Treatment Effect Heterogeneity in Regression Discontinuity Designs: Methodology and the `rdhte` Package

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Introduction

Motivation

- Many policies have effects that differ across individuals or groups.
- Understanding these **heterogeneous treatment effects** (HTE) is essential for:
  - Fairness and equity analysis
  - Targeted interventions
  - Policy design
- This project develops formal tools to study HTE in the **regression discontinuity** (RD) design
  - Method/theory to augment standard local polynomials with additional covariates
  - Fully feature package `rdhte`, complements `rdrobust`

References

- CCFPT (2025) "Treatment Effect Heterogeneity in Regression Discontinuity Designs"
- `rdhte`: Software implementing all results, see `https://rdpackages.github.io/rdhte/`

Background: Standard Sharp Regression Discontinuity

Model & Parameter of Interest

- Each unit i gets a continuous **score** X_i. (*test score, index value, vote share*)
- Cutoff x_0 (I'll set $x_0 = 0$ most of the time)
- Treatment status: $T_i = \begin{cases} 0 & \text{if } X_i < x_0 \\ 1 & \text{if } X_i \geq x_0 \end{cases}$
- Potential outcomes $Y_i(0)$ and $Y_i(1)$; observed Y_i
- Parameter of interest = average treatment effect at the cutoff (not the usual ATE)

$$\tau_{\texttt{SRD}} = \mathbb{E}[Y_i(1) - Y_i(0)|X_i = x_0]$$

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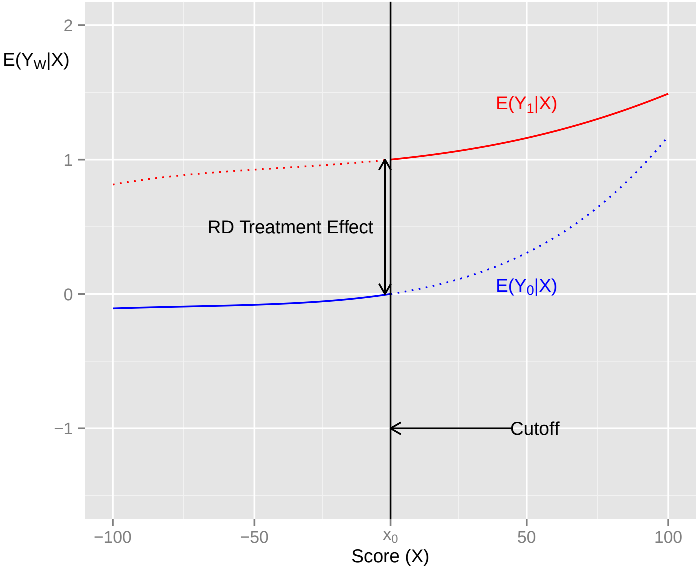
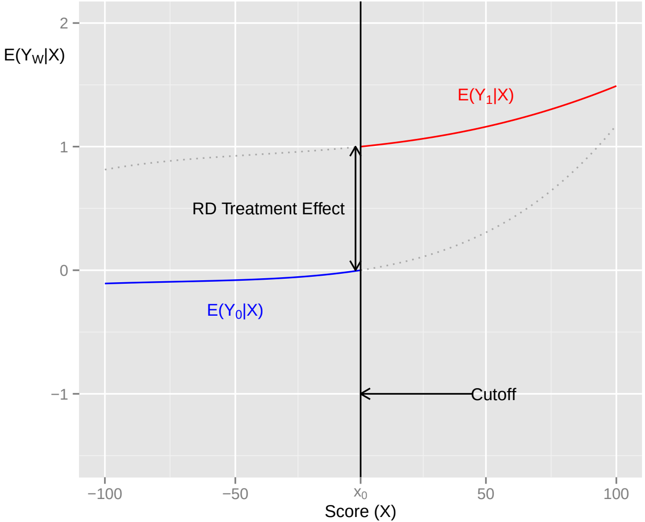


$$\tau_{\texttt{SRD}} = \underbrace{\mathbb{E}[Y_i(1) - Y_i(0) | X_i = x_0]}_{\text{Unobservable}} = \underbrace{\lim_{x \downarrow x_0} \mathbb{E}[Y_i | X_i = x]}_{\text{Observable}} - \underbrace{\lim_{x \uparrow x_0} \mathbb{E}[Y_i | X_i = x]}_{\text{Observable}}$$



