

UNIVERSIDADE FEDERAL  
DE MINAS GERAIS

# **ilt, ownchild, leslie: Mortality, Fertility and Demographic Projections Using Stata**

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# Goals

1. Demonstrate ilt, a software to estimate single-state intercensal life tables and residual migration
2. Introduce ownchild, to estimate fertility patterns and trends in the last 15 years
3. Illustrate leslie, a program for cohort-component and multistate projections



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# Iterative intercensal single-decrement life tables using Stata

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1.

**Abstract.** One way to estimate mortality in countries with incomplete data is to utilize intercensal methods, which do not require model life tables and provide accurate results even in the presence of age distortions and death underregistration. In this article, I revisit three of these techniques (census based, death distribution, and an iterative procedure) and introduce `ilt`, a command to calculate single-decrement life tables and the net flow of migrants by age. The required inputs are two age-specific population distributions and the average number of deaths between them. The empirical example draws on data from Vietnam, but the methods are extendable to any context and period.



# Estimating Mortality with Incomplete Data: A Comparison of Intercensal Methods

This iterative loop creates corrected population growth rates, reducing bias from life tables from flawed population data affected by migration and reporting data.

## The Challenge: Standard Methods Have Key Weaknesses



### Method 1: Census-Based

Estimates mortality using two successive population counts but is sensitive to migration and age distortions.



### Method 2: Death-Distribution

Uses population counts and recorded deaths. Less sensitive but still affected by migration.



### Corrects for Migration and Census Coverage Gaps

The final life tables are less affected by residual migration and differential census enumeration.

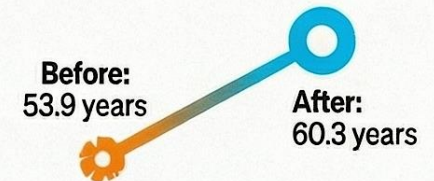
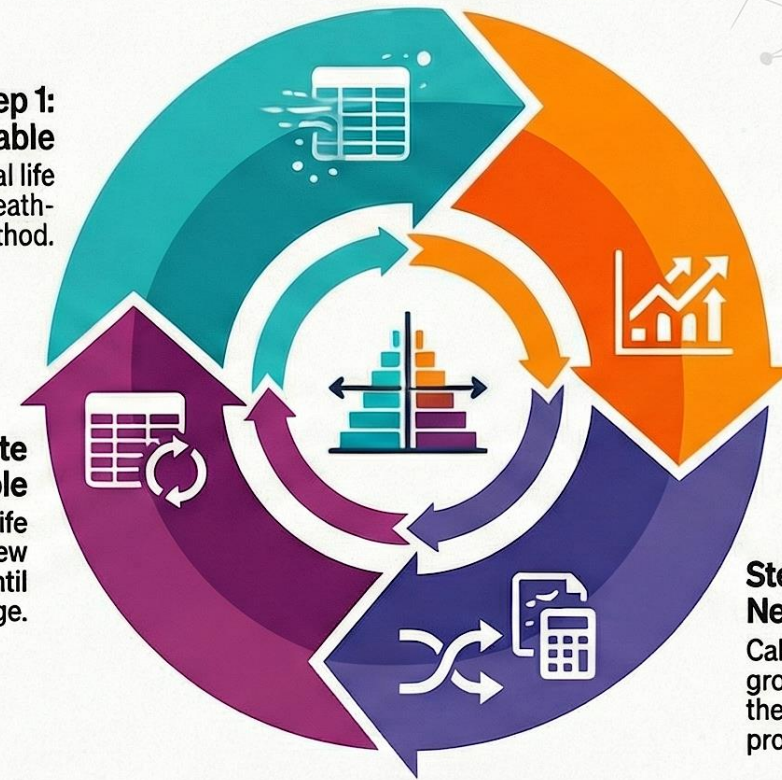
## The Solution: A 4-Step Cycle Reconciles the Methods to Correct for Errors

**Step 1: Initial Life Table**  
Create an initial life table using the Death-Distribution method.

**Step 4: Recalculate Life Table**  
Recalculate the life table with the new rates. Repeat until results converge.

**Step 2: Project Baseline Population**  
Use this table to project the baseline population forward in time.

**Step 3: Calculate New Growth Rates**  
Calculate new, corrected growth rates between the original and new projected populations.



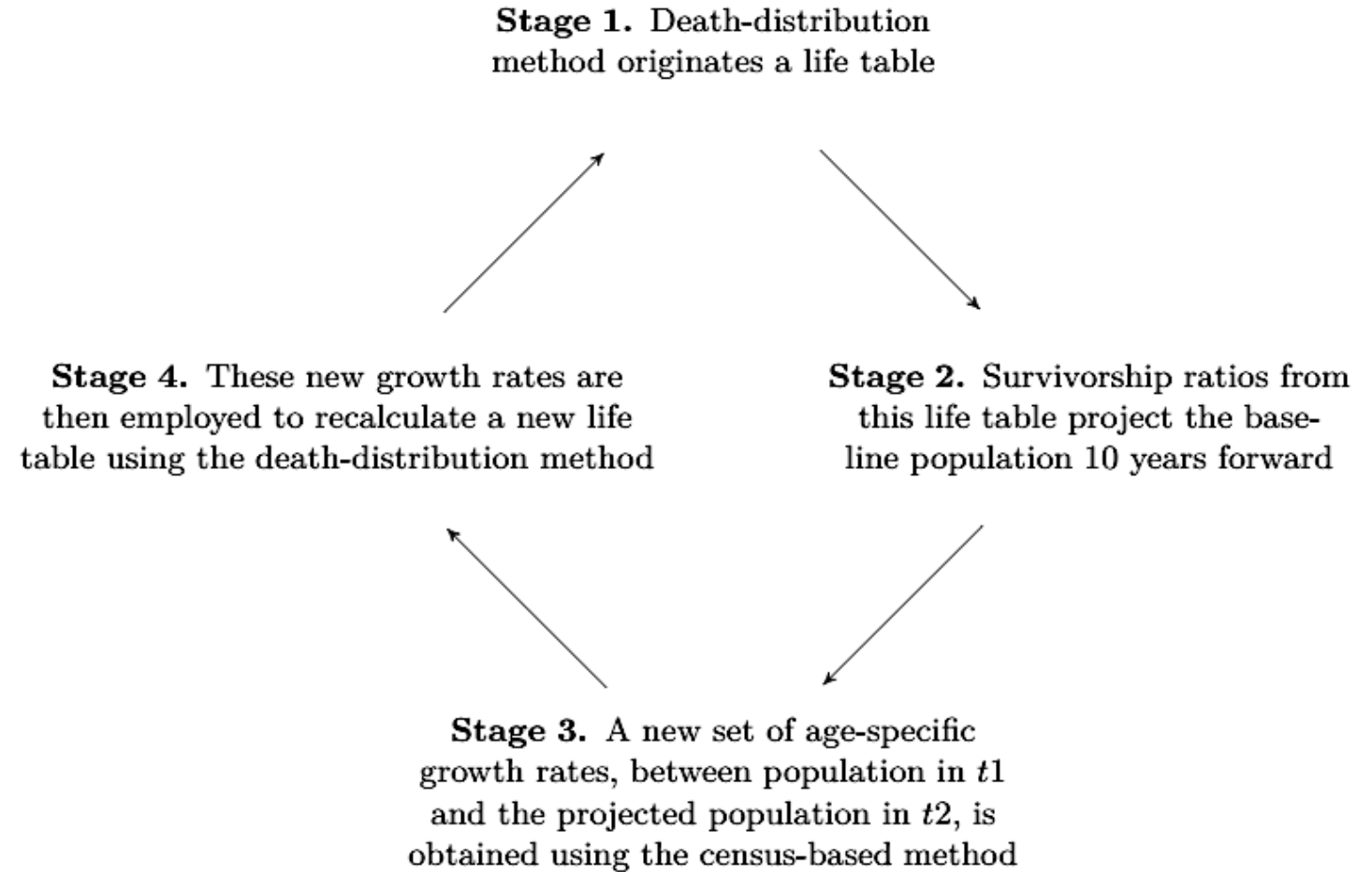
**Case Study Impact (Vietnam 1979-1989)**  
The iterative process raised the estimated life expectancy at age 5 from 53.9 to 60.3 years.



