rbiprobit: Recursive bivariate probit estimation and decomposition of marginal effects

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net install rbiprobit, from("https://raw.githubusercontent.com/cobanomics/rbiprobit/main")
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Motivation

Effects of Interest

1. What we want
   ▶ Estimate: Effect of binary or treatment variable on binary outcome variable
   ▶ Treatment variable itself is endogenous
   ▶ Unobservables may correlate with treatment and outcome equation
   ▶ Compute average treatment effect
   ▶ Compute average marginal effect for covariates

2. What doesn’t work:
   ▶ margins gives incorrect treatment effect
   ▶ margins gives incorrect average marginal effect for covariates
   ▶ ivprobit inappropriate; treatment variable is binary

3. What we need
   ▶ Correct Estimation of a RBPM
   ▶ Considering recursive nature of the model for postestimation commands
Contribution

A new Stata Command

- `rbiprobit` estimates RBPMs like `biprobit` or `cmp`
- `rbiprobit` accounts for recursive nature in postestimation commands
  - predict and `predictnl`
  - `rbiprobit margdec`
  - `rbiprobit tmeffects`
- `rbiprobit margdec` incorporates `margins` command, enabling
  - Decomposition of average marginal effects of covariates
  - Standard errors using the delta method
- `rbiprobit tmeffects` incorporates `margins` command, enabling
  - Different definitions of treatment effects
  - Standard errors using the delta method
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Recursive bivariate probit model

The Model

A structural model with endogenous explanatory treatment variable $y_2$ correlated with the unobservables

$$
y_1^* = x' \beta + \alpha y_2 + \epsilon_1 , y_1 = 1[y_1^* > 0]
$$

$$
y_2^* = z' \gamma + \epsilon_2 , y_2 = 1[y_2^* > 0]
$$

with \( \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} \sim \mathcal{N} \left[ \begin{pmatrix} 0 \\ 0 \end{pmatrix} , \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right] \)

- correlation between $\epsilon_1$ and $\epsilon_2$ induces the endogeneity
- $x$ and $z$ can share some or all covariates
- Greene (2018) notes that endogenous nature of $y_2$ can be ignored
- Han and Lee (2019): estimates are at best weakly identified if $x = z$
Recursive bivariate probit model

Treatment Effects

1. Average Treatment Effect (ATE)

\[
ATE = \frac{1}{n} \sum_{i=n}^{n} \Phi(x_i' \beta + \alpha) - \Phi(x_i' \beta)
\]

2. Average Treatment Effect on the Treated (ATET)

\[
ATET = \frac{1}{n_2} \sum_{i=1}^{n_2} \Phi \left( \frac{x_i' \beta + \alpha - \rho z_i' \gamma}{\sqrt{1 - \rho^2}} \right) - \Phi \left( \frac{x_i' \beta - \rho z_i' \gamma}{\sqrt{1 - \rho^2}} \right) \quad \forall y_{2i} = 1
\]

3. Average Treatment Effect on Conditional Probability (ATEC)

\[
ATEC = \frac{1}{n} \sum_{i=1}^{n} \frac{\Phi_2(x_i' \beta + \alpha, z_i' \gamma, \rho)}{\Phi(z_i' \gamma)} - \frac{\Phi_2(x_i' \beta - z_i' \gamma, -\rho)}{\Phi(-z_i' \gamma)}
\]
Decomposition of Marginal Effects

Joint and Conditional Probabilities

- Covariate $d$ appears in both $x$ and $z$
- Decomposition of total marginal effects on the probabilities (except marginal probabilities) are then

1. Continuous Variables (see Greene, 2018)

\[
\text{ME} = \frac{\partial \Pr}{\partial \begin{pmatrix} x_d \\ z_d \end{pmatrix}} = \frac{\partial \Pr}{\partial x_d} + \frac{\partial \Pr}{\partial z_d}
\]

- direct effect
- indirect effect

2. Discrete Variables (see Hasebe, 2013; Edwards et al., 2019)

\[
\text{ME} = \left[ \Pr |_{x_d=1} - \Pr |_{x_d=0} \right] + \left[ \Pr |_{z_d=1} - \Pr |_{z_d=0} \right]
\]

- direct effect
- indirect effect
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Basic Syntax

```
rbiprobit depvar [=] [indepvars] [if] [in]
    , endogenous(depvar_en [=] [indepvars_en] [, enopts]) [options]
```

