#### METHODOLOGICAL ISSUES AND STRATEGIES FOR USING AND MEASURING PACE IN REGRESSION ANALYSES

SCHOOL OF GLOBAL

**PUBLIC HEALTH** 

Melody S. Goodman, PhD Associate Professor of Biostatistics Associate Dean for Research NYU School of Global Public Health



2 INTRODUCTION - CONTEXT

# HEALTH DISPARITIES

- Differences in health that:
  - Are systematic and plausibly avoidable
  - Influenced by policies
  - Put socially disadvantaged groups at further disadvantage with respect to their health
- NIH definition of health disparities:
  - "Differences in the incidence, prevalence, mortality, and burden of diseases and other adverse health conditions that exist among specific population groups in the U.S."

## HEALTHCARE DISPARITIES



"Racial or ethnic differences in the quality of healthcare that are <u>not</u> due to access-related factors or clinical needs, patient preferences, and appropriateness of intervention."

## RACE ON BIRTH RECORDS BEFORE 1989

Father	Mother	Baby*
White	White	White
Black	Black	Black
Hawaiian	Hawaiian	Hawaiian
Asian	Asian	Asian
White	Black	Black
Black	White	Black
Black	Asian	Black
White	Asian	Asian
Black	Hawaiian	Hawaiian

\*Not reported on birth records but used for other statistical purposes



## RACE: WHAT IT IS, WHAT IT IS NOT

"Racial categories reflect social and ideological conventions, not meaningful natural distinctions"

Race is a man-made social construct

**Source**: Krieger N. Discrimination and Health Inequities. In: Berkamn LF, Kawachi I, Glymour MM, editors. *Soc. Epidemiol.* Second Edition. New York: Oxford University Press; 2014.



## SALIENCE OF RACE IN STATISTICAL ANALYSIS

Race is salient to statistical analysis because:

- Early science  $\rightarrow$  flawed racial thinking (e.g., race as biology).
  - This thinking continues to creep into contemporary discourses and research
- Vulnerable populations include disproportionate numbers of racial and ethnic minorities
  - Minorities have poorer health and worse access to care
  - The interventions that address these problems often are colorblind—that is, they provide individuals with immediate care, but they are not intended to address broader racial factors that may undergird the disparities.
- Racialization is salient whenever the purpose of research is to understand how racial factors influence disease distributions
  - Research on racial differences may be either problematic or constructive
  - To advance racist hypotheses; many foundational statistical methods were generated by eugenicists
  - To contest racist research
  - To identify social determinants of disparities





## DATA DRIVEN APPROACHES

"Data never speak for themselves. It is the questions we pose (and those we fail to ask) as well as our theories, concepts and ideas that bring a narrative and meaning to marginal distributions, correlations, regression coefficients, and statistics of all kinds."

- Lawrence Bobo 2004



# CODING RACE/ETHNICITY VARIABLES IN REGRESSION ANALYSIS



- Self-report
- Other-report
  - Visual
  - Objective
- Familial factors
- Nativity
- Identity
- Social class

#### **Sample Survey Question**

Which of the following best describes your race?

- 1. African American or Black
- 2. Asian American or Asian
- 3. White or Caucasian
- 4. Other

From: NYC Low-Income Housing, Neighborhoods and Health Study



## RACE IS A CATEGORICAL (NOMINAL) VARIABLE

- Categorical variables require special attention in regression analysis because, unlike dichotomous or continuous variables, they cannot by entered into the regression equation just as they are
- Categorical variables need to be recoded into a series of variables which can be entered into the regression model
- There are a variety of coding systems that can be used when coding categorical variables
- Ideally, you would choose a coding system that reflects the comparisons that you want to make
- We often recode categorical variables in regression analysis using dummy variables
  - Not the only coding scheme that you can use

 $\label{eq:stats.idre.ucla.edu/stata/webbooks/reg/chapter5/regression-with-statachapter-5-additional-coding-systems-for-categorical-variables-in-regressionanalysis/$ 

## CHOSE YOUR CODING SYSTEM APPROPRIATELY

- By deliberately choosing a coding system, you can obtain comparisons that are most meaningful for testing your hypotheses.
- If you want to compare each level to the next higher level, you will want to use "forward difference" coding
- Regardless of the coding system you choose, the test of the overall effect of the categorical variable (i.e., the overall effect of race) will remain the same.

<u>Source</u>: https://stats.idre.ucla.edu/stata/webbooks/reg/chapter5/regression-with-statachapter-5-additional-coding-systems-for-categorical-variables-in-regressionanalysis/

## TYPES OF CONTRASTS & THE COMPARISON MADE

Name of contrast	Comparison made
Simple Coding	Compares each level of a variable to the reference level
Forward Difference Coding	Adjacent levels of a variable (each level minus the next level)
<b>Backward Difference Coding</b>	Adjacent levels of a variable (each level minus the prior level)
<b>Deviation Coding</b>	Compares deviations from the grand mean
<u>Statistical Analyst</u> <u>Defined Coding</u>	Statistical Analyst defined contrast

Choose a coding system that yields comparisons that make the most sense for testing your hypotheses



Source: https://stats.idre.ucla.edu/stata/webbooks/reg/chapter5/regression-with-statachapter-5-additional-coding-systems-for-categorical-variables-in-regressionanalysis/

#### NOTES ON CODING SYSTEMS

- Represent planned comparisons and not post hoc comparisons
  - They are comparisons that you plan to do before you begin analyzing your data
  - Not comparisons that you think of once you have seen the results of preliminary analyses
- Some forms of coding make more sense with ordinal categorical variables than with nominal categorical variables.
- Because simple effect coding compares the mean of the dependent variable for each level of the categorical variable to the mean of the dependent variable at for the reference level, it makes sense with a nominal variable.
- However, it may not make as much sense to use a coding scheme that tests the linear effect of race.

#### EXAMPLE DATASET

- **b** Data from NYC community health survey
- k https://www1.nyc.gov/site/doh/data/data-sets/community-health-surveypublic-use-data.page
- We will focus on the categorical variable **newrace**, which has four levels (1 = Non-Hispanic White, 2 = Black (non-Hispanic), 3 = Hispanic and 4 = Asian)
  - × This is how race was categorized in the dataset (limitation of secondary analysis)
  - 💥 we will use **body mass index (bmi)** as our dependent variable
- Although our example uses a variable with four levels, these coding systems work with variables that have more or fewer categories
- No matter which coding system you select, you will always have one fewer recoded variables than levels of the original variable
  - X In our example, the race variable has four levels so we will have three new variables
  - X A variable corresponding to the final level of the categorical variables would be redundant and therefore unnecessary.

