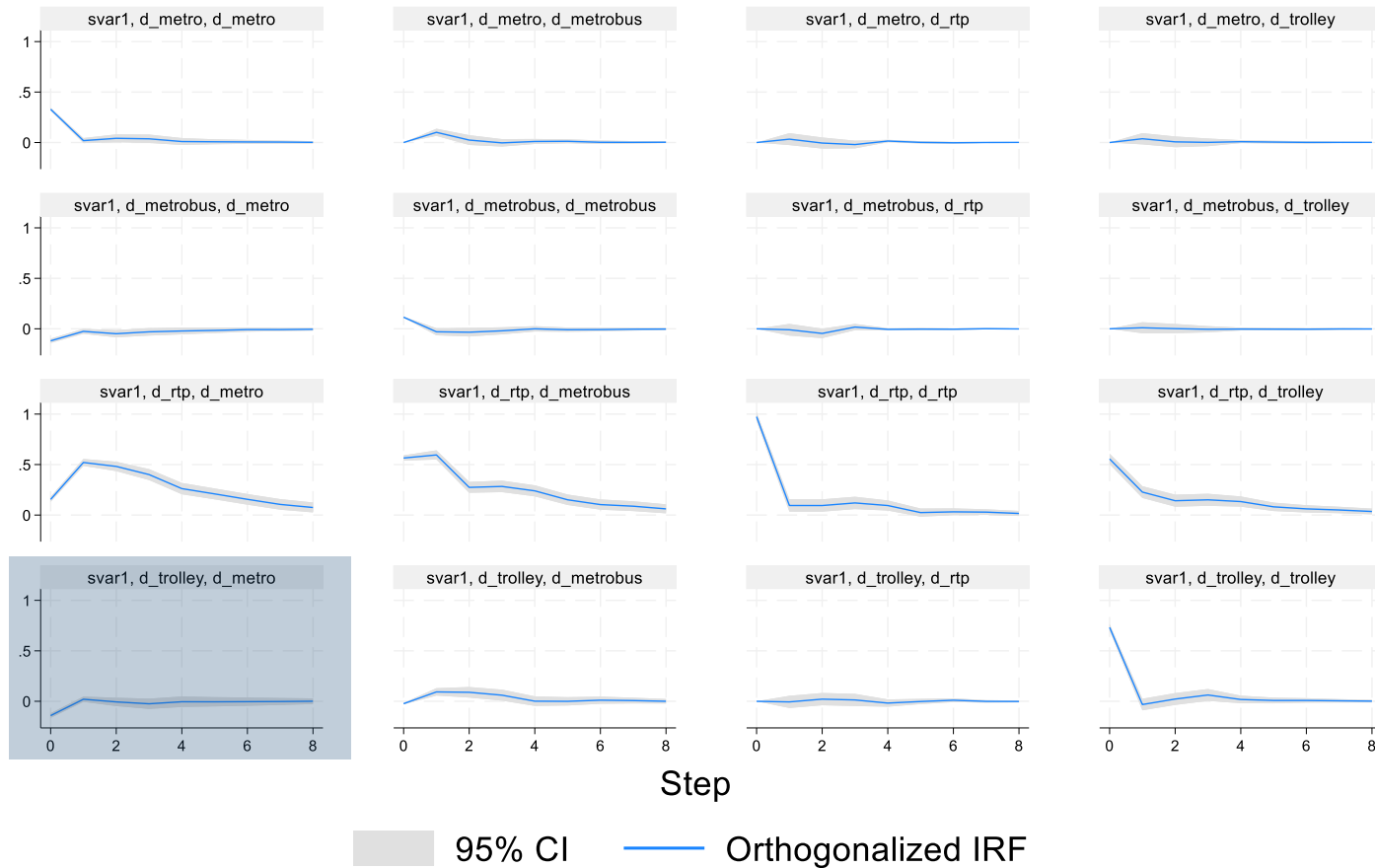
Graphs by irfname, impulse variable, and response variable