- `depvar_en` automatically added to outcome equation as factor-variable
- `rbiprobit` implemented as an `lf1 ml` evaluator
- `depvar` and `depvar_en` have to be binary (current version)
- Factor variables and time-series operators allowed
- `rbiprobit` postestimation available for features after estimation
## rbiprobit Output

```
. webuse class10, clear
(Class of 2010 profile)
. rbiprobit graduate = income i.roommate i.hsgpagrp ///
> , endog(program = i.campus i.scholar income i.hsgpagrp)
```

### Univariate Probits for starting values

Comparison: log likelihood = -2673.8688

### Recursive Bivariate Probit Regression

- Number of obs = 2,500
- Wald chi2(12) = 964.07
- Prob > chi2 = 0.0000

| Coef. | Std. Err. | z    | P>|z| | [95% Conf. Interval] |
|-------|-----------|------|-----|-----------------------|
| graduate        |           |      |     |                       |
| 1.program      | .3522094  | .1770159 | 1.99 | 0.047 | .0052646 | .6991542 |
| income         | .1434782  | .0142911 | 10.04 | 0.000 | .1154681 | .1714882 |
| roommate       |           |      |     |                       |
| yes            | .267713   | .0588568 | 4.55 | 0.000 | .1523559 | .3830701 |
| hsgpagrp       |           |      |     |                       |
| 2.5-2.9        | .9451679  | .1357869 | 6.96 | 0.000 | .6790305 | 1.211305 |
| 3.0-3.4        | 1.939513   | .147325 | 13.16 | 0.000 | 1.650761 | 2.228264 |
| 3.5-4.0        | 6.535829   | 127.5038 | 0.05 | 0.959 | -243.367 | 256.4387 |
| _cons          | -2.076232  | .2181295 | -9.52 | 0.000 | -2.503758 | -1.648706 |
| program        |           |      |     |                       |
| campus         |           |      |     |                       |
| yes            | .7465297  | .0747092 | 9.99 | 0.000 | .6001024 | .8929569 |
| scholar        |           |      |     |                       |
| yes            | .9007975  | .0579886 | 15.53 | 0.000 | .787142  | 1.014453 |
| income         | -.0785837 | .0096477 | -8.15 | 0.000 | -.0974928 | -.0596746 |
| hsgpagrp       |           |      |     |                       |
| 2.5-2.9        | .0586754  | .1099653 | 0.53 | 0.594 | -.1568526 | .2742035 |
| 3.0-3.4        | .0651845  | .1152074 | 0.57 | 0.572 | -.1606179 | .2909869 |
| 3.5-4.0        | -.0970995 | .1780755 | -0.55 | 0.586 | -.4461211 | .2519222 |
| _cons          | -.4441949 | .1276995 | -3.48 | 0.001 | -.6944812 | -.1939085 |
| /atanrho       |           |      |     |                       |
|                | .4138925  | .118934 | 3.48 | 0.001 | .1807862 | .6499888 |
| rho            | .3917727  | .1006793 | 3.86 | 0.000 | .188842  | .5696461 |

Wald test of rho=0: chi2(1) = 12.1105  Prob > chi2 = 0.00005
```
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Postestimation Commands

Predictions

```
predict [type] newvar [if] [in] [, statistic ]
```

### statistic

- **p11** \( \Pr(\text{depvar} = 1, \text{depvar}_{en} = 1) \); the default
- **p10** \( \Pr(\text{depvar} = 1, \text{depvar}_{en} = 0) \)
- **p01** \( \Pr(\text{depvar} = 0, \text{depvar}_{en} = 1) \)
- **p00** \( \Pr(\text{depvar} = 0, \text{depvar}_{en} = 0) \)
- **pmarg1** \( \Pr(\text{depvar} = 1) \); marginal success probability for outcome eq.
- **pmarg2** \( \Pr(\text{depvar}_{en} = 1) \); marginal success probability for endogenous eq.
- **pcond1** \( \Pr(\text{depvar} = 1 \mid \text{depvar}_{en} = 1) \)
- **pcond2** \( \Pr(\text{depvar}_{en} = 1 \mid \text{depvar} = 1) \)
- **xb1** linear prediction for outcome eq.
- **xb2** linear prediction for endogenous eq.
- **stdp1** standard error of the linear prediction for outcome eq.
- **stdp2** standard error of the linear prediction for endogenous eq.
Postestimation Commands

Margins and Treatment Effects

**rbiprobit margdec [if] [in] [, response_options options]**

**rbiprobit tmeffects [if] [in] [, tmeffect(effecttype) options]**

**rbiprobit margdec options**

- **effect(effecttype)**
  - specify type of effect; effecttype may be total, direct, or indirect; default is total

- **predict(pred_opt)**
  - estimate margins for predict, pred_opt
  - multiple predict not applicable

- **dydx(varlist)**
  - estimate marginal effect of variables in varlist

**rbiprobit tmeffects options**

- **tmeffect(effecttype)**
  - specify type of effect; effecttype may be ate, atet, or atec; default is ate
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Post-Estimation: predict

Comparison: biprobit vs. rbiprobit

