

Bayesian multilevel modeling in Stata

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You can download the slides and other materials here:
<https://tinyurl.com/StataBMM22>

Bayesian MLM in Stata

bayes:

<code>mixed</code>	<code>xtreg</code>
<code>metobit</code>	<code>xtlogit</code>
<code>meintreg</code>	<code>xtprobit</code>
<code>melogit</code>	<code>xtologit</code>
<code>meprobit</code>	<code>xtoprobit</code>
<code>mecloglog</code>	<code>xtmlogit</code>
<code>meoprobit</code>	<code>xtpoisson</code>
<code>mepoisson</code>	<code>xtnbreg</code>
<code>menbreg</code>	
<code>meglm</code>	
<code>mestreg</code>	

bayesmh

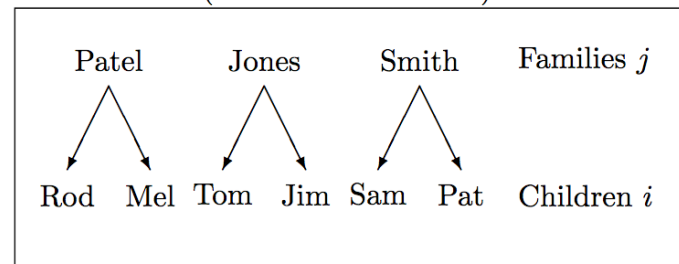
Overview

- What are multilevel models?
- Why Bayesian?
- Analysis example
 - `mixed`
 - `bayes: mixed`
 - `bayesmh`
- More complex examples
 - Three-level models
 - Nonlinear multilevel models
 - Multivariate multilevel model

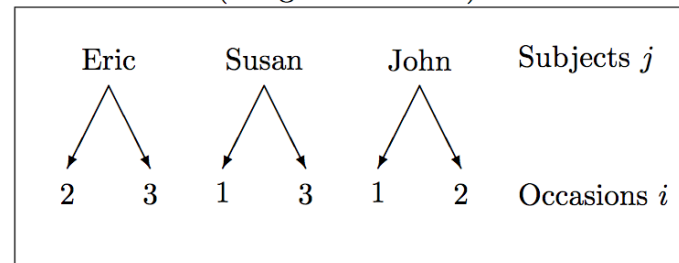
What are multilevel models (MLMs)?

- Hierarchical linear models (HLM)
- Mixed-effects models
- Mixed models
- Random-effects models
- Variance-component models

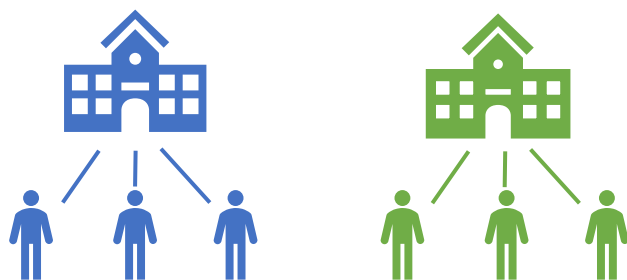
Children nested in families
(Cross-sectional data)



Occasions nested in subjects
(Longitudinal data)