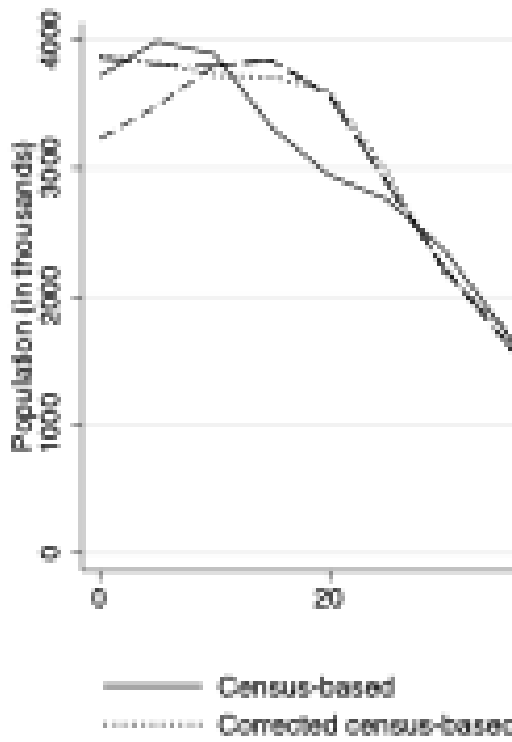
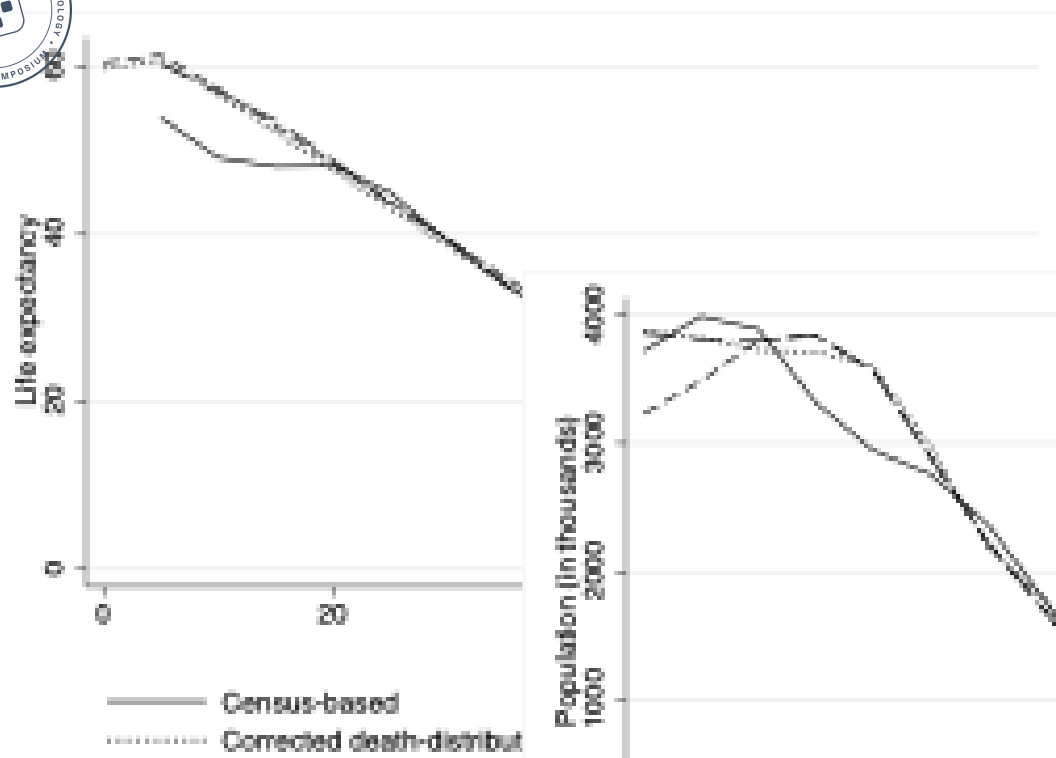
# Input

```
. use pop_vietnam
. list, noobs separator(17)
```

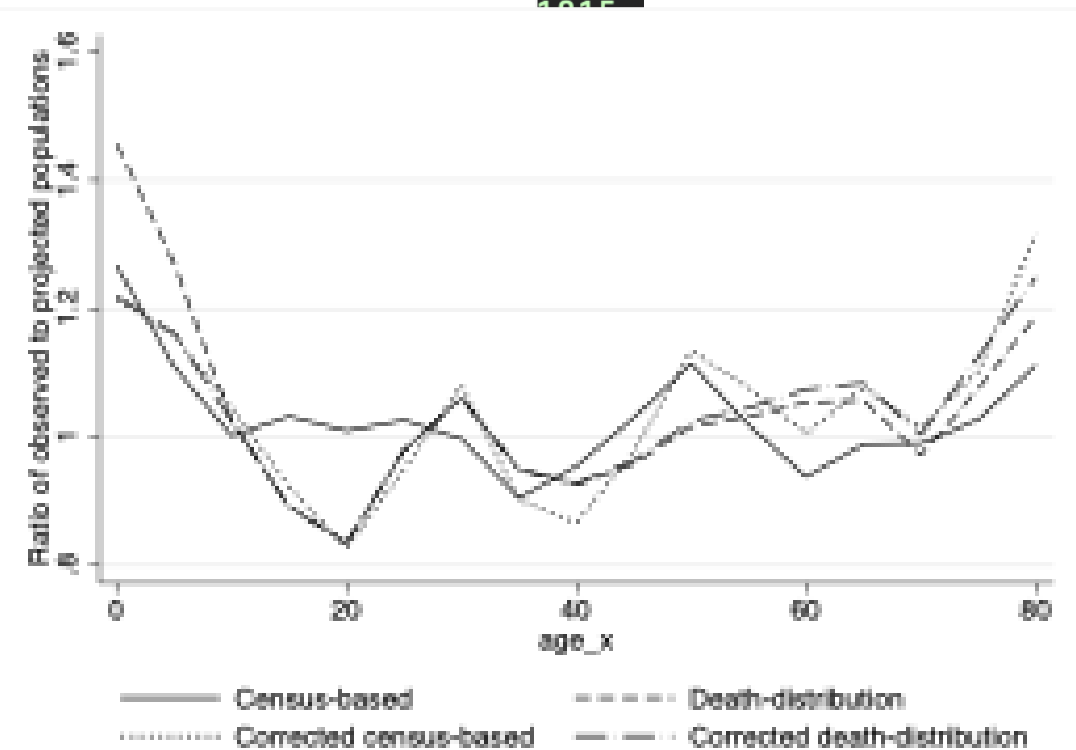
age_x	N_t1	N_t2	D
0	3946224	4710423	48508
5	3928795	4430179	8029
10	3632555	3898298	3928
15	2954333	3427357	3783
20	2281171	2974282	3856
25	1742277	2832160	3469
30	1177320	2361692	3053
35	966580	1604918	3093
40	919291	1048246	3345
45	994602	877589	4836
50	825356	866821	6215
55	680996	918363	9138
60	540920	725079	12070
65	419164	533445	13645
70	284003	329167	14310
75	183222	215510	14357
80	103773	145637	14787



Ta  
a



30	.0636382	6598.26		
35	.0558127	9011.04		
40	.0206004	11796.5		
45	-.0087106	17569.3		
50	.002413	22226.5		
55	.0254806	35040.28	1838399	6782353
60	.0221368	52134.31	1620463	4943954
65	.0160103	64834.7	1328040	3323491
70	.0143962	73364.7	982541.7	1995451
75	.004346	77136.57	606288.5	1012910
80	.0116553	82689.42	406621.1	406621.1





# The own-children method of fertility estimation using Stata

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2.

**Abstract.** In this article, I present the `ownchild` command, which calculates age-specific fertility rates using an advanced version of the own-children method, originally proposed by Grabill and Cho (1965, *Demography* 2: 50–73). `ownchild` provides a graphical representation of average fertility patterns by age over the last 15 years, generates weighted estimates for population subgroups, enhances accuracy by restricting calculations to biological connections between children and their mothers, and delivers 15 reproductive measures. These measures include total and net fertility rates, mean age at childbearing, the percentage of teenage pregnancies, the proportion of childless women, the percentage of unmatched children, and the replacement level of fertility. I demonstrate the capabilities of `ownchild` using 2010 Brazilian Census microdata sourced from Integrated Public Use Microdata Series-International to calculate fertility rates by race.



# Estimating Fertility Without Birth Records: The Own-Children Method



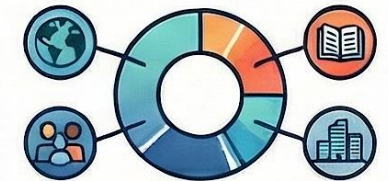
## OVERCOMES INCOMPLETE BIRTH RECORDS

A powerful alternative when vital registration systems are missing or unreliable.



## COST-EFFECTIVE & DATA-EFFICIENT

Uses existing census data to create 15 years of detailed fertility trends.



## ENABLES SUBGROUP ANALYSIS

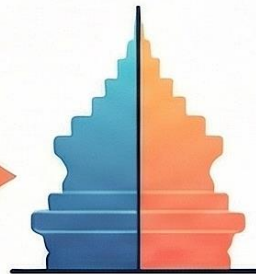
Allows fertility estimation for specific groups by race, education, or location.

## HOW IT WORKS: A 3-STEP PROCESS



### 1. LINK CHILDREN TO MOTHERS

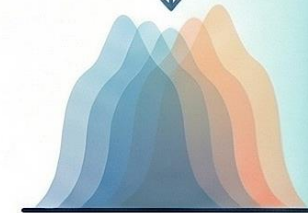
Identifies and counts mother-child pairs within households from survey data.



PRESENT  
SURVEY DATA



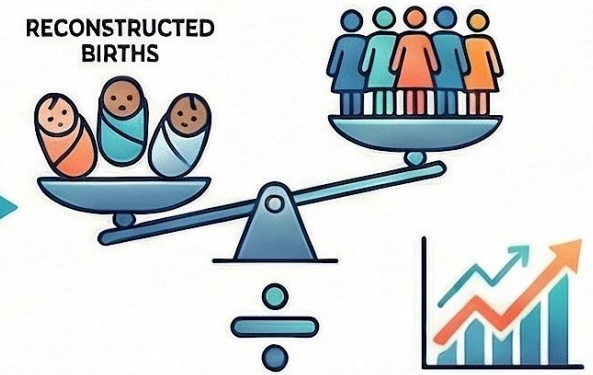
MORTALITY DATA



ESTIMATED  
PAST POPULATIONS

### 2. APPLY "REVERSE SURVIVAL"

Uses mortality data to estimate past populations of women and children.



RECONSTRUCTED  
BIRTHS

### 3. CALCULATE FERTILITY RATES

Divides the reconstructed number of births by the number of women for each past year.

## CORE ASSUMPTIONS

The method's accuracy relies on a few key assumptions. These include accurate age reporting, low migration, and correct mother-child links.



ACCURATE AGE  
REPORTING



LOW  
MIGRATION



CORRECT  
MOTHER-CHILD  
LINKS



# Inputs

1. Cumulative survival probabilities for children from 0 to 15 years;
2. Cumulative survival probabilities for women from 15 to 65 years;
3. A data extract from IPUMS containing at least the following variables:
  - `country, year, serial, pernum,`
  - `momloc`, `stepmom`,
  - `age, sex`
  - `(perwt, race etc.)`

# Output

```
. use br2010_sample.dta, clear
(2010 Brazilian Census, 5% sample stratified by race, IPUMS-I)
. ownchild [iw=perwt], place(Brazil) year(2010) summary
Estimated 3-year fertility rates, by five-year age group
```

Age group	1996-1998	1999-2001	2002-2004	2005-2007	2008-2010
15-19	.0854704	.0821683	.0808985	.075596	.0729358
20-24	.1465717	.1381307	.1333658	.122683	.1158171
25-29	.1268478	.1171481	.1148365	.1067211	.1018005
30-34	.0859292	.0817193	.0778396	.0732433	.0705107
35-39	.0458244	.0454338	.0433897	.0405855	.0385478
40-44	.0195403	.0169631	.015793	.0147041	.0141398
45-49	.0047814	.004989	.0043621	.0035982	.0032523
TFR	2.574826	2.432761	2.352426	2.185656	2.085019

Note: These estimates consider 3368720 women.