Observations: Variables:		8,781 147		
Variable	Storage	Display	Value	
name	type	format	label	Variable label
cid	double	%12.0g		Unique identifier across all years of CHS
strata	double	%12.0g		Variance stratum - Collapsed
survey	double	%12.0g		Survey number
wt21_dual	double	%12.0g		CHS 2020 Survey weight
wt21_dual_q1	double	%12.0g		CHS 2020 Long Survey Weight
strata_q1	double	%12.0g		Variance stratum - Collapsed half sample
qxvers	double	%12.0g		Questionnaire Version. QXVERS = 1 (LONG VERSION) QXVERS = 2 (SHORT VERSION)
mood1	double	%12.0g	mood1	Q5.1 During the past 30 days, how often did you feel So sad that nothing coul

# EXAMINE THE DATA

Before considering any analyses, let's examine the dataset



Unweighted: tabulate race, summarize (bmi)

Race/ethnic itv			
variable of	Summary of Bo	ody Mass Index	(kg/sq
interviewed	Mean	Std. dev.	Freq.
White	26.35302	5.62598	2,752
Black	28.834652	6.5778025	1,759
Hispanic	28.631466	7.052452	2,357
Asian/PI	24.535502	4.9037819	1,275
Total	27.264005	6.3805962	8,143



- Calculate the mean of the dependent variable, bmi, for each level of race
- This will help in interpreting the output from regression analyses



## EXAMINE THE KEY VARIABLES

Survey: Mean estimation

Number of strata = 159 Number of PSUs = 8,143 Number of obs = 8,143 Population size = 5,988,401 Design df = 7,984

<ul> <li>Calculate the mean of the</li> </ul>
dependent variable, <b>bmi</b> , for
each level of <b>race</b>

 This will help in interpreting the output from regression analyses

al]
079
681
835
735
a 0 6 8 7

svyset cid [pweight=wt21\_dual], strata(strata) vce(linearized) singleunit(missing)
Weighted:svy: mean bmi, over(newrace)

#### DUMMY CODING

- It is a way to make the categorical variable into a series of dichotomous variables
  - Variables can have a value of zero or one only
  - For all but one of the levels of the categorical variable, a new variable will be created that has a value of one for each observation at that level and zero for all others.
- In our example using the variable race
  - The first dummy variable (x1) will have a value of one for each observation in which race is Black, and zero for all other observations.
  - Likewise, we create x2 to be 1 when the person is Hispanic, and 0 otherwise
  - **x3** is 1 when the person is Asian, and 0 otherwise.
- The level of the categorical variable that is coded as zero in all of the new variables is the reference level, or the level to which all of the other levels are compared.
  - In our example, white is the reference level because this is what is typically done (doesn't mean it's ideal)
  - You can select any level of the categorical variable as the reference level.

#### DUMMY CODING EXAMPLE

Level of race	New variable 1 (x1)	New variable 2 (x2)	New variable 3 (x3)
l (White)	0	0	0
2 (Black)	1	0	0
3 (Hispanic)	0	1	0
4 (Asian)	0	0	1

The coefficient for x1 is the mean of the dependent variable for group 1 minus the mean of the dependent variable for the omitted group.

- ▶ The coefficient for x1 is the mean of BMI for the Black group minus the mean of BMI for the White group
- The coefficient for x2 would be the mean of BMI for the Hispanic group minus the mean of BMI for the White group
- The coefficient for x3 would be the mean of BMI for the Asian group minus the mean of BMI for the White group.

Source: https://stats.idre.ucla.edu/spss/faq/coding-systems-for-categorical-variables-in-regression-analysis/

## DUMMY VARIABLE REGRESSION OUTPUT (UNWEIGHTED)

regress bmi i.newrace

Source	SS	df	MS	Numb - E(3	per of obs	=	8,143 179 06	Race/ethnic	
Model	20522.7022	3	6840.90072	Prot	) > F	=	0.0000	ity variable of	Summary of Bo
Residual	310954.466	8,139	38.2054879	• R-sc - Adj	juared R-squared	=	0.0619 0.0616	person interviewed	Mean
Total	331477.168	8,142	40.7120079	Root	MSE	=	6.1811	White Black Hispanic	26.35302 28.834652 28.631466
bmi	Coefficient	Std. err.	t	P> t	[95% co	nf.	interval]	Asian/PI	24.535502
newrace						<u> </u>		Total	27.264005
Black	2.481632	.1886871	13.15	0.000	2.11175	7	2.851507		
Hispanic	2.278446	.173471	13.13	0.000	1.93839	8	2.618493		
Asian/PI	-1.817518	.2093989	-8.68	0.000	-2.22799	4	-1.407043		
_cons	26.35302	.1178253	223.66	0.000	26.1220	5	26.58399		



## DUMMY VARIABLE REGRESSION OUTPUT (WEIGHTED)

svy: regress bmi i.newrace

Survey: Linear regression

Survey: Mean estimation

Mean

26.32592 28.4591

28.43682

24.32051

Number of str Number of PSU	ata = 159 s = 8,143			Numb Popu Desi F(3,	er of obs = lation size = gn df = 7982) =	= 8,143 = 5,988,401 = 7,984 = 77.90	Number of strata Number of PSUs	a = 159 = 8,143
				Prob R-sq	> F = uared =	= 0.0000 = 0.0533		Ме
bmi	Coefficient	Linearized std. err.	t	P> t	[95% conf	. interval]	c.bmi@newrace White Black Hispanic	26.325 28.45 28.436
newrace							Asian/PI	24.320
Black	2.133186	.3019247	7.07	0.000	1.541335	2.725037		
Hispanic	2.110904	.2628895	8.03	0.000	1.595571	2.626236		
Asian/PI	-2.00541	.2876277	-6.97	0.000	-2.569236	-1.441585		
_cons	26.32592	.1708285	154.11	0.000	25.99105	26.66079		2

#### EFFECT CODING

- Other coding systems use more values than just zero and one
- Allows you to make other types of comparisons
- Unlike dummy coding, effect coding allows you to assign different weights to the various levels
  of the categorical variable.
- The "rule" in dummy coding is that only values of zero and one are valid
- The "rule" in effect coding is that all of the values in any new variable must sum to zero
- Which level is assigned a positive or negative value is not very important
  - 0 1 -1 0 is the same as 0 -1 1 0
  - Both of these codings compare the second and the third levels of the variable
  - However, the sign of the coefficient would change.