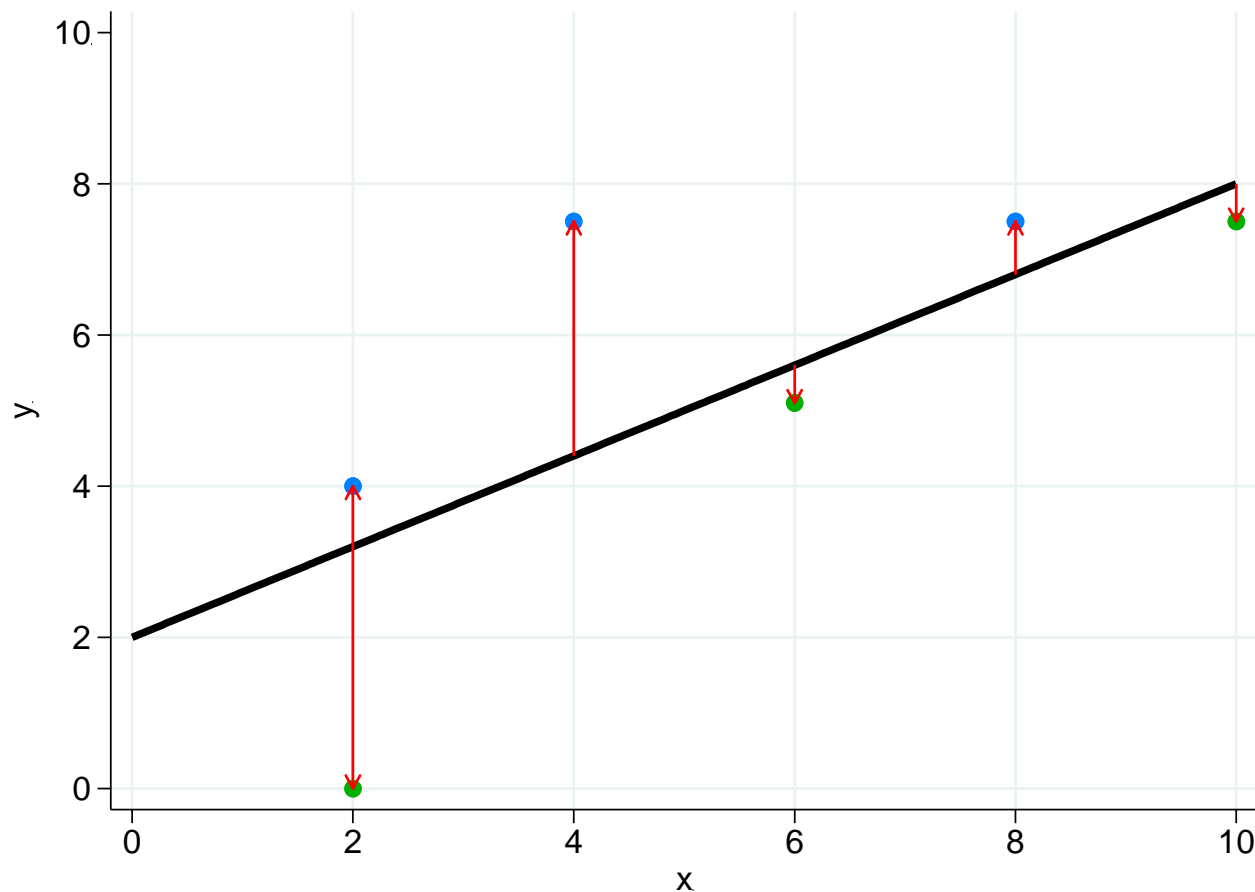


Nested data



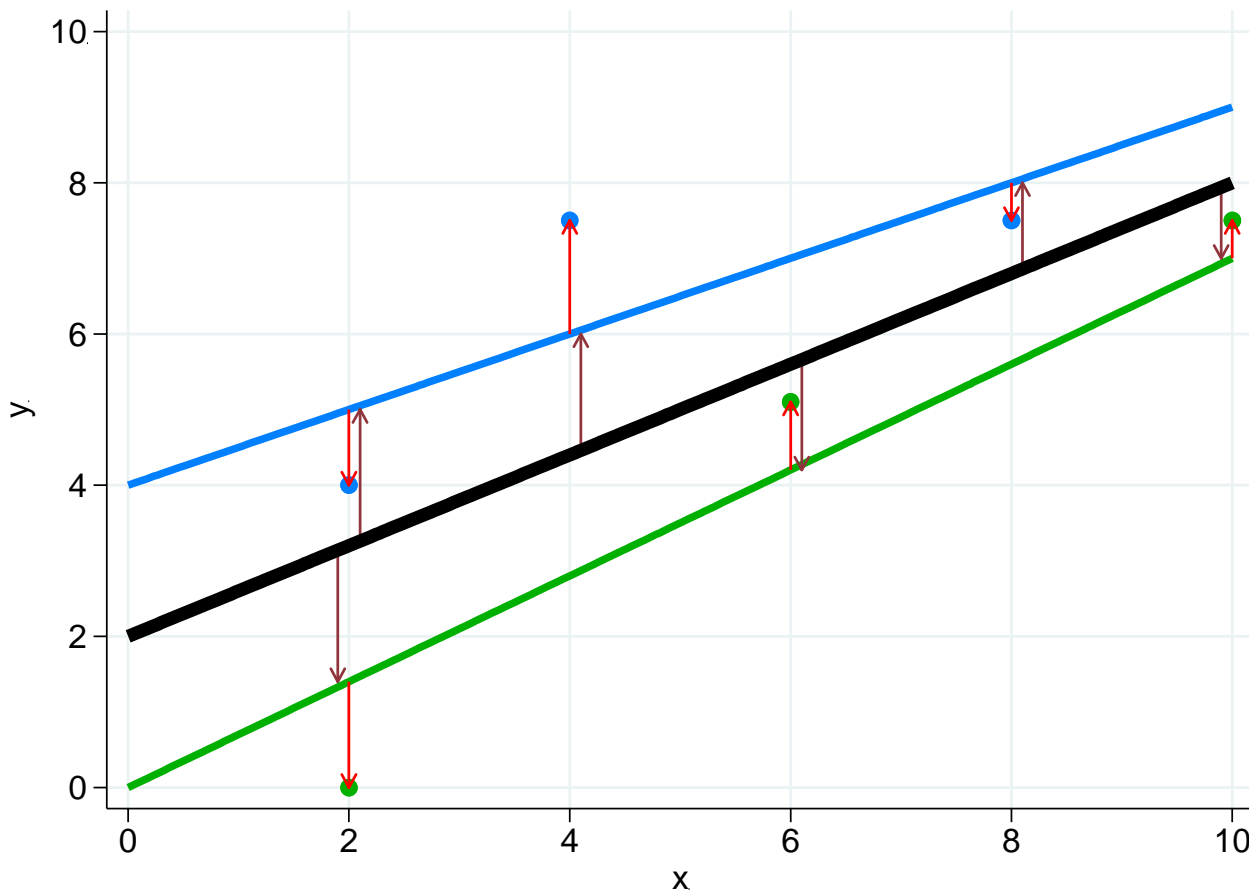
Linear regression

$$y_i = \beta_0 + \beta_1 x_i + e_i$$

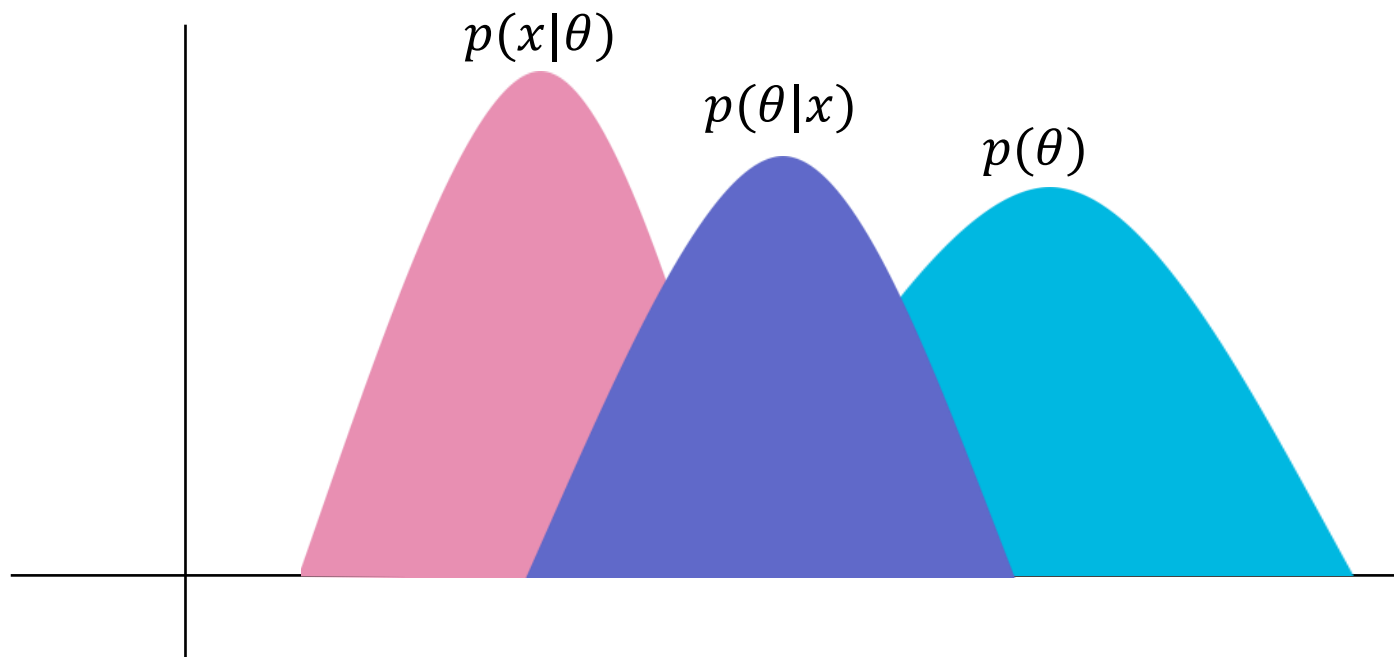


Multilevel model

$$y_{ij} = \gamma_0 + \gamma_1 x_{ij} + u_{0j} + u_{1j} x_{ij} + e_{ij}$$



What is Bayesian analysis?



Bayesian: the probability of the model parameters given the data

- Data are fixed
- Parameters are random

Frequentist: the probability of the data given the model parameters

- Parameters are fixed
- Data are random

What is Bayesian analysis?

Bayesian: the probability of the model parameters given the data

- Data are fixed
- Parameters are random



Random effects are estimated jointly along with the model parameters

Frequentist: the probability of the data given the model parameters

- Parameters are fixed
- Data are random



Random effects are integrated out to estimate the model parameters; they can be predicted after estimation

Why Bayesian?

- Estimate random effects jointly along with the model parameters, thus providing correct standard errors
- Estimate more complex models
 - Many random effects, crossed effects
 - Nonlinear relationships and nonnormal error
- Incorporate prior knowledge
- Better small-sample inference

Example

How does number of hours spent on homework influence math scores in high school students?



Example

```
. use math  
. describe
```

Contains data from math.dta

Observations: 519

Variables: 4

21 Feb 2022 14:12

Variable name	Storage type	Display format	Value label	Variable label
schid	float	%9.0g		School ID
homework	float	%9.0g		Time spent on math homework each week
sctype	float	%9.0g	public	School type
math	float	%9.0g		Math score

```
. label list public
```

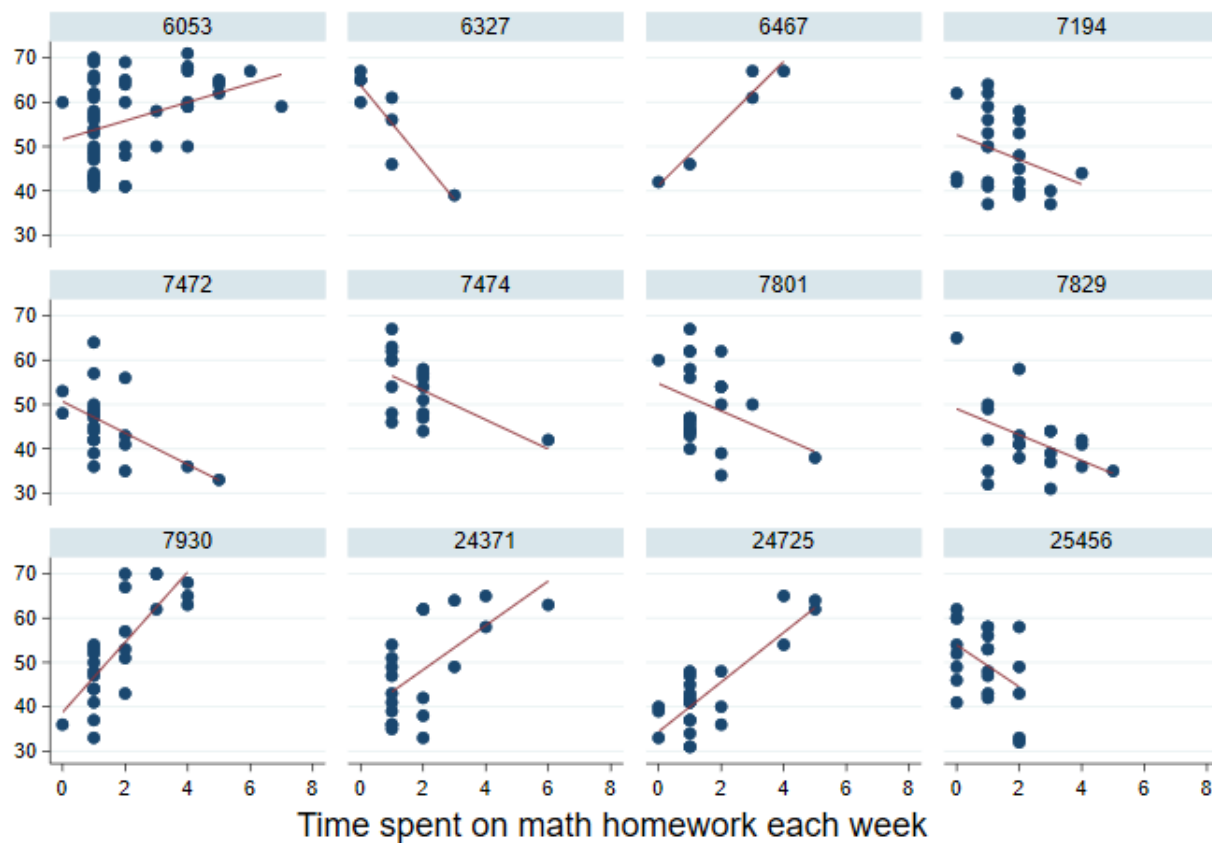
public:

0 Private

1 Public

Example

```
. twoway (scatter math homework) (lfit math homework), by(schid)
```



Graphs by School ID

MLMs with mixed

```
. mixed math homework i.schtype || schid: homework, covariance(unstructured)
```

math	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
homework	1.982092	.895697	2.21	0.027	.2265586	3.737626
schtype						
Private	0 (base)					
Public	-4.08201	1.895363	-2.15	0.031	-7.796853	-.367167
_cons	49.06494	2.112821	23.22	0.000	44.92389	53.20599

Random-effects parameters	Estimate	Std. err.	[95% conf. interval]	
schid: Unstructured				
var(homework)	16.37279	5.695054	8.280255	32.37439
var(_cons)	56.24904	19.24056	28.77087	109.9708
cov(homework,_cons)	-25.99454	9.645101	-44.89859	-7.090486
var(Residual)	53.34282	3.471612	46.95467	60.60007

MLMs with mixed

```
. mixed math homework i.schtype || schid: homework, covariance(unstructured)  
. estat sd
```

Random-effects parameters	Estimate	Std. err.	[95% conf. interval]	
schid: Unstructured				
sd(homework)	4.04633	.7037307	2.877543	5.68985
sd(_cons)	7.499936	1.282715	5.363848	10.4867
corr(homework,_cons)	-.8565706	.0640156	-.941546	-.6691179
sd(Residual)	7.303617	.2376639	6.852348	7.784605

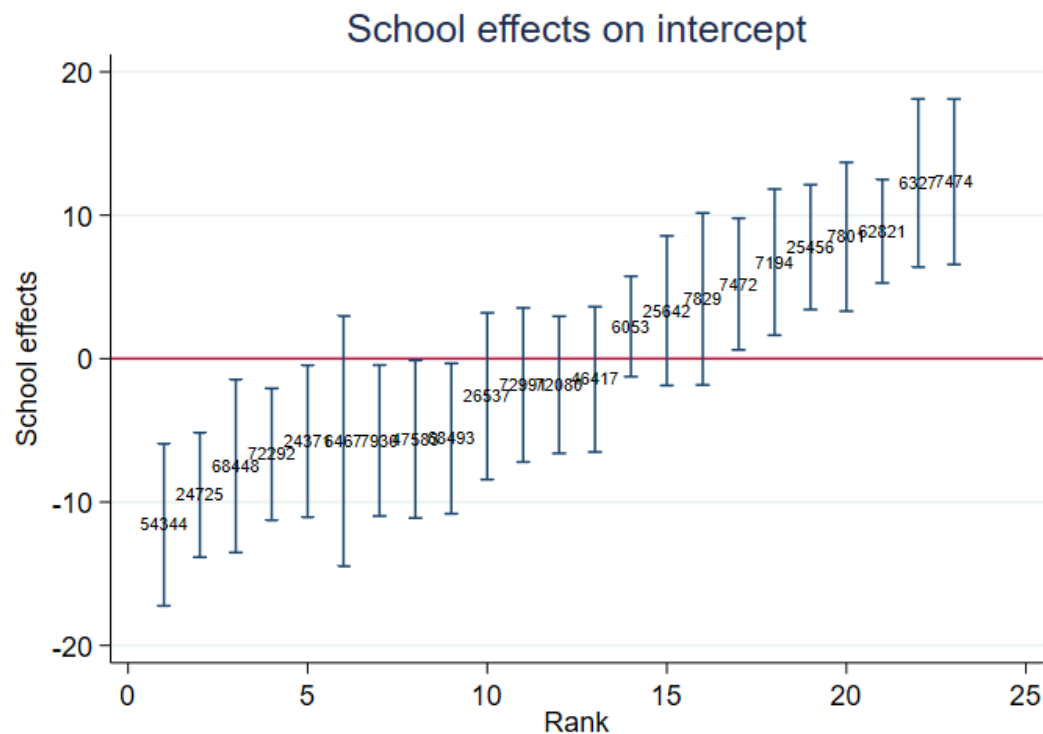
MLMs with mixed

- ```
. mixed math homework i.schtype || schid: homework, covariance(unstructured)
. estat sd
. predict u*, reffects reses(s*)
```
- Calculates best linear unbiased predictions (BLUPs) of the random effects conditional on the model parameters



# Random effects

- . predict u\*, reffects reses(s\*)
- . egen tagme = tag(schid)
- . egen rank = rank(u2) if tagme
- . serrbar u2 s2 rank if tagme, scale(1.96)



# Bayesian MLMs with bayes :

```
. bayes, rseed(317) melabel: /
> mixed math homework i.schtype || schid: homework, covariance(unstructured)
```

Bayesian multilevel regression  
Metropolis-Hastings and Gibbs sampling

Group variable: schid

MCMC iterations = 12,500  
Burn-in = 2,500  
MCMC sample size = 10,000  
Number of groups= 23  
Obs per group:  
min = 5  
avg = 22.6  
max = 67  
Number of obs = 519  
Acceptance rate = .7085  
Efficiency: min = .001621  
avg = .1043  
max = .4937

Log marginal-likelihood

|      |          | Mean     | Std. dev. | MCSE    | Median  | Equal-tailed<br>[95% cred. interval] |         |
|------|----------|----------|-----------|---------|---------|--------------------------------------|---------|
| math |          |          |           |         |         |                                      |         |
|      | homework | 2.200887 | 1.0217    | .253736 | 2.17371 | .2195924                             | 4.03408 |
|      | schtype  |          |           |         |         |                                      |         |

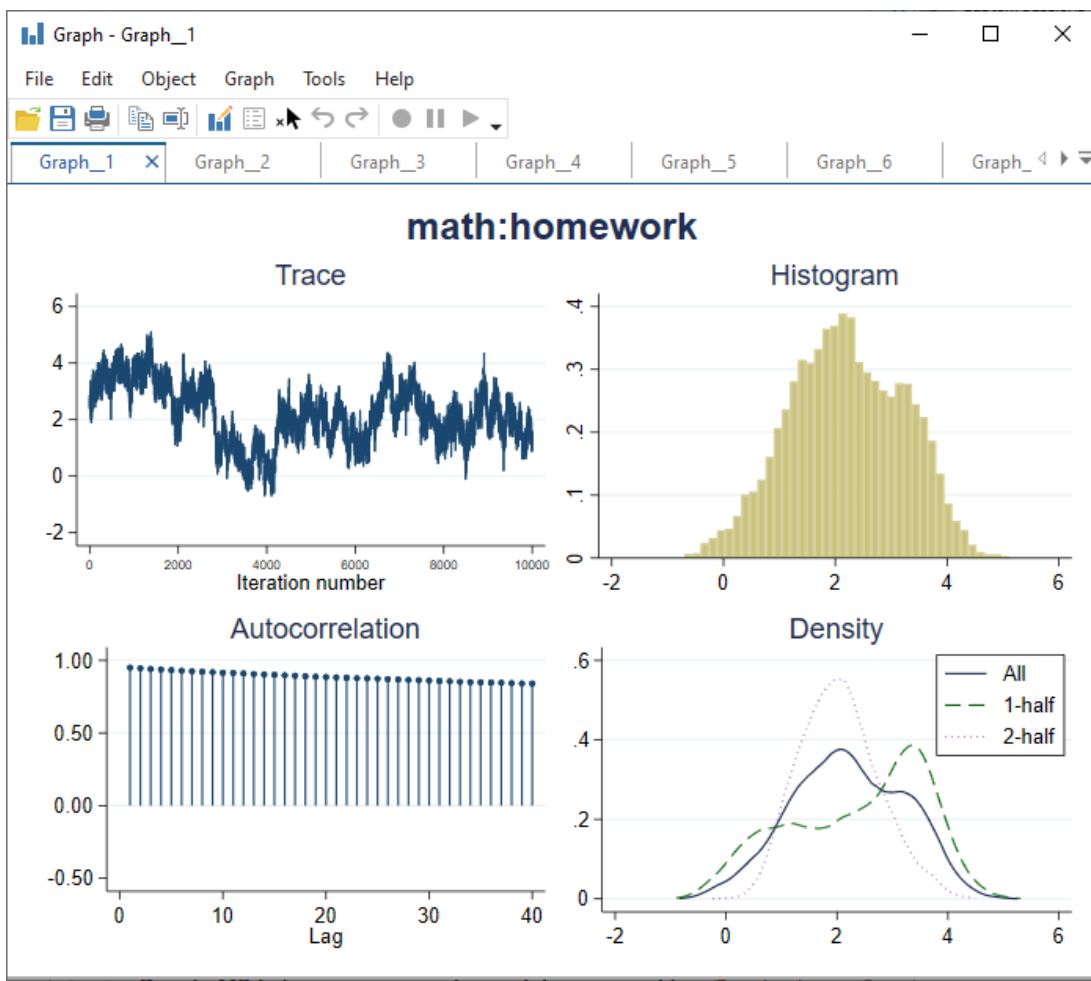
# Bayesian MLMs with bayes:

```
. bayes, melabel hpd showreffects
```

