Multilevel meta-analysis

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Overview

- Standard meta-analysis
- Multilevel meta-analysis
    - Fit a random-intercepts model
    - Fit a random-slope model
    - Assess heterogeneity at different levels
    - Perform sensitivity analysis

Standard meta-analysis

Meta-analysis

- This is a statistical technique for combining the results from several similar studies.
- The goal is to provide a single estimate of the effect of interest.
- If results vary widely across studies, the goal is then to understand the inconsistencies in the results.

Student achievement data

- Consider a series of studies that examined whether students performed better under a modified school calendar, with frequent breaks, as opposed to the traditional schedule (Cooper et al. 2003).
- Each study was performed in a different school.
- The effect size is the standardized mean difference in performance.

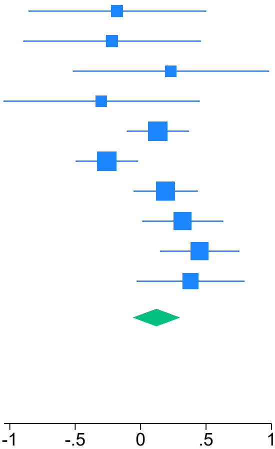
Modified school calendar data

```
. list study stdmdiff se if _n < 5 | _n > 52, sep(0) ab(20)
```

	study	stdmdiff	se
1.	1	-.18	.34351128
2.	2	-.22	.34351128
3.	3	.23	.37947332
4.	4	-.3	.37947332
53.	53	.12	.29495762
54.	54	.61	.28635642
55.	55	.04	.25884358
56.	56	-.05	.25884358

- Positive values indicate that students on the modified calendar performed better than students on the traditional calendar

Visualizing the effect sizes



Study	Effect size with 95% CI	Weight (%)
Study 1	-0.18 [-0.85, 0.49]	5.28
Study 2	-0.22 [-0.89, 0.45]	5.28
Study 3	0.23 [-0.51, 0.97]	4.54
Study 4	-0.30 [-1.04, 0.44]	4.54
Study 5	0.13 [-0.10, 0.36]	15.01
Study 6	-0.26 [-0.49, -0.03]	15.01
Study 7	0.19 [-0.05, 0.43]	14.75
Study 8	0.32 [0.02, 0.62]	12.75
Study 9	0.45 [0.15, 0.75]	12.94
Study 10	0.38 [-0.03, 0.79]	9.91
Overall	0.12 [-0.06, 0.30]	

Heterogeneity: $\tau^2 = 0.04$, $I^2 = 58.09\%$, $H^2 = 2.39$

Test of $\theta_i = \theta_j$: $Q(9) = 21.67$, $p = 0.01$

Test of $\theta = 0$: $z = 1.31$, $p = 0.19$

Meta-analysis goals

- The department of education needs to decide whether they should implement the modified schedule
- Our goal is to report a single estimate of the standardized mean difference in performance
  - We'll assume that the effect sizes are independent across studies.
- If we observe substantial variation across the studies, we instead focus on trying to explain this variation
- Perhaps the design of the test, rigor of the academic curriculum, or some other study-level covariates can explain the discrepancies

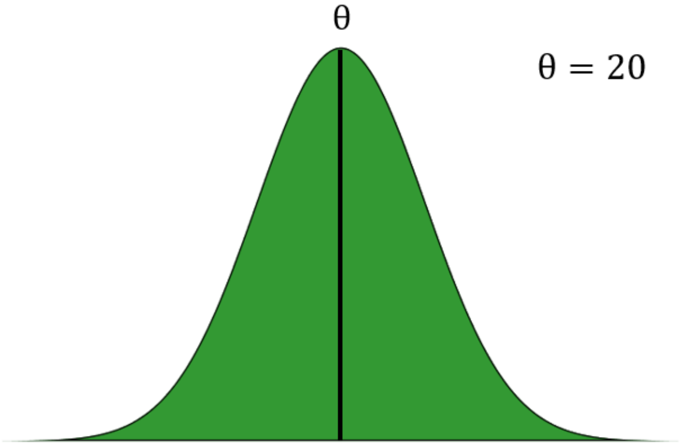
Random effects meta-analysis model

- K independent studies, each reports:
  - an estimate, $\hat{\theta}_j$, of the true (unknown) effect size θ_j
  - an estimate, $\hat{\sigma}_j$, of its standard error