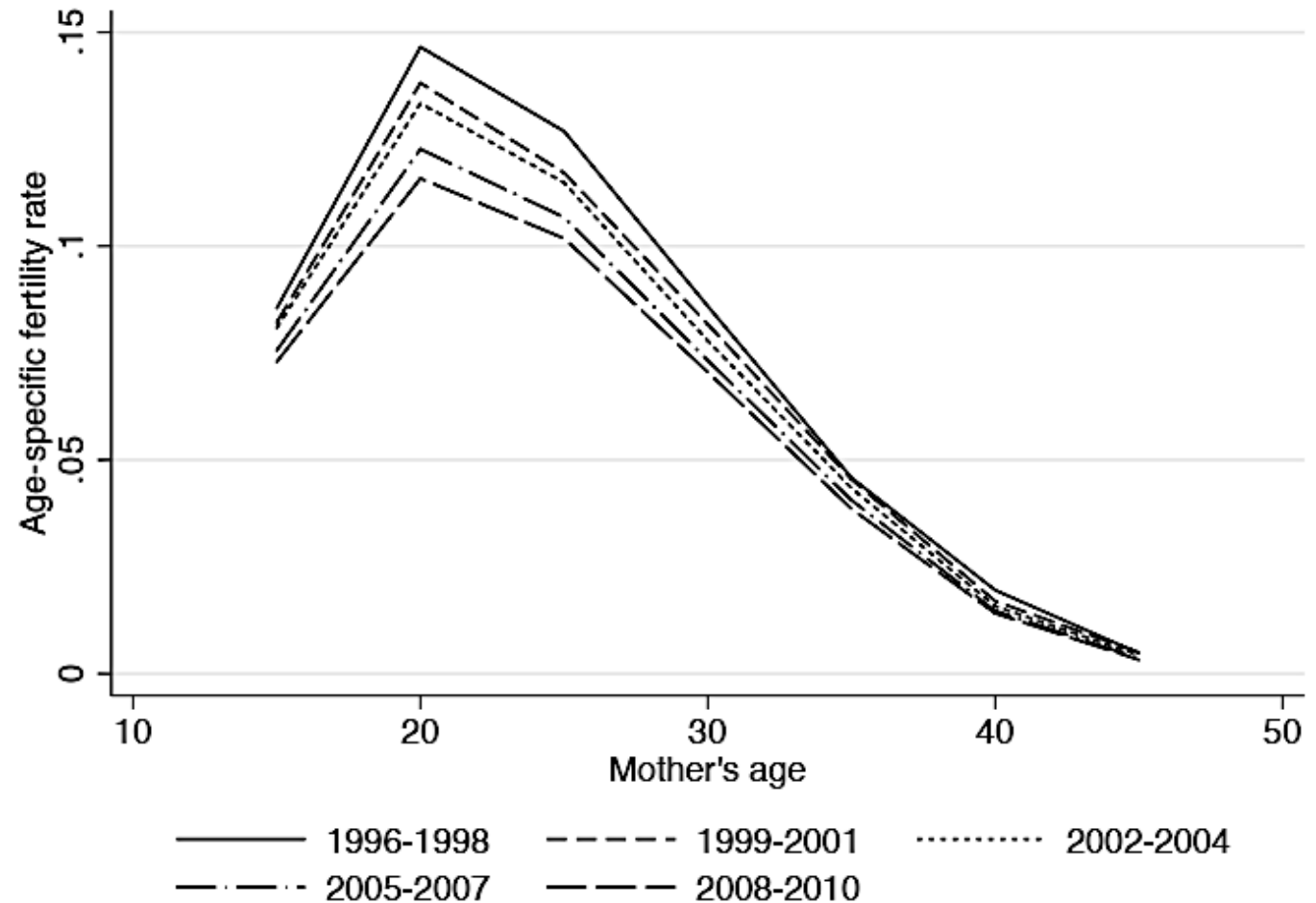
# Output

## Summary measures, Brazil, 2010

Measure	Value
Mean age of the population	32.1182
Mean age of childbearing	26.7967
Crude birth rate(CBR)	0.0359
Gross reproduction rate(GRR)	1.0184
Net reproduction rate(NRR)	1.0098
% change in TFR(mean)	5.1283
% change in TFR(sd)	1.5923
Female population	4.872e+06
% of social mothers *	38.8622
% of biological mothers	37.3477
% of mothers(10-19 years)	0.4995
% of mothers(after 45 years)	0.3100
% childless women	62.6523
% of unmatched children **	13.4907
Replacement level of TFR	2.0647

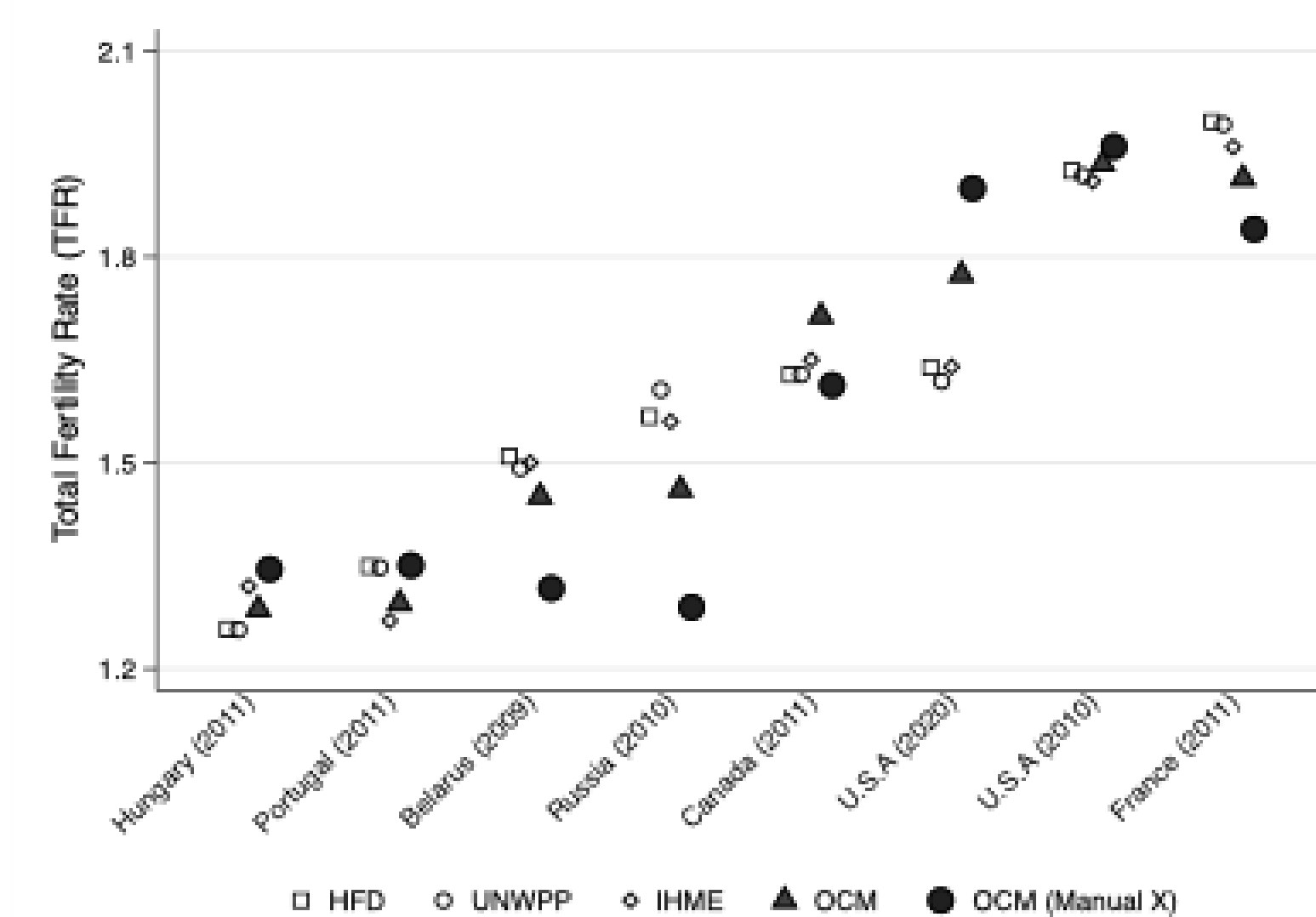
\* Includes stepmother, biological, and adopted mothers.

\*\* Children with unidentified mother in the most current estimate.





# Validation





# leslie: Demographic Projections by Age, Sex and Multistate Categories Using Matrix Models (Work in Progress - NOT FOR CITATION)

