

# Regression standardization with time-to-event data to estimate marginal measures of association and causal effects using the `standsurv` command

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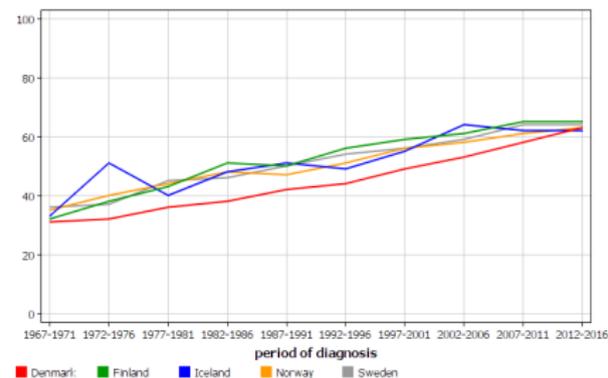


# Standardized Relative/Net Survival

Colon: Male

5-year age standardised relative survival, age at diagnosis 0-89

%



- Relative survival removes differential other cause mortality.
- Age standardization removes differences in age distribution at diagnosis.

# Regression Standardization

- 1 Fit a statistical model incorporating exposure,  $X$ , and confounders,  $Z$ .
  - 2 Predict outcome for all individuals assuming they are all exposed ( $X = 1$ ).
  - 3 Take mean to give marginal estimate of outcome.
  - 4 Repeat for unexposed ( $X = 0$ ).
  - 5 Take the difference in means to form contrasts.
- Key point is the distribution of confounders,  $Z$ , is the same for the exposed and unexposed.
  - If model is sufficient for confounding control then such contrasts can be interpreted as causal effects.
  - Also known as direct/model based standardization. G-formula (with no time-dependent confounders)[1].

# But.... margins does this?

- `margins` does regression standardization, so why not use this?
- It is an excellent command, but does not do what I wanted for survival data.
- In particular, extensions to competing risks and relative survival.

# Marginal survival time

- With survival data

$X$  - is a binary exposure: 0 (unexposed) and 1 (exposed).

$T$  - is a survival time.

$T^0$  - is the potential survival time if  $X$  is set to 0.

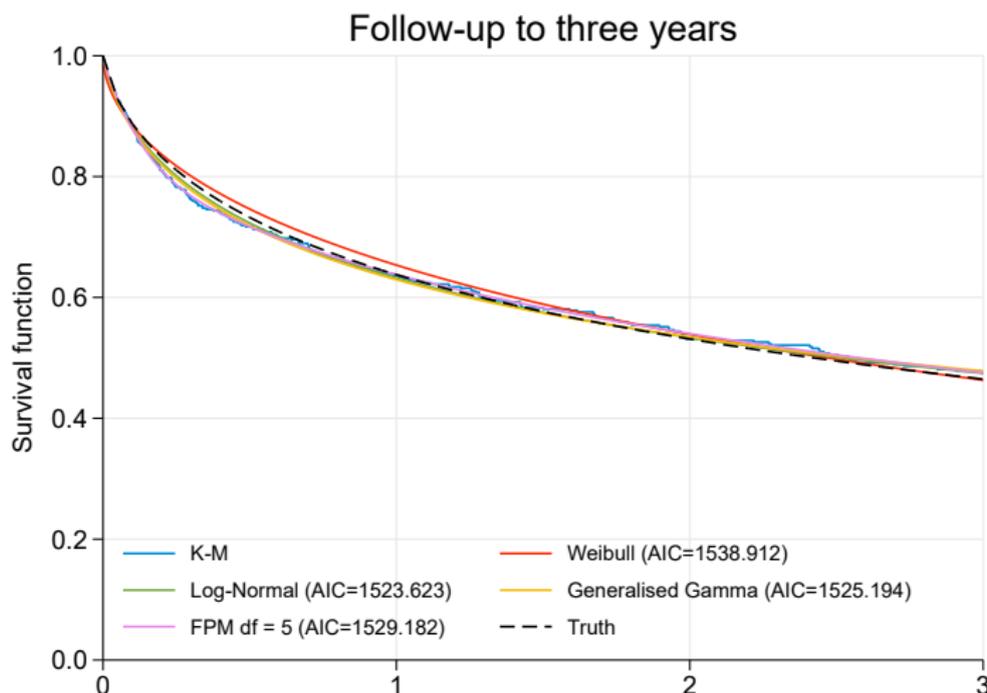
$T^1$  - is the potential survival time if  $X$  is set to 1.

- The average causal difference in mean survival time

$$E[T^1] - E[T^0]$$

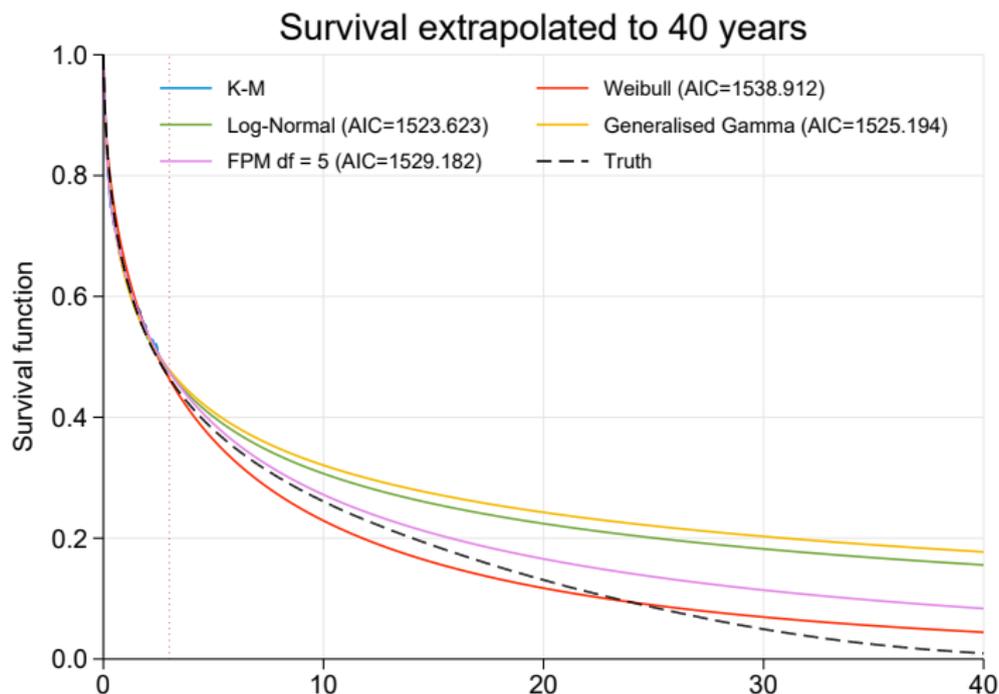
- This is what **stteffects** can estimate.
- We often have limited follow-up and calculating the mean survival requires extrapolation and makes very strong distributional assumptions.

# Problems with extrapolation [2]



Need to extrapolate to obtain mean survival

# Problems with extrapolation [2]



Need to extrapolate to obtain mean survival

# Marginal Survival functions

- Rather than use mean survival we can define our causal effect in terms of the marginal survival function.

$$E[T^1 > t] - E[T^0 > t]$$

- We can limit  $t$  within observed follow-up time.
- For confounders,  $\mathbf{Z}$ , we can write this as,

$$E_{\mathbf{Z}}[S(t|X = 1, \mathbf{Z})] - E_{\mathbf{Z}}[S(t|X = 0, \mathbf{Z})]$$

- Note that this is the expectation over the distribution of  $\mathbf{Z}$ .