RD Estimation & Inference

Estimating $\tau_{\texttt{SRD}}$

- Nonparametric regression problem
- Use **local polynomial regression**, usually local linear
- Choose a **bandwidth** to determine how local
- Conduct inference with **robust bias correction (RBC)**
- Industry standard `rdrobust` package

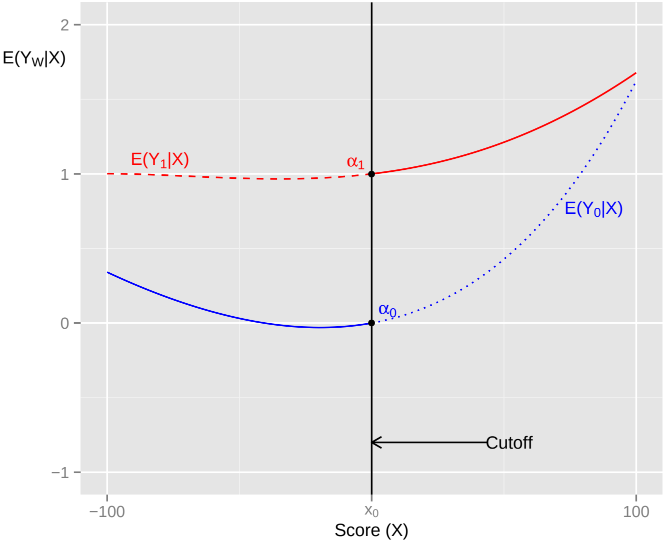
As a regression

- After kernel weighting, run a linear regression:

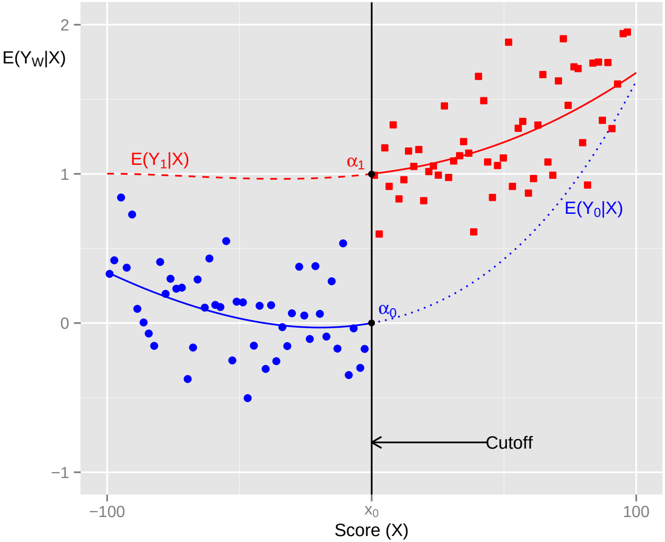
$$\dot{Y}_i = \dot{\mu} + \dot{\tau} T_i + \dot{\omega}_1 X_i + \dot{\omega}_2 T_i X_i.$$

- Same as separate regressions on either side ($\dot{\tau} = \dot{\alpha}_1 - \dot{\alpha}_0$)
- Cannot conduct inference using this linear regression, need RBC