## SIMPLE REGRESSION CODING

Level of race	New variable l (sxl)	New variable 2 (sx2)	New variable 3 (sx3)
l (White)	-1/4	-1/4	-1/4
2 (Black)	3/4	-1/4	-1/4
3 (Hispanic)	-1/4	3/4	-1/4
4 (Asian)	-1/4	-1/4	3/4

- Level 1 is the reference level
- sxl compares level 2 to level 1
- sx2 compares level 3 to level 1
- sx3 compares level 4 to level 1.
- For sxl the coding is 3/4 for level 2, and -1/4 for all other levels.
- for sx2 the coding is 3/4 for level 3, and -1/4 for all other levels,
- for sx3 the coding is 3/4 for level 4, and -1/4 for all other levels.

Source: https://stats.idre.ucla.edu/stata/webbooks/reg/chapter5/regression-with-statachapter-5-additional-coding-systems-for-categorical-variables-in-regressionanalysis/

#### GENERAL RULE FOR SIMPLE CODING

Level of grouping variable	New variable l (xl)	New variable 2 (x2)	New variable 3 (x3)
Group l	-1 / k	-1 / k	-1 / k
Group 2	(k-1) / k	-1 / k	-1 / k
Group 3	-1 / k	(k-l) / k	-1 / k
Group k	-1 / k	-1 / k	(k-l) / k

- It may not intuitive that this regression coding scheme yields these comparisons
  - Follow this general rule to obtain simple comparisons
- General rule for creating a simple coding scheme using regression coding
  - Where k is the number of levels of the categorical variable
  - In this example, k = 4

Source: https://stats.idre.ucla.edu/stata/webbooks/reg/chapter5/regression-with-statachapter-5-additional-coding-systems-for-categorical-variables-in-regressionanalysis/

#### SIMPLE CODING REGRESSION MODEL INTERPRETATION (UNWEIGHTED)

The results of simple coding are very similar to dummy coding in that each level is compared to the reference level

Source Model Residual	SS 20522.7022 310954.466	df 3 8,139	MS 6840.90072 38.2054879	Number of obs F(3, 8139) Prob > F R-squared Adj R-squared	= = = =	8,143 179.06 0.0000 0.0619 0.0616	Race/ethnic ity variable of person interviewed	Summary of Bo Mean
bmi	Coefficient	8,142 Std. err.	40./1200/9 t F	ROOT MSE 	= onf. i	.nterval]	White Black Hispanic	26.35302 28.834652 28.631466
sx1 sx2 sx3 _cons	2.481632 2.278446 -1.817518 27.08866	.1886871 .173471 .2093989 .0714918	13.15 13.13 -8.68 378.91	0.0002.11170.0001.93830.000-2.22790.00026.948	57 98 94 - 52	2.851507 2.618493 1.407043 27.2288	Asian/PI Total	24.535502

- Level 1 (White) is the reference level
- regress bmi sx1 sx2 sx3
- x1 compares level 2 (Black) to level 1 = 28.83-26.35 = 2.48
  - X2 compares level 3 (Hispanic) to level 1 = 28.63-26.35= 2.28
  - X3 compares level 4 (Asian) to level 1 = 24.54-26.35 = -1.81



#### SIMPLE CODING REGRESSION MODEL INTERPRETATION (WEIGHTED)

The results of simple coding are very similar to dummy coding in that each level is compared to the reference level Survey: Linear regression

Number of strata = 159 Number of PSUs = 8,143

Number of obs	=	8,143
Population size	=	5,988,401
Design df	=	7,984
F(3, 7982)	=	77.90
Prob > F	=	0.0000
R-squared	=	0.0533

Survey: Mean estimation

Number of strata = 159 Number of PSUs = 8,143

bmi Co	pefficient	Linearized	t	P> t	[95% conf.	intervall		Mean
sx1 sx2 sx3	2.133186 2.110904 -2.00541	.3019247 .2628895 .2876277	7.07 8.03 -6.97	0.000 0.000 0.000	1.541335 1.595571 -2.569236	2.725037 2.626236 -1.441585	c.bmi@newrace White Black Hispanic	26.32592 28.4591 28.43682

• Level 1 (White) is the reference level

svy: regress bmi sx1 sx2 sx3

- x1 compares level 2 (Black) to level 1 = 28.46-26.33 = 2.13
- X2 compares level 3 (Hispanic) to level 1 = 28.44-26.33= 2.11
- X3 compares level 4 (Asian) to level 1 = 24.32-26.33 = -2.01



Source: https://stats.idre.ucla.edu/stata/webbooks/reg/chapter5/regression-with-statachapter-5-additional-coding-systems-for-categorical-variables-in-regressionanalysis/

#### FORWARD DIFFERENCE CODING

- The mean of the dependent variable for one level of the categorical variable is compared to the mean of the dependent variable for the next (adjacent) level.
- First comparison compares the mean of BMI for level 1 with the mean of BMI for level 2 of race
  - White minus Black
- The second comparison compares the mean of **BMI** for level 2 minus level 3
  - Black minus Hispanic
- The third comparison compares the mean of **BMI** for level 3 minus level 4
  - Hispanic minus Asian

28

#### FORWARD DIFFERENCE REGRESSION CODING

- For the first comparison, the first and second levels are compared
  - fxl is coded  $3/\overline{4}$  for level 1 and the other levels are coded -1/4
- For the second comparison where level 2 is compared with level 3
  - fx2 is coded 1/2 1/2 -1/2 -1/2
- For the third comparison where level 3 is compared with level 4
  - **fx3** is coded 1/4 1/4 1/4 -3/4

Level of race	New variable 1 (fx1)	New variable 2 (fx2)	New variable 3 (fx3)
	Level 1 v. Level 2	Level 2 v. Level 3	Level 3 v. Level 4
l (White)	3/4	1/2	1/4
2 (Black)	-1/4	1/2	1/4
3 (Hispanic)	-1/4	-1/2	1/4
4 (Asian)	-1/4	-1/2	-3/4