# Estimation

- Fit a survival model for exposure  $X$  and confounders  $Z$ .
- Predict survival function for each individual setting  $X = x$  and then average.
- Force everyone to be exposed and then unexposed.

$$\frac{1}{N} \sum_{i=1}^N \hat{S}(t|X = 1, Z = z_i) - \frac{1}{N} \sum_{i=1}^N \hat{S}(t|X = 0, Z = z_i)$$

- Use their observed covariate pattern,  $Z = z_i$ .
- `standsurv` will perform these calculation.

# What can standsurv do?

- standsurv will obtain standardized survival curves and related measures over the study population (or a subset).
- Can treat some covariates as fixed (`at()` option).
- Implemented for `streg`, `stpm2` and `strcs` models.
- Linear and non-linear function of marginal estimates.
- Weights (useful for external standardization & mediation analysis)
- I will describe the use of standsurv in three frameworks.
  - Standard survival
  - Competing risks
  - Relative survival

# Parametric models

- We make a lot of use of flexible parametric survival models[3].
- The flexibility comes from the use of splines to model the effect of time
- Can model on the log cumulative hazard (`stpm2`)[4] or log hazard scale (`strcs`)[5].

$$\ln[H(t)] = s(\ln(t)|k_0) + \mathbf{X}\beta$$

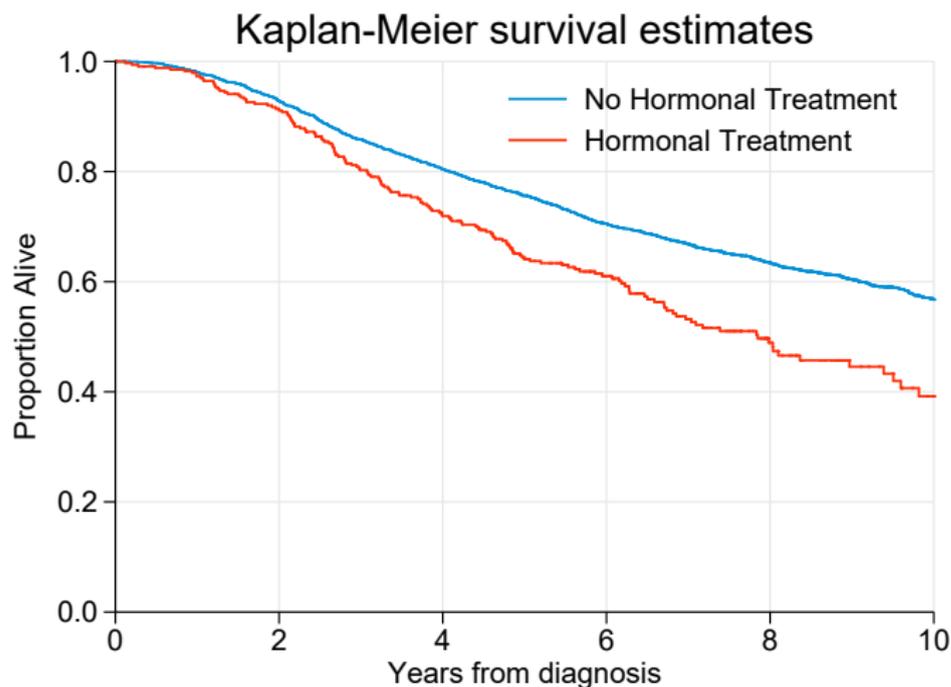
$$\ln[h(t)] = s(\ln(t)|k_0) + \mathbf{X}\beta$$

- Shown to capture complex shapes for hazard functions[6, 7].
- Can fit proportional hazards models, but easy to relax this assumption.
- `standsurv` also works with `streg` models.

# Example

- I will use the Rotterdam breast cancer data: 2,982 women diagnosed with primary breast cancer.
- Observational study, but interest lies in comparing those taking and not taking hormonal therapy ([hormon](#)).
- Outcome is all-cause mortality.
- In a simplified analysis I will consider the following confounders.
  - [age](#) Age at diagnosis
  - [enodes](#) Number of positive lymph nodes (transformed)
  - [pr\\_1](#) Progesterone receptors (fmol/l) (transformed)

# Kaplan-Meier curves



Number at risk

No treatment	2643	2436	2083	1668	1188	660
Treatment	339	307	231	141	63	25

## Confounders

```
. tabstat age nodes pr, by(hormon)
```

```
Summary statistics: mean
```

```
by categories of: hormon (Hormonal therapy)
```

hormon	age	nodes	pr
no	54.09762	2.326523	168.706
yes	62.54867	5.719764	108.233
Total	55.05835	2.712274	161.8313

- Those taking treatment tend to be older and have more severe disease.

Unadjusted 1.54 (95% CI 1.30 to 1.82)

Adjusted 0.79 (95% CI 0.66 to 0.94)

- Strong confounding.
- From the adjusted model we can predict the survival for any combination of covariates.

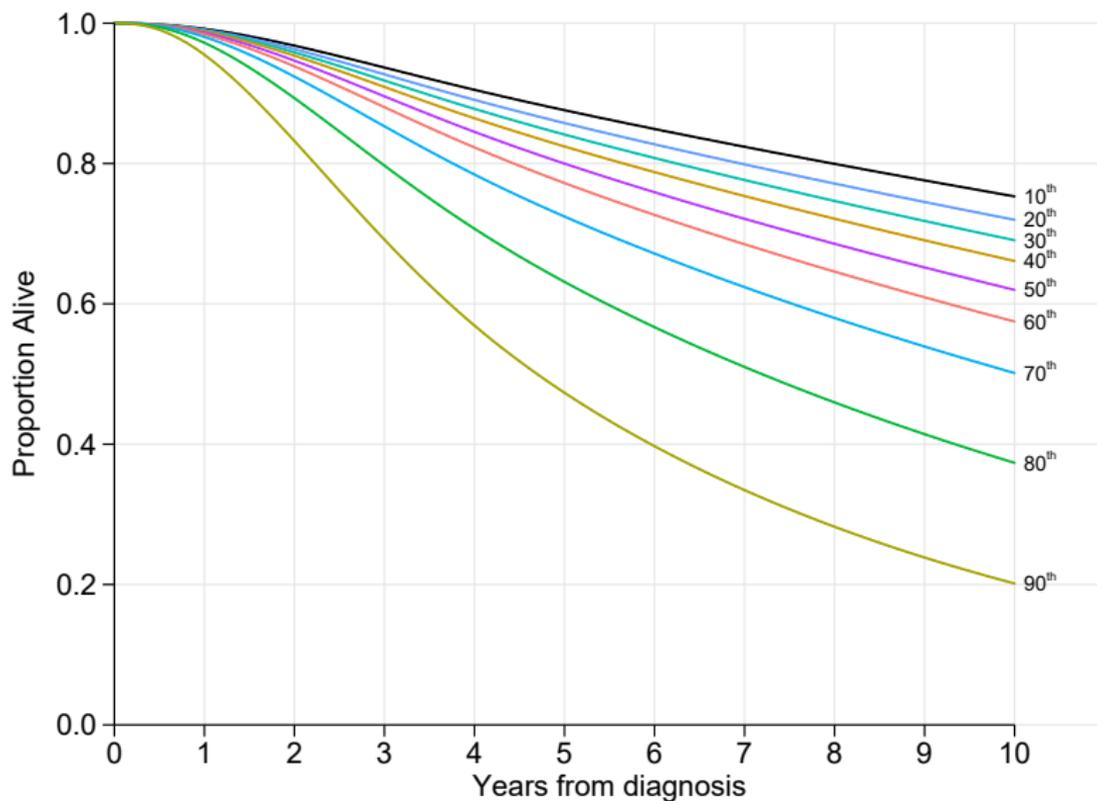
# Adjusted Hazard Ratio

```
. stpm2 hormon age enodes pr_1, scale(hazard) df(4) eform nolog cformat(%3.2f)
Log likelihood = -2668.4925                Number of obs   =       2,982
```

	exp(b)	Std. Err.	z	P> z	[95% Conf. Interval]	
xb						
hormon	0.79	0.07	-2.60	0.009	0.66	0.94
age	1.01	0.00	5.53	0.000	1.01	1.02
enodes	0.11	0.01	-22.40	0.000	0.09	0.14
pr_1	0.91	0.01	-7.46	0.000	0.88	0.93
_rcs1	2.63	0.07	34.67	0.000	2.49	2.78
_rcs2	1.18	0.03	6.08	0.000	1.12	1.25
_rcs3	1.02	0.02	1.36	0.175	0.99	1.05
_rcs4	1.00	0.01	-0.47	0.639	0.98	1.01
_cons	1.10	0.18	0.60	0.546	0.80	1.51