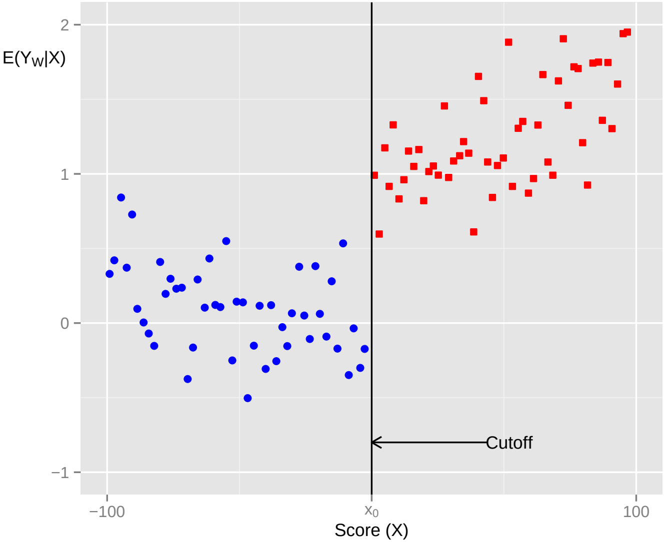
Local Estimation



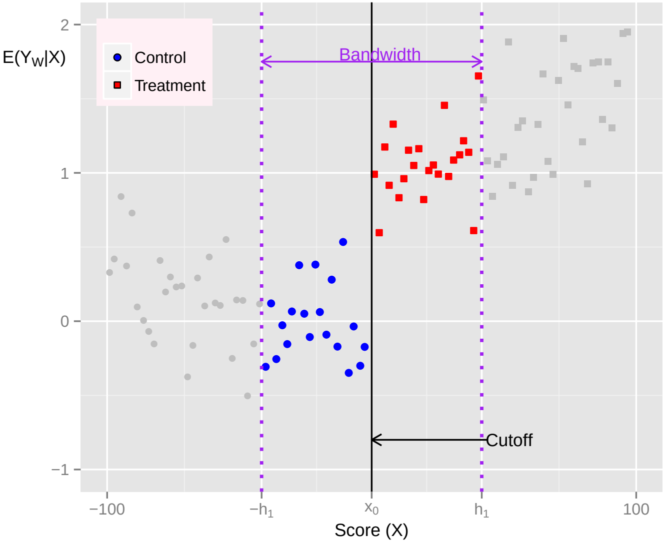
Local Estimation



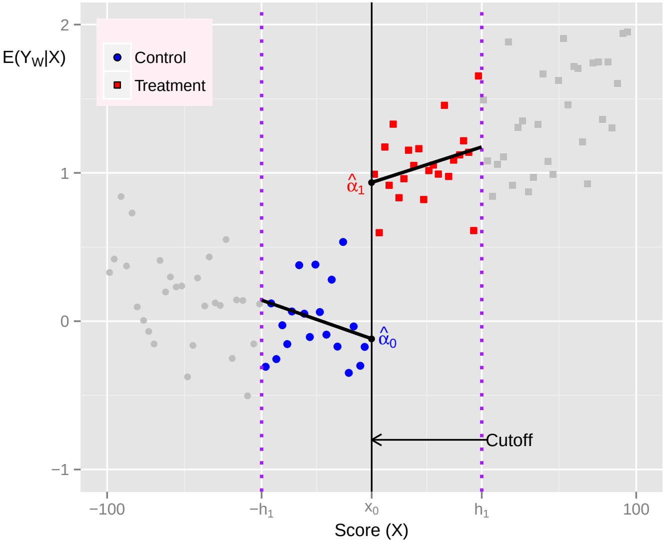
Local Estimation



Local Estimation



Local Estimation



Adding Covariates – Efficiency Only

Prior methodology work: add covariates for efficiency

- CCFT (2019) "Regression Discontinuity Designs Using Covariates"
- Pre-treatment variables $\mathbf{Z}_i$
- Estimating equation for `rdrobust`:

$$\widetilde{Y}_i = \widetilde{\mu} + \widetilde{\tau} T_i + \widetilde{\omega}_1 X_i + \widetilde{\omega}_2 T_i X_i + \widetilde{\boldsymbol{\gamma}}' \mathbf{Z}_i$$

- **Not** from separate fits on either side, common coefficient on $\mathbf{Z}$
- Key: nonparametric in X_i, like standard RD, with a projection in $\mathbf{Z}_i$ for feasibility
- Still estimating the average effect $\widetilde{\tau} \to_p \tau_{\texttt{SRD}}$
- No heterogeneity!

Heterogeneity in RD?