29

Source: https://stats.idre.ucla.edu/stata/webbooks/reg/chapter5/regression-with-statachapter-5-additional-coding-systems-for-categorical-variables-in-regressionanalysis/

#### GENERAL RULE FOR FORWARD DIFFERENCE CODING

- The general rule for forward difference regression coding scheme,
- Where k is the number of levels of the categorical variable (in this case k = 4)

Groups	New variable 1 (x1)	New variable 2 (x2)	New variable 3 (x3)
	Group 1 v. Group 2	Group 2 v. Group 3	Group 3 v. Group 4
Group l	(k-1)/k	(k-2)/k	(k-3)/k
Group 2	-1/k	(k-2)/k	(k-3)/k
Group 3	-1/k	-2/k	(k-3)/k
Group 4	-1/k	-2/k	-3/k

-30

#### FORWARD DIFFERENCE CODING REGRESSION OUTPUT (UNWEIGHTED)

regress bmi fx1 fx2 fx3

Source	SS	df	MS	Num	per of ob	s =	8,143
Model	20522.7022	3	6840.9007	- F(3) 2 Prol	, 8139) 5 > F	=	1/9.06
Residual	310954.466	8,139	38.205487	– Adj	quared R-square	= d =	0.0619
Total	331477.168	8,142	40.7120079	9 Roo <sup>-</sup>	t MSE	=	6.1811
bmi	Coefficient	Std. err.	t	P> t	[95%	conf.	interval]
fx1	-2.481632	.1886871	-13.15	0.000	-2.851	507	-2.111757
fx2	.2031861	.1947547	1.04	0.297	1785	828	.584955
fx3	4.095964	.2148824	19.06	0.000	3.67	474	4.517189
_cons	27.08866	.0714918	378.91	0.000	26.94	852	27.2288



#### FORWARD DIFFERENCE CODING REGRESSION MODEL INTERPRETATION (UNWEIGHTED)

Sour

Residu

- With this coding system, adjacent levels of the categorical variable are compared
- You can see the regression coefficient for fxl is the mean of **BMI** for level 1 (White) minus the mean of **BMI** for level 2 (Black)
  - 26.35 28.83 = -2.48, which is statistically significant (p<0.001)
- The regression coefficient for fx2 is the mean of **BMI** for level 2 (Black) minus the mean of **BMI** for level 3 (Hispanic)
  - The calculation of the contrast coefficient would be 28.83 - 28.63 = 0.2, which is not statistically significant (p=0.297)
- The regression coefficient for fx3 is the mean of **BMI** for level 3 (Hispanic) minus the mean of **BMI** for level 4 (Asian)
  - 28.63-24.54 = 4.09, a statistically significant difference (p < 0.001)

))			Race/et variabl	hnic ity e of	Summa	ary	of B(
			ре	rson			
			intervi	ewed		I	Mean
			W	hite	26	5 <b>.</b> 3!	5302
			В	lack	28	.834	4652
			Hisp	anic	28	.63	1466
			Asia	n/PI	24	.53	5502
			Т	otal	27	.264	4005
ource	SS	df	MS	Numbe	er of obs	=	8,143
Mada 1	20522 7022	 -	6040 00072	- F(3,	8139)	=	179.06
ridual	20522.7022	3 8 139	38 2054870	2 Prod 9 R-sau	> F Jared	=	0.0000
	510554.400		50.2054075	- Adj A	R-squared	=	0.0616
Total	331477.168	8,142	40.7120079	Root	MSE	=	6.1811
bmi	Coefficient	Std. err.	t	P> t	[95% co	onf.	interval]
fx1	-2.481632	.1886871	-13.15	0.000	-2.8515	07	-2.111757
fx2	.2031861	.1947547	1.04	0.297	178582	28	.584955
fx3	4.095964	.2148824	19.06	0.000	3.674	74	4.517189
_cons	27.08866	.0714918	378.91	0.000	26,948	52	27.2288



#### FORWARD DIFFERENCE CODING REGRESSION OUTPUT (WEIGHTED)

svy: regress bmi fx1 fx2 fx3

Survey: Linear regression

Number of strata = 159 Number of PSUs = 8,143

Number of obs	=	8,143
Population size	=	5,988,401
Design df	=	7,984
F(3, 7982)	=	77.90
Prob > F	=	0.0000
R-squared	=	0.0533

bmi	Coefficient	Linearized std. err.	t	P> t	[95% conf.	interval]
fx1	-2.133186	.3019247	-7.07	0.000	-2.725037	-1.541335
fx2	.0222823	.3192645	0.07	0.944	6035594	.648124
fx3	4.116314	.3060669	13.45	0.000	3.516343	4.716285
_cons	26.88559	.1071415	250.94	0.000	26.67556	27.09561



#### FORWARD DIFFERENCE CODING REGRESSION MODEL INTERPRETATION Survey: Mean estimation (WEIGHTED) Number of strata = 159

- With this coding system, adjacent levels of the categorical variable are compared
- You can see the regression coefficient for fxl is the mean of BMI for level 1 (White) minus the mean of BMI for level 2 (Black)
  - 26.32 28.45 = -2.13, which is statistically significant (p<0.001)</li>
- The regression coefficient for fx2 is the mean of BMI for level 2 (Black) minus the mean of BMI for level 3 (Hispanic)
  - The calculation of the contrast coefficient would be 28.459 – 28.437= 0.022, which is not statistically significant (p=0.944)
- The regression coefficient for fx3 is the mean of BMI for level 3 (Hispanic) minus the mean of BMI for level 4 (Asian)
  - 28.44-24.32 =4.12, a statistically significant difference (p<0.001)</li>

	Mea
c.bmi@newrace	
White	26.3259
Black	28.459
Hispanic	28.4368
Asian/PI	24.3205

Number of PSUs = 8,143

Survey: Linear regression

Number	of	strata	=	159
Number	of	PSUs	=	8,143

Number of obs	=	8,143
Population size	=	5,988,401
Design df	=	7,984
F(3, 7982)	=	77.90
Prob > F	=	0.0000
R-squared	=	0.0533

bmi	Coefficient	Linearized std. err.	t	P> t	[95% conf.	interval]
fx1 fx2 fx2	-2.133186 .0222823	.3019247	-7.07 0.07	0.000 0.944	-2.725037 6035594	-1.541335 .648124
_cons	4.116314 26.88559	.1071415	250.94	0.000	26.67556	27.09561