|           |          | Mean      | Std. dev. | MCSE    | Median    | HPD<br>[95% cred. interval] |           |
|-----------|----------|-----------|-----------|---------|-----------|-----------------------------|-----------|
| math      | homework | 2.200887  | 1.0217    | .253736 | 2.17371   | .2372925                    | 4.037823  |
|           | schtype  |           |           |         |           |                             |           |
|           | 0        | (base)    |           |         |           |                             |           |
|           | 1        | -4.117564 | 1.930991  | .16501  | -4.161968 | -7.930175                   | -.2308442 |
|           | _cons    | 48.81127  | 2.159534  | .397155 | 48.81032  | 44.61455                    | 52.96314  |
| U0[schid] |          |           |           |         |           |                             |           |
|           | 6053     | 2.380443  | 2.641693  | .386327 | 2.314307  | -2.907423                   | 7.183145  |
|           | 6327     | 12.27766  | 3.517942  | .318514 | 12.12157  | 6.278201                    | 19.85371  |
|           | 6467     | -5.118423 | 4.663207  | .338852 | -4.89623  | -14.38878                   | 3.934085  |
|           | 7194     | 6.727625  | 3.284143  | .48035  | 6.655871  | -.040091                    | 13.34809  |
|           | 7472     | 5.386686  | 2.972176  | .462614 | 5.239304  | -.207199                    | 11.51072  |
|           | 7474     | 12.60276  | 3.50846   | .468233 | 12.5518   | 5.866633                    | 19.55605  |
|           | 7801     | 8.702496  | 3.232788  | .485823 | 8.57521   | 2.676306                    | 15.35657  |
|           | 7829     | 4.332392  | 3.692502  | .510892 | 4.26765   | -2.241447                   | 12.03113  |

# Diagnostics

```
. bayesgraph diagnostics _all
```



Graph options

trace

ac

histogram

kdensity

cusum

matrix

# Diagnostics

```
. bayesstats ess
```

Efficiency summaries      MCMC sample size =      10,000

|             | ESS     | Corr. time | Efficiency |
|-------------|---------|------------|------------|
| math        |         |            |            |
| homework    | 16.21   | 616.76     | 0.0016     |
| schtype     |         |            |            |
| 0           | (base)  |            |            |
| 1           | 136.94  | 73.02      | 0.0137     |
| _cons       | 29.57   | 338.22     | 0.0030     |
| schid       |         |            |            |
| U:Sigma_1_1 | 812.39  | 12.31      | 0.0812     |
| U:Sigma_2_1 | 708.64  | 14.11      | 0.0709     |
| U:Sigma_2_2 | 663.64  | 15.07      | 0.0664     |
| e.math      |         |            |            |
| sigma2      | 4937.08 | 2.03       | 0.4937     |

# MCMC settings

- `nchains(1)`
- `burnin(2500)`
- `mcmcsize(10000)`
- `thinning(1)`
- `block(param [, blockopts])`
- `initial({param} inits)`

# MCMC settings

```
. search bayesparallel
. bayesparallel, nproc(3):
> bayes, rseed(317) burnin(5000) mcmcsize(15000) nchains(3):
> mixed math homework i.schtype || schid: homework, cov(un)
```

Simulating multiple chains ...

Done.

# MCMC settings

```
. search bayesparallel
. bayesparallel, nproc(3):
> bayes, rseed(317) burnin(5000) mcmcsize(15000) nchains(3):
> mixed math homework i.schtype || schid: homework, cov(un)
. bayes, melabel
```

Bayesian multilevel regression  
Metropolis-Hastings and Gibbs sampling

Group variable: schid

```
Number of chains = 3
Per MCMC chain:
 Iterations = 20,000
 Burn-in = 5,000
 Sample size = 15,000
Number of groups= 23
Obs per group:
 min = 5
 avg = 22.6
 max = 67
Number of obs = 519
Avg acceptance rate = .7075
Avg efficiency: min = .003347
 avg = .08935
 max = .4027
Max Gelman-Rubin Rc = 1.022
```

Log marginal-likelihood



# Diagnostics

```
. bayesstats ess
```

```
Efficiency summaries Number of chains = 3
 MCMC sample size = 45,000
```

|             | ESS      | Corr. time | Efficiency |
|-------------|----------|------------|------------|
| math        |          |            |            |
| homework    | 150.60   | 298.81     | 0.0033     |
| schtype     |          |            |            |
| 0           | (base)   |            |            |
| 1           | 988.46   | 45.53      | 0.0220     |
| _cons       | 285.18   | 157.79     | 0.0063     |
| schid       |          |            |            |
| U:Sigma_1_1 | 2710.29  | 16.60      | 0.0602     |
| U:Sigma_2_1 | 2851.40  | 15.78      | 0.0634     |
| U:Sigma_2_2 | 3038.14  | 14.81      | 0.0675     |
| e.math      |          |            |            |
| sigma2      | 18122.55 | 2.48       | 0.4027     |

# Diagnostics

```
. bayesstats grubin
```

Gelman–Rubin convergence diagnostic

|                     | Rc       |
|---------------------|----------|
| math                |          |
| homework            | 1.021691 |
| schtype             |          |
| 0                   | (base)   |
| 1                   | 1.003556 |
| _cons               | 1.006634 |
| schid               |          |
| var(homework)       | 1.004815 |
| var(_cons)          | 1.002796 |
| cov(homework,_cons) | 1.00415  |
| var(Residual)       | 1.000185 |

Convergence rule:  $Rc < 1.1$

# Default priors

. bayes

Multilevel structure

---

schid

{U0}: random intercepts

{U1}: random coefficients for homework

---

Model summary

---

Likelihood:

math ~ normal(xb\_math,{e.math:sigma2})

Priors:

{math:homework 1.schtype \_cons} ~ normal(0,10000) (1)

{U0 U1} ~ mvnormal(2,{U:Sigma,m}) (1)

{e.math:sigma2} ~ igamma(.01,.01)

Hyperprior:

{U:Sigma,m} ~ iwishart(2,3,I(2))

---

(1) Parameters are elements of the linear form xb\_math.

# Default priors

- Scalar parameters:  $\sim N(0, 10000)$
- Variance parameters:  $\sim IG(0.01, 0.01)$
- Covariance parameters:  $\sim IW(d+1, I/d)$

## Change default hyperparameters:

- Scalar parameters: `normalprior(sd)`
- Variance parameters: `igammaprior(shape scale)`
- Covariance parameters: `iwishartprior(d [S])`

# Available priors

- `normal(mu, var)`
- `t(mu, sigma2, df)`
- `lognormal(mu, var)`
- `lnormal(mu, var)`
- `uniform(a, b)`
- `gamma(alpha, beta)`
- `igamma(alpha, beta)`
- `exponential(beta)`
- `beta(a, b)`
- `poisson(mu)`
- `laplace(mu, beta)`
- `cauchy(loc, beta)`
- `chi2(df)`
- `pareto(alpha, beta)`
- `jeffreys(d)`
- `mvnormal(d, mean, Sigma)`
- `mvnexchangeable(d, mean, var, rho)`
- `mvnindependent(d, mean, vars)`
- `mvnidentity(d, mean, var)`
- `mvnscaled(d, mean, A, {var})`
- `zellnersg(d, g, mean, {var})`
- `dirichlet(a_1, a_2, ..., a_d)`
- `wishart(d, df, V)`
- `iwishart(d, df, V)`
- `bernoulli(p)`
- `geometric(p)`
- `index(p1, ..., pk)`
- `flat`
- `density(f)`
- `logdensity(logf)`

# Model comparison

```
. bayes, rseed(317) saving(m1):
> mixed math homework i.schtype || schid: homework, covariance(unstructured)
. estimates store m1

. bayes, rseed(317) saving(m2) normalprior(5):
> mixed math homework i.schtype || schid: homework, covariance(unstructured)
. estimates store m2

. bayes, rseed(317) saving(m3) iwishartprior(10):
> mixed math homework i.schtype || schid: homework, covariance(unstructured)
. estimates store m3

. bayes, rseed(317) saving(m4) prior({e.math:sigma2}, jeffreys):
> mixed math homework i.schtype || schid: homework, covariance(unstructured)
. estimates store m4
```