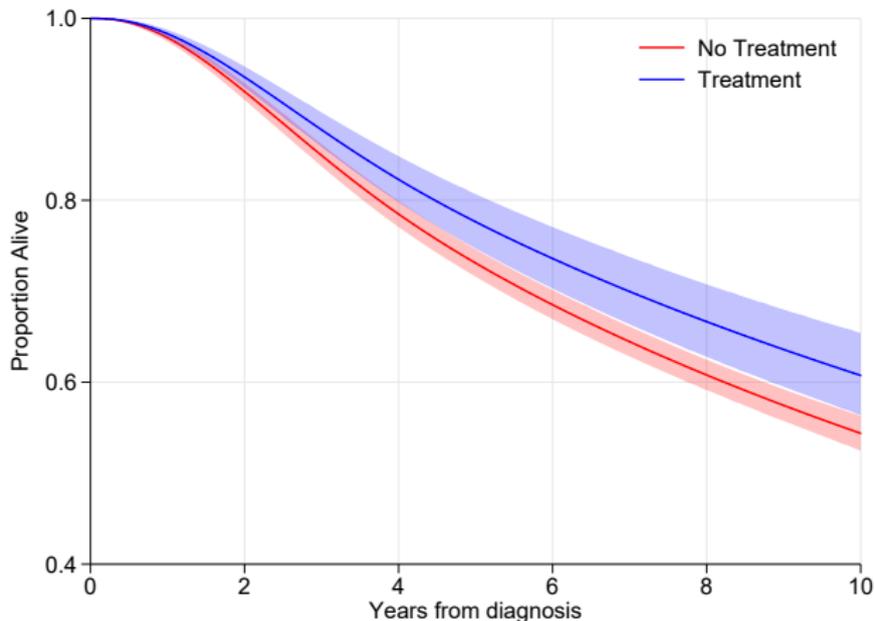
Note: Estimates are transformed only in the first equation.

# Predicted survival functions (centiles)



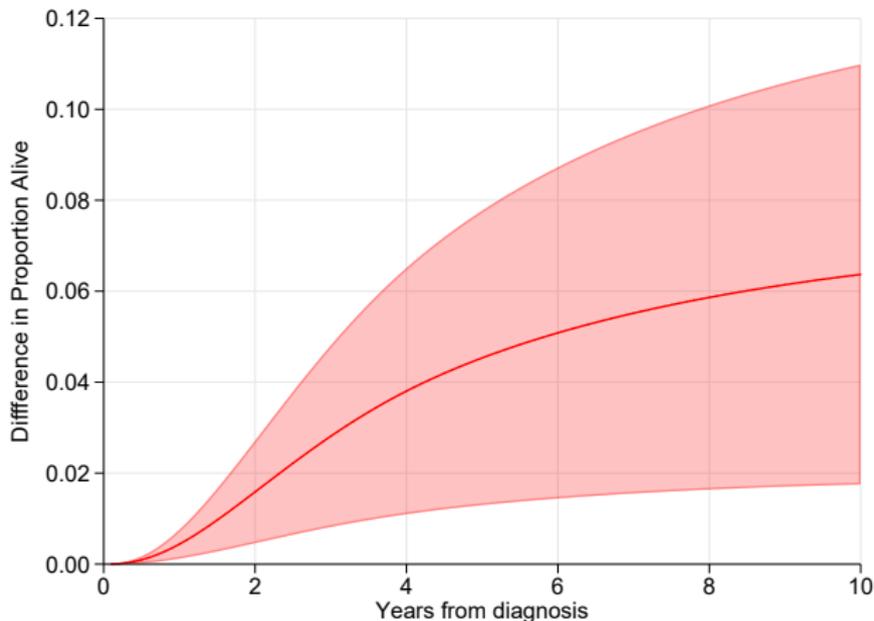
# Marginal survival functions using standsurv

```
. range tt 0 10 101  
. standsurv, at1(hormon 0) at2(hormon 1) timevar(tt) ci ///  
> contrast(difference) atvar(ms_hor0 ms_hor1) contrastvar(ms_diff)
```

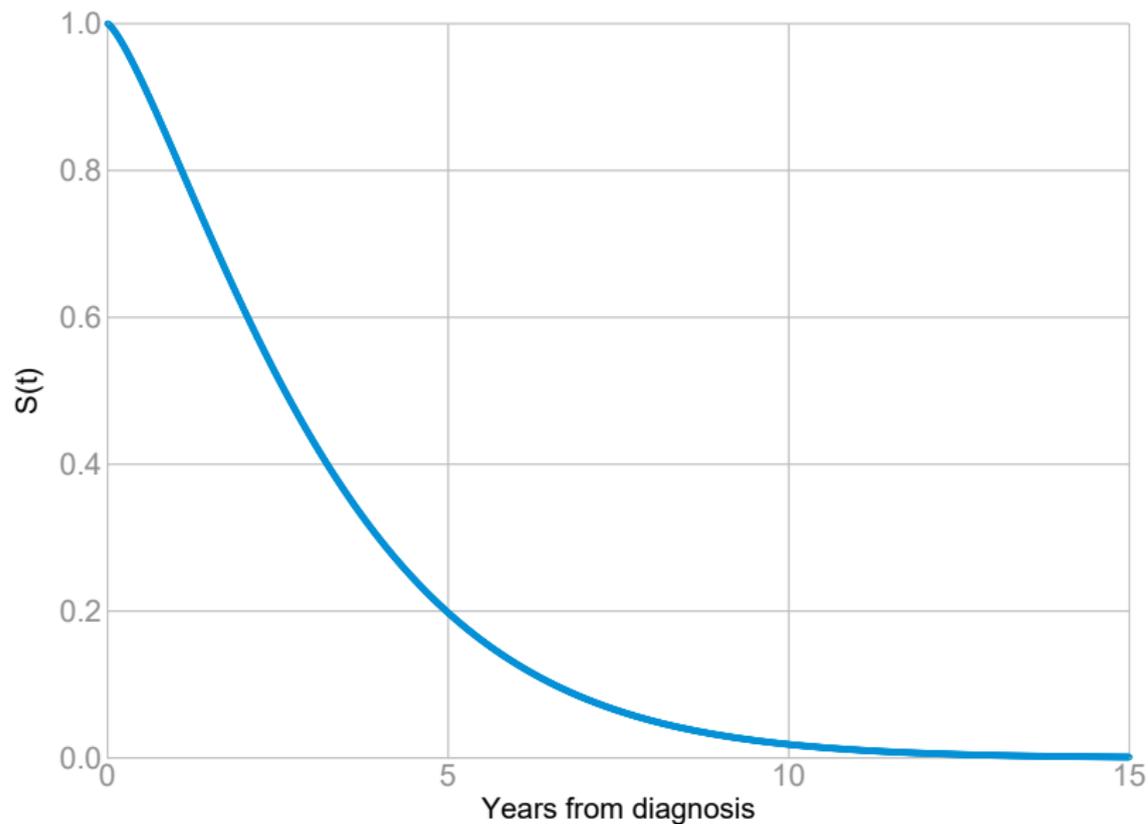


# Standardized Survival Difference

```
. range tt 0 10 101  
. standsurv, at1(hormon 0) at2(hormon 1) timevar(tt) ci ///  
> contrast(difference) atvar(ms_hor0 ms_hor1) contrastvar(ms_diff)
```