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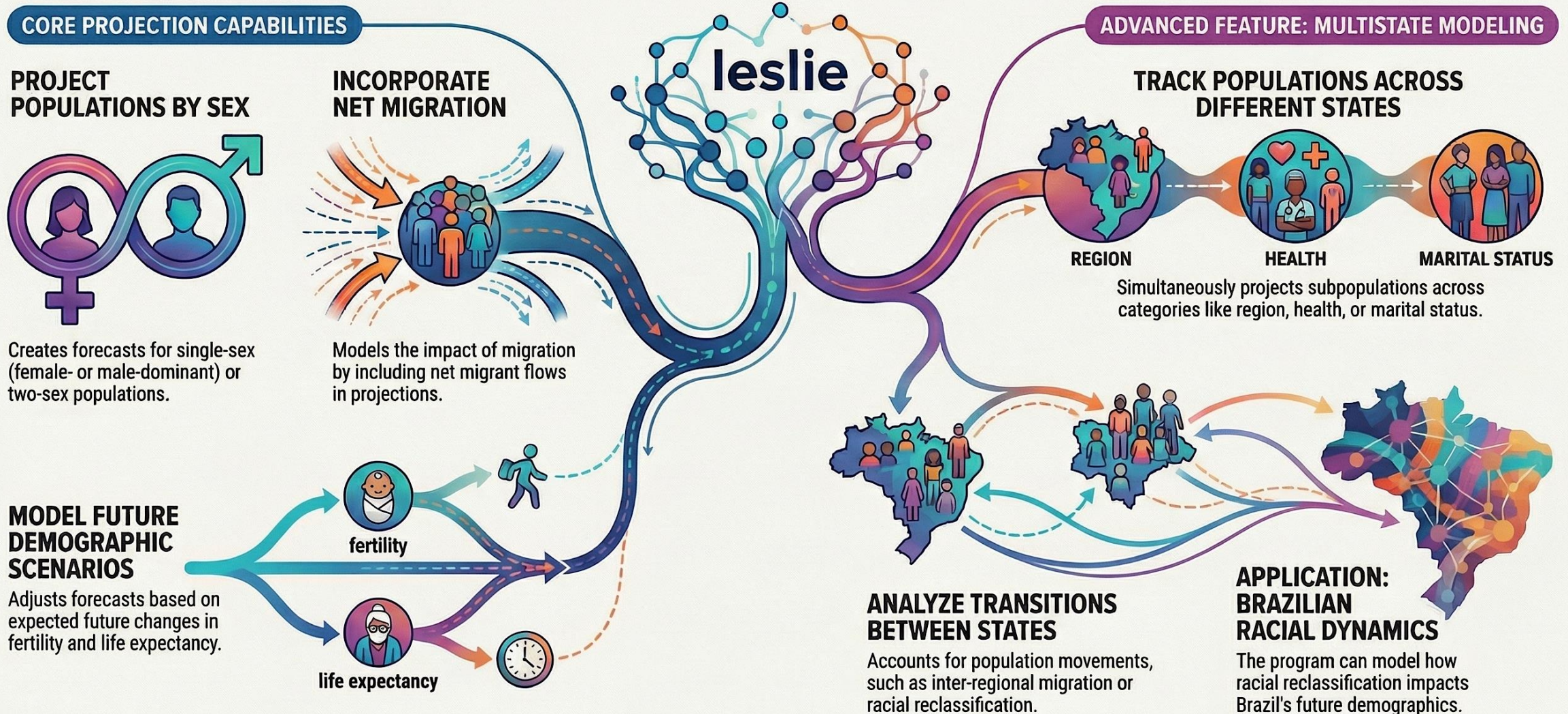
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## Abstract

This article introduces **leslie**, a program for projecting populations subject to schedules of mortality, fertility, and net migration rates. The program computes the future size and age distributions by sex or other dimensions of interest such as region, health, marital, labor market, and racial statuses. It optionally generates stable-equivalent age structures and state-specific summary measures such as life expectancy at birth, total, gross and net fertility rates, intrinsic rates, time to stability, population momentum, mean ages at death, and the mean age of the population. The program supports one- and two-sex populations, open or closed to migration. Empirical examples use data from Sweden for general projections and from Brazil for multistate projections involving racial reclassification. The software can be applied to any context and time period, making it a versatile resource for demographic analysis.



# FORECASTING POPULATIONS: AN OVERVIEW OF THE `LESLIE` PROGRAM





# Why Matrix Models in Demography?

Dr. Patrick Holt Leslie (1900-1972)



VOLUME XXXIII, PART III

NOVEMBER 1945

## ON THE USE OF MATRICES IN CERTAIN POPULATION MATHEMATICS

By P. H. LESLIE, *Bureau of Animal Population, Oxford University*

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### 1. INTRODUCTION

If we are given the age distribution of a population on a certain date, we may require to know the age distribution of the survivors and descendants of the original population at successive intervals of time, supposing that these individuals are subject to some given age-specific rates of fertility and mortality. In order to simplify the problem as much as possible, it will be assumed that the age-specific rates remain constant over a period of time, and the female population alone will be considered. The initial age distribution may be entirely arbitrary; thus, for instance, it might consist of a group of females confined to only one of the age classes.

# Projection Inputs

```
use sweden_1993_female.dta
```

```
l, noobs sep(18)
```

age	p1993	L	f
0	293395	497807	0
5	248369	497356	0
10	240012	497054	0
15	261346	496576	.010238
20	285209	495803	.081414
25	314388	494959	.144546
30	281290	493914	.109054
35	286923	492412	.04325
40	304108	490188	.007676
45	324946	486586	.000218
50	247613	480921	0
55	211351	472204	0
60	215141	458494	0
65	221763	437310	0
70	223507	404547	0
75	183654	352535	0
80	141865	272331	0
85	112257	256542	0

## 1. Population distribution by age

Ideally from a census (e.g. IPUMS, [Human Mortality Database](#), World Bank, [United Nations](#), National Statistical Agencies), already correct for coverage and underreporting

## 2. Person-years from estimated life table

Human mortality database, UN World Population Prospects, or estimated using alternative methods (indirect or iterative)

## 3. Age-specific fertility rates

From the [Human Fertility Database](#), or estimated using alternative methods (Brass' P/F, Own-children method)

## 4. Optionally: Net migration rates

Residually calculated using alternative methods (using previous population distributions or ilt, Muniz 2023). The [United Nations](#) also has total estimates for the net number of migrants and crude net migration rate

# How a Leslie Matrix Works

```
use sweden_1993_female.dta
```

```
l, noobs sep(18)
```

age	p1993	L	f
0	293395	497807	0
5	248369	497356	0
10	240012	497054	0
15	261346	496576	.010238
20	285209	495803	.081414
25	314388	494959	.144546
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60	215141	458494	0
65	221763	437310	0
70	223507	404547	0
75	183654	352535	0
80	141865	272331	0
85	112257	256542	0



Future Swedish female populations are projected as:

M

0	0	.01	.12	.29	.32	.19	.06	.01	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	.99	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	.99	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	.98	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	.97	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	.95	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.92	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.87	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.77
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.52
																	.52

$$M * p^{1993} = p^{1998}$$

$$M * p^{1998} = p^{2003}$$

... ..

$$M * p^{t+n-5} = p^{t+n}$$



# Example outputs from `leslie`

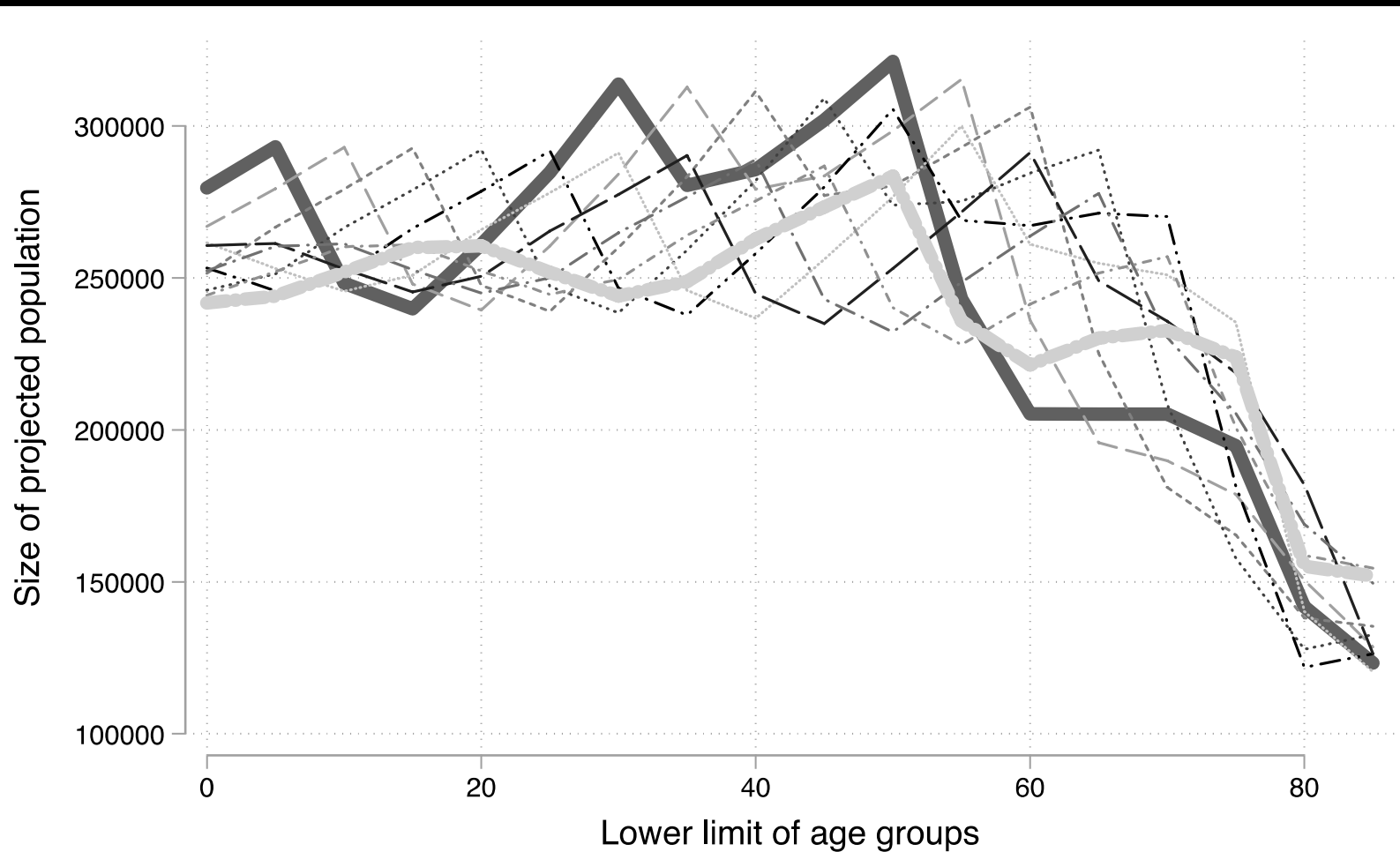
## One-Sex Model (female dominant)

## Without and with migration

. leslie, period(10)

Projected population between ages x and x+5

Age x	c1
0	279603
5	293129
10	248218
15	239781
20	260939
25	284723
30	313724
35	280435
40	285627
45	301873
50	321163
55	243125
60	205215
65	205201
70	205149
75	194771
80	141872
85	123268
Total	4427815