$$\hat{\theta}_j = \theta + u_j + \epsilon_j$$

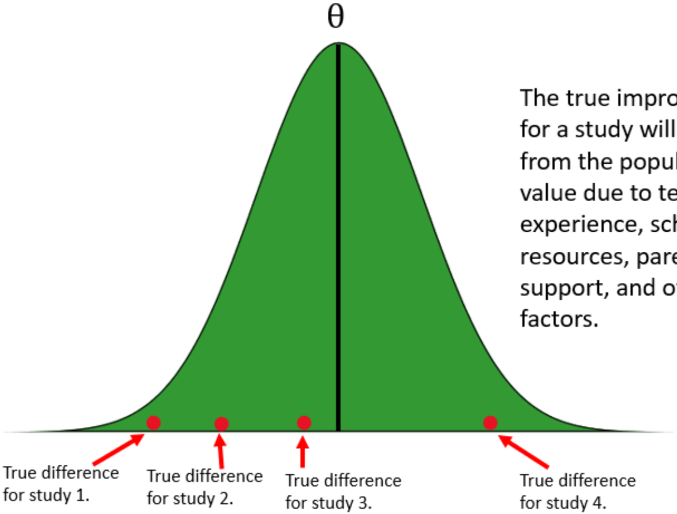
  for $j = 1, 2, \ldots, K$, where $\epsilon_j \sim \mathcal{N}(0, \hat{\sigma}_j^2)$ and $u_j \sim \mathcal{N}(0, \tau^2)$.
  The ϵ_js are the sampling errors and the u_js are the random effects
- The estimate of the overall effect size is the mean of the distribution of effect sizes, $\theta_{pop} = \mathbb{E}(\theta_j)$.

True difference in performance in the population



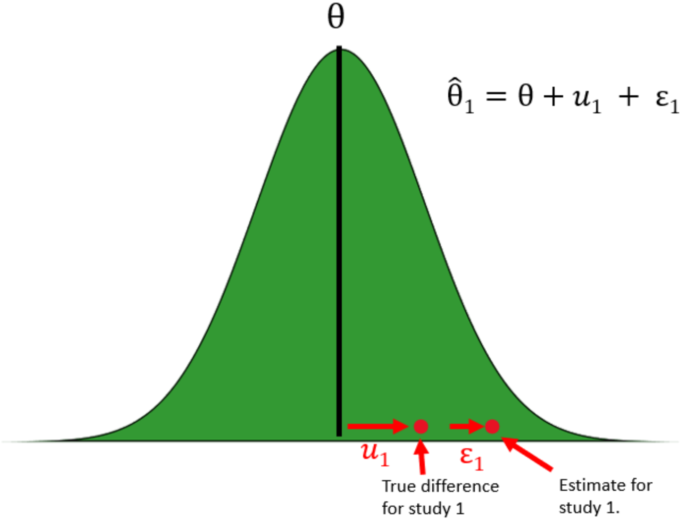
Suppose that the modified school calendar improves student performance by 20 points.

Underlying values for each study



The true improvement for a study will vary from the population value due to teachers' experience, school resources, parental support, and other factors.

Estimate of the difference for study 1



Random effects

- The random effect, u_1, is the deviation of the study's true effect size from the overall mean, θ (Borenstein et al., 2009)
- The sampling error, ϵ_1, is the deviation of the observed effect size from the study's true effect size
- We're assuming the true effects are normally distributed, and we only have a sample of studies from the population of interest

Standard meta analysis

```
. use schoolcal3, clear
(Effect of modified school calendar on student achievement)

. quietly: meta set stdmdiff se

. meta summarize, nostudies

  Effect-size label: Effect size
        Effect size: stdmdiff
          Std. err.: se

Meta-analysis summary                                Number of studies =     56
Random-effects model                                 Heterogeneity:
Method: REML                                                     tau2 =  0.0884
                                                               I2 (%) =   94.70
                                                                   H2 =   18.89

------------------------------------------------------------------------------
             |   Estimate   Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
       theta |   .1279321   .0438703     2.92   0.004     .0419479    .2139162
------------------------------------------------------------------------------
Test of homogeneity: Q = chi2(55) = 578.86                    Prob > Q = 0.0000
```

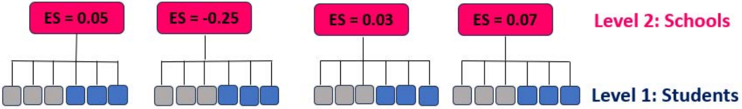
Multilevel meta-analysis

Multilevel data

- Previously, we assumed that the effect sizes were independent across studies
- This assumption wouldn't be valid if the schools in our dataset were nested within districts, because the estimated effect size for schools within the same district would likely be correlated
- If our meta-analytic data have a multilevel (hierarchical) structure, we can perform multilevel meta-analysis to account for the correlation between effect sizes in the same group

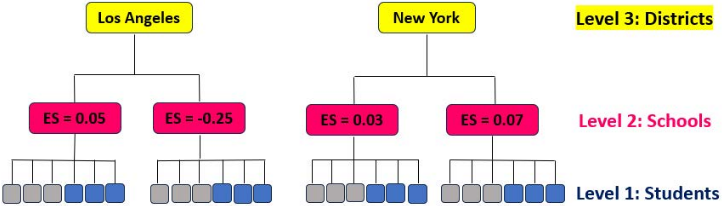
Standard meta-analysis as a two-level model

- Here we see the effect size reported by each study



Three-level model

- Now suppose that multiple studies belong to the same district
- Schools belonging to the same district will be more similar in terms of demographics and socioeconomical factors, resulting in a correlation between results within a district