# Model comparison

```
. bayesstats ic m*, diconly
```

Deviance information criterion

|    | DIC      |
|----|----------|
| m1 | 3581.631 |
| m2 | 3582.73  |
| m3 | 3584.731 |
| m4 | 3582.641 |

# Postestimation

```
. estimates restore m1
. bayesstats summary (corr:{U:Sigma_2_1}/sqrt({U:Sigma_1_1}*{U:Sigma_2_2}))
```

Posterior summary statistics

MCMC sample size = 10,000

corr : { U:Sigma\_2\_1 } /sqrt( { U:Sigma\_1\_1 } \* { U:Sigma\_2\_2 } )

|      | Mean      | Std. dev. | MCSE    | Median    | Equal-tailed<br>[95% cred. interval] |           |
|------|-----------|-----------|---------|-----------|--------------------------------------|-----------|
| corr | -.8470514 | .0702993  | .002396 | -.8600276 | -.9437647                            | -.6738007 |



# Postestimation

```
. bayestest interval {math:1.schtype}, upper(0)
```

```
Interval tests MCMC sample size = 10,000
```

```
prob1 : {math:1.schtype} < 0
```

Options

upper()

lower()

joint

|       | Mean  | Std. dev. | MCSE     |
|-------|-------|-----------|----------|
| prob1 | .9764 | 0.15181   | .0081267 |

# Bayesian MLMs with bayesmh

```
bayes: mixed math homework i.schtype || schid: homework,
covariance(unstructured)
```

```
bayesmh math homework i.schtype U0[schid] U1[schid]#c.homework,
likelihood(normal({e.math:sigma2}))
prior({math:}, normal(0,10000))
prior({e.math:sigma2}, igamma(0.01,0.01))
prior({U0 U1}, mvnormal(2,0,0,{Sigma,matrix}))
prior({Sigma,m}, iwishart(2,3,I(2)))
```

# Alternative covariance structures

- `mvnnormal(d, mean, Sigma)`
- `mvnexchangeable(d, mean, var, rho)`
- `mvnindependent(d, mean, vars)` <sup>Default</sup>
- `mvnidentity(d, mean, var)`

- `mvn0(d, Sigma)`
- `mvn0exchangeable(d, var, rho)`
- `mvn0independent(d, vars)` <sup>Default</sup>
- `mvn0identity(d, var)`

# Identity covariance

```
bayes: mixed math homework i.schtype || schid: homework,
covariance(identity)
```

$$\{U0 \ U1\} \sim \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}$$

```
bayesmh math homework i.schtype U0[schid] U1[schid]#c.homework,
likelihood(normal({e.math:sigma2}))
prior({math:}, normal(0,10000))
prior({e.math:sigma2}, igamma(0.01,0.01))
prior({U0 U1}, mvn0identity(2,{Sigma}))
prior({Sigma}, igamma(0.01,0.01))
```

# Independent covariance

```
bayes: mixed math homework i.schtype || schid: homework
```

$$\{U0 \ U1\} \sim \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$$

```
bayesmh math homework i.schtype U0[schid] U1[schid]#c.homework,
likelihood(normal({e.math:sigma2}))
prior({math:}, normal(0,10000))
prior({e.math:sigma2}, igamma(0.01,0.01))
```

# Exchangeable covariance

```
bayes: mixed math homework i.schtype || schid: homework,
covariance(exchangeable)
```

$$\{U0 \ U1\} \sim \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix}$$

```
bayesmh math homework i.schtype U0[schid] U1[schid]#c.homework,
likelihood(normal({e.math:sigma2}))
prior({math:}, normal(0,10000))
prior({e.math:sigma2}, igamma(0.01,0.01))
prior({U0 U1}, mvn0exchangeable(2,{Sigma},{rho}))
prior({Sigma}, igamma(0.01,0.01))
prior({rho}, uniform(-1,1))
```

# Unstructured covariance

```
bayes: mixed math homework i.schtype || schid: homework,
covariance(unstructured)
```

$$\{U0 \ U1\} \sim \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 \\ \sigma_{12}^2 & \sigma_{22}^2 \end{bmatrix}$$

```
bayesmh math homework i.schtype U0[schid] U1[schid]#c.homework,
likelihood(normal({e.math:sigma2}))
prior({math:}, normal(0,10000))
prior({e.math:sigma2}, igamma(0.01,0.01))
prior({U0 U1}, mvn0(2,{Sigma,m}))
prior({Sigma,m}, iwishart(2,3,I(2)))
```

# Priors for random effects

```
bayesmh math homework i.schtype U0[schid] U1[schid]#c.homework,
likelihood(normal({e.math:sigma2}))
prior({math:}, normal(0,10000))
prior({e.math:sigma2}, igamma(0.01,0.01))
prior({U0 U1}, t(0,{Sigma},{df}),split)
prior({Sigma}, igamma(0.01,0.01))
Prior({df}, uniform(1,500))
```



# Gibbs sampling

```
. bayesmh math homework i.schtype U0[schid] U1[schid]#c.homework, ///
> likelihood(normal({e.math:sigma2})) ///
> prior({math:}, normal(0,10000)) ///
> prior({e.math:sigma2}, igamma(0.01,0.01)) ///
> prior({U0 U1}, mvnormal(2,0,0,{Sigma,matrix})) ///
> prior({Sigma,m}, iwishart(2,3,I(2))) ///
> block({math:}, gibbs) ///
> block({U0 U1} {e.math:sigma2} {Sigma,m}, split gibbs) ///
> rseed(317)
```

Model summary

---

Likelihood:

$\text{math} \sim \text{normal}(\text{xb\_math}, \{e.\text{math}:\text{sigma2}\})$

Priors:

$\{\text{math}:\text{homework } 1.\text{schtype } \_cons\} \sim \text{normal}(0,10000)$  (1)

$\{e.\text{math}:\text{sigma2}\} \sim \text{igamma}(0.01,0.01)$

$\{U0[\text{schid}] U1[\text{schid}]\} \sim \text{mvnormal}(2,0,0,\{\text{Sigma},m\})$  (1)

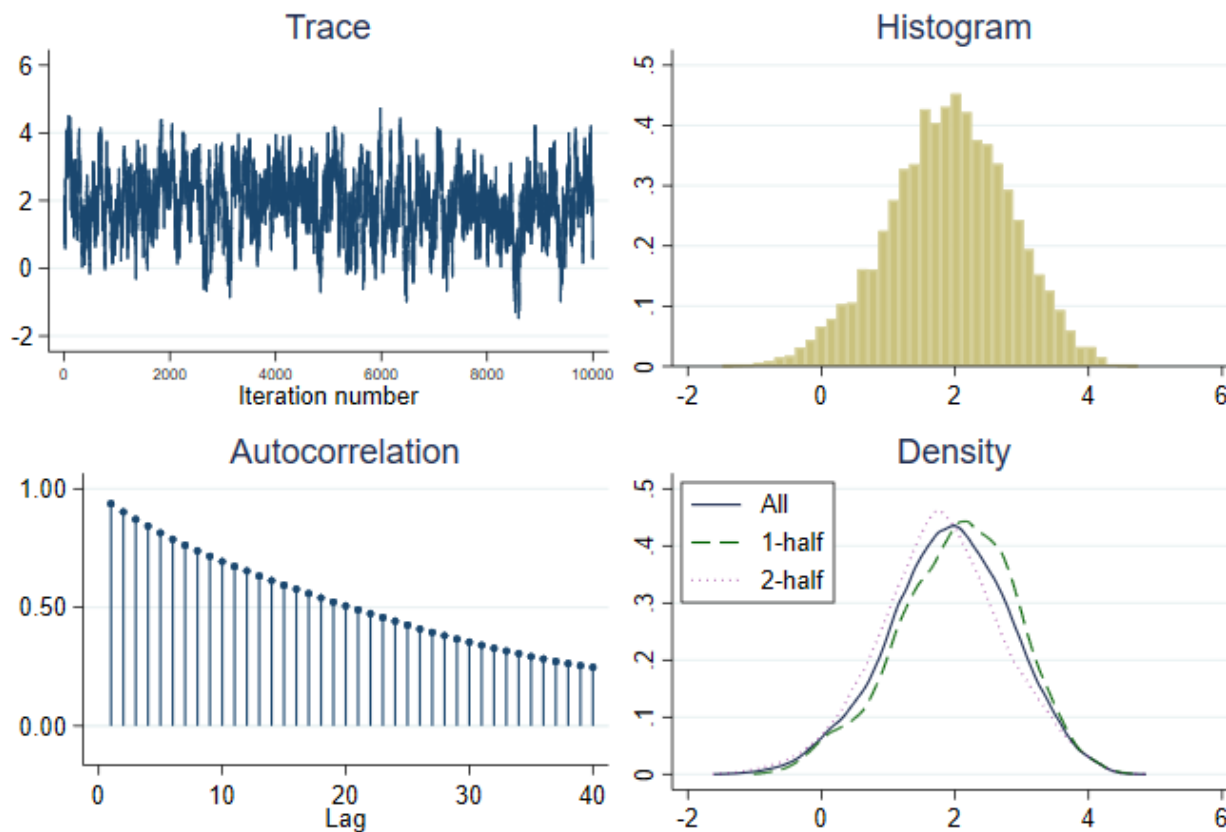
Hyperprior:

$\{\text{Sigma } m\} \sim \text{iwishart}(2,3,I(2))$

# Diagnostics

```
. bayesgraph diagnostics {math:homework}
```

**math:homework**



# Diagnostics

```
. bayesstats ess
```

Efficiency summaries      MCMC sample size =      10,000

|           | ESS     | Corr. time | Efficiency |
|-----------|---------|------------|------------|
| math      |         |            |            |
| homework  | 205.55  | 48.65      | 0.0206     |
| schtype   |         |            |            |
| Private   | (base)  |            |            |
| Public    | 679.75  | 14.71      | 0.0680     |
| _cons     | 369.80  | 27.04      | 0.0370     |
| Sigma_1_1 | 4041.74 | 2.47       | 0.4042     |
| Sigma_2_1 | 3994.31 | 2.50       | 0.3994     |
| Sigma_2_2 | 3349.70 | 2.99       | 0.3350     |
| e.math    |         |            |            |
| sigma2    | 7646.01 | 1.31       | 0.7646     |

# Random effects

```
. bayesmh, showreffects
. bayesstats summary {U0}
```

Posterior summary statistics

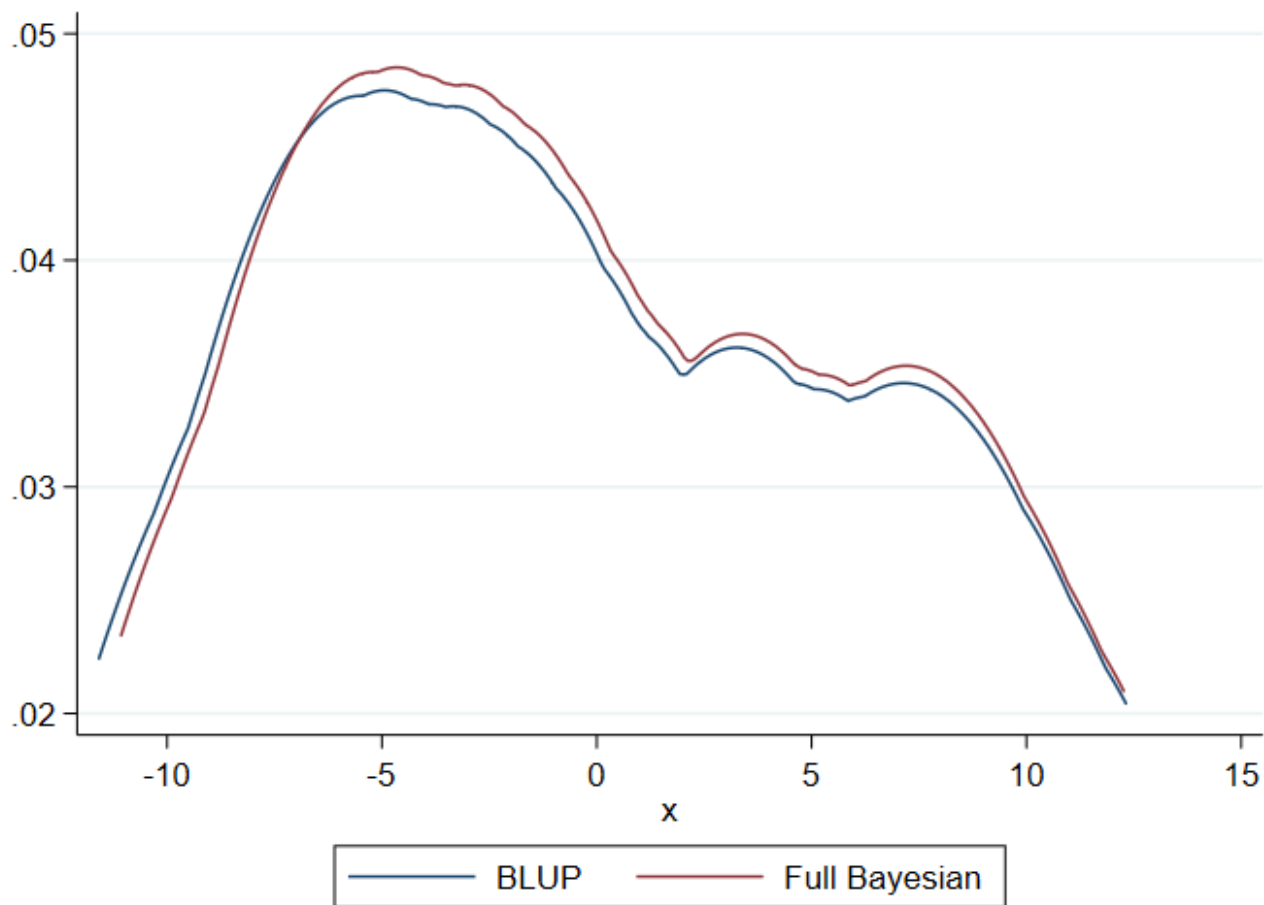
MCMC sample size = 10,000

| U0[schid] | Mean      | Std. dev. | MCSE    | Median    | Equal-tailed<br>[95% cred. interval] |          |
|-----------|-----------|-----------|---------|-----------|--------------------------------------|----------|
| 6053      | 2.425225  | 2.598754  | .106485 | 2.378599  | -2.629702                            | 7.608975 |
| 6327      | 12.12415  | 3.472318  | .095326 | 12.07392  | 5.424737                             | 19.00017 |
| 6467      | -5.457836 | 4.688123  | .126723 | -5.381636 | -14.86095                            | 3.543554 |
| 7194      | 6.872519  | 2.965798  | .093126 | 6.863353  | 1.091618                             | 12.7788  |
| 7472      | 5.337342  | 2.813943  | .093274 | 5.314509  | -.2015624                            | 10.91311 |
| 7474      | 12.27824  | 3.35497   | .095961 | 12.16729  | 5.899553                             | 18.97457 |
| 7801      | 8.475593  | 3.039685  | .101188 | 8.486299  | 2.412335                             | 14.42537 |
| 7829      | 4.233715  | 3.38201   | .107448 | 4.264049  | -2.410927                            | 10.92189 |
| 7930      | -5.368508 | 3.031667  | .101591 | -5.376529 | -11.34424                            | .5399448 |
| ⋮         |           |           |         |           |                                      |          |

```
. svmat r(summary), names(u2b_)
```

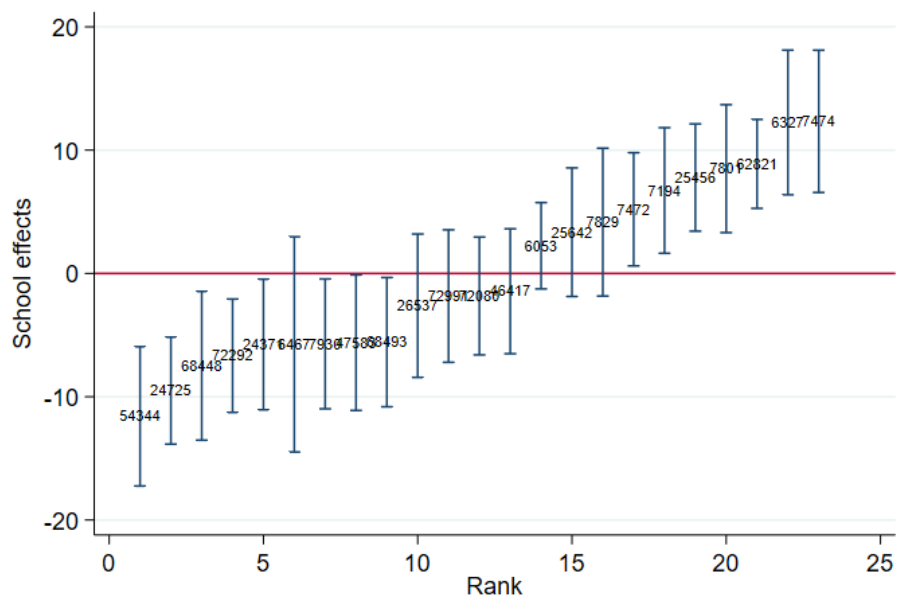
# Random effects

```
. twoway (kdensity u2 if tagme) (kdensity u2b_1)
```