#### BACKWARD DISTANCE CODING

- The mean of the dependent variable for one level of the categorical variable is compared to the mean of the dependent variable for the prior adjacent level.
- In our example, the first comparison variable compares the mean of BMI for level 2 (Black) with the mean of BMI for level 1 (White) of race
  - **bxl** is coded -3/4 for level 1 while the other levels are coded -1/4.
- The second comparison variable compares the mean of BMI for level 3 (Hispanic) minus level 2 (Black)
  - **bx2** is coded -1/2 -1/2 1/2 1/2
- The third comparison variable compares the mean of BMI for level 4 (Asian) minus level 3 (Hispanic)
  - **bx3** is coded -1/4 -1/4 -1/4 3/4.

#### BACKWARD DISTANCE REGRESSION CODING

<b>Race/Ethnicity</b>	New variable 1 (bx1)	New variable 2 (bx2)	New variable 3 (bx3)
	<b>Black v. White</b>	Hispanic v. Black	Asian v. Hispanic
White	- 3/4	-1/2	-1/4
Black	1/4	-1/2	-1/4
Hispanic	1/4	1/2	-1/4
Asian	1/4	1/2	3/4

Source: https://stats.idre.ucla.edu/stata/webbooks/reg/chapter5/regression-with-statachapter-5-additional-coding-systems-for-categorical-variables-in-regressionanalysis/

#### GENERAL RULE FOR BACKWARD DISTANCE CODING

- The general rule for the backward distance regression coding scheme,
- Where k is the number of levels of the categorical variable (in this case, k = 4)

Groups	New variable 1 (x1)	New variable 2 (x2)	New variable 3 (x3)		
	Group 1 v. Group 2	Group 2 v. Group 3	Group 3 v. Group 4		
Group 1	-(k-1)/k	-(k-2)/k	-(k-3)/k		
Group 2	l/k	-(k-2)/k	-(k-3)/k		
Group 3	l/k	2/k	-(k-3)/k		
Group 4	l/k	2/k	3/k		

Source: https://stats.idre.ucla.edu/stata/webbooks/reg/chapter5/regression-with-statachapter-5-additional-coding-systems-for-categorical-variables-in-regressionanalysis/

#### BACKWARD DISTANCE CODING REGRESSION OUTPUT (UNWEIGHTED)

regress bmi bx1 bx2 bx3

Source	SS	df	MS	Number of obs	=	8,143
			· · · · · · · · · · · · · · · · · · ·	F(3, 8139)	=	179.06
Model	20522.7022	3	6840.90072	Prob > F	=	0.0000
Residual	310954.466	8,139	38.2054879	R-squared	=	0.0619
				Adj R-squared	=	0.0616
Total	331477.168	8,142	40.7120079	Root MSE	=	6.1811

bmi	Coefficient	Std. err.	t	P> t	[95% conf.	interval]
bx1	2.481632	.1886871	13.15	0.000	2.111757	2.851507
bx2	2031861	.1947547	-1.04	0.297	584955	.1785828
bx3	-4.095964	.2148824	-19.06	0.000	-4.517189	-3.67474
_cons	27.08866	.0714918	378.91	0.000	26.94852	27.2288

#### BACKWARD DISTANCE CODING REGRESSION MODEL INTERPRETATION (UNWEIGHTED)

- With this coding system, adjacent levels of the categorical variable are compared

  Each level compared to the prior level

  The regression coefficient for **bx1** is the mean of **BMI** for level 2 (Black) minus the mean of **BMI** for level 1 (White)
  - 28.83-26.35 = 2.48; A statistically significant difference (p<0.001)</li>
- The regression coefficient for bx2 is the mean of BMI for level 3 (Hispanic) minus the mean of BMI for level 2 (Black)
  - 28.63 28.83 = -0.2; Not a statistically significant difference (p=0.297)
- The regression coefficient for bx3 is the mean of BMI for level 4 (Asian) minus the mean of BMI for level 3 (Hispanic)
  - 24.54 28.63 = -4.09; A statistically significant difference (p<0.001)</li>

ulvali	j v j			Race/ethnic		
				ity		
				variable of	Summa	ary of Bu
				person		
				interviewed		Mean
				White	20	6.35302
				Black	28	.834652
				Hispanic	28	.631466
				Asian/PI	24	.535502
				Total	27	.264005
Source	SS	df	MS	Number of c	obs =	8,143
	· · · · · · · · · · · · · · · · · · ·			F(3, 8139)	=	179.06
Model	20522.7022	3	6840.90072	Prob > F	=	0.0000
Residual	310954.466	8,139	38.2054879	R-squared	=	0.0619
Total	331/177 168	8 1/12	10 7120079	- Adj R-squar	red =	0.0616
TOCAL	551477.100	0,142	40.7120075	KOUT HSE	_	0.1011
bmi	Coefficient	Std. err.	t	P> t  [95%	conf.	interval]
bx1	2.481632	.1886871	13.15	0.000 2.11	.1757	2.851507
bx2	2031861	.1947547	-1.04	0.29758	4955	.1785828
bx3	-4.095964	.2148824	-19.06	0.000 -4.51	.7189	-3.67474
_cons	27.08866	.0714918	378.91	0.000 26.9	4852	27.2288
	1					



#### BACKWARD DISTANCE CODING REGRESSION OUTPUT (WEIGHTED)

svy: regress bmi bx1 bx2 bx3

Survey: Linear regression

Number of strata = 159 Number of PSUs = 8,143

Number of obs	=	8,143
Population size	=	5,988,401
Design df	=	7,984
F(3, 7982)	=	77.90
Prob > F	=	0.0000
R-squared	=	0.0533

bmi	Coefficient	Linearized std. err.	t	P> t	[95% conf.	interval]
bx1	2.133186	.3019247	7.07	0.000	1.541335	2.725037
bx2	0222823	.3192645	-0.07	0.944	648124	.6035594
bx3	-4.116314	.3060669	-13.45	0.000	-4.716285	-3.516343
_cons	26.88559	.1071415	250.94	0.000	26.67556	27.09561