Heterogeneity analysis is common in research

- In practice, researchers often explore whether RD effects differ by gender, income, region, etc.
- Mostly ad hoc, methods vary widely and are not always valid

Conditional (Local) Average Treatment Effect

- Pre-treatment variables $\mathbf{W}_i$ (different from $\mathbf{Z}_i$)
- Parameter of interest is

$$\kappa(\mathbf{w}) = \mathbb{E}[Y_i(1) - Y_i(0) | X_i = 0, \mathbf{W}_i = \mathbf{w}].$$

This paper & package

- Formalize how to estimate heterogeneous treatment effects (HTE) in RD
- Simplify and unify empirical practice, add rigor but keep interpretability
- Maintain key RD idea & empirical feasibility: localization is based on X
- Package uses `regress` at heart, so it is fast, reliable, and has all the options

Main Ideas

- Localize in X_i, not in $\mathbf{W}_i$
- Full nonparametric estimation is...
  - Unappealing in practice: curse of dimensionality, many bandwidth choices, small samples
  - Perhaps not in the spirit of RD, only nonparametric in X
- Natural extension of `rdrobust`, keep all the `rdrobust` advantages

Our Estimation Method

$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\theta} T_i + \widehat{\boldsymbol{\lambda}}' \mathbf{W}_i + \widehat{\boldsymbol{\xi}}' T_i \mathbf{W}_i + \widehat{\omega}_1 X_i + \widehat{\omega}_2 T_i X_i + \widehat{\boldsymbol{\omega}}_3' X_i \mathbf{W}_i + \widehat{\boldsymbol{\omega}}_4' T_i X_i \mathbf{W}_i$$

- Heterogeneous effect at some value $\mathbf{W} = \mathbf{w}$

$$\widehat{\kappa}(\mathbf{w}) = \widehat{\theta} + \widehat{\boldsymbol{\xi}}' \mathbf{w}$$

- The "intercept" $\widehat{\theta}$ is the baseline effect, the "slope" $\widehat{\boldsymbol{\xi}}$ captures how the TE change with $\mathbf{W}$
- Localize with a bandwidth h, weight with a kernel $K(\cdot)$, then run `reg`
- Add other efficiency covariates $\mathbf{Z}_i$ if you want
- Inference using robust bias correction (don't grab standard errors from `reg`!)

Main Results

Most important: When does $\widehat{\kappa}(\mathbf{w})$ capture HTE?

- Requires that potential outcomes follow a functional coefficient model:

$$\mathbb{E}[Y_i(t)|X_i = x, \mathbf{W}_i = \mathbf{w}] = \beta_t(x) + \boldsymbol{\delta}_t(x)'\mathbf{w}$$

- Automatically holds for dummy variables (e.g., gender, region)
- For continuous $\mathbf{W}$ it's a mild semiparametric assumption
  - The $\mathbf{W}$ vector can include bins, polynomials, etc

Estimation/Inference Results: All the usual good stuff

- MSE expansion and bandwidth selection
  - Important to vary bandwidth (e.g. different size subgroups)
- Consistent standard errors (including clustering)
- Valid inference via RBC
  - Confidence intervals for $\kappa(\mathbf{w})$
  - Tests across subgroups $\kappa(\mathbf{w}_1) = \kappa(\mathbf{w}_2)$

Empirical Illustration

Setting & data

- Brazilian mayoral elections & political turnover
- Klasnja & Titiunik (2017) *APSR* / Akhtari, Moreira, & Trucco (2022) *AER*
- RD design uses close elections
- Comparing municipalities where the incumbent party barely loses (resulting in political turnover) to those where the incumbent party barely wins (no turnover)
- Akhtari, Moreira, & Trucco find that turnover $\Rightarrow$ expansion of municipal bureaucracy

Heterogeneity – Example for Today

- Running variable X = vote margin
- Outcome Y = replacement of school headmasters
- Heterogeneity variable $\mathbf{W}$: municipal income
  - Discretize income into buckets?
  - Use as continuous?