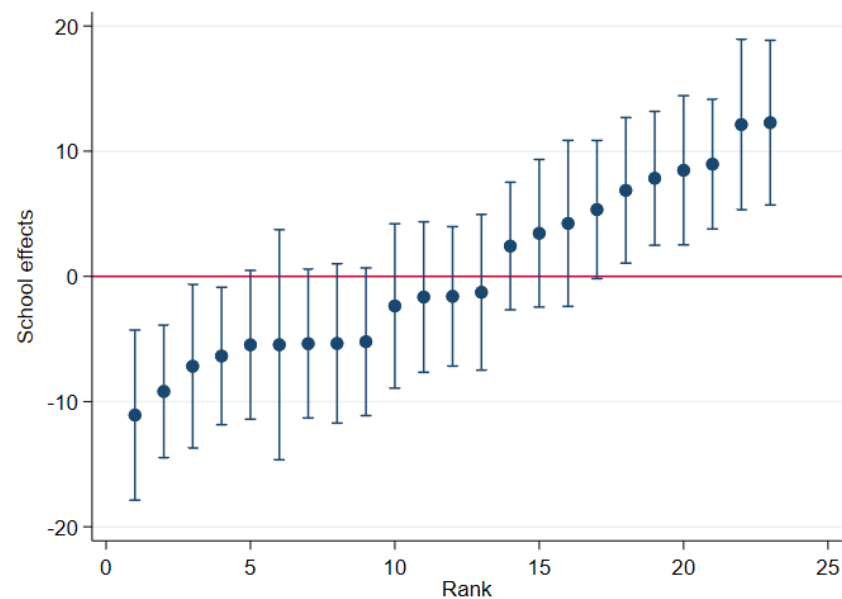


# Random effects

BLUP



Full Bayesian



# Model checking

$$PPP = \text{prob}(T(\mathbf{y}_{sim}, \boldsymbol{\theta}) \geq T(\mathbf{y}_{obs}, \boldsymbol{\theta}))$$

$$p(\boldsymbol{\theta}|\mathbf{y}_{obs}) \quad p(\mathbf{y}_{sim}|\mathbf{y}_{obs})$$

$$\boldsymbol{\theta}^1 \longrightarrow \mathbf{y}_{sim}^1 \longrightarrow T(\mathbf{y}_{sim}^1, \boldsymbol{\theta}^1)$$

$$\boldsymbol{\theta}^2 \longrightarrow \mathbf{y}_{sim}^2 \longrightarrow T(\mathbf{y}_{sim}^2, \boldsymbol{\theta}^2)$$

$$\vdots \qquad \qquad \vdots$$

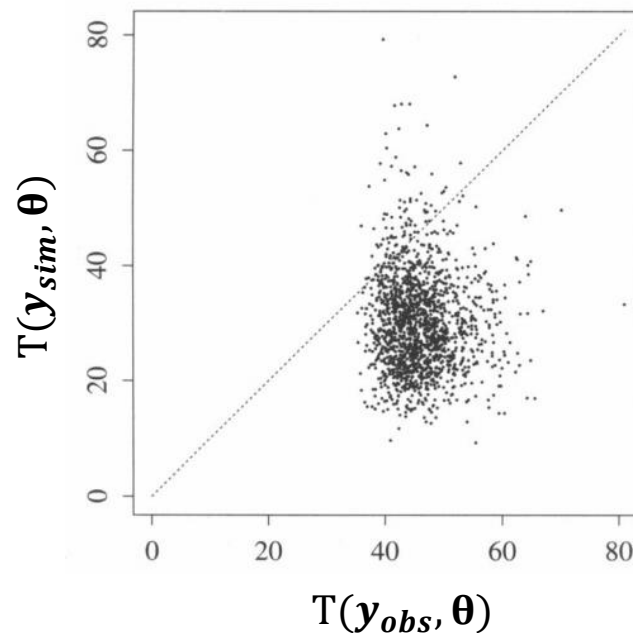
$$\boldsymbol{\theta}^J \longrightarrow \mathbf{y}_{sim}^J \longrightarrow T(\mathbf{y}_{sim}^J, \boldsymbol{\theta}^J)$$

$$\boldsymbol{\theta}^1 \longrightarrow T(\mathbf{y}_{obs}, \boldsymbol{\theta}^1)$$

$$\boldsymbol{\theta}^2 \longrightarrow T(\mathbf{y}_{obs}, \boldsymbol{\theta}^2)$$

$$\vdots$$

$$\boldsymbol{\theta}^J \longrightarrow T(\mathbf{y}_{obs}, \boldsymbol{\theta}^J)$$



# Model checking

```
. bayesmh, saving(bmh)
. bayespredict {_ysim}, saving(prdata)
. bayesstats ppvalues (@max({_resid})) using prdata
```

Posterior predictive summary      MCMC sample size =      10,000

| T           | Mean     | Std. dev. | E(T_obs) | P(T>=T_obs) |
|-------------|----------|-----------|----------|-------------|
| _resid1_max | 22.35586 | 2.773591  | 21.27055 | .6135       |

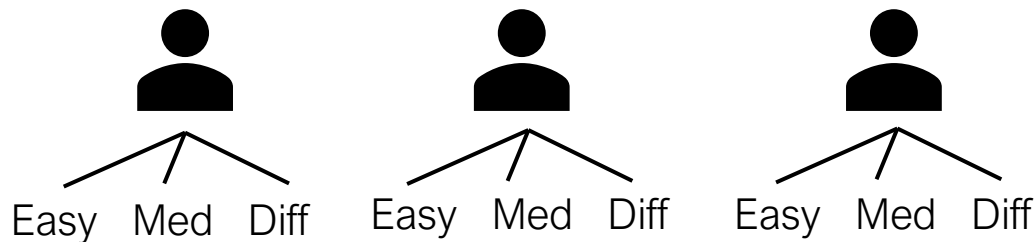
Note: P(T>=T\_obs) close to 0 or 1 indicates lack of fit.



# Available likelihoods

- `normal(var)`
- `t(sigma2, df)`
- `lognormal(var)`
- `lnormal(var)`
- `exponential`
- `mvnormal(Sigma)`
- `probit`
- `logit`
- `logistic`
- `binomial(n)`
- `oprobit`
- `ologit`
- `poisson`
- `stexponential`
- `stgamma(lns)`
- `stloglogistic(lns)`
- `stlognormal(lnstd)`
- `stweibull(lnp)`

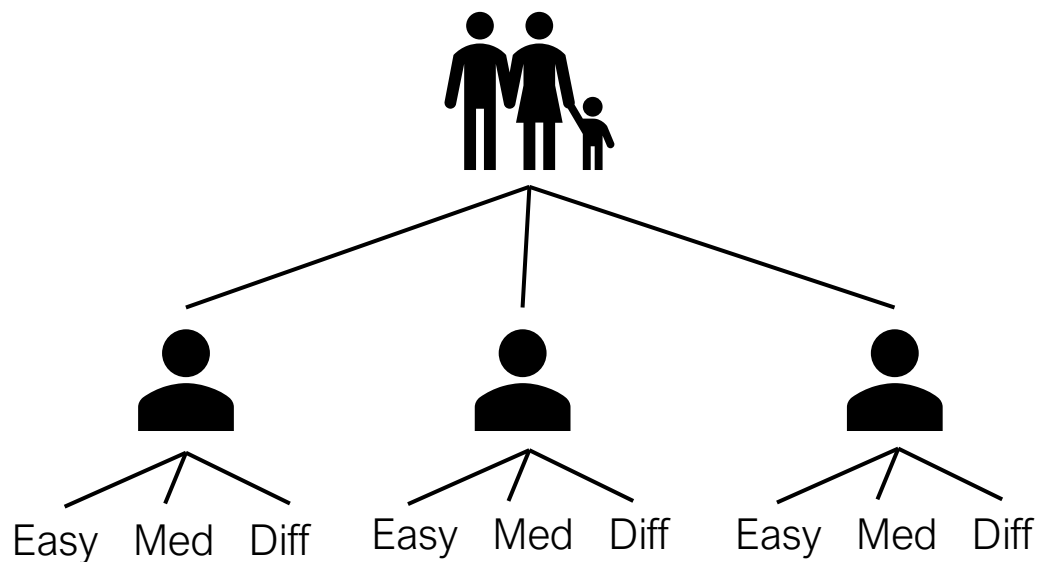
# Logistic MLM



```
bayes: melogit taskdone i.difficulty || subject:
```

```
bayesmh taskdone i.difficulty U0[subject],
likelihood(logistic)
prior({taskdone:}, normal(0,10000))
prior({U0}, normal(0,{Sigma}))
prior({Sigma}, igamma(0.01,0.01))
```