- Here we see how studies are grouped by district

Modified school calendar data

```
. describe

Contains data from schoolcal3.dta
 Observations:            56                  Effect of modified school calendar on student achievement
    Variables:            10                  20 Sep 2023 12:02
                                              (_dta has notes)
------------------------------------------------------------------------------------------------------
Variable      Storage   Display    Value
    name         type    format    label      Variable label
------------------------------------------------------------------------------------------------------
district        int     %12.0g                District ID
school          byte    %9.0g                 School ID
study           byte    %12.0g                Study ID
stdmdiff        double  %10.0g                Standardized difference in means of achievement test scores
var             double  %10.0g                Within-study variance of stdmdiff
year            int     %12.0g                Year of the study
se              double  %10.0g                Within-study standard-error of stdmdiff
year_c          byte    %9.0g                 Year of the study centered around 1990
mean_exp        float   %9.0g                 Mean teacher experience
class_size      float   %9.0g                 Mean class size
------------------------------------------------------------------------------------------------------
Sorted by: stdmdiff
```

Modified school calendar data

```
. list district school study stdmdiff mean_exp in 1/11, sepby(district)
```

	district	school	study	stdmdiff	mean_exp
1.	11	1	1	-.18	6.394918
2.	11	2	2	-.22	1.820014
3.	11	3	3	.23	7.86858
4.	11	4	4	-.3	8.369441
5.	12	1	5	.13	10.48499
6.	12	2	6	-.26	10.73829
7.	12	3	7	.19	2.892403
8.	12	4	8	.32	6.689758
9.	18	1	9	.45	5.5483
10.	18	2	10	.38	13.40538
11.	18	3	11	.29	3.927117

Multilevel meta-analysis model

By performing a multilevel meta-analysis, we can

- estimate the effect size more precisely by accounting for the dependence between observations within a group
- assess the heterogeneity between schools within a district and between districts
- estimate how each district varies from the overall mean
  - This will help us decide whether the modified calendar should be applied to some districts and not others

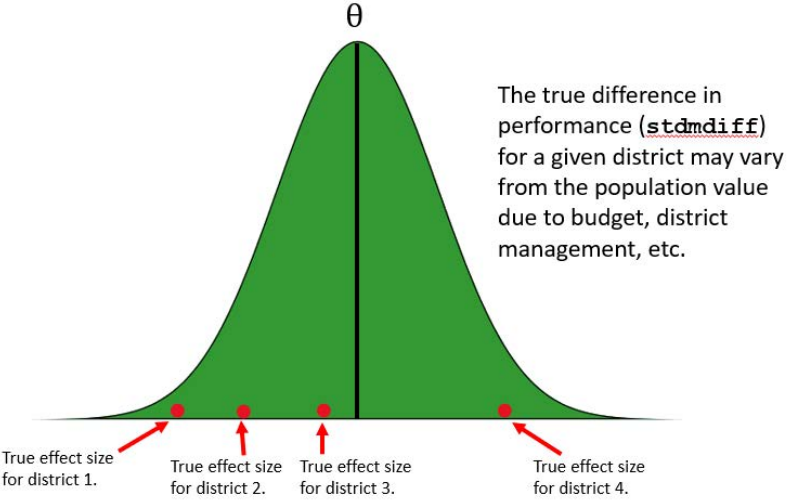
Multilevel meta-analysis model

We'll fit a three-level random-intercepts model

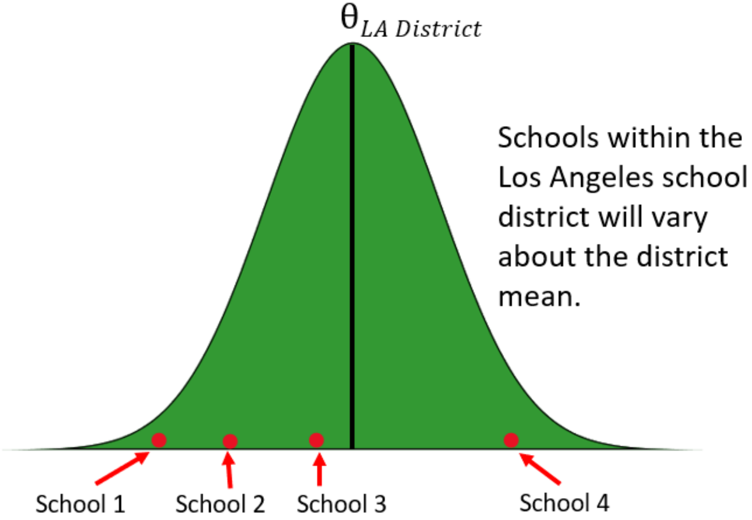
$$\hat{\theta}_{jk} = \theta + u_j^{(3)} + u_{jk}^{(2)} + \epsilon_{jk}$$

where $u_j^{(3)} \sim \mathcal{N}(0,\, \tau_3^2)$, $u_{jk}^{(2)} \sim \mathcal{N}(0,\, \tau_2^2)$, and $\epsilon_{jk} \sim \mathcal{N}(0,\, \hat{\sigma}_{jk}^2)$. Note that j represents the third level (district), k represents the second level (school within district), and ϵ_{jk} represents the sampling errors.