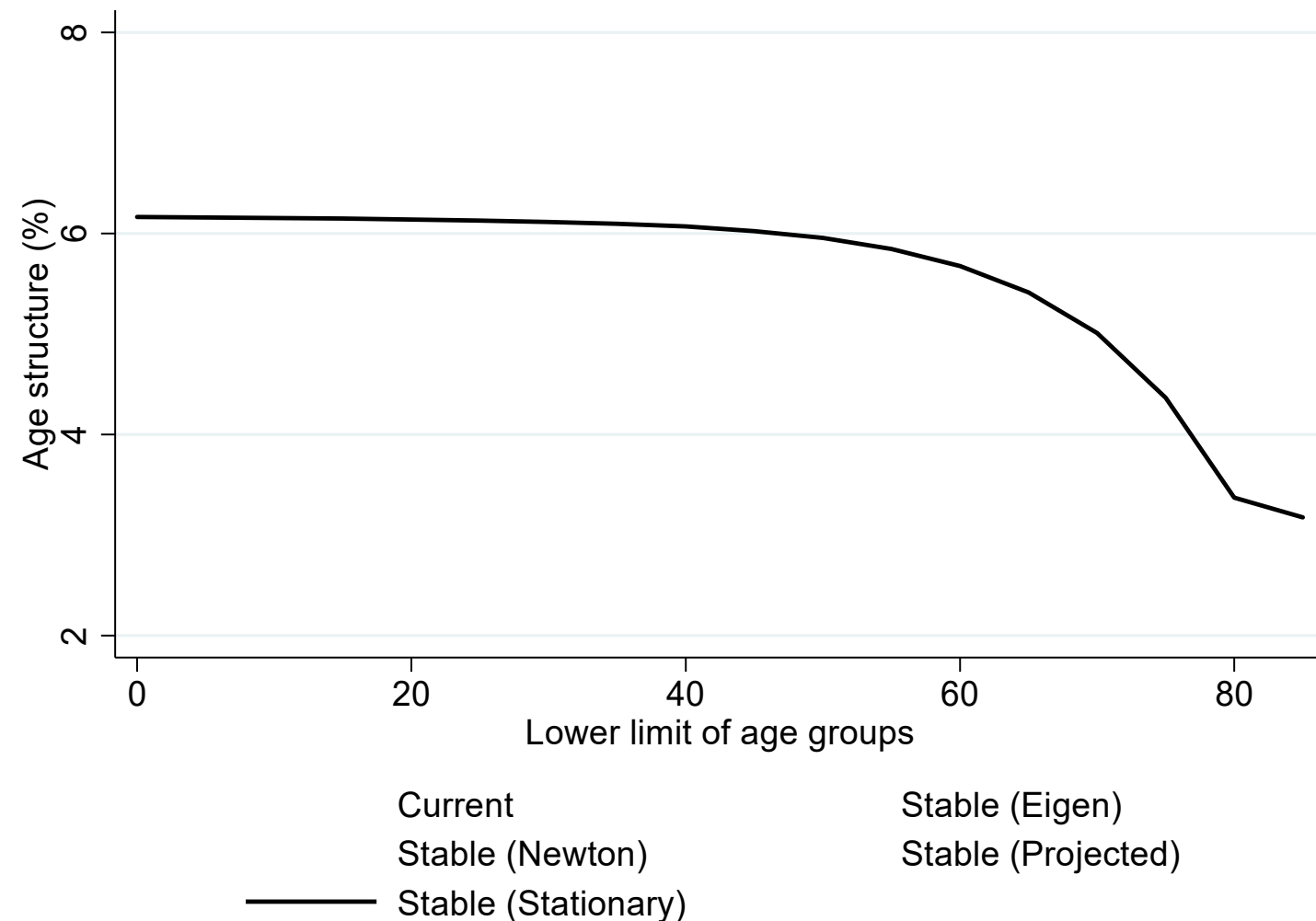


c9	c10
244243	241731
252074	244022
260235	251920
260932	259985
252267	260525
244549	251838
249547	244033
264047	248788
275323	262854
286782	273300
240162	283443
228013	235809
241304	221393
251561	230155
256960	232714
200773	223923
158684	155096
154492	151913
4321946	4273441

. leslie, period(10) stable

Age structure of current and stable-equivalent populations: percent between ages x and x+5

Age x	Current	Eigen
0	6.6724	5.7969
5	5.6484	5.8366
10	5.4584	5.8783
15	5.9435	5.9181
20	6.4862	5.9547
25	7.1498	5.9907
30	6.3971	6.0243
35	6.5252	6.0526
40	6.9160	6.0720
45	7.3899	6.0741
50	5.6312	6.0499
55	4.8066	5.9863
60	4.8928	5.8575
65	5.0433	5.6302
70	5.0830	5.2487
75	4.1767	4.6094
80	3.2263	3.5883
85	2.5530	3.4315



\*Convergence to stability achieved after about 140 years.





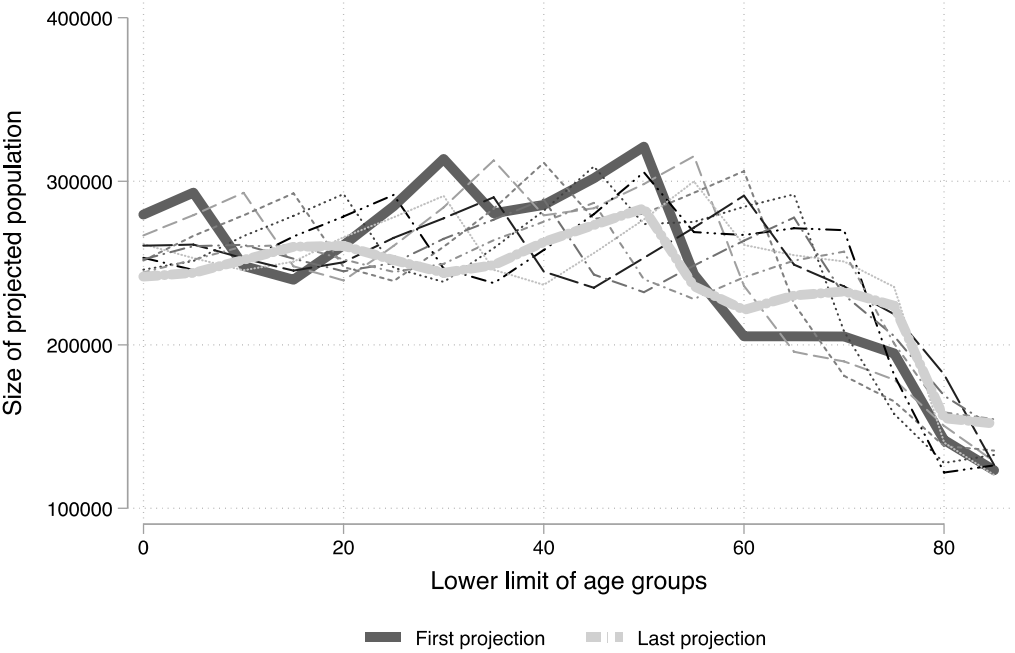
```
. leslie, period(10) summary
```

### Summary measures

Measure	Value
Life expectancy at birth(e0)	80.7754
Mean age at death	78.6059
Mean age of the population	40.7141
Mean length of generation	28.9917
Mean age at maternity	28.9701
Mean age at maternity*	29.0132
Total fertility rate(TFR)	1.9820
Gross reproduction rate(GRR)	0.9668
Net reproduction rate(NRR)	0.9562
Crude birth rate(CBR)	0.0130
Intrinsic birth rate(b)	0.0116
Intrinsic death rate(d)	0.0131
Intrinsic growth rate(r)	-0.0015
Keyfitz's delta	0.0560
Years to stability	140.0000
Total age dependency ratio(%)	60.9308
Years of positive growth	15.0000
Proj./obs. when decline starts	1.0081
Proj./obs. in stability	0.5343
Momentum (Frauenthal 1975)	0.9584
Momentum (Preston-Guillot 1997)	1.0035