#### BACKWARD DISTANCE CODING REGRESSION MODEL INTERPRETATION (WEIGHTED)

Mean

<ul> <li>With this coding system, adjacent levels of the categorical variable are compared</li> <li>Each level compared to the prior level</li> </ul>			c.t	omi@new Wh Bl	vrace nite .ack	26.325 28.45	592 591
<ul> <li>The regression coefficient for bxl is the mean of BMI for level 2 (Black) minus the mean of BMI for level 1 (White)</li> </ul>				Hispa Asiar	nic n/PI	28.436	582 951
<ul> <li>28.46-26.33 = 2.13A statistically significant difference (p&lt;0.001)</li> </ul>	Survey: Linea Number of str	r regression ata = 159			Numb	er of obs =	= 8,143
<ul> <li>The regression coefficient for bx2 is the mean of BMI for level 3 (Hispanic) minus the mean of BMI for level 2 (Black)</li> <li>28.44 - 28.46 = -0.02; Not a statistically</li> </ul>	Number of PSU:	5 = 8,143			Popu Desi F(3, Prob R-sq	lation size = gn df = 7982) = > F = Juared =	= 5,988,401 = 7,984 = 77.90 = 0.0000 = 0.0533
significant difference (p=0.944)	bmi	Coefficient	Linearized std. err.	t	P> t	[95% conf.	. interval]
<ul> <li>The regression coefficient for bx3 is the mean of BMI for level 4 (Asian) minus the mean of BMI for level 3 (Hispanic)</li> </ul>	bx1 bx2 bx3 _cons	2.133186 0222823 -4.116314 26.88559	.3019247 .3192645 .3060669 .1071415	7.07 -0.07 -13.45 250.94	0.000 0.944 0.000 0.000	1.541335 648124 -4.716285 26.67556	2.725037 .6035594 -3.516343 27.09561
<ul> <li>24.32 – 28.44 = -4.12; A statistically significant difference (p&lt;0.001)</li> </ul>							41

#### **DEVIATION CODING**

- This coding system compares the mean of the dependent variable for a given level to the mean of the dependent variable for the all levels of the variable.
  - In our example, the first comparison variable compares level 2 (Blacks) to all levels of **race**
  - The second new variable compares level 3 (Hispanics) to all levels of **race**
  - The third comparison variable compares level 4 (Asians) to all levels of race
- The regression coding is accomplished by assigning
  - 1 to level 2 for the first comparison (because level 2 is the level to be compared to all)
  - 1 to level 3 for the second comparison (because level 3 is to be compared to all)
  - 1 to level 4 for the third comparison (because level 4 is to be compared to all)
  - Note that a -1 is assigned to level 1 for all three comparisons (because it is the level that is never compared to the other levels) and all other values are assigned a 0

#### **DEVIATION REGRESSION CODING**

<b>Race/Ethnicity</b>	New variable 1 (dxl)	New variable 2 (dx2)	New variable 3 (dx3)		
	Black v. Mean	Hispanic v. Mean	Asian v. Mean		
White	-1	-1	-1		
Black	1	0	0		
Hispanic	0	1	0		
Asian	0	0	1		

## DEVIATION CODING REGRESSION OUTPUT (UNWEIGHTED)

regress bmi dx1 dx2 dx3

Source	SS	df	MS	Number of obs	=	8,143
			<u> </u>	F(3, 8139)	=	179.06
Model	20522.7022	3	6840.90072	Prob > F	=	0.0000
Residual	310954.466	8,139	38.2054879	R-squared	=	0.0619
			· · · · · · · · · · · · · · · · · · ·	Adj R-squared	=	0.0616
Total	331477.168	8,142	40.7120079	Root MSE	=	6.1811

br	i	Coefficient	Std. err.	t	P> t	[95% conf.	interval]
d×	1	1.745992	.1263767	13.82	0.000	1.498261	1.993723
d×	2	1.542806	.1149598	13.42	0.000	1.317455	1.768157
dx	3	-2.553158	.141752	-18.01	0.000	-2.831028	-2.275288
_cor	IS	27.08866	.0714918	378.91	0.000	26.94852	27.2288



#### DEVIATION CODING REGRESSION COEFFICIENTS (UNWEIGHTED)

- The regression coefficient for **dx1** is the mean for level 2 (Black) minus the grand mean.
  - However, this grand mean is not the overall mean of the dependent variable that you would get from taking the mean (27.24) of all of the observations
  - Rather, it is the mean of means of the dependent variable at each level of the categorical variable
  - (26.35+28.83+28.63+24.54) / 4 = 27.088
  - The regression coefficient is then 28.83 27.088 = 1.74; this difference is statistically significant (p<0.001)

						Race/ethnic ity variable of	Summary of B
						interviewed	Mean
Variable	Obs	Mean	Std. dev.	Min	Max	White	26.35302
bmi	8,423	27.23568	6.352999	7.933049	82.16812	Black Hispanic Asian/PI	28.834652 28.631466 24.535502
						Total	27.264005

#### **DEVIATION CODING REGRESSION INTERPRETATION (UNWEIGHTED)**

regress bmi dx1 dx2 dx3

- The coefficient for dx2 is the mean for level 3 (Hispanic) of race minus the overall mean
  - 28.63-27.088 = 1.54; this difference is statistically significant (p<0.001)</li>
- The coefficient for dx3 is the mean for level 4 (Asian) of race minus the overall mean
  - 24.54 27.088 = -2.55; this difference is statistically significant (p<0.001)</li>

Source	SS	df	MS	Num	ber of obs	=	8,143
				– F(3	, 8139)	=	179.06
Model	20522.7022	3	6840.90072	2 Pro	b > F	=	0.0000
Residual	310954.466	8,139	38.2054879	9 R-s	quared	=	0.0619
		· · · · · · · · · · · · · · · · · · ·		- Adj	R-squared	=	0.0616
Total	331477.168	8,142	40.7120079	9 Roo	t MSE	=	6.1811
bmi	Coefficient	Std. err.	t	P> t	[95% coi	nf.	interval]
dx1	1.745992	.1263767	13.82	0.000	1.498262	1	1.993723
dx2	1.542806	.1149598	13.42	0.000	1.31745	5	1.768157
dx3	-2.553158	.141752	-18.01	0.000	-2.831028	8	-2.275288
_cons	27.08866	.0714918	378.91	0.000	26.94852	2	27.2288