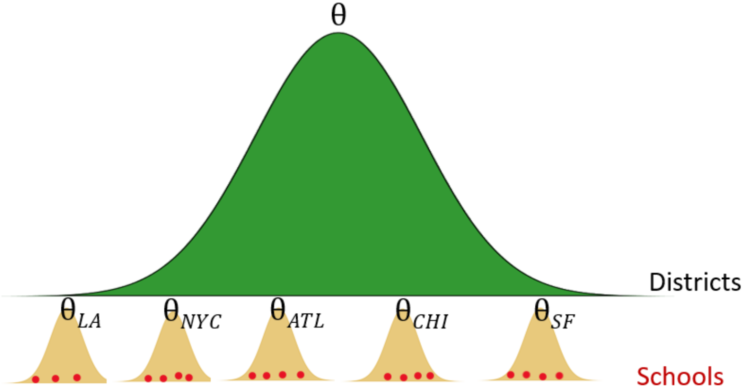
True difference in performance in the population



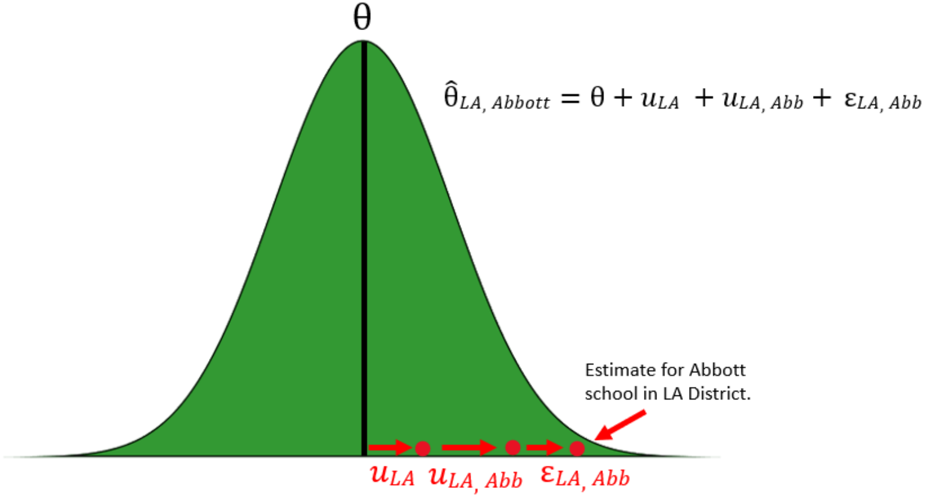
Schools within a district



Schools and districts



Estimate of the difference for study 1



Syntax for multilevel random-intercepts meta-regression

- We can specify the effect-size standard errors

  . `meta multilevel` *depvar* [*indepvars*] , `relevels(`*levels*`)` `essevariable(`*varname*`)`

- or the effect-size variances

  . `meta multilevel` *depvar* [*indepvars*] , `relevels(`*levels*`)` `esvarvariable(`*varname*`)`

  Option `relevels()` specifies the grouping structure, beginning with the highest level.

Three-level meta-analysis

```
. meta multilevel stdmdiff, relevels(district school) essevariable(se) nolog
Multilevel REML meta-analysis                          Number of obs = 56
        Grouping information
        ------------------------------------------------------------
                       |   No. of       Observations per group
        Group variable |   groups    Minimum    Average    Maximum
        ---------------+--------------------------------------------
              district |       11          3        5.1         11
                school |       56          1        1.0          1
        ------------------------------------------------------------
                                                       Wald chi2(0)   =   .
Log restricted-likelihood = -7.9587239                 Prob > chi2    =   .
------------------------------------------------------------------------------
    stdmdiff | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
       _cons |   .1847132   .0845559     2.18   0.029     .0189866    .3504397
------------------------------------------------------------------------------
Test of homogeneity: Q_M = chi2(55) = 578.86          Prob > Q_M = 0.0000
-----------------------------------------
  Random-effects parameters |   Estimate
----------------------------+------------
district: Identity          |
                   sd(_cons) |   .2550724
----------------------------+------------
school: Identity            |
                   sd(_cons) |   .1809324
-----------------------------------------
```

Assess variability among effect sizes

```
. estat heterogeneity

Method: Cochran
Joint:
  I2 (%) = 90.50

Method: Higgins–Thompson
district:
  I2 (%) = 63.32

school:
  I2 (%) = 31.86

Total:
  I2 (%) = 95.19
```

Higgins–Thompson heterogeneity statistics

- Higgins–Thompson I^2 statistics for level 3 (district) and level 2 (school)

$$I^2_{District} = \frac{\hat{\tau}^2_D}{\hat{\tau}^2_D + \hat{\tau}^2_S + s^2_{HT}}$$

$$I^2_{School} = \frac{\hat{\tau}^2_S}{\hat{\tau}^2_D + \hat{\tau}^2_S + s^2_{HT}}$$

where $\hat{\tau}^2_D$ and $\hat{\tau}^2_S$ are the estimated variances for the district and school levels, and s^2_{HT} is the summary measure of the level 1 variances.

Comparing the three and two level models

- Two-level model:

$$\hat{\theta}_j = \theta + u_j^{(2)} + \epsilon_j$$

- Three-level model:

$$\hat{\theta}_{jk} = \theta + u_j^{(3)} + u_{jk}^{(2)} + \epsilon_{jk}$$

where $u_j^{(3)} \sim \mathcal{N}(0,\, \tau_3^2)$ and $u_{jk}^{(2)} \sim \mathcal{N}(0,\, \tau_2^2)$.