```
. webuse class10, clear
(Class of 2010 profile)

. qui: rbiprobit graduate = income i.roommate i.hsgpagrp ///
> , endog(program = i.campus i.scholar income i.hsgpagrp)

. predict p11r, p11

. qui: biprobit (graduate = income i.roommate i.hsgpagrp i.program) ///
> (program = i.campus i.scholar income i.hsgpagrp)

. predict p11b, p11

. compare p11r p11b
```

<table>
<thead>
<tr>
<th></th>
<th>count</th>
<th>minimum</th>
<th>average</th>
<th>maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>p11r&lt;p11b</td>
<td>678</td>
<td>-.0000178</td>
<td>-.0000104</td>
<td>-1.49e-08</td>
</tr>
<tr>
<td>p11r=p11b</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p11r&gt;p11b</td>
<td>1821</td>
<td>2.98e-08</td>
<td>.026773</td>
<td>.1206536</td>
</tr>
<tr>
<td>jointly defined</td>
<td>2500</td>
<td>-.0000178</td>
<td>.0194987</td>
<td>.1206536</td>
</tr>
<tr>
<td>total</td>
<td>2500</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Post-Estimation: margdec

Continuous Covariate: Total Average Marginal Effects

```
. rbiprobit margdec, dydx(income) predict(p11) effect(total)

Average marginal effects Number of obs = 2,500
Model VCE : OIM
Expression : Pr(graduate=1,program=1), predict(p11)
dy/dx w.r.t. : income

<table>
<thead>
<tr>
<th></th>
<th>Delta-method</th>
<th></th>
<th></th>
<th>[95% Conf. Interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dy/dx</td>
<td>Std. Err.</td>
<td>z</td>
<td>P&gt;</td>
</tr>
<tr>
<td>income</td>
<td>.0032146</td>
<td>.002856</td>
<td>1.13</td>
<td>0.260</td>
</tr>
</tbody>
</table>
```

Average marginal effects
Number of obs = 2,500
Model VCE : OIM
Expression : Pr(graduate=1,program=1), predict(p11)
dy/dx w.r.t. : income

```
Post-Estimation: rbiprobit margdec

Continuous Covariate: Direct Average Marginal Effects

```
. rbiprobit margdec, dydx(income) predict(p11) effect(direct)
```

Average marginal effects Number of obs = 2,500
Model VCE : OIM

Expression : Pr(graduate=1,program=1), predict(p11)
dy/dx w.r.t. : income

<table>
<thead>
<tr>
<th></th>
<th>Delta-method</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dy/dx</td>
<td>Std. Err.</td>
<td>z</td>
<td>P&gt;</td>
<td>z</td>
<td></td>
</tr>
<tr>
<td>income</td>
<td>.0207027</td>
<td>.0017927</td>
<td>11.55</td>
<td>0.000</td>
<td>.0171891</td>
<td>.0242163</td>
</tr>
</tbody>
</table>
Post-Estimation: rbiprobit margdec

Continuous Covariate: Indirect Average Marginal Effects

. rbiprobit margdec, dydx(income) predict(p11) effect(indirect)

Average marginal effects Number of obs = 2,500
Model VCE : OIM

Expression : Pr(graduate=1,program=1), predict(p11)
dy/dx w.r.t. : income

|                | Delta-method
|----------------|---------------------
| dy/dx  Std. Err. |        z    P>|z|    [95% Conf. Interval] |
| income | -0.0174881 0.00214 -8.17 0.000 -0.0216825 -0.0132937 |

log close
Post-Estimation: rbiprobit tmeffects

Average Treatment Effect

```
. rbiprobit tmeffects, tmeffect(ate)
Treatment effect                                Number of obs     =      2,500
Model VCE    : OIM
Expression   : Pr(graduate=1), predict(pmarg1)
Effect       : Average treatment effect
dydx w.r.t.  : 1.program

|            Delta-method                             |
| dy/dx   Std. Err.    z   P>|z|   [95% Conf. Interval] |
-------------+-----------------------------------------------|
 ate        | .0981233    .0476266    2.06  0.039    .0047769    .1914697   |
```
Post-Estimation: marginsplot