Empirical Analysis

Key Empirical Findings

- Large heterogeneity: effect is much stronger in **low-income municipalities**
- Formal inference confirms **statistically significant** heterogeneity
- Continuous analysis (using income directly) agrees with discretized results

Implementation

- Illustrate `rdhte` and post estimation
- Visualize results

Continuous Heterogeneity

- Linear in Income

```
. rdhte Y X, covs_hte(W) vce(hc2 cluster_id_var)

Sharp RD Heterogeneous Treatment Effects: Continuous.

      Cutoff c = 0 | Left of c  Right of c           Number of obs =      26099
-------------------+----------------------           BW type       =      mserd
     Number of obs |     13689       12410           Kernel        = Triangular
Eff. Number of obs |      7404        7188           VCE method    =        HC2
    Order est. (p) |         1           1
    Order bias (q) |         2           2
       BW est. (h) |     0.151       0.151


Outcome: Y. Running variable: X.
--------------------------------------------------------------------------------
                   | Point        | Robust Inference
                 W | Estimate     | z-stat     P>|z|     [95% Conf. Interval]
-------------------+------------------------------------------------------------
                 T | 0.472        | 5.669      0.000       0.307      0.632
             T#c.W | -0.217       | -2.847     0.004      -0.381     -0.070
--------------------------------------------------------------------------------
   (Std. err. adjusted for 2515 clusters in cluster_id_var)
```

Continuous Heterogeneity

- Quadratic in Income
- STATA's factor/interaction syntax is allowed (`c.`, `i.`, `etc`)

```
. rdhte Y X, covs_hte(c.W##c.W) vce(hc2 cluster_id_var)

Sharp RD Heterogeneous Treatment Effects: Continuous.

      Cutoff c = 0 | Left of c  Right of c            Number of obs =      26099
-------------------+----------------------            BW type       =      mserd
     Number of obs |     13689       12410            Kernel        = Triangular
Eff. Number of obs |      7404        7188            VCE method    =        HC2
    Order est. (p) |         1           1
    Order bias (q) |         2           2
       BW est. (h) |     0.151       0.151


Outcome: Y. Running variable: X.
------------------------------------------------------------------------------------
                       | Point        | Robust Inference
             c.W##c.W  | Estimate     | z-stat      P>|z|      [95% Conf. Interval]
-----------------------+------------------------------------------------------------
                     T | 0.625        | 3.447       0.001        0.248      0.903
                 T#c.W | -0.586       | -1.202      0.229       -1.269      0.304
             T#c.W#c.W | 0.161        | 0.600       0.548       -0.255      0.480
------------------------------------------------------------------------------------
   (Std. err. adjusted for 2515 clusters in cluster_id_var)
```

Discrete Heterogeneity

- Split Income at the Median
- Linear combination testing requires our `rdhte_lincom`

```
. rdhte Y X, covs_hte(i.W_med) vce(cluster cluster_id_var)

Sharp RD Heterogeneous Treatment Effects: Subgroups.

      Cutoff c = 0 | Left of c  Right of c            Number of obs =      26099
-------------------+----------------------            BW type       =      mserd
     Number of obs |     13689       12410            Kernel        = Triangular
    Order est. (p) |         1           1            VCE method    =    Cluster
    Order bias (q) |         2           2


Outcome: Y. Running variable: X.
--------------------------------------------------------------------------------------------------------------------
            | Point        | Robust Inference
   i.W_med  | Estimate     | z-stat      P>|z|      [95% Conf. Interval]       Nh-       Nh+         h-        h+
------------+-------------------------------------------------------------------------------------------------------
   0.W_med  | 0.373        | 7.771       0.000        0.286       0.479        5126      4328        0.187     0.187
   1.W_med  | 0.133        | 1.634       0.102        -0.022      0.246        2844      3232        0.160     0.160
--------------------------------------------------------------------------------------------------------------------
   (Std. err. adjusted for 2667 clusters in cluster_id_var)
```

Discrete Heterogeneity

- Split Income at the Median
- Linear combination testing requires our `rdhte_lincom`

```
. rdhte_lincom 0.W_med - 1.W_med

RD Heterogeneous Treatment Effects: Linear combinations of parameters

( 1)  0.W_med - 1.W_med = 0

--------------------------------------------------------------------------------
                Y1 |    Coefficient    [95% Conf. Interval]     P>|z|
-------------------+------------------------------------------------------------
               (1) |     .23998       0.105       0.436        0.001
--------------------------------------------------------------------------------
```