- j represents the third level (district)
- k represents the second level (school within district)

Comparing the three and two level models

- We want to test whether there is a nonnegligible amount of heterogeneity between the districts
- Essentially, we're testing whether $\tau_3^2 = 0$

Comparing the three and two level models

- First, we store our results from the previous model

```
. meta multilevel stdmdiff, ///
 relevels(district school) essevariable(se)
. estimates store full_model
```

- We now fit a two-level model with study as the second level

```
. meta multilevel stdmdiff, ///
 relevels(study) essevariable(se)
. estimates store standard_model
```

Likelihood-ratio test

```
. lrtest full_model standard_model

Likelihood-ratio test
Assumption: standard_model nested within full_model

 LR chi2(1) =  17.77
Prob > chi2 = 0.0000

Note: The reported degrees of freedom assumes the null hypothesis is not on the boundary of
      the parameter space. If this is not true, then the reported test is conservative.
Note: LR tests based on REML are valid only when the fixed-effects specification is identical
      for both models.
```

- We prefer the full model with district and school-level random effects

Fit a two-level model

- We want to test whether there is a nonnegligible amount of heterogeneity between the schools within a district
- First, we store our results from the previous model

```
. meta multilevel stdmdiff, ///
 relevels(district school) essevariable(se)
. estimates store full_model
```

- We now fit a two-level model with district as the second level

```
. meta multilevel stdmdiff, ///
 relevels(district) essevariable(se)
. estimates store district_re
```

Likelihood-ratio test

```
. lrtest full_model district_re

Likelihood-ratio test
Assumption: district_re nested within full_model

 LR chi2(1) =  48.52
Prob > chi2 = 0.0000

Note: The reported degrees of freedom assumes the null hypothesis is not on the boundary of
      the parameter space. If this is not true, then the reported test is conservative.
Note: LR tests based on REML are valid only when the fixed-effects specification is identical
      for both models.
```

Estimation method

- We can fit our models using maximum likelihood (ML) or restricted maximum likelihood (REML).
- Restricted maximum likelihood is also known as residual maximum likelihood, and it is the default method because it produces unbiased estimates.
- Note that for REML the likelihood does not depend on the fixed-effects component of the model.

Information criteria

```
. estimates stats full_model district_re standard_model, n(55)
```

Akaike's information criterion and Bayesian information criterion

Model	N	ll(null)	ll(model)	df	AIC	BIC
full_model	55	.	-7.958724	3	21.91745	27.93945
district_re	55	.	-32.21648	2	68.43295	72.44762
standard_m~l	55	.	-16.8455	2	37.691	41.70566

Preparing table of estimation results

```
. etable, estimates(full_model district_re standard_model)
```

	stdmdiff	stdmdiff	stdmdiff
Intercept	0.185	0.196	0.128
	(0.085)	(0.090)	(0.044)
sd(_cons)	0.255	0.288	
	(0.070)	(0.068)	
sd(_cons)	0.181		
	(0.031)		
(output omitted)			
sd(_cons)			0.297
			(0.034)
Number of observations	56	56	56

Customizing and exporting the table

```
. etable, estimates(full_model district_re standard_model) ///
>  equations(stdmdiff district school) showeq ///
>  mstat(N) mstat(aic) mstat(bic) column(index) ///
>  export(mytable.docx)
```

	1	2	3
Standardized difference in means of achievement test scores			
Intercept	0.185	0.196	0.128
	(0.085)	(0.090)	(0.044)
District ID			
sd(_cons)	0.255	0.288	
	(0.070)	(0.068)	
School ID			
sd(_cons)	0.181		
	(0.031)		
Number of observations	56	56	56
AIC	21.92	68.43	37.69
BIC	27.99	72.48	41.74

(collection ETable exported to file mytable.docx)

Sensitivity analysis

- Suppose we're interested in exploring how different magnitudes of the school-level variation impact our estimates of the standardized mean difference and the district-level variation.
- To answer this question, we'll refit our model, each time setting the random-effects standard deviations for the school level to a different value.

Random-intercepts standard deviations

```
. meta multilevel stdmdiff, ///
 relevels(district school, sd(.  0.01)) esse(se)
. estimates store fixsd1
. meta multilevel stdmdiff, ///
 relevels(district school, sd(.  0.18)) esse(se)
. estimates store fixsd2
. meta multilevel stdmdiff, ///
 relevels(district school, sd(.  0.60)) esse(se)
. estimates store fixsd3
```

Comparing effect sizes

```
. estimates table _all, stats(sd2) keep(stdmdiff:_cons) b(%8.3f) se(%8.3f)
```

Variable	fixsd1	fixsd2	fixsd3
_cons	0.196	0.185	0.123
	0.090	0.085	0.083
sd2	0.010	0.180	0.600

Legend: b/se

Accessing stored random-effects standard deviations

```
. meta multilevel stdmdiff, relevels(district school, sd(. 0.01)) ///
>  essevariable(se) nogroup nolog noheader nofetable

-----------------------------------------------
  Random-effects parameters  |   Estimate
-----------------------------+-----------------
district: Identity           |
                  sd(_cons)  |   .2876359
-----------------------------+-----------------
school: Custom               |
                  sd(_cons)  |        .01*
-----------------------------------------------
(*) fixed during estimation

. matrix list e(b)

e(b)[1,3]
      stdmdiff:   lns1_1_1:    lnsig_e:
         _cons        _cons       _cons
y1   .19597049   -1.2460597           0

. display exp(-1.2460597)
.28763594
```