# Mean survival time



# Mean survival time



# Restricted mean survival time (5 years)



# Restricted mean survival time (RMST)

## restricted mean survival time

$$RMST(t^*) = E[\min(T, t^*)]$$

$$RMST_s(t^* | X = x, Z) = E_Z \left[ \int_0^{t^*} S(t | X = x, Z) \right]$$

and is estimated by

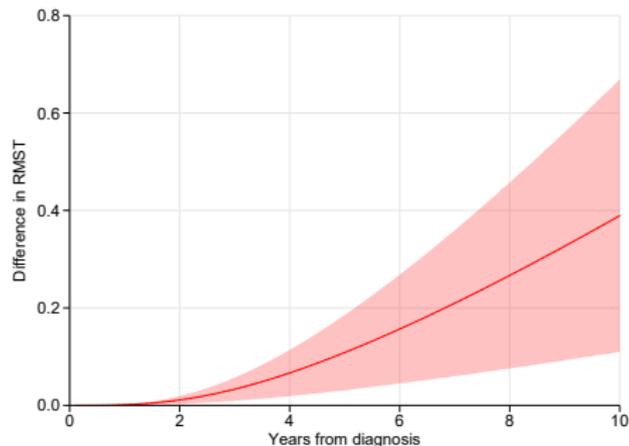
$$\widehat{RMST}_s(t^* | X = x, Z) = \frac{1}{N} \sum_{i=1}^N \int_0^{t^*} S(t | X = x, Z = z_i)$$

- we can then take differences or ratios.
- Various authors suggest a better causal effect than HR[11]

# Difference in standardized RMST

```
. standsurv, at1(hormon 0) at2(hormon 1) timevar(tt) ci rmst ///  
> contrast(difference) atvar(rmst_hor0 rmst_hor1) contrastvar(rmst_diff)  
. list rmst_hor0 rmst_hor1 rmst_diff* if tt==10, noobs
```

rmst_hor0	rmst_hor1	rmst_diff	rms-f_lci	rms-f_uci
7.5505209	7.9399486	.38942772	.11008298	.66877246



# Centiles of the marginal survival function

$$E_Z [S(t_p | X = x, Z)] = \alpha$$

This is done through root finding (using Brent's root finder) by solving for  $t_p$ ,

$$\frac{1}{N} \sum_{i=1}^N S(t_p | X = x, Z = z_i) - \alpha = 0$$

- Can perform contrasts, e.g. difference in median of marginal survival functions.
- Use `centile` option.

# Hazard of the marginal survival function

- Apply standard transformation from survival to hazard of marginal survival function.

## Marginal hazard function

$$h(t) = -\frac{d}{dt} \log (E_Z [S(t|X = x, Z)])$$

and is estimated by,

$$\hat{h}_s(t) = \frac{1}{N} \frac{\sum_{i=1}^N S(t|X = x, Z = z_i) h(t|X = x, Z = z_i)}{\sum_{i=1}^N S(t|X = x, Z = z_i)}$$

- Note this is very different from the mean of the hazard functions.
- Can perform contrasts to get marginal hazard ratios (or differences).
- Use the **hazard** option.

# User defined functions

- We may need other transformations of standardized functions.
- Use `userfunction()` option for this.
- For example, in survival studies the attributable fraction is defined as,

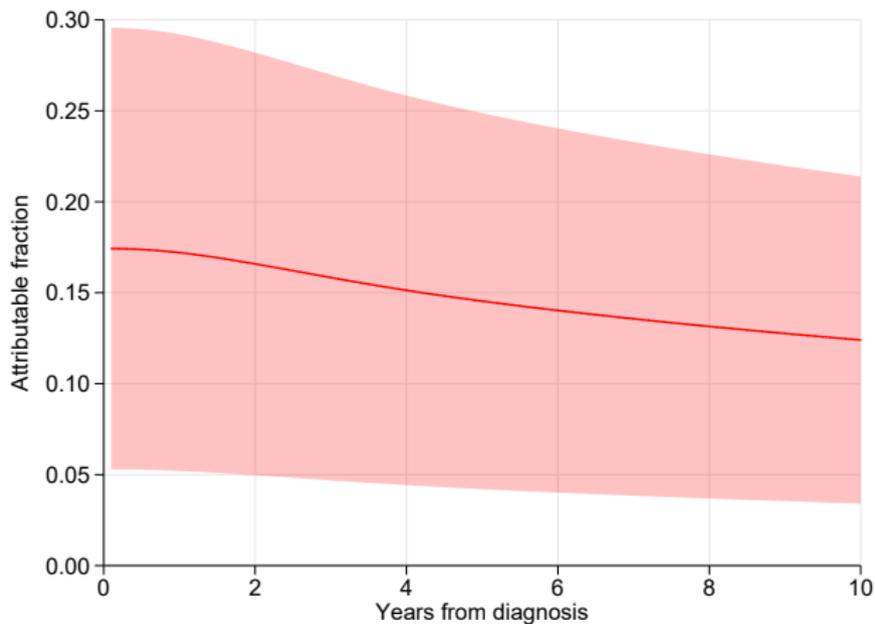
$$AF(t) = \frac{E[F(t|X, Z)] - E[F(t|X = 0, Z)]}{E[F(t|X, Z)]}$$

## User function

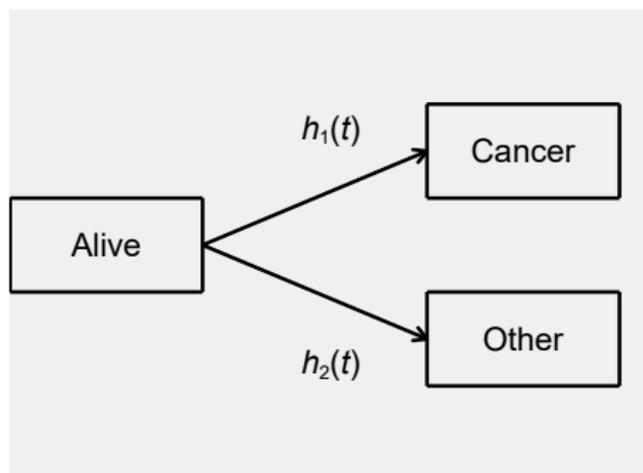
```
mata:  
function calcAF(at)  
{  
  // at2 is F(t|unexposed,Z)  
  // at1 is F(t|X,Z)  
  return((at[1] - at[2])/at[1])  
}
```

# Example of user defined function

```
. standsurv, at1(.) at2(hormon 1) ci failure ///  
> timevar(tt) userfunction(calcAF) userfunctionvar(AF)
```



# Competing risks



Separate models for each cause, e.g.

$$h_1(t|\mathbf{Z}) = h_{0,1}(t) \exp(\beta_1 \mathbf{Z})$$

$$h_2(t|\mathbf{Z}) = h_{0,2}(t) \exp(\beta_2 \mathbf{Z})$$

# Two types of probability

- We may be interested in cause-specific survival/failure.

## (1) *In the absence of other causes (net)*

$$F_k(t) = 1 - S_k(t) = P(T_k \leq t) = \int_0^t S_k(u)h_k(u)du$$

- We may be interested in cumulative incidence functions.

## (2) *In the presence of other causes (crude)*

$$CIF_k(t) = P(T \leq t, \text{event} = k) = \int_0^t S(u)h_k(u)du$$

- Both are of interest - depends on research question.
- (1) Needs conditional independence assumption to interpret as net probability of death.

# Description of Example

- 39,625 patients with bladder cancer in England (2000-2012).
- Death due to cancer and other causes.
- Covariates age, sex and deprivation in five groups.
- Restrict here to most and least deprived.