## **DEVIATION CODING REGRESSION OUTPUT (WEIGHTED)**

svy: regress bmi dx1 dx2 dx3

Survey: Linear regression

Number of strata = 159 Number of PSUs = 8,143

Number of obs	=	8,143
Population size	=	5,988,401
Design df	=	7,984
F(3, 7982)	=	77.90
Prob > F	=	0.0000
R-squared	=	0.0533

bmi	Coefficient	Linearized std. err.	t	P> t	[95% conf.	interval]
dx1	1.573516	.2060685	7.64	0.000	1.169568	1.977464
dx2	1.551234	.1776651	8.73	0.000	1.202964	1.899504
dx3	-2.56508	.195688	-13.11	0.000	-2.94868	-2.181481
_cons	26.88559	.1071415	250.94	0.000	26.67556	27.09561



## DEVIATION CODING REGRESSION COEFFICIENTS (WEIGHTED)

- The regression coefficient for dxl is the mean for level 2 (Black) minus the grand mean.
  - However, this grand mean is not the overall mean of the dependent variable that you would get from taking the mean (27.09) of all of the observations
  - Rather, it is the mean of means of the dependent variable at each level of the categorical variable
  - (26.33+28.46+28.44+24.32) / 4 = 26.89
  - The regression coefficient is then 28.46 26.89 = 1.57

Survey: Mean e	estimation					Mean
Number of stra Number of PSUs	ata = 159 5 = 8,423	Numbo Popu Desi	er of obs = lation size = gn df =	8,423 6,137,719 8,264	c.bmi@newrace White Black	26.32592 28.4591
	Mean	Linearized std. err.	[95% conf.	interval]	Hispanic Asian/PI	28.43682 24.32051
bmi	27.09084	.1046812	26.88564	27.29604		



## **DEVIATION CODING REGRESSION INTERPRETATION (WEIGHTED)**

svy: regress bmi dx1 dx2 dx3

Survey: Linear regression

- The coefficient for dx2 is the mean for level 3 (Hispanic) of race minus the overall mean
  - 28.44-26.89 = 1.55

Number of strata = 159 Number of PSUs = 8,143

Number of obs	=	8,143
Population size	=	5,988,401
Design df	=	7,984
F(3, 7982)	=	77.90
Prob > F	=	0.0000
R-squared	=	0.0533

 The coefficient for dx3 is the mean for level 4 (Asian) of race minus the overall mean

• 24.32 - 26.89 = -2.57

60685 7.64 0.0 76651 8.73 0.0 95688 -13.11 0.0	0001.1695681.9774640001.2029641.899504000-2.94868-2.181481
	50685 7.64 0.0 76651 8.73 0.0 95688 -13.11 0.0 71415 250.94 0.0



## STATISTICAL ANALYST DEFINED CODING

- You can create your own regression coding system based on the comparisons you are interested in making
- Suppose we want to make the following three comparisons:
  - 1) level 1 (White) to level 4 (Asian)
    - In order to compare level 1 to level 4, we use the contrast coefficients 1 0 0 -1
  - 2) level 2 (Black) to levels 1 and 4 (White and Asian)
    - To compare level 2 to levels 1 and 4 we use the contrast coefficients -1/2 1 0 -1/2
  - 3) levels 2 and 3 (Black and Hispanic) to levels 1 and 4 (White and Asian)
    - To compare levels 2 and 3 with levels 1 and 4 we use the contrast coefficients 1/2 -1/2 1/2 1/2

## **KEY POINTS: CONTRASTS**

- For the first contrast, we are comparing level 1 to level 4, and the contrast coefficients are 1 0 0 -1
- The levels associated with the contrast coefficients with opposite signs are being compared
- The mean of the dependent variable is multiplied by the contrast coefficient
- Hence, levels 2 and 3 are not involved in the comparison
  - They are multiplied by zero and "dropped out"
- You will also notice that the contrast coefficients sum to zero
  - This is necessary
  - If the contrast coefficients do not sum to zero, the contrast is not estimable
- Which level of the categorical variable is assigned a positive or negative value is not terribly important
  - 10-10 is the same as -1010 in that both of these codings compare the first and the third levels of the variable

51

However, the sign of the regression coefficient would change

#### DETERMINE THE REGRESSION CODING SYSTEM

- For methods 1 and 2 it was quite easy to translate the comparisons we wanted to make into contrast codings, but it is not as easy to translate the comparisons we want into a regression coding scheme.
- If we know the contrast coding system, then we can convert that into a regression coding system
- We place the three contrast codings we want into the matrix c and then perform a set of matrix operations on c, yielding the matrix x.
  - matrix input  $c = (1 \ 0 \ 0 \ -1 \ -.5 \ 1 \ 0 \ -.5 \ .5 \ -.5 \ .5)$

```
matrix list c
```

```
matrix x = c'*inv(c*c')
```

• matrix list x

Г —	. 7				x[4,	3]		
c[3	<b>,</b> 4]					r1	r2	r3
	c1	c2	c3	c4	c1	.5	0	.5
r1	1	0	0	-1	c2	0	1	.5
r2	5	1	0	5	с3	0	-1	-1.5
r3	.5	5	5	.5	c4	5	0	.5

#### STATISTICAL ANALYST DEFINED CODING REGRESSION OUTPUT (UNWEIGHTED)

regress bmi ax1 ax2 ax3

Source	SS	df	MS	Numb	er of obs	s =	8,143
Model	20522.7022	3	6840.90072	2 Prot	$\rightarrow$ F	=	0.0000
	510954.400		50.2054075	- Adj	R-squared	d =	0.0616
Total	331477.168	8,142	40.7120079	9 Root	: MSE	=	6.1811
bmi	Coefficient	Std. err.	t	P> t	[95% 0	conf.	interval]
ax1	1.817518	.2093989	8.68	0.000	1.4070	043	2.227994
ax2	3.390391	.1807816	18.75	0.000	3.0360	013	3.744769
ax3	-3.288798	.1429836	-23.00	0.000	-3.5690	083	-3.008514
_cons	27.08866	.0714918	378.91	0.000	26.948	852	27.2288