Graph results from rbiprobit margdec and rbiprobit tmeffects

```
qui: rbiprobit margdec, dydx(income roommate hsgpagrp) predict(pl1) effect(direct)
marginsplot
Variables that uniquely identify margins: _deriv

qui: rbiprobit tmeffects, tmeffect(ate)
marginsplot
Variables that uniquely identify margins:
```
Post-Estimation: marginsplot

Graph results from rbiprobit margdec

![Graph of Average Marginal Effects with 95% CIs](image-url)
Post-Estimation: marginsplot

Graph results from rbiprobit tmeffects

Treatment Effect of ate with 95% CIs
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Future Additions

Estimation and Post-Estimation Options

1. Estimation Options
   - Weights
   - Model and SE options
   - Reporting options
   - Maximization options

2. Post-Estimation Options
   - Appropriate margins options (at(), contrast, etc.)
   - Weights
   - SE options
   - Reporting options
   - Maximization options

3. Post-Estimation Commands
   - bphltest
   - scoregof
Thank you

Version 1.0.0 available

`net install rbiprobit, from("https://raw.githubusercontent.com/cobanomics/rbiprobit/main/"`)`

GitHub: github.com/cobanomics/rbiprobit
Email: mustafa.coban@iab.de
Web: mustafacoban.de
References


Appendix

Predictions of Interest

1. Joint Probabilities

\[
\begin{align*}
\Pr(y_1 = 1, y_2 = 1|x, z) &= \Phi_2(x' \beta + \alpha, z' \gamma, \rho) \\
\Pr(y_1 = 1, y_2 = 0|x, z) &= \Phi_2(x' \beta, -z' \gamma, -\rho) \\
\Pr(y_1 = 0, y_2 = 1|x, z) &= \Phi_2(-x' \beta + \alpha, z' \gamma, -\rho) \\
\Pr(y_1 = 0, y_2 = 0|x, z) &= \Phi_2(-x' \beta, -z' \gamma, \rho)
\end{align*}
\]

2. Conditional Probabilities

\[
\begin{align*}
\Pr(y_1 = 1|y_2 = 1, x, z) &= \frac{\Phi_2(x' \beta + \alpha, z' \gamma, \rho)}{\Phi(z' \gamma)} \\
\Pr(y_2 = 1|y_1 = 1, x, z) &= \frac{\Phi_2(x' \beta + \alpha, z' \gamma, \rho)}{\Phi(x' \beta + \alpha)}
\end{align*}
\]
Appendix
Predictions of Interest

3. Marginal Probabilities

\[
\Pr(y_1 = 1|x) = \Phi(x' \beta + \alpha y_2)
\]
\[
\Pr(y_2 = 1|z) = \Phi(z' \gamma)
\]

4. Unconditional Mean Function (see Blasch et al., 2019; Alrasheed, 2019)

\[
E[y_1|x, z] = \Pr(y_2 = 1|z) \cdot E[y_1|y_2 = 1, x, z] \\
+ \Pr(y_2 = 0|z) \cdot E[y_1|y_2 = 0, x, z] \\
= \Pr(y_1 = 1, y_2 = 1|x, z) + \Pr(y_1 = 1, y_2 = 0|x, z) \\
= \Phi_2(x' \beta + \alpha, z' \gamma, \rho) + \Phi_2(x' \beta, -z' \gamma, -\rho)
\]
Appendix

Manual Changes of Dependent Variables for Predictions

. qui: rbiprobit graduate = income i.roommate i.hsgpagrp ///
> , endog(program = i.campus i.scholar income i.hsgpagrp)

. predict p11r, p11

. qui: biprobit (graduate = income i.roommate i.hsgpagrp i.program) ///
> (program = i.campus i.scholar income i.hsgpagrp)

. replace graduate = 1
(972 real changes made)

. replace program = 1
(1,148 real changes made)

. predict p11b, p11

. compare p11r p11b

---------- difference ----------
count       minimum      average     maximum
------------------------------------------------------------------------
p11r<p11b                   1033     -.0000178    -8.81e-06   -1.49e-08
p11r=p11b                       7
p11r>p11b                    1460      2.98e-08     .0000105     .000084
----------
jointly defined              2500     -.0000178     2.47e-06     .000084
----------
total                        2500
Incorrect Standard Errors after margins

```
. qui: rbiprobit graduate = income i.roommate i.hsgpagr //
   > , endog(program = i.campus i.scholar income i.hsgpagr)

. rbiprobit margdec, dydx(income) predict(p11) effect(total)
```

Average marginal effects                        Number of obs     =      2,500
Model VCE    : OIM
Expression   : Pr(graduate=1,program=1), predict(p11)
dy/dx w.r.t. : income