IRF graphs



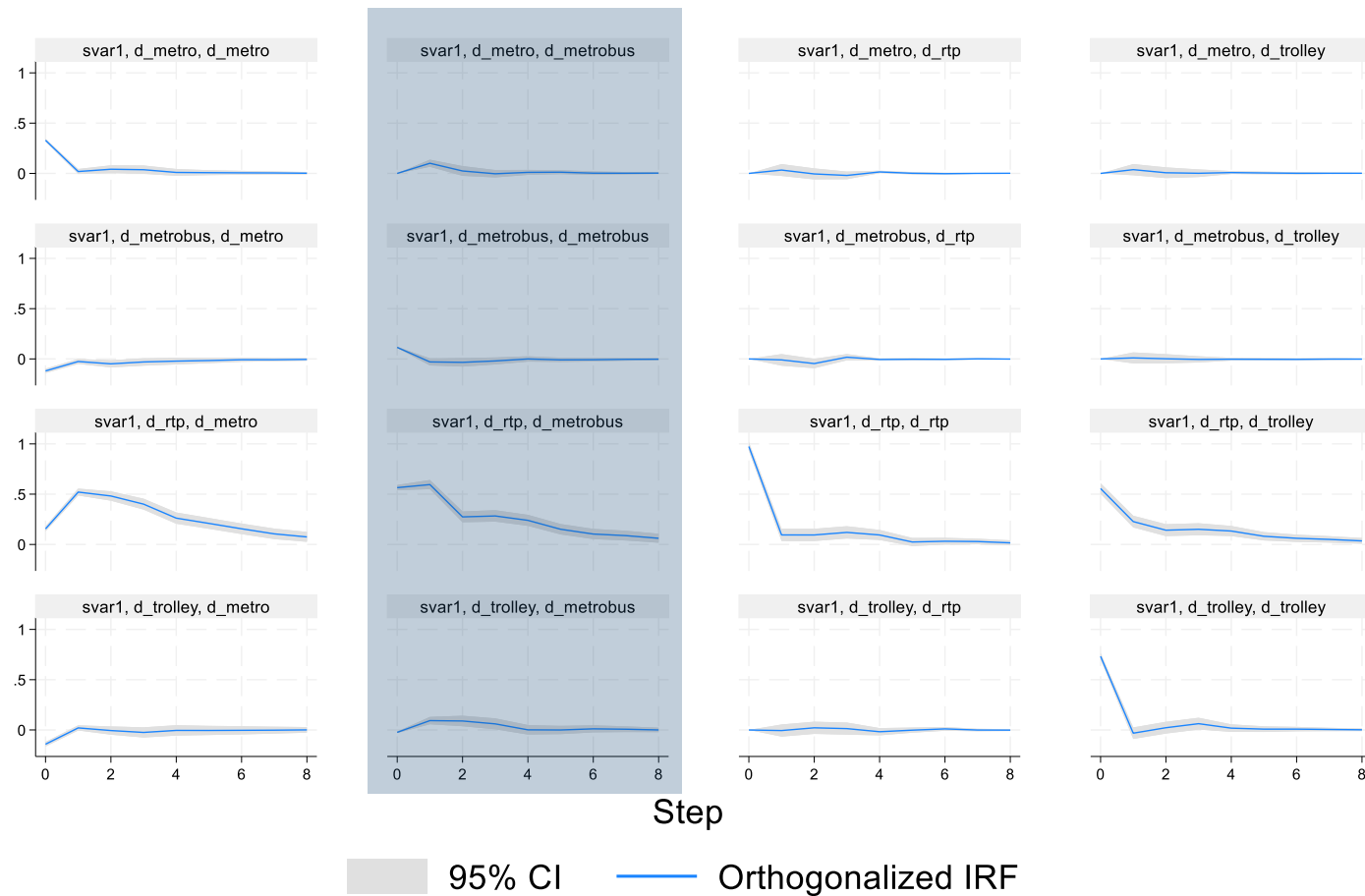
Graphs by irfname, impulse variable, and response variable