#### STATISTICAL ANALYST DEFINED CODING INTERPRETATION (UNWEIGHTED)

- The coefficient for axl (-1.82) corresponds to the first contrast comparing level 1 (White) to level 4 (Asian) of race
  - The coefficient is the mean of level 1 (White) of **BMI** minus the mean for level 4 (Asian) of **BMI**
  - 26.35-24.54 = 1.8
  - p-value<0.001 (statistically significant)</p>
- The coefficient for ax2 is 3.39 which is the mean of level 2 (Black) minus the mean of level 1 and level 4 (White and Asian)

$$28.83 - \frac{(26.35 + 24.54)}{2} = 28.83 - 25.445 = 3.39$$

- This difference is statistically significant, p<0.001
- The coefficient for ax3 is -3.29 which is the mean of levels 1 and 4 (White and Asian) minus the mean of levels 2 and 3 (Black and Hispanic) = (26.35 + 24.54) (28.83 + 28.63)

2

- 25.445-28.73 = -3.39
  - This contrast is statistically significant, p<0.001

ity variable of	Summary of Bu
person interviewed	Mean
White Black Hispanic Asian/PI	26.35302 28.834652 28.631466 24.535502
Total	27.264005

Race/ethnic

#### STATISTICAL ANALYST DEFINED CODING REGRESSION OUTPUT (WEIGHTED)

svy: regress bmi ax1 ax2 ax3

Survey: Linear regression

Number of strata = 159 Number of PSUs = 8,143

Number of obs	=	8,143
Population size	=	5,988,401
Design df	=	7,984
F(3, 7982)	=	77.90
Prob > F	=	0.0000
R-squared	=	0.0533

bmi	Coefficient	Linearized std. err.	t	P> t	[95% conf.	interval]
ax1	2.00541	.2876277	6.97	0.000	1.441585	2.569236
ax2	3.135891	.2872458	10.92	0.000	2.572814	3.698968
ax3	-3.12475	.2147752	-14.55	0.000	-3.545765	-2.703734
_cons	26.88559	.1071415	250.94	0.000	26.67556	27.09561



#### STATISTICAL ANALYST DEFINED CODING INTERPRETATION (WEIGHTED)

- The coefficient for axl (2.0) corresponds to the first contrast comparing level 1 (White) to level 4 (Asian) of race
  - The coefficient is the mean of level 1 (White) of **BMI** minus the mean for level 4 (Asian) of **BMI**
  - 26.33-24.32 = 2.0
  - p-value<0.001 (statistically significant)</pre>
- The coefficient for ax2 is 3.14 which is the mean of level 2 (Black) minus the mean of level 1 and level 4 (White and Asian)

• 
$$28.46 - \frac{(26.33 + 24.32)}{2} = 28.46 - 25.325 = 3.14$$

- This difference is statistically significant, p<0.001
- The coefficient for ax3 is -3.12 which is the mean of levels 1 and 4 (White and Asian) minus the mean of levels 2 and 3 (Black and Hispanic) = (26.33 + 24.32) (28.46 + 28.44)

2

- 25.33-28.45 = -3.12
  - This contrast is statistically significant, p<0.001</li>

		Mean
2 (Black)	c.bmi@newrace White Black Hispanic	26.32592 28.4591 28.43682
	Asian/PI	24.32051

## USING FRACTIONS IN CONTRASTS

- In the contrast coefficients for the second and third comparisons, notice that in both cases we use fractions that sum to one (or minus one)
  - Note they do not have to sum to one (or minus one)
- You may wonder why we would use fractions like -1/2 1 0 -1/2 instead of whole numbers such as -1 2 0 -1.
- While -1/2 1 0 -1/2 and -1 2 0 -1 both compare level 2 with levels 1 and 4
  - Both give you the same t-value and p-value for the regression coefficient
  - The regression coefficients would be different, as would their interpretation
- The coefficient for the -1/2 1 0 -1/2 contrast is the mean of level 2 minus the mean of the means for levels 1 and 4
  - $28.46 \frac{(26.33 + 24.32)}{2} = 28.46 25.325 = 3.14$

	Mean
c.bmi@newrace	
White	26.32592
Black	28.4591
Hispanic	28.43682
Asian/PI	24.32051

## USING FRACTIONS IN CONTRASTS CONTINUED

- Alternatively, you can multiply the contrasts by the mean of the dependent variable for each level of the categorical variable:
  - $-1/2 \times 26.33 + 1 \times 28.46 + 0 \times 28.44 + -1/2 \times 24.32 = -13.165 + 28.46 12.16 = 3.14$

		Mean
These are equivalent ways of thinking about how the contrast coefficient is calculated and give the same value	c.bmi@newrace	26 32592
	WIIILE	20.52592
	Black	28.4591
By comparison, the coefficient for the -1 20-1 contrast is two times the	Hispanic	28.43682
mean for level 2 minus the means of the dependent variable for levels	Asian/PI	24.32051
l and 4		

- 2\*28.46 (26.33 + 24.32) = 56.92-50.65 = 6.27
- Which is the same as -1\* 26.33 + 2\*28.46 + 0\*28.44- 1\* 24.32
- =-26.33+56.92-24.32=6.27
- Note that the regression coefficient using the contrast coefficients -1 2
   0 -1 is twice the regression coefficient obtained when -1/2 1 0 -1/2 is used

#### THINK CRITICALLY ABOUT MODELING RACE IN REGRESSION

- It is insufficient to control for race in a regression model without thinking critically about what that means
  - Race is not a modifiable factor
- How will race/ethnicity be measured and analyzed?
  - Use appropriate scales validated in your population of interested
- How is race being used in the presentation of the results?
  - Is race a proxy for something else?
- Is it racism and not race?
  - Think about how racism is operating and could impact the results of your research study
  - If race is social and not biological is race really operating at the individual level?





SCHOOL OF GLOBAL PUBLIC HEALTH

## QUESTIONS?

Melody S. Goodman, PhD Associate Dean for Research Associate Professor of Biostatistics NYU School of Global Public Health



melody.goodman@nyu.edu



Biostatistics for Clinical and Public Health Research

Melody S. Goodman

R



#### @goodmanthebrain



Public Health Research Methods for Partnerships and Practice

CRC CRC Press

Edited by Melody S. Goodman and Vetta Sanders Thompson