Comparing random-effects standard deviations for districts

```
. estimates table _all, stats(sd2) keep(lns1_1_1:_cons) b(%8.3f) eform
```

Variable	fixsd1	fixsd2	fixsd3
_cons	0.288	0.255	0.000
sd2	0.010	0.180	0.600

Predictions of random effects

```
. qui: meta multilevel stdmdiff, relevels(district school) esse(se)
. predict double u3 u2, reffects reses(se_u3 se_u2, diagnostic)
. by district, sort: generate tolist = (_n==1)
. list district u3 se_u3 if tolist
```

	district	u3	se_u3
1.	11	-.18998595	.07071818
5.	12	-.08467077	.13168501
9.	18	.1407273	.11790486
12.	27	.24064814	.13641505
16.	56	-.1072942	.13633364
20.	58	-.23650899	.15003184
31.	71	.5342778	.12606073
34.	86	-.2004695	.1499012
42.	91	.05711692	.14284823
48.	108	-.14168396	.13094894
53.	644	-.01215679	.10054689

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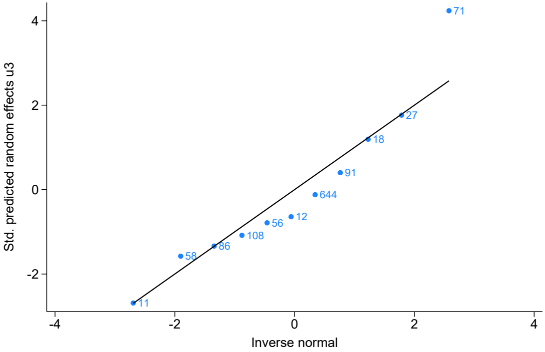
Normal quantile plot

```
. generate double ustan3 = u3/se_u3

. qnorm ustan3 if tolist, mlabel(district)
```



Three-level meta-analysis and meta-regression

- Multilevel meta-analysis; random intercepts for district (3) and school (2):

$$\hat{\theta}_{jk} = \theta + u_j^{(3)} + u_{jk}^{(2)} + \epsilon_{jk}$$

Three-level meta-analysis and meta-regression

- Multilevel meta-analysis; random intercepts for district (3) and school (2):

$$\hat{\theta}_{jk} = \theta + u_j^{(3)} + u_{jk}^{(2)} + \epsilon_{jk}$$

- Multilevel meta-regression; random intercepts and a moderator

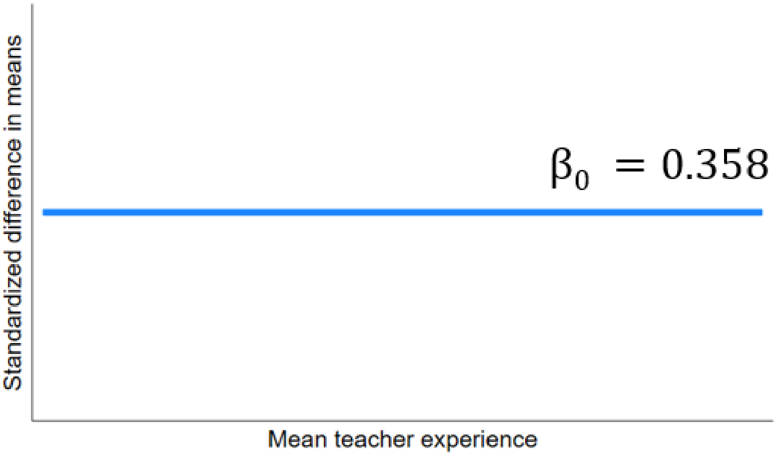
$$\hat{\theta}_{jk} = \beta_0 + \beta_1 exper_{jk} + u_j^{(3)} + u_{jk}^{(2)} + \epsilon_{jk}$$

  - One general effect of `exper` for all studies

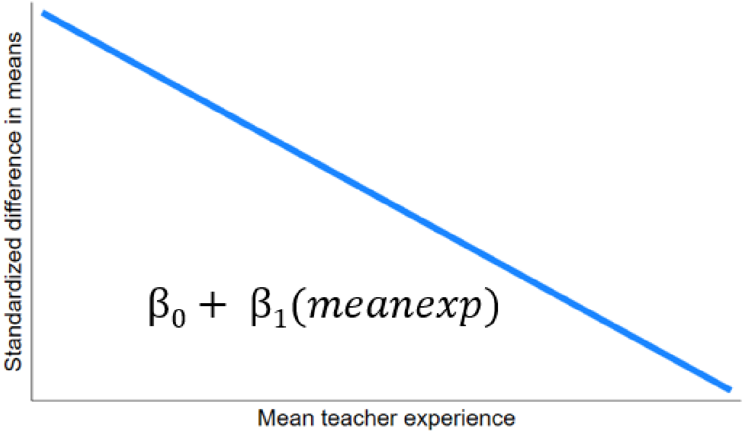
Three-level meta-analysis and meta-regression