IRF graphs



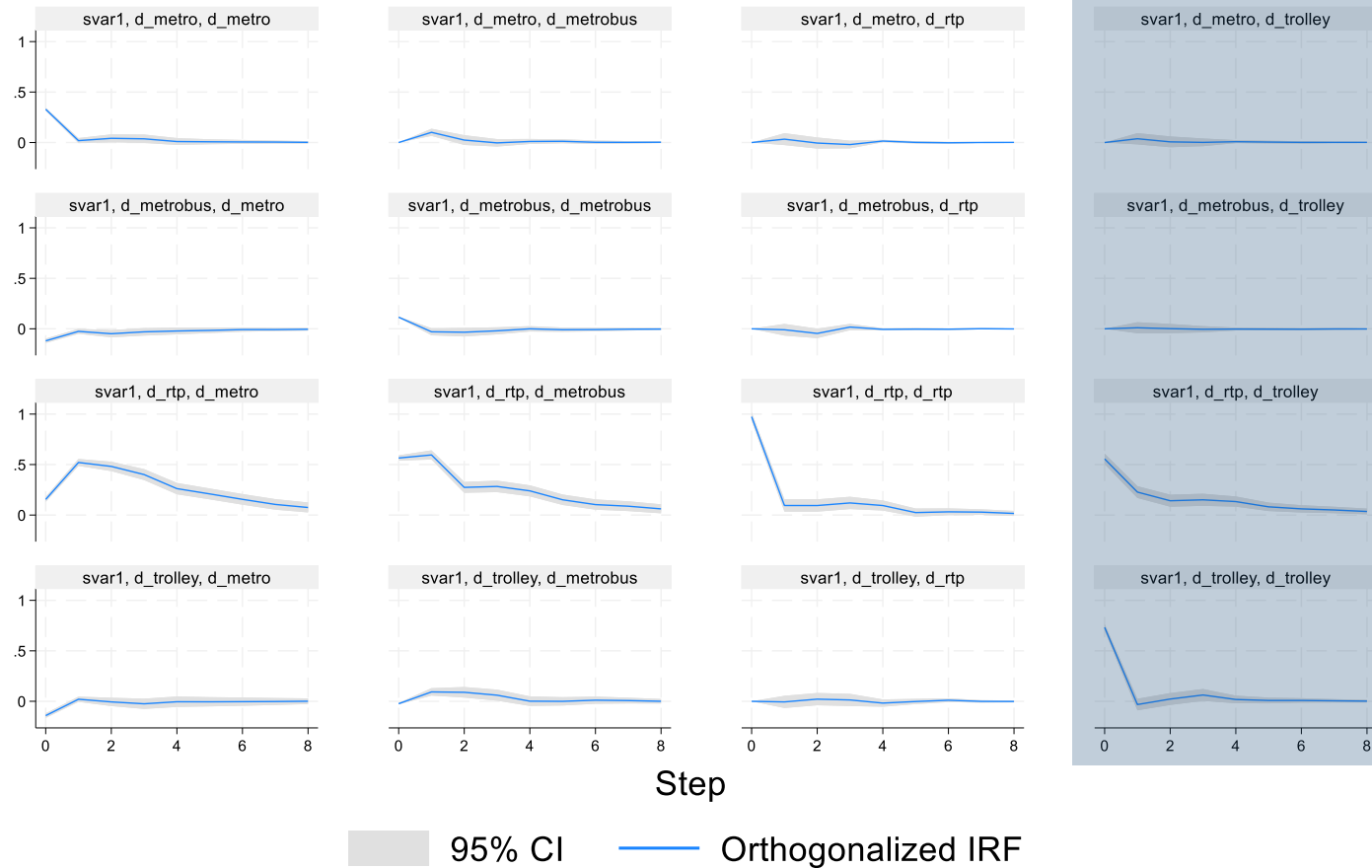
Graphs by irfname, impulse variable, and response variable