## Models

- Flexible parametric (Royston-Parmar) models[3]
- Separate model for cancer and other causes.
- Age modelled using splines (3 df)
- 2-way interactions
- Time-dependent effects for all covariates.

# Two separate cause-specific models

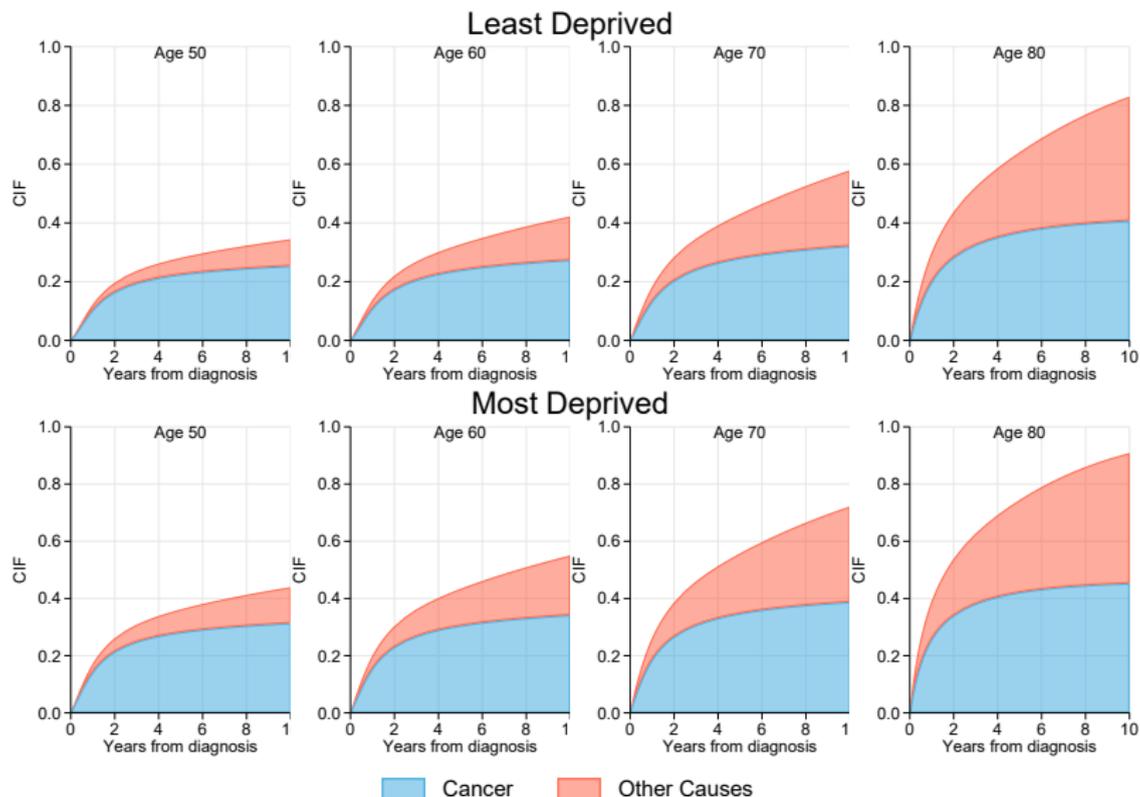
## Cancer Model

```
stset dod, failure(status==1) exit(time min(dx+365.24*10,mdy(12,31,2013))) ///  
        origin(dx) id(patid) scale(365.24)  
  
stpm2 dep5 male agerics* dep_agerics*, df(5) scale(hazard)           ///  
        tvc(agerics* male dep5) dftvc(3)  
  
estimates store cancer
```

## Other Cause Model

```
stset dod, failure(status==2) exit(time min(dx+365.24*10,mdy(12,31,2013))) ///  
        origin(dx) id(patid) scale(365.24)  
  
stpm2 dep5 male agerics* dep_agerics*, df(5) scale(hazard)           ///  
        tvc(agerics* male dep5) dftvc(3)  
  
estimates store cancer
```

# Conditional cause-specific CIFs (Females)



# Standardized cause-specific CIF

- Probability of death in the presence of other causes.
- We can standardize the cause-specific CIF in the same way.
- These requires combining  $K$  different models

$$E_Z [CIF_k(t|X = x, Z)]$$

$$\frac{1}{N} \sum_{i=1}^N \int_0^t \hat{S}(u|X = x, Z = z_i) \hat{h}_k(u|X = x, Z = z_i) du$$

- Calculate for  $X=1$  and  $X=0$  and then obtain contrast.
- Can be interpreted as causal effects under assumptions[12].

# Using `standsurv`

- Take mean of 39,625 CIFs where all individuals forced to be unexposed.
- Take mean of 39,625 CIFs where all individuals forced to be exposed.

# Using standsurv

- Take mean of 39,625 CIFs where all individuals forced to be unexposed.
- Take mean of 39,625 CIFs where all individuals forced to be exposed.

```
. standsurv, crmodels(cancer other) timevar(tt) cif ci          ///  
  at1(dep5 0 dep_agercs1 0 dep_agercs2 0 dep_agercs3 0)      ///  
  at2(dep5 1 dep_agercs1=agercs1 dep_agercs2=agercs2 dep_agercs3=agercs3) ///  
  contrast(difference)                                       ///  
  atvar(CIF_s_dep1 CIF_s_dep5))                             ///  
  contrastvar(CIF_diff)
```

# Using standsurv

- Take mean of 39,625 CIFs where all individuals forced to be unexposed.
- Take mean of 39,625 CIFs where all individuals forced to be exposed.

```
. standsurv, crmodels(cancer other) timevar(tt) cif ci          ///  
  at1(dep5 0 dep_agercs1 0 dep_agercs2 0 dep_agercs3 0)      ///  
  at2(dep5 1 dep_agercs1=agercs1 dep_agercs2=agercs2 dep_agercs3=agercs3) ///  
  contrast(difference)                                       ///  
  atvar(CIF_s_dep1 CIF_s_dep5)                               ///  
  contrastvar(CIF_diff)
```

# Using standsurv

- Take mean of 39,625 CIFs where all individuals forced to be unexposed.
- Take mean of 39,625 CIFs where all individuals forced to be exposed.

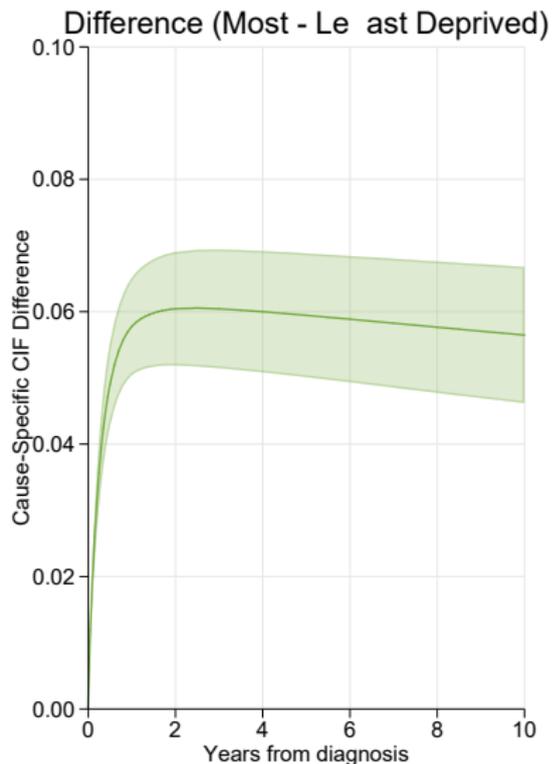
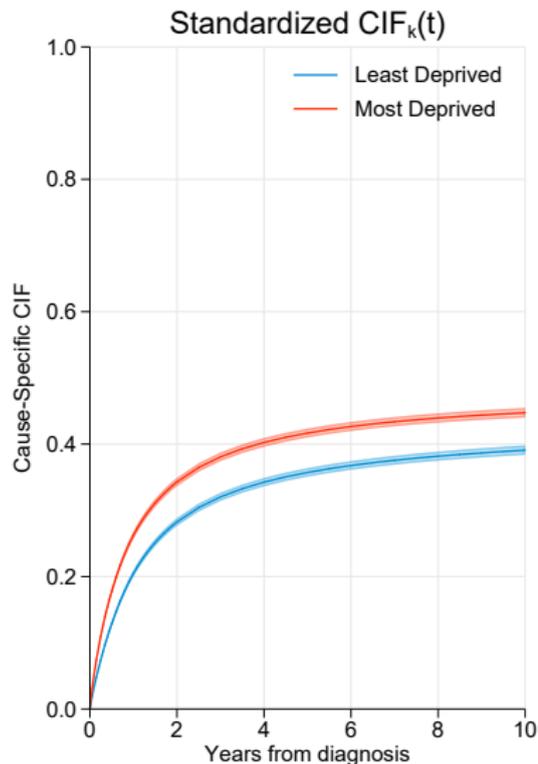
```
. standsurv, crmodels(cancer other) timevar(tt) cif ci          ///  
  at1(dep5 0 dep_agercs1 0 dep_agercs2 0 dep_agercs3 0)      ///  
  at2(dep5 1 dep_agercs1=agercs1 dep_agercs2=agercs2 dep_agercs3=agercs3) ///  
  contrast(difference)                                       ///  
  atvar(CIF_s_dep1 CIF_s_dep5)                               ///  
  contrastvar(CIF_diff)
```