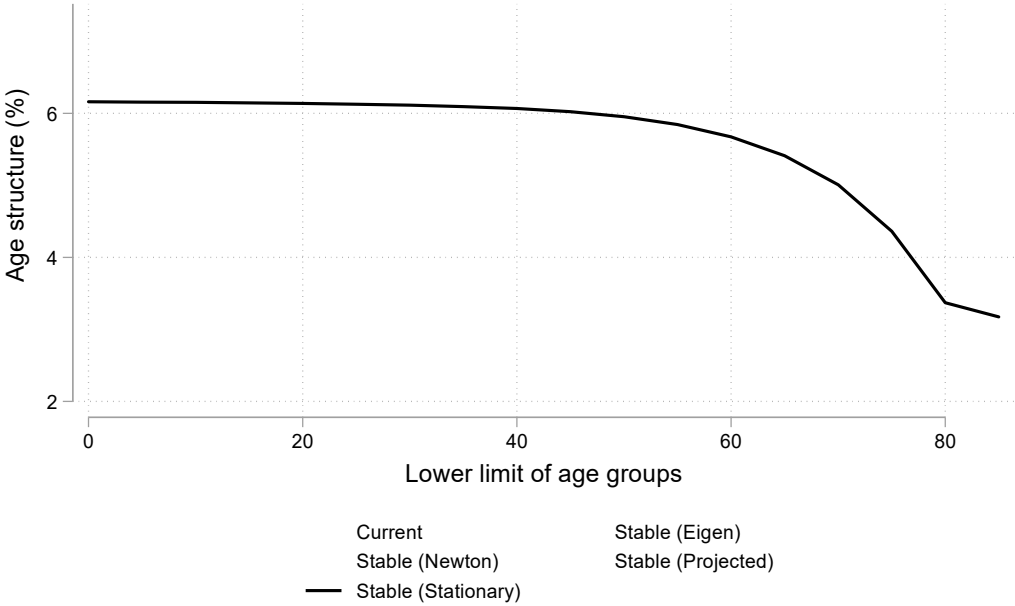
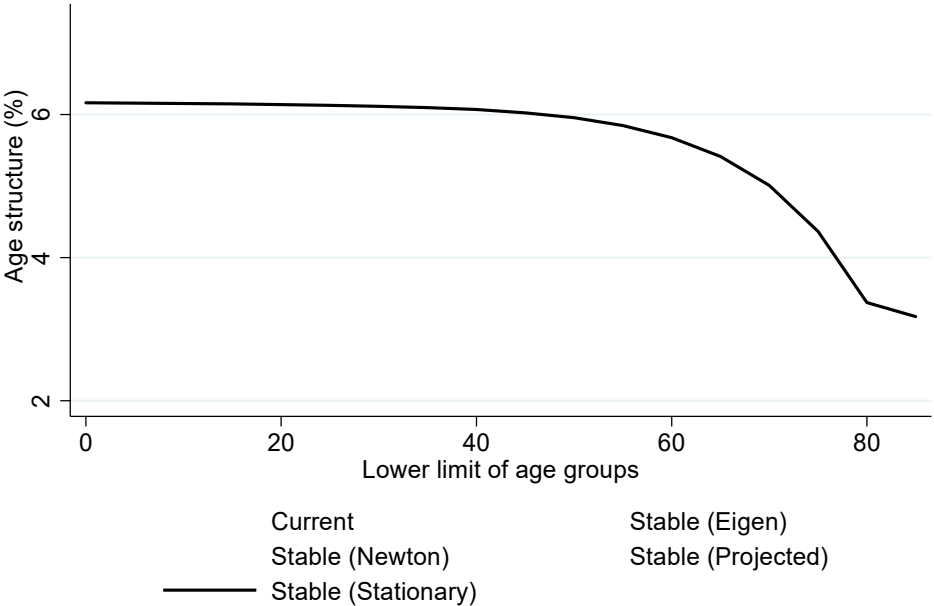
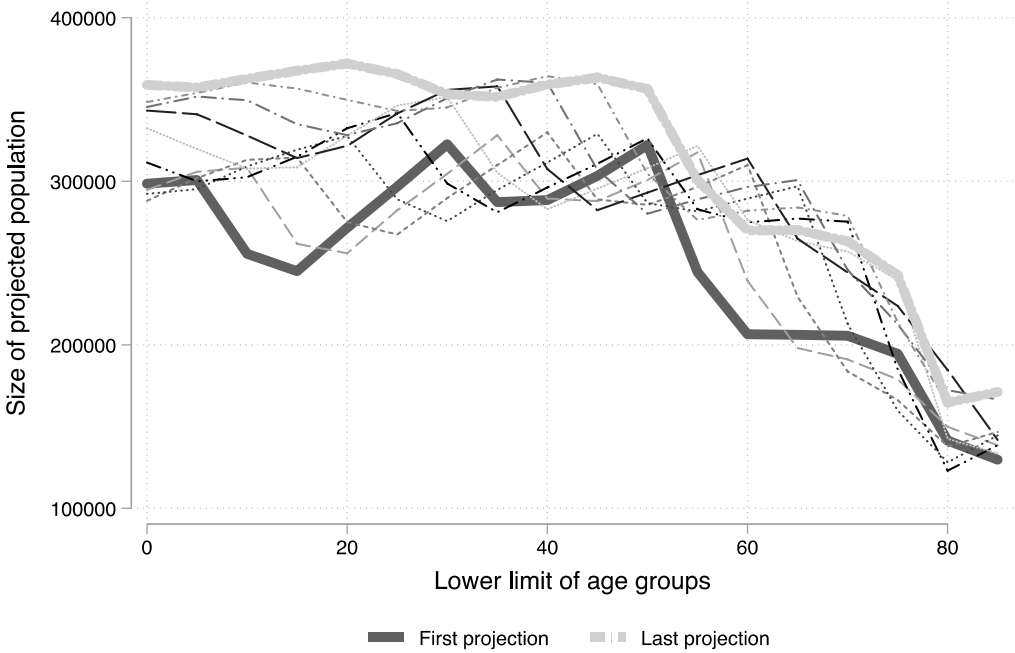
\*In the stable population.

Sem migração



Com migração (líquida)

`.leslie, p(10) mig1(sweden_1988_female.dta) s gropt(sch(plotplain))`



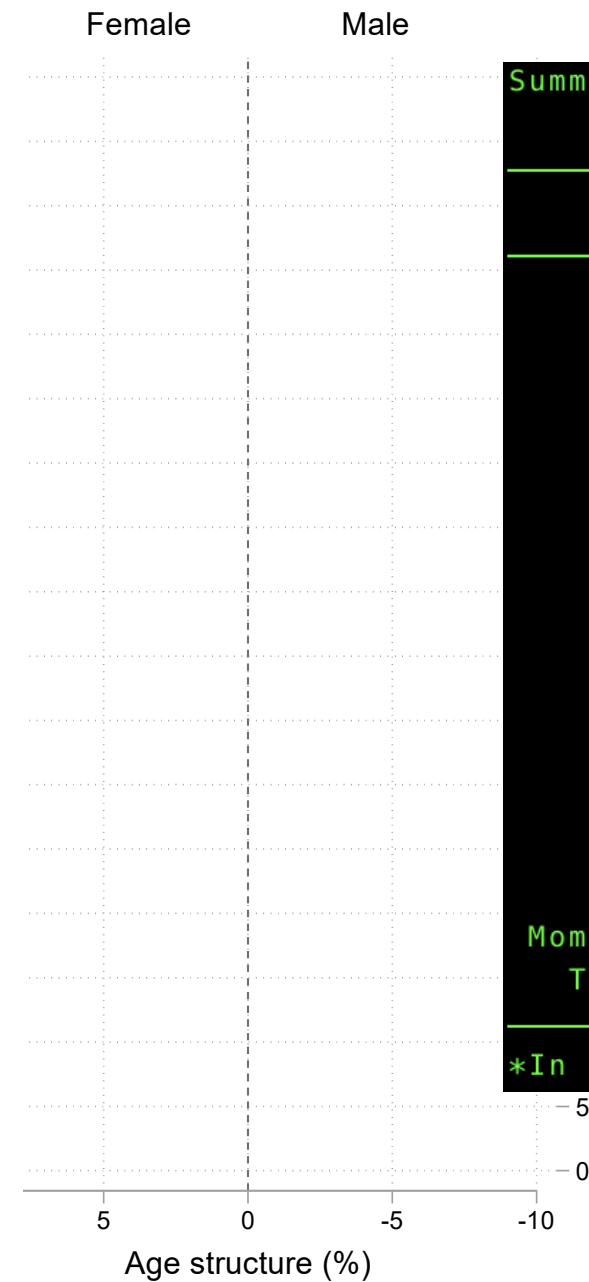
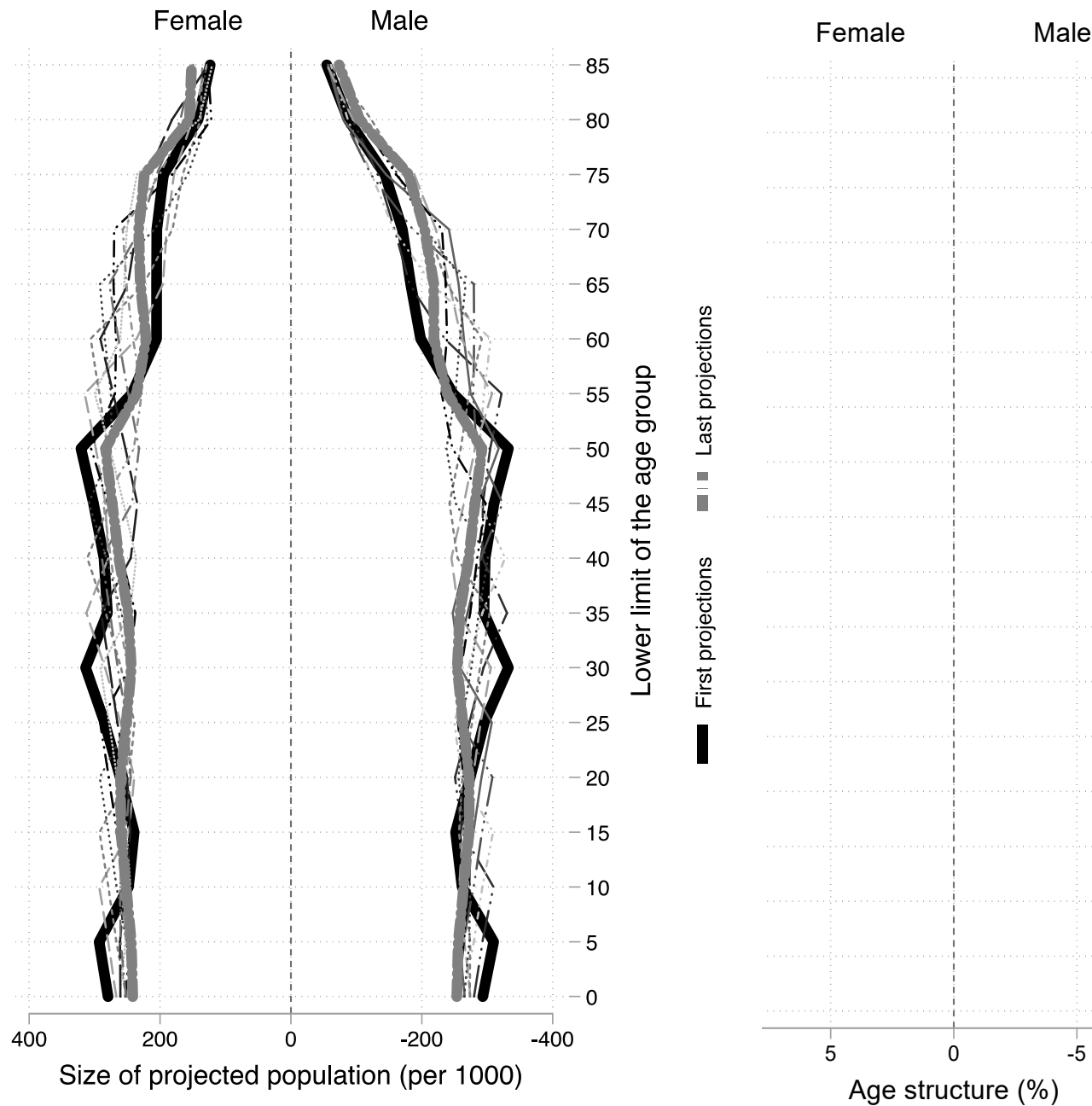


# Example outputs from `leslie`

## Two-Sex Model

Simultaneous projections for both sexes without migration

```
. use sweden_1993_bothsexes
. leslie, p(10) two stable summary gropt(sch(plotplain))
```



Summary measures		
Measure	Female	Male
Life expectancy at birth(e0)	80.7754	75.4926
Mean age at death	78.6059	74.1943
Mean age of the population	40.7141	38.0280
Mean length of generation	28.9917	28.9581
Mean age at maternity	28.9701	28.9567
Mean age at maternity*	29.0132	28.9594
Total fertility rate(TFR)	1.9820	1.9820
Gross reproduction rate(GRR)	0.9668	1.0152
Net reproduction rate(NRR)	0.9562	0.9972
Crude birth rate(CBR)	0.0130	0.0147
Intrinsic birth rate(b)	0.0116	0.0132
Intrinsic death rate(d)	0.0131	0.0133
Intrinsic growth rate(r)	-0.0015	-0.0001
Keyfitz's delta	0.0560	0.0640
Momentum (Frauenthal 1975)	0.9584	0.9976
Momentum (Preston-Guillot 1997)	1.0035	1.0135
Total age dependency ratio(%)	60.9308	52.3304

\*In the stable population.