- Multilevel meta-analysis; random intercepts for district (3) and school (2):
$$\hat{\theta}_{jk} = \theta + u_j^{(3)} + u_{jk}^{(2)} + \epsilon_{jk}$$
- Multilevel meta-regression; random intercepts and a moderator
$$\hat{\theta}_{jk} = \beta_0 + \beta_1 exper_{jk} + u_j^{(3)} + u_{jk}^{(2)} + \epsilon_{jk}$$
  - One effect of `exper` for all studies
- Multilevel meta-regression with random slopes
$$\hat{\theta}_{jk} = \beta_0 + \beta_1 exper_{jk} + u_{0j}^{(3)} + u_{1j}^{(3)} exper_{jk} + u_{jk}^{(2)} + \epsilon_{jk}$$
  - One effect of `exper` specific to each district

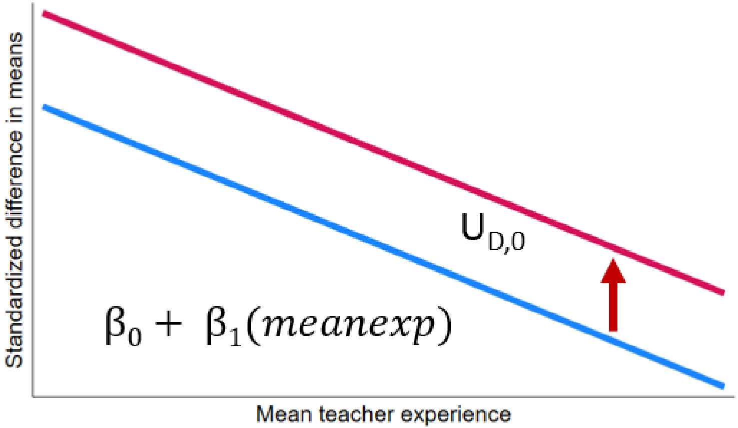
Multilevel meta-analysis



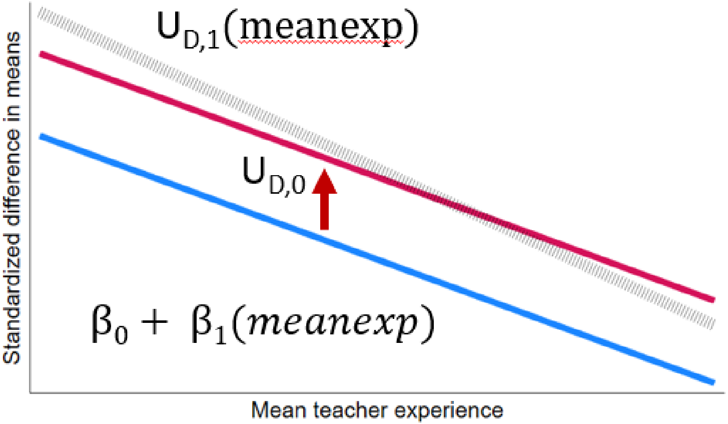
Multilevel meta-regression



Multilevel meta-regression



Multilevel meta-regression with a random slope



Models with random slopes

- To fit models with random slopes we need `meta meregress`

```
. meta meregress stdmdiff x1 || district:  x1 || school:, esse(se)
```

- The random slope for `x1` allows the effect of `x1` to vary across districts
- Just like with `meta multilevel`, we can specify the effect-size standard errors

  `. meta meregress` *depvar fe_equation* `||` *re_equation* [`||` *re_equation*] `, essevariable(`*varname*`)`

- or the effect-size variances

  `. meta meregress` *depvar fe_equation* `||` *re_equation* [`||` *re_equation*] `, esvarvariable(`*varname*`)`

Three-level meta-regression with random slopes

```
. meta meregress stdmdiff mean_exp ///
>   || district: mean_exp ///
>   || school:, essevariable(se) nolog nogroup

Multilevel REML meta-regression                   Number of obs =      56
                                                  Wald chi2(1)  =    8.37
Log restricted-likelihood = -3.3635425            Prob > chi2   = 0.0038

------------------------------------------------------------------------------
    stdmdiff | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
    mean_exp |  -.0262054    .009058    -2.89   0.004    -.0439587   -.0084521
       _cons |   .3580009   .0981127     3.65   0.000     .1657036    .5502983
------------------------------------------------------------------------------
Test of homogeneity: Q_M = chi2(54) = 558.47              Prob > Q_M = 0.0000

-----------------------------------------
  Random-effects parameters |   Estimate
----------------------------+------------
district: Independent       |
              sd(mean_exp)  |   .0156308
                 sd(_cons)  |   .2605429
----------------------------+------------
school: Identity            |
                 sd(_cons)  |    .146955
-----------------------------------------
```