# Using standsurv

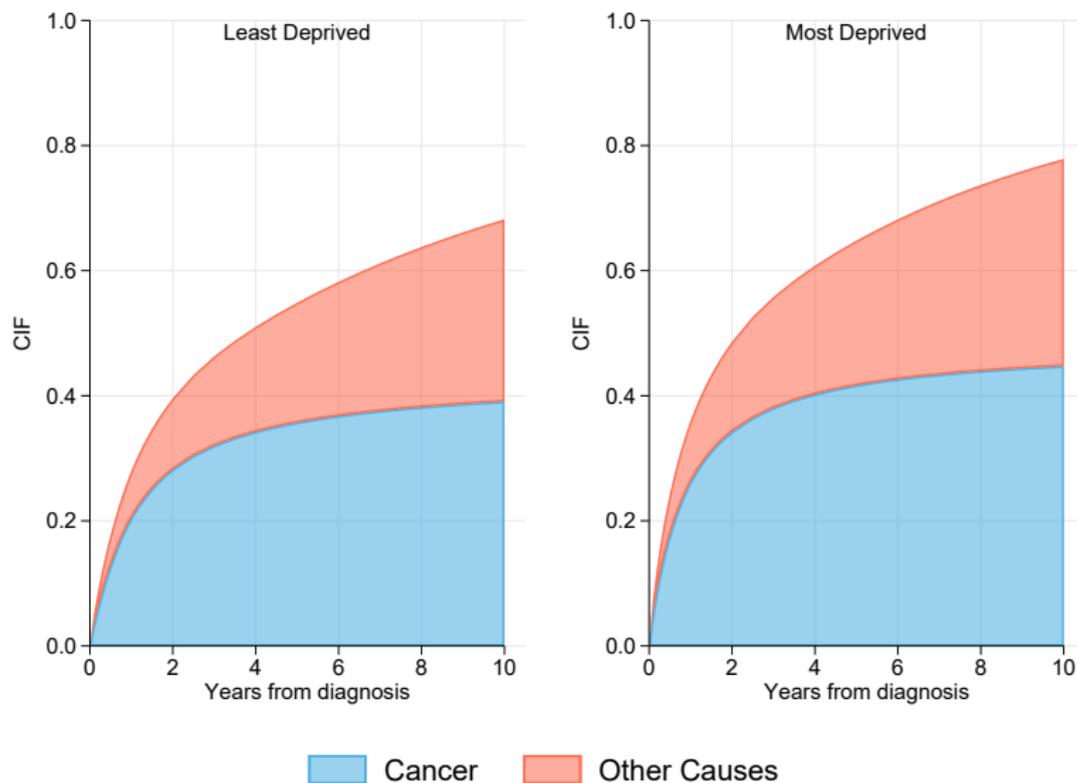
- Take mean of 39,625 CIFs where all individuals forced to be unexposed.
- Take mean of 39,625 CIFs where all individuals forced to be exposed.

```
. standsurv, crmodels(cancer other) timevar(tt) cif ci          ///  
  at1(dep5 0 dep_agercs1 0 dep_agercs2 0 dep_agercs3 0)      ///  
  at2(dep5 1 dep_agercs1=agercs1 dep_agercs2=agercs2 dep_agercs3=agercs3) ///  
  contrast(difference)                                       ///  
  atvar(CIF_s_dep1 CIF_s_dep5)                               ///  
  contrastvar(CIF_diff)
```

# Standardized cause-specific CIF



# Stacked standardized cause-specific CIF



# Competing Risks - extensions

- Can also obtain area under standardized CIF which gives a standardized version of the expected years of life lost (Andersen 2013[13]). Use `cif` and `rmft options`. See Mozumder *et al* 2021 [14].
- Various causal measures in competing risks described in Young *et al* 2020[15] can be estimated using `standsurv`.
- Separable effects can also be estimated (Stensrud *et al* 2020)[16].
- Can also use user-defined functions, e.g. Standardized attributable fraction in competing risks setting.
- Different parametric models can be used for different causes.
- Different time scales can be used for different causes (e.g. attained age / time from diagnosis).

# Relative Survival

- Relative survival models used with large population cancer registry data when cause of death not available or not reliable.

$$h(t|X, Z) = h^*(t|X, Z) + \lambda(t|X, Z)$$

- $h(t|X, Z)$  - All-cause mortality rate
- $h^*(t|X, Z)$  - Expected mortality rate
- $\lambda(t|X, Z)$  - Excess mortality rate

- Expected mortality rates obtained from national life tables.
- On survival scale.

$$S(t|X, Z) = S^*(t|X, Z)R(t|X, Z)$$

- The equivalent of a CIF is known as a crude probability in the relative survival framework.