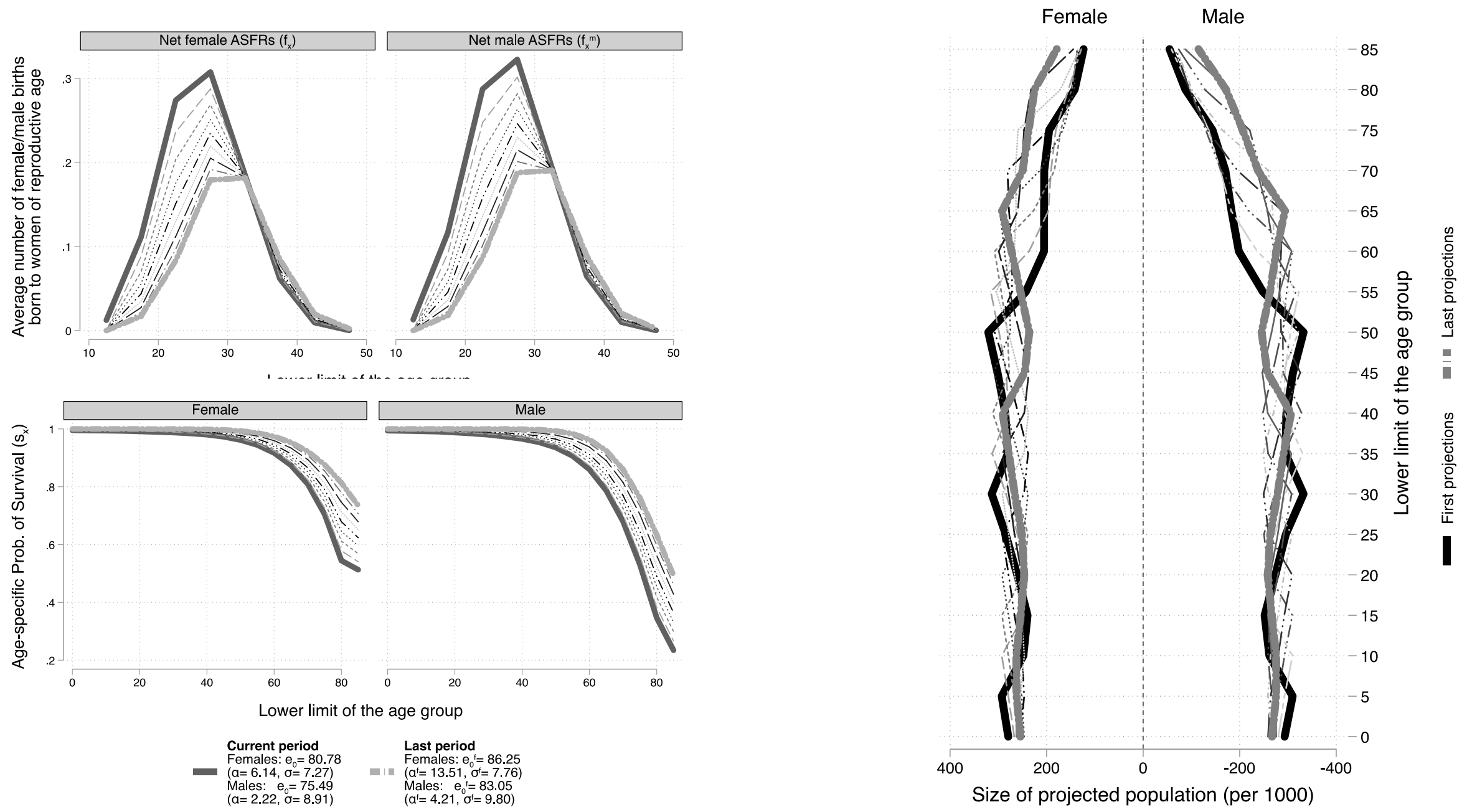


# Example outputs from `leslie`

## Two-Sex Model

Simultaneous projections for both sexes with variable fertility and mortality schedules

```
. leslie, p(8) two base(1993) place(Sweden) surv(. .) gr(sch(plotplain))
```



Note: Life expectancy at birth ( $e_0$ ) estimated as the area under the survival curve ( $S_x$ ).