Display variance components

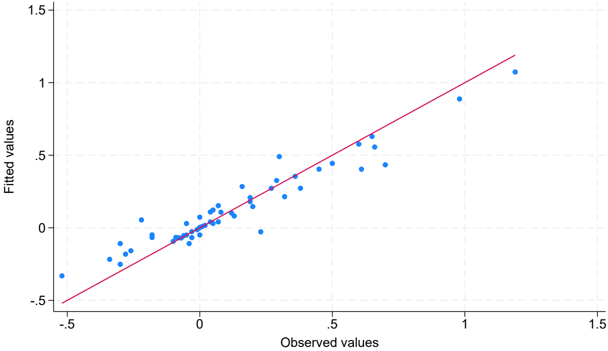
```
. estat sd, variance
```

Random-effects parameters	Estimate
district: Independent	
var(mean_exp)	.0002443
var(_cons)	.0678826
school: Identity	
var(_cons)	.0215958

Checking model fit

```
. predict double fit, fitted

. twoway (scatter fit stdmdiff) (function y = x, range(stdmdiff))
```



Random-effects covariance structures

unstructured $\Sigma = \begin{bmatrix} \sigma_{11} & & \\ \sigma_{21} & \sigma_{22} & \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$

independent $\Sigma = \begin{bmatrix} \sigma_{11} & & \\ 0 & \sigma_{22} & \\ 0 & 0 & \sigma_{33} \end{bmatrix}$

exchangeable $\Sigma = \begin{bmatrix} \sigma_{11} & & \\ \sigma_{21} & \sigma_{11} & \\ \sigma_{21} & \sigma_{21} & \sigma_{11} \end{bmatrix}$

identity $\Sigma = \begin{bmatrix} \sigma_{11} & & \\ 0 & \sigma_{11} & \\ 0 & 0 & \sigma_{11} \end{bmatrix}$

Specifying the random-effects covariance structure

```
. meta meregress stdmdiff mean_exp ///
>   || district: mean_exp, covariance(exchangeable) ///
>   || school:, essevariable(se) nolog nogroup

Multilevel REML meta-regression                     Number of obs =      56
                                                    Wald chi2(1)  =    2.62
Log restricted-likelihood = -10.569892              Prob > chi2   =  0.1054

------------------------------------------------------------------------------------
          stdmdiff | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------------+----------------------------------------------------------------
          mean_exp |  -.0203673   .0125786    -1.62   0.105    -.0450208    .0042862
             _cons |   .3051209   .0695575     4.39   0.000     .1687908    .4414511
------------------------------------------------------------------------------------
Test of homogeneity: Q_M = chi2(54) = 558.47           Prob > Q_M = 0.0000

------------------------------------------------------
  Random-effects parameters |   Estimate
----------------------------+-------------------------
district: Exchangeable      |
         sd(mean_exp _cons) |   .0263766
        corr(mean_exp,_cons)|   .9999999
----------------------------+-------------------------
school: Identity            |
                   sd(_cons)|   .2195733
------------------------------------------------------
```

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Exchangeable covariance structure

```
. meta meregress stdmdiff mean_exp class_size ///
> || district: mean_exp class_size, covariance(exchangeable) ///
> || school:, essevariable(se) var matlog nolog nogroup noheader

------------------------------------------------------------------------------
    stdmdiff | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
    mean_exp |  -.0132714   .0042179    -3.15   0.002    -.0215384   -.0050044
  class_size |   .0251681   .0029628     8.49   0.000     .0193611    .0309751
       _cons |  -.3344917   .0521791    -6.41   0.000    -.4367608   -.2322226
------------------------------------------------------------------------------
Test of homogeneity: Q_M = chi2(53) = 126.71             Prob > Q_M = 0.0000

------------------------------------------------
  Random-effects parameters  |         Estimate
-----------------------------+------------------
district: Exchangeable       |
    var(mean_exp.._cons)(1)  |         .0000461
    cov(mean_exp.._cons)(1)  |         -.000023
-----------------------------+------------------
school: Identity             |
                  var(_cons) |         8.15e-09
------------------------------------------------
(1) mean_exp class_size _cons
```

Random-effects covariance matrix

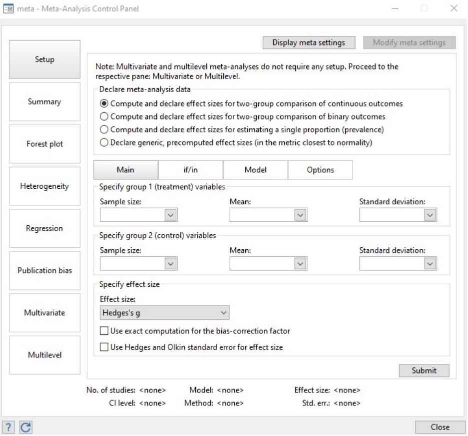
```
. estat recovariance

Random-effects covariance matrix for level district

             |  mean_exp   class_s~e       _cons
-------------+------------------------------------
    mean_exp |  .0000461
  class_size |  -.000023    .0000461
       _cons |  -.000023    -.000023    .0000461

Random-effects covariance matrix for level school

             |     _cons
-------------+-----------
       _cons |  8.15e-09
```

Summary

Summary

- Today, we learned how to do the following in Stata:
  - Perform meta-regression with effect sizes that have hierarchical structures.
  - Assess heterogeneity at different levels of the hierarchy.
  - Create and export a table of estimation results.

Perform meta-analysis using Stata's graphical interface



Resources

- Overview of meta-analysis features in Stata
- Video tutorial on performing meta-analysis in Stata
- Stata Meta-Analysis Reference Manual

References

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