Discrete Heterogeneity

- Split Income at the Quartiles
- STATA's `test` command works fine

```
. rdhte Y X, covs_hte(i.W_qrt) vce(hc2 cluster_id_var)

Sharp RD Heterogeneous Treatment Effects: Subgroups.

      Cutoff c = 0 | Left of c  Right of c            Number of obs =      26099
-------------------+----------------------            BW type       =      mserd
     Number of obs |     13689       12410            Kernel        = Triangular
    Order est. (p) |         1           1            VCE method    =        HC2
    Order bias (q) |         2           2


Outcome: Y. Running variable: X.
---------------------------------------------------------------------------------------------------------------
            | Point        | Robust Inference
   i.W_qrt  | Estimate     | z-stat      P>|z|      [95% Conf. Interval]       Nh-      Nh+        h-        h+
------------+--------------------------------------------------------------------------------------------------
   0.W_qrt  | 0.416        | 5.710       0.000       0.256       0.524       2371     1983       0.151     0.151
   1.W_qrt  | 0.334        | 4.457       0.000       0.207       0.531       2244     1888       0.166     0.166
   2.W_qrt  | 0.164        | 0.793       0.428      -0.150       0.353       1134     1180       0.151     0.151
   3.W_qrt  | 0.102        | 1.330       0.183      -0.051       0.265       1723     2116       0.175     0.175
---------------------------------------------------------------------------------------------------------------
   (Std. err. adjusted for 2557 clusters in cluster_id_var)
```

Discrete Heterogeneity

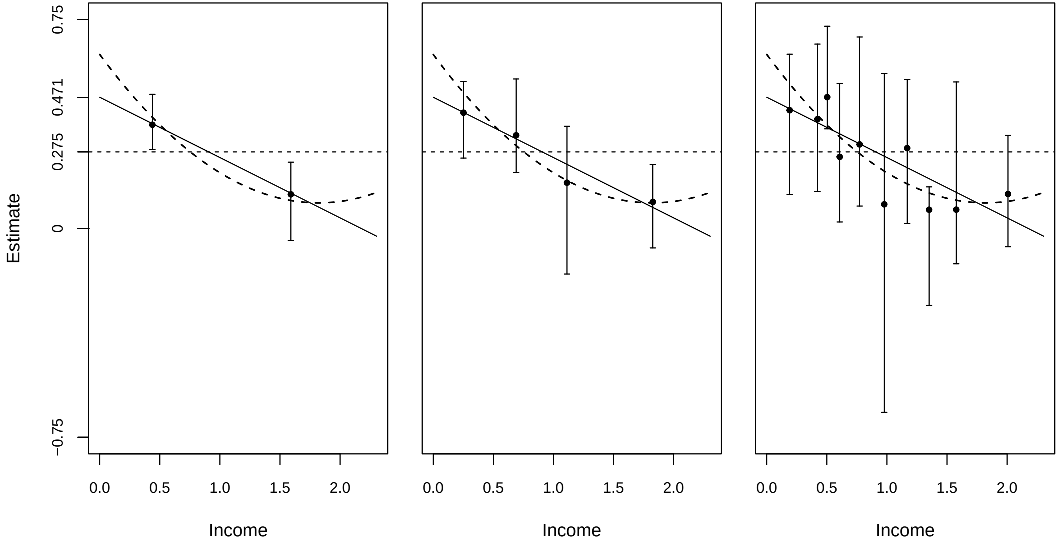
- Split Income at the Quartiles
- `STATA`'s `test` command works fine

```
. test (2.W_qrt = 3.W_qrt) (0.W_qrt = 1.W_qrt)

 ( 1)  2.W_qrt - 3.W_qrt = 0
 ( 2)  0bn.W_qrt - 1.W_qrt = 0

           chi2(  2) =    0.04
         Prob > chi2 =    0.9805
```

Graphical Version



Conclusion

- Heterogeneous effects are key for modern policy evaluation
- The paper formalizes how to estimate and test them in RD settings
- Maintains RD's simplicity while adding rigor and interpretability
- `rdhte` implements all the results
- `https://rdpackages.github.io/rdhte/`
    - Package, papers, replication files