# Melanoma Example

## Relative Survival Model

```
stpm2 dep5 ageracs* , scale(hazard) df(5) tvc(dep5 ageracs*) dftvc(3) bhazard(rate)
```

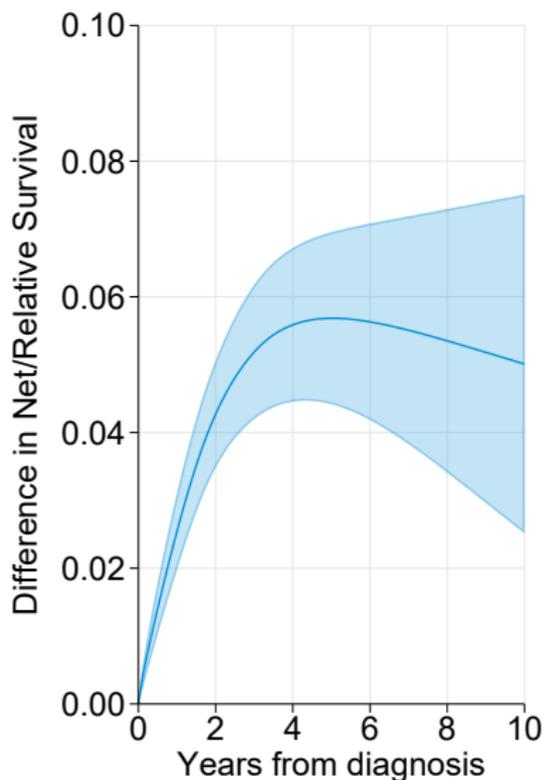
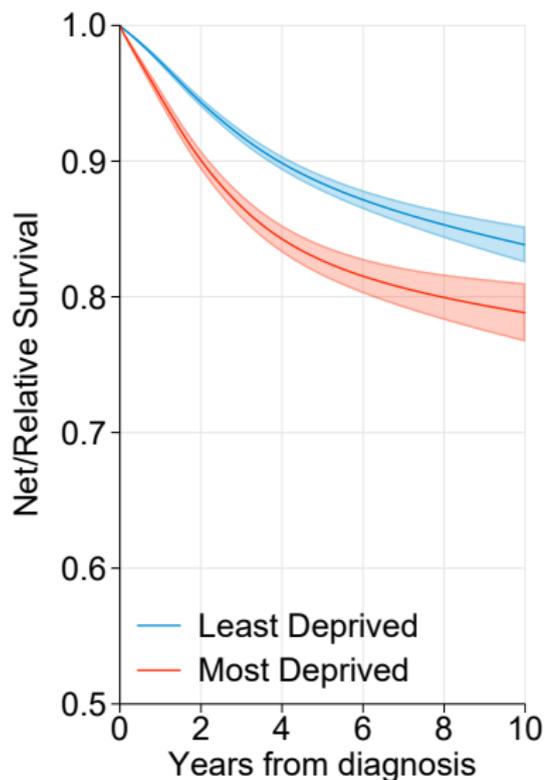
$$\bar{R}(t|X = x, Z) = \frac{1}{N} \sum_{i=1}^N w_i R_i(t|X = x, Z = z_i)$$

- The weights,  $w_i$ , enables standardization to external population through up- or down-weighting relative to a reference population.

## Standardized Relative Survival

```
standsurv, timevar(tt) ci                                     ///  
  at1(dep5 0 ageracs1_dep5 0 ageracs2_dep5 0 ageracs3_dep5 0)  ///  
  at2(dep5 1 ageracs1_dep5=ageracs1 ageracs2_dep5=ageracs2 ageracs3_dep5=ageracs3)  ///  
  indweights(wt)                                             ///  
  contrast(difference)                                       ///  
  atvar(R_dep5 R_dep1)                                       ///  
  contrastvar(R_diff)                                        ///
```

# Standardized Relative Survival

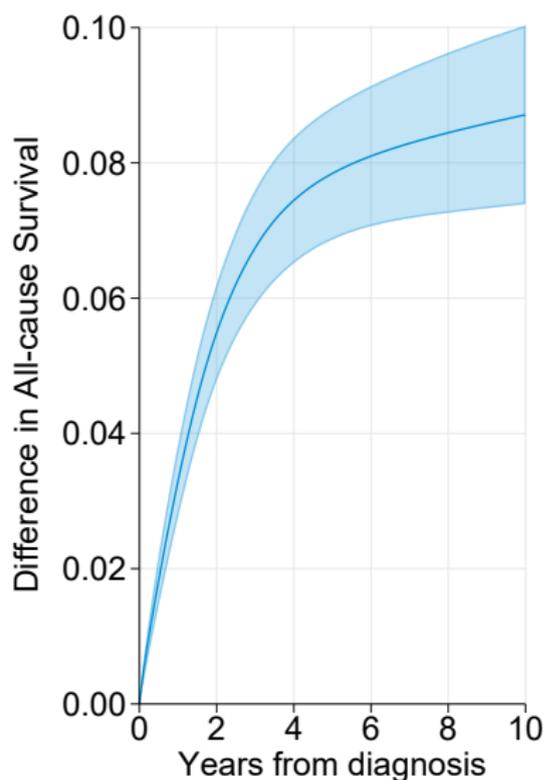
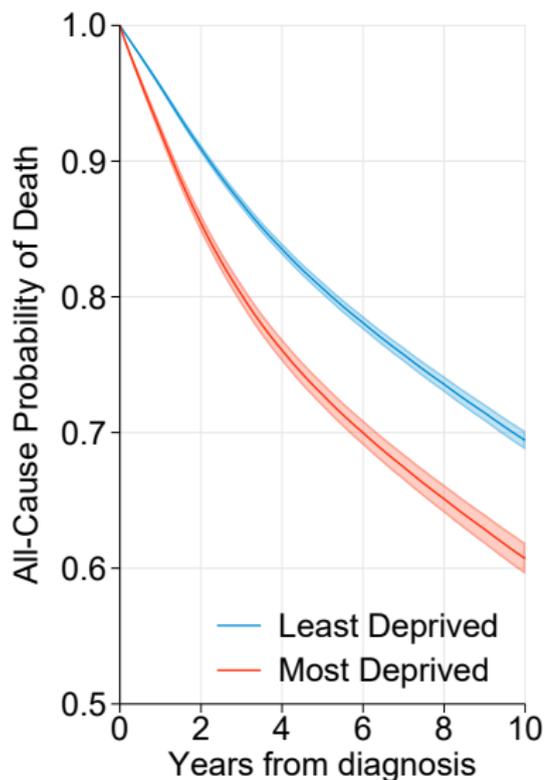


# All-cause Survival

$$\bar{S}(t|X = x, \mathbf{Z}) = \frac{1}{N} \sum_{i=1}^N S^*(t|X = x, \mathbf{Z} = z_i) R_i(t|X = x, \mathbf{Z} = z_i)$$

```
standsurv, timevar(tt) ci                                     ///
  at1(dep5 0 agercs1_dep5 0 agercs2_dep5 0 agercs3_dep5 0)    ///
  at2(dep5 1 agercs1_dep5=agercs1 agercs2_dep5=agercs2 agercs3_dep5=agercs3) ///
  expsurv(using(popmort_uk_regions_2017.dta)                  ///
  datediag(dx)                                              ///
  ageddiag(agediag)                                        ///
  pmrate(rate)                                            ///
  pmage(age)                                              ///
  pmyear(year)                                            ///
  pmother(sex dep region)                                 ///
  pmaxyear(2016)                                         ///
  at1(dep 1)                                              ///
  at2(dep 5))                                             ///
  contrast(difference)                                       ///
  atvar(S_dep5 S_dep1)                                       ///
  contrastvar(S_diff)
```

# Standardized All-cause Survival

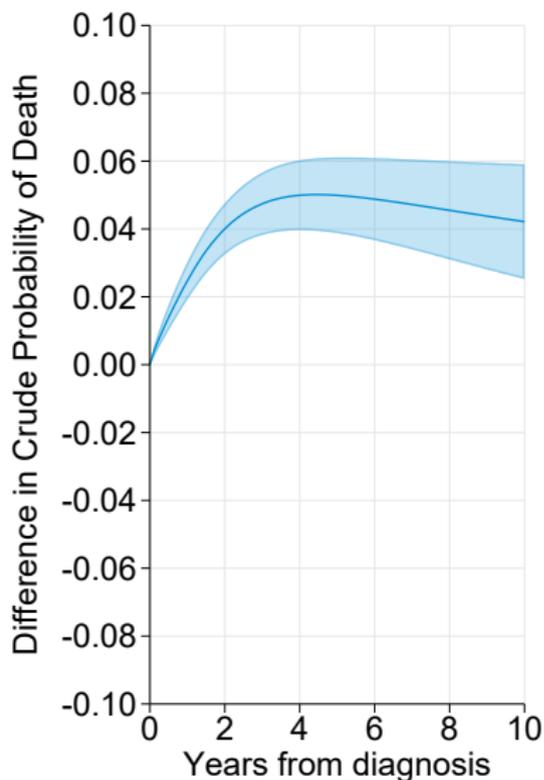
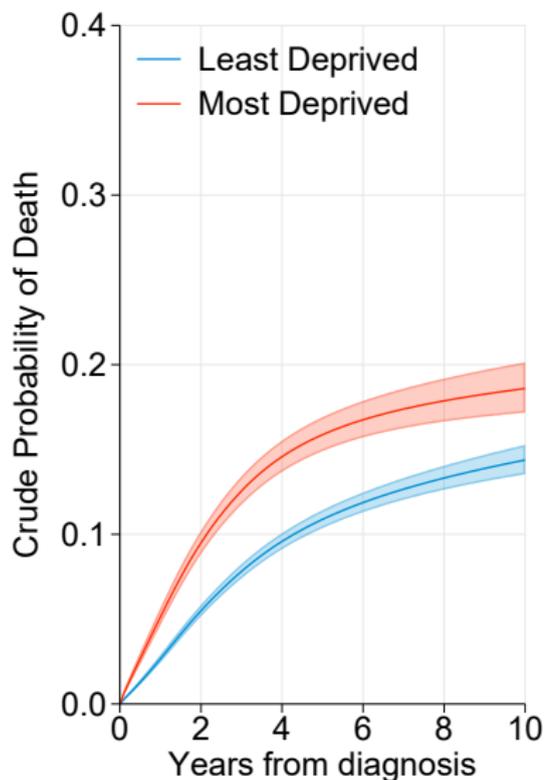