# Three-level logistic MLM

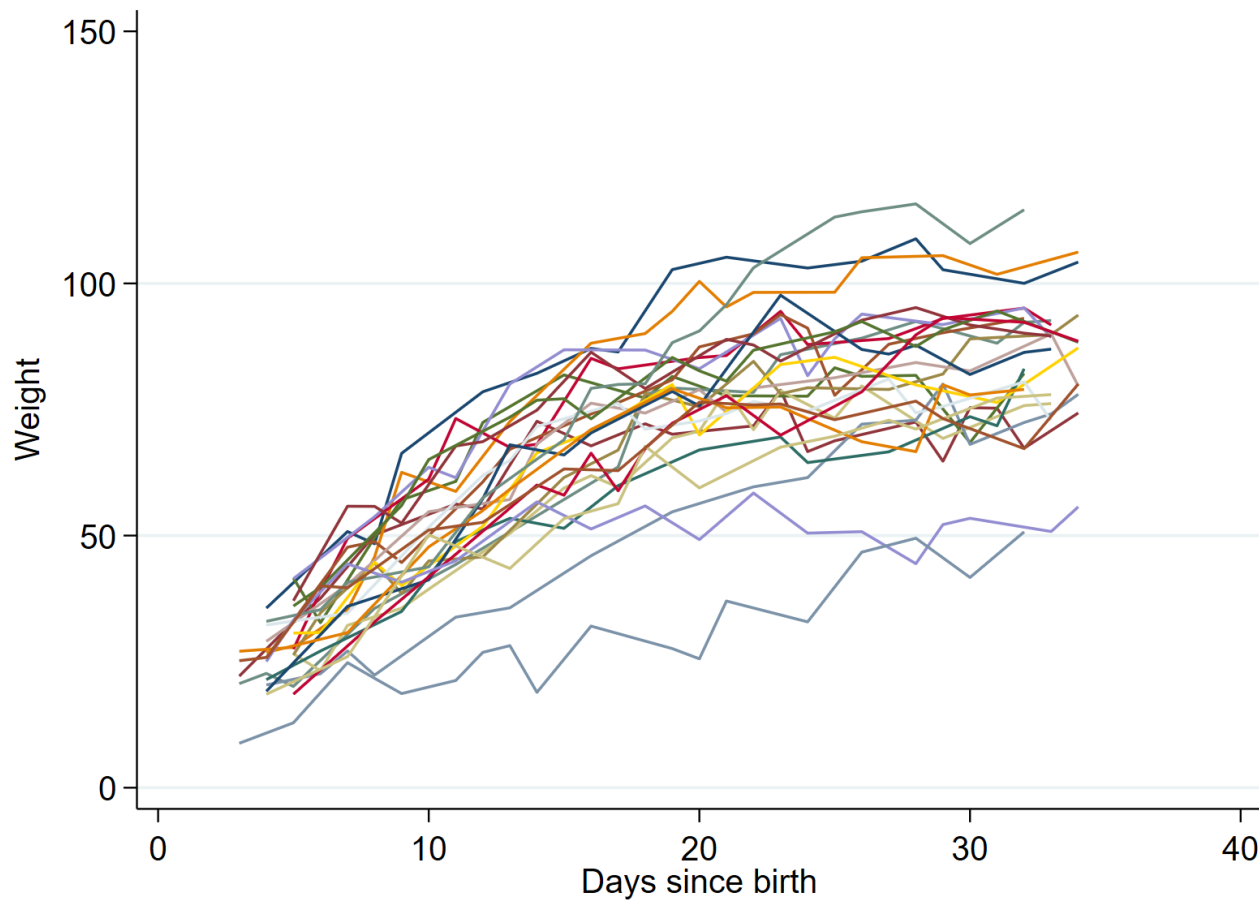


# Three-level logistic MLM

```
bayes: melogit taskdone i.difficulty || family: || subject:
```

```
bayesmh taskdone i.difficulty U0[subject] R01[family>subject],
likelihood(logistic)
prior({taskdone:}, normal(0,10000))
prior({U0}, normal(0,{U_Sigma}))
prior({U_Sigma}, igamma(0.01,0.01))
prior({R01}, normal(0,{R_Sigma}))
prior({R_Sigma}, igamma(0.01,0.01))
```

# Nonlinear MLM

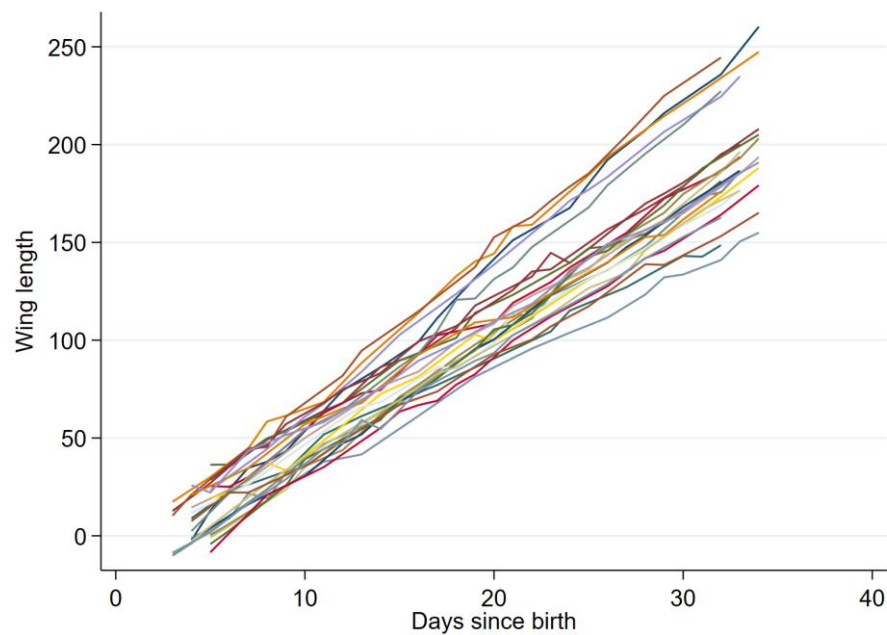
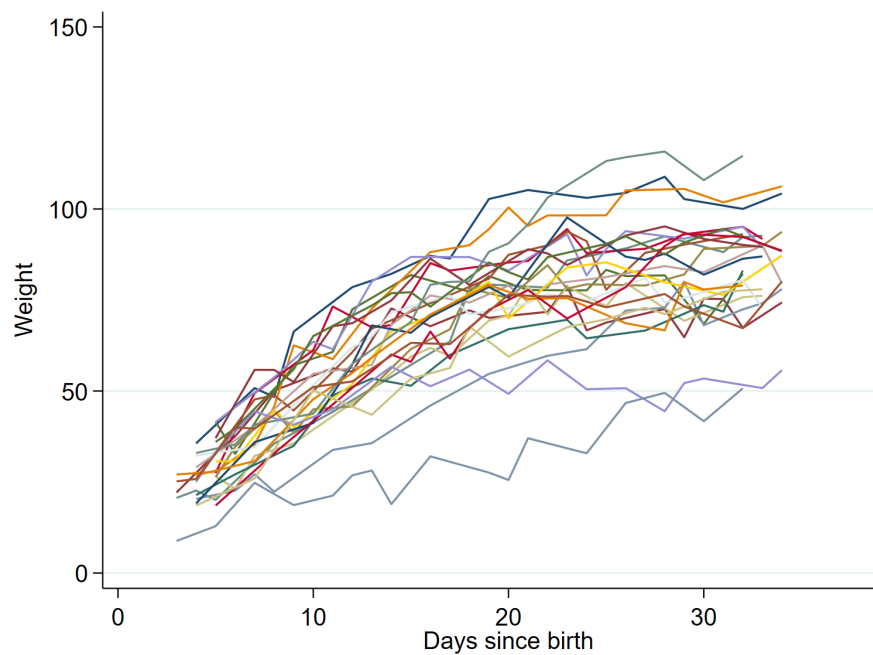


# Nonlinear MLM

```
menl weight = ({C[id]}/(1+{d}*{C[id]}*exp(-{B[id]}*time)))
```

```
bayesmh weight = ({C[id]}/(1+{d}*{C[id]}*exp(-{B[id]}*time))),
likelihood(normal({e.weight:sigma2}))
prior({C B}, mvnormal(4,{c},{b},{Sigma,m}))
prior({c b}, normal(0, 100))
prior({e.weight:sigma2}, igamma(0.01,0.01))
prior({Sigma,m}, iwishart(2,3,I(2)))
```

# Multivariate MLM



# Multivariate MLM

```
bayesmh (wing = ({U[id]} + time*{V[id]}))
(weight = ({C[id]}/(1+{d}*{C[id]}*exp(-{B[id]}*time)))),
likelihood(mvnormal({e.sigma,m}))
prior({U V C B}, mvnormal(4,{u},{v},{c},{b},{Sigma,m}))
prior({u v c b}, normal(0, 100))
prior({Sigma0,m}, iwishart(2,3,I(2)))
prior({Sigma,m}, iwishart(4,5,I(4)))
prior({d}, exp(1))
```



# Thank you!

# Questions?

You can download the slides and other materials here:

<https://tinyurl.com/StataBMM22>

You can contact tech support at [tech-support@stata.com](mailto:tech-support@stata.com)