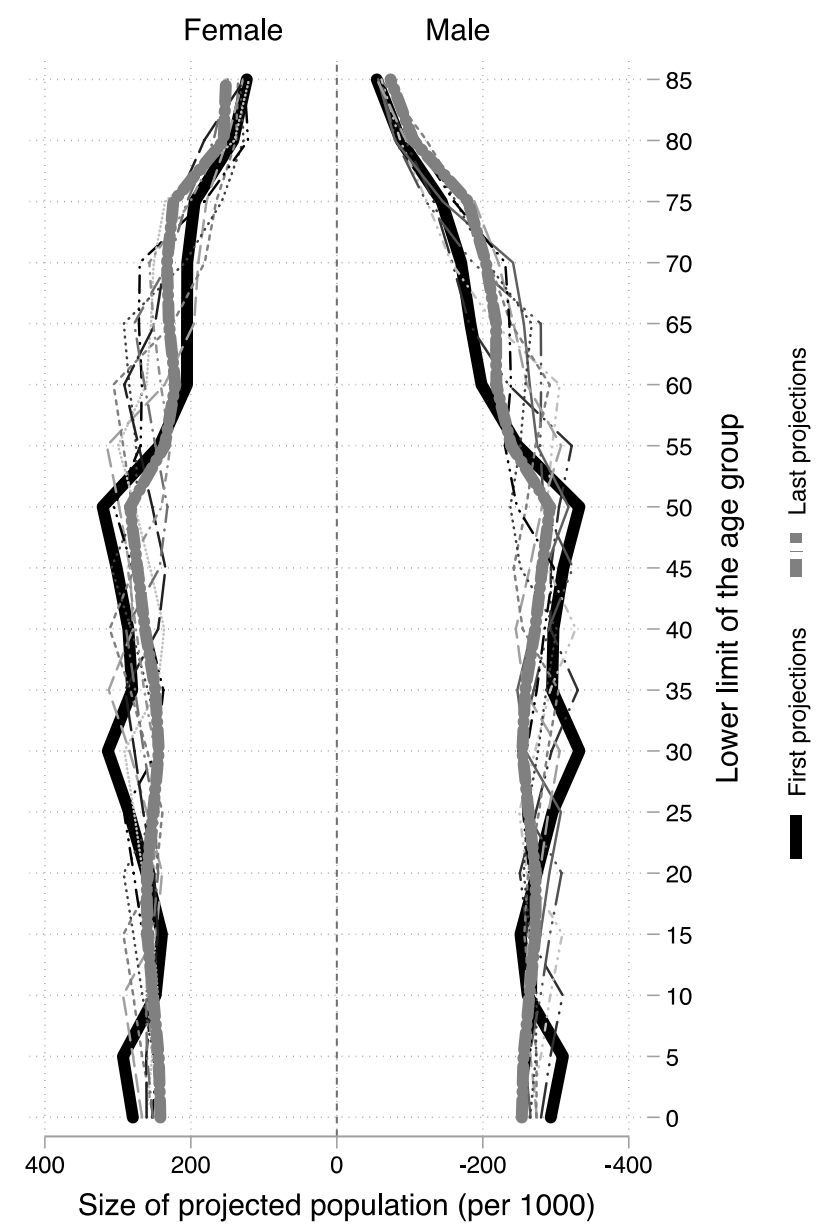
# Example outputs from `leslie`

## Two-Sex Model

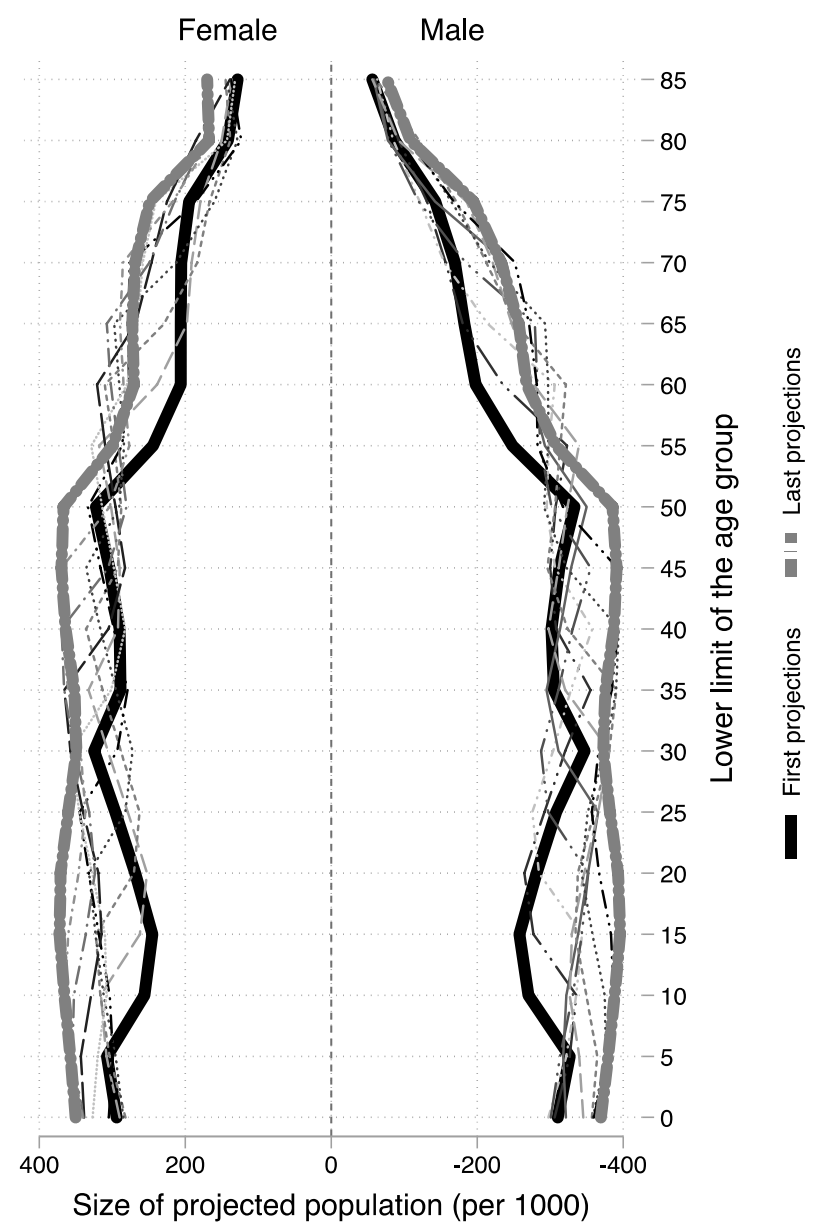
Simultaneous projections for both sexes **with net migration**



```
. use sweden_1993_bothsexes
. leslie, p(10) two gropt(sch(plotplain))
```



```
. leslie, p(10) two stable gropt(sch(plotplain))
migl1(sweden_1988_bothsexes.dta) mig2
```

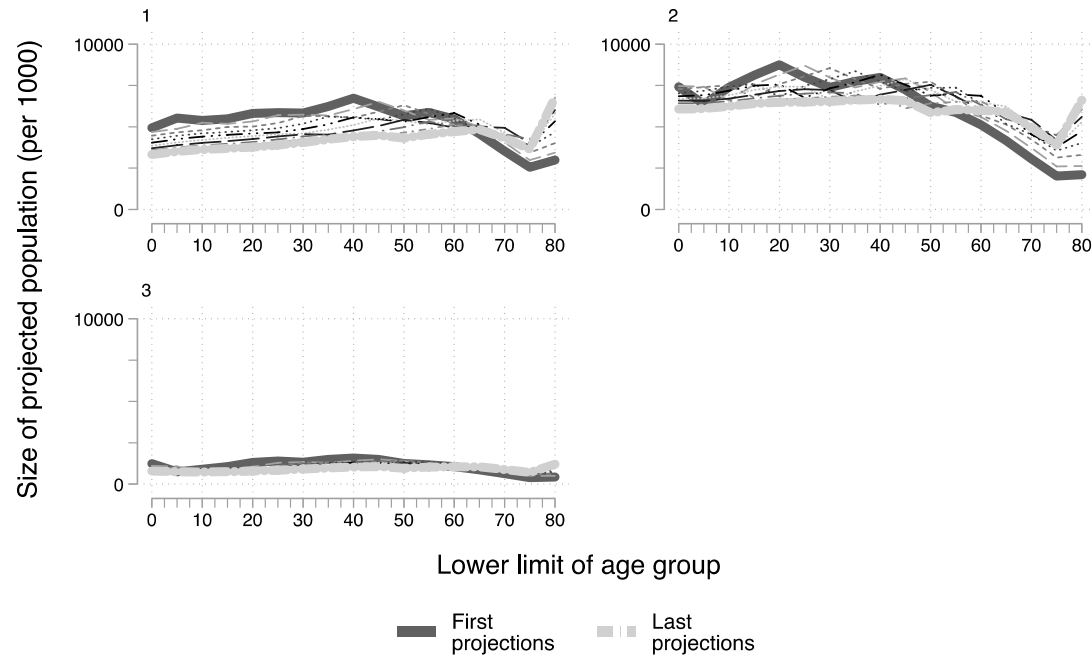




# Example outputs from `leslie`

## Multistate projections

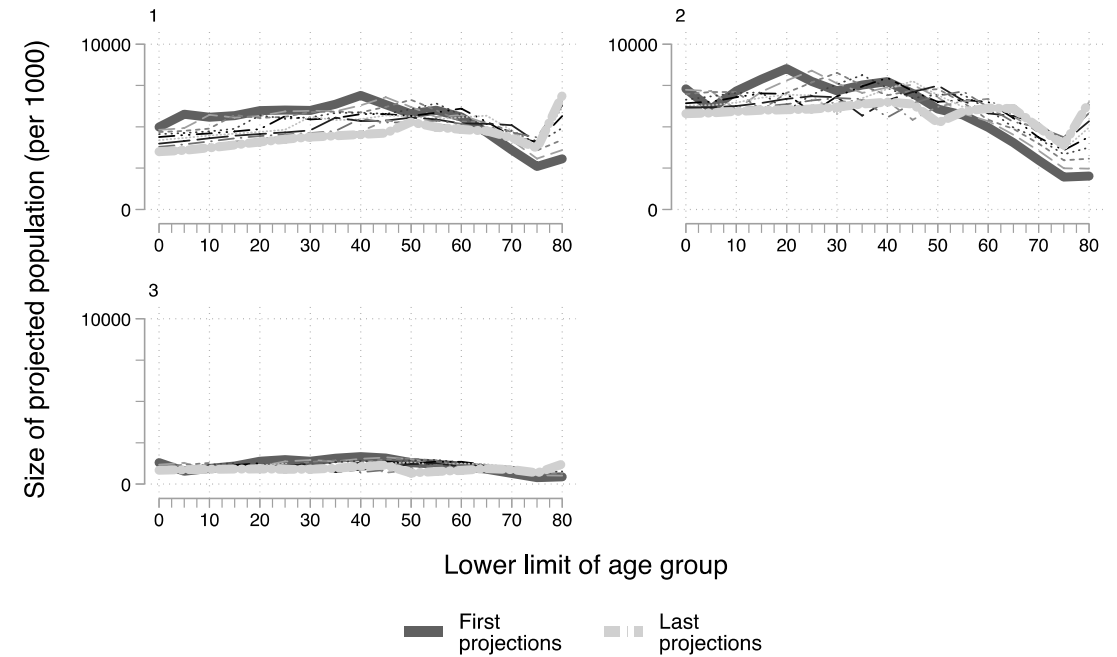
```
. leslie, period(10) multistate  
gropt(sch(plotplain))
```



Graphs by 1 if White, 2 if Brown, 3 if Black

WHITES: 72.17 millions in 50 years  
BROWNS: 103.44 millions  
BLACKS: 15.54 millions

```
. leslie, period(10) multistate  
gropt(sch(plotplain)) nomobility
```



Graphs by 1 if White, 2 if Brown, 3 if Black

WHITES: 75.78 millions in 50 years  
BROWNS: 99.85 million  
BLACKS: 15.4 million



. leslie, period(10) summary multistate

Summary measures for the total population

Measure	Value
Intrinsic growth rate(r)	<b>-0.0054</b>
Years to stability	<b>165.0000</b>
Years of positive growth	<b>20.0000</b>
Proj./obs. when decline starts	<b>1.0118</b>
Proj./obs. in stability	<b>0.3480</b>

Summary measures by state of occupancy

Measure	State 1	State 2	State 3
ASD life expectancy*	<b>73.2226</b>	<b>69.0408</b>	<b>68.1007</b>
Mean age at transfer	<b>37.9228</b>	<b>37.3629</b>	<b>36.6759</b>
Mean age of the population	<b>37.9781</b>	<b>34.4137</b>	<b>37.6788</b>
Total fertility rate(TFR)	<b>1.6867</b>	<b>1.9624</b>	<b>1.8812</b>
Gross reproduction rate(GRR)	<b>0.8228</b>	<b>0.9573</b>	<b>0.9177</b>
Crude birth rate(CBR)	<b>0.0235</b>	<b>0.0310</b>	<b>0.0299</b>
Keyfitz's delta	<b>0.3071</b>	<b>0.2384</b>	<b>0.4577</b>
Total age dependency ratio(%)	<b>52.6470</b>	<b>48.6681</b>	<b>36.3411</b>

```
. leslie, period(10) summary multistate
```

Life expectancies at birth by state: sojourn time in state j (in columns) given occupancy of state i (in rows)

State-specific e0	State 1	State 2	State 3
State 1	29.3693	25.5404	25.3834
State 2	35.8267	39.0787	36.8941
State 3	5.5084	5.6309	8.6117
Total	70.7044	70.2500	70.8892

# Software Capabilities Summary

1. Use **leslie** to achieve efficiency, agility, and reliability in producing projections. The program has the advantage of integrating, while at the same time maintaining the independence of demographic components.
2. Use **leslie** to investigate counterfactual scenarios of fertility and survival variation, with and without migration, and with or without interstate migration.
3. Use **leslie** to calculate equivalent stable populations and key demographic indicators (TFR,  $e_0$ , NRR, momentum, mean ages, intrinsic rates, dependency ratio etc.).



THANK  
YOU!!





# Five key references:

Leslie, P. H. 1945. On the use of matrices in certain population mathematics. *Biometrika* 33(3): 183-212.

Rogers, A. 1968. Matrix analysis of interregional population growth and distribution. 1<sup>st</sup> ed. Berkeley and Los Angeles, California: University of California Press.

Rogers, A. 1995. Multiregional demography: principles, methods and extensions. England: John Wiley and Sons Ltd.

**Muniz, J. 2020. Multistate life tables using Stata. *The Stata Journal* 20(3): 721-745.**

**Muniz, J. 2023. Iterative intercensal single-decrement life tables using Stata. *The Stata Journal* 23: 813-834.**