IRF graphs



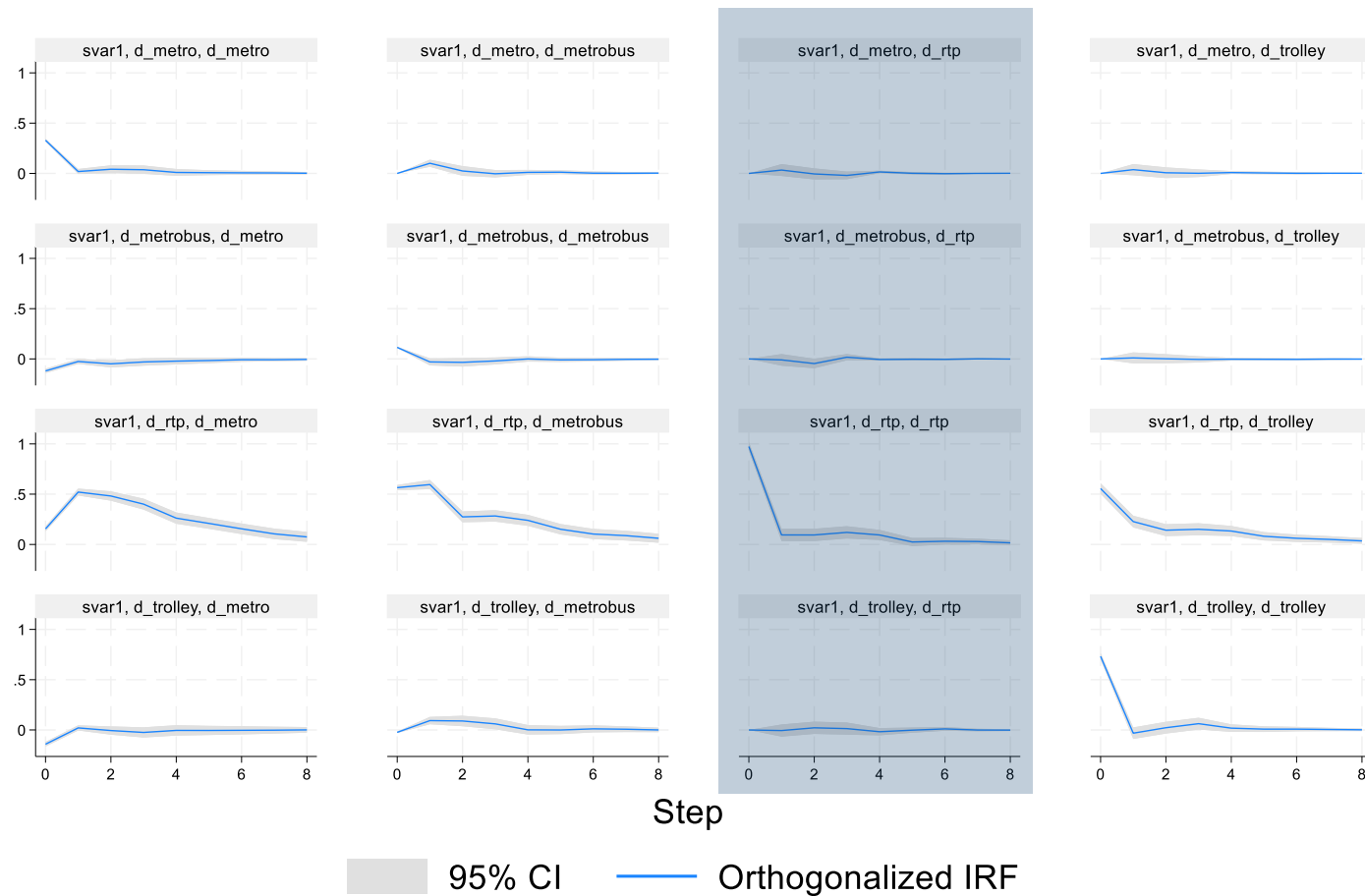
Graphs by irfname, impulse variable, and response variable

IRF graphs



Graphs by irfname, impulse variable, and response variable

IRF graphs



Graphs by irfname, impulse variable, and response variable

Thank you for joining us!