# Standardized Crude Probabilities

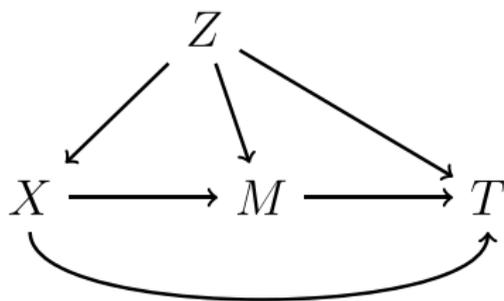
$$\bar{F}_c(t|X = x, \mathbf{Z}) = \frac{1}{N} \sum_{i=1}^N \int_0^t S^*(u|X = x, \mathbf{Z} = \mathbf{z}_i) R(u|X = x, \mathbf{Z} = \mathbf{z}_i) \lambda(u|X = x, \mathbf{Z} = \mathbf{z}_i),$$

```
standsurv, crudeprob timevar(tt) ci ///  
  at1(dep5 0 agercs1_dep5 0 agercs2_dep5 0 agercs3_dep5 0) ///  
  at2(dep5 1 agercs1_dep5=agercs1 agercs2_dep5=agercs2 agercs3_dep5=agercs3) ///  
  expsurv(using(popmort_uk_regions_2017.dta) ///  
    datediag(dx) ///  
    agediag(agediag) ///  
    pmrate(rate) ///  
    pimage(age) ///  
    pmyear(year) ///  
    pmother(sex dep re) ///  
    pmmaxyear(2016) ///  
    at1(dep 1) ///  
    at2(dep 5)) ///  
  contrast(difference) ///  
  atvar(CP_dep5 CP_dep1) ///  
  contrastvar(CP_diff)
```

# Standardized Crude Probabilities of Death



# Mediation Analysis in Relative Survival Framework



$$\widehat{\text{NDE}}_{RS} = \frac{1}{N} \sum_{i=1}^N \sum_m \hat{R}(t|X = 1, \mathbf{Z}_2 = \mathbf{z}_{2i}, M = m) \hat{P}(M = m|X = 0, \mathbf{Z}_2 = \mathbf{z}_{2i})$$
$$- \frac{1}{N} \sum_{i=1}^N \sum_m \hat{R}(t|X = 0, \mathbf{Z}_2 = \mathbf{z}_{2i}, M = m) \hat{P}(M = m|X = 0, \mathbf{Z}_2 = \mathbf{z}_{2i})$$

$$\widehat{\text{NIE}}_{RS} = \frac{1}{N} \sum_{i=1}^N \sum_m \hat{R}(t|X = 1, \mathbf{Z}_2 = \mathbf{z}_{2i}, M = m) \hat{P}(M = m|X = 1, \mathbf{Z}_2 = \mathbf{z}_{2i})$$
$$- \frac{1}{N} \sum_{i=1}^N \sum_m \hat{R}(t|X = 1, \mathbf{Z}_2 = \mathbf{z}_{2i}, M = m) \hat{P}(M = m|X = 0, \mathbf{Z}_2 = \mathbf{z}_{2i})$$

# Mediation Analysis in Relative Survival Framework

```
// Natural Indirect Effect
standsurv, failure timevar(tt)   ///
  at1(dep5 1 stage2 0 stage3 0 stage4 0 st2dep5 0 st3dep5 0 st4dep5 0, atindweights(p11)) ///
  at2(dep5 1 stage2 1 stage3 0 stage4 0 st2dep5 1 st3dep5 0 st4dep5 0, atindweights(p12)) ///
  at3(dep5 1 stage2 0 stage3 1 stage4 0 st2dep5 0 st3dep5 1 st4dep5 0, atindweights(p13)) ///
  at4(dep5 1 stage2 0 stage3 0 stage4 1 st2dep5 0 st3dep5 0 st4dep5 1, atindweights(p14)) ///
  at5(dep5 1 stage2 0 stage3 0 stage4 0 st2dep5 0 st3dep5 0 st4dep5 0, atindweights(p01)) ///
  at6(dep5 1 stage2 1 stage3 0 stage4 0 st2dep5 1 st3dep5 0 st4dep5 0, atindweights(p02)) ///
  at7(dep5 1 stage2 0 stage3 1 stage4 0 st2dep5 0 st3dep5 1 st4dep5 0, atindweights(p03)) ///
  at8(dep5 1 stage2 0 stage3 0 stage4 1 st2dep5 0 st3dep5 0 st4dep5 1, atindweights(p04)) ///
  lincom(1 1 1 1 -1 -1 -1 -1) lincomvar(t_nie) ci
```

# Mediation Analysis in Relative Survival Framework

```
// Natural Indirect Effect
standsurv, failure timevar(tt) ///
  at1(dep5 1 stage2 0 stage3 0 stage4 0 st2dep5 0 st3dep5 0 st4dep5 0, atindweights(p11)) ///
  at2(dep5 1 stage2 1 stage3 0 stage4 0 st2dep5 1 st3dep5 0 st4dep5 0, atindweights(p12)) ///
  at3(dep5 1 stage2 0 stage3 1 stage4 0 st2dep5 0 st3dep5 1 st4dep5 0, atindweights(p13)) ///
  at4(dep5 1 stage2 0 stage3 0 stage4 1 st2dep5 0 st3dep5 0 st4dep5 1, atindweights(p14)) ///
  at5(dep5 1 stage2 0 stage3 0 stage4 0 st2dep5 0 st3dep5 0 st4dep5 0, atindweights(p01)) ///
  at6(dep5 1 stage2 1 stage3 0 stage4 0 st2dep5 1 st3dep5 0 st4dep5 0, atindweights(p02)) ///
  at7(dep5 1 stage2 0 stage3 1 stage4 0 st2dep5 0 st3dep5 1 st4dep5 0, atindweights(p03)) ///
  at8(dep5 1 stage2 0 stage3 0 stage4 1 st2dep5 0 st3dep5 0 st4dep5 1, atindweights(p04)) ///
  lincom(1 1 1 1 -1 -1 -1 -1) lincomvar(t_nie) ci
```

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RESEARCH PAPER

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## Understanding disparities in cancer prognosis: An extension of mediation analysis to the relative survival framework

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Code available as online Appendix

# Relative Survival Framework Extensions

- Common to use weights (`indweights()`) to standardize to external population.
- Reference adjusted measures - using expected mortality rates from external population[17].
- Incorporate inverse probability weights into model to get doubly robust standardization.
- Marginal measures of life expectancy.

- Regression standardisation is a simple and underused tool with survival data.
- As long as we can predict survival function, models can be as complex as we like (non-linear effects, non-proportional hazards, interactions with exposure etc.)
- Marginal estimates also used in validation of prognostic models

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