<table>
<thead>
<tr>
<th>Delta-method</th>
</tr>
</thead>
<tbody>
<tr>
<td>dy/dx</td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td>income</td>
</tr>
</tbody>
</table>

```
. margins, dydx(income) predict(p11)
```

Average marginal effects                        Number of obs     =      2,500
Model VCE    : OIM
Expression   : Pr(graduate=1,program=1), predict(p11)
dy/dx w.r.t. : income

<table>
<thead>
<tr>
<th>Delta-method</th>
</tr>
</thead>
<tbody>
<tr>
<td>dy/dx</td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td>income</td>
</tr>
</tbody>
</table>
Appendix

Discrete Covariate: Direct Average Marginal Effects

```
. rbiprobit margdec, dydx(hsgpaggrp) predict(p11) effect(direct)

Average marginal effects                        Number of obs     =      2,500
Model VCE    : OIM

Expression   : Pr(graduate=1,program=1), predict(p11)
dy/dx w.r.t. : 25.hsgpaggrp 30.hsgpaggrp 35.hsgpaggrp

------------------------------------------------------------------------------
|            Delta-method
|      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  hsgpgagrp   |
  2.5-2.9     |   .1821001   .0234585     7.76   0.000     .1361223    .2280779
  3.0-3.4     |   .3297082   .0236584    13.94   0.000     .2833386    .3760777
  3.5-4.0     |    .386345   .0231423    16.69   0.000     .3409869    .4317032
-------------+----------------------------------------------------------------

Note: dy/dx for factor levels is the discrete change from the base level.
```
Appendix

Discrete Covariate: Indirect Average Marginal Effects

```
Appendix

Discrete Covariate: Indirect Average Marginal Effects

. rbiprobit margdec, dydx(hsgpagrp) predict(p11) effect(indirect)

Average marginal effects                        Number of obs     =      2,500
Model VCE    : OIM
Expression   : Pr(graduate=1,program=1), predict(p11)
dy/dx w.r.t. : 25.hsgpagrp 30.hsgpagrp 35.hsgpagrp

------------------------------------------------------------------------------
|             Delta-method             |
|            dy/dx   Std. Err.      z    P>|z|    [95% Conf. Interval] |
|-------------+-----------------+-------+------+-----------------+-----+-----+-----------------|
| hsgpagrp    |                 |
| 2.5-2.9     | 0.0131266       | 0.0246783 | 0.53 | 0.595          | -0.035242 | 0.0614953 |
| 3.0-3.4     | 0.0145757       | 0.0258243 | 0.56 | 0.572          | -0.036039 | 0.0651905 |
| 3.5-4.0     | -0.0219075      | 0.0402565 | -0.54| 0.586          | -0.1008087| 0.0569938 |
------------------------------------------------------------------------------

Note: dy/dx for factor levels is the discrete change from the base level.
```
Appendix

Average Treatment Effect on the Treated

```
. rbiprobit tmeffects, tmeffect(atet)

Treatment effect                                Number of obs     =      1,352
Model VCE    : OIM
Expression   : normal(graduate=1|program=1) - normal(graduate=1|program=0)
Effect       : Average treatment effect on the treated
dydx w.r.t.  : 1.program

------------------------------------------------------------------------------
|            Delta-method
|      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  atet |   .1033448   .0489003     2.11   0.035     .0075019    .1991877
------------------------------------------------------------------------------
```

August 5-6, 2021 2021 Stata Conference Mustafa Coban
## Appendix

### Average Treatment Effect on the Conditional Probability

```stata
.rbiprobit tmeffects, tmeffect(atec)
```

<table>
<thead>
<tr>
<th>Treatment effect</th>
<th>Number of obs</th>
<th>= 2,500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model VCE</td>
<td>OIM</td>
<td></td>
</tr>
<tr>
<td>Expression: Pr(graduate=1</td>
<td>program=1)-Pr(graduate=1</td>
<td>program=0), predict(pcond1)- ict(pcond10)</td>
</tr>
<tr>
<td>Effect: Average treatment effect on conditional probability</td>
<td></td>
<td></td>
</tr>
<tr>
<td>dydx w.r.t.: 1.program</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Delta-method</th>
<th></th>
<th></th>
<th>[95% Conf. Interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dy/dx</td>
<td>Std. Err.</td>
<td>z</td>
<td>P&gt;</td>
</tr>
<tr>
<td>atece</td>
<td>.2765848</td>
<td>.0164366</td>
<td>16.83</td>
<td>0.000</td>
</tr>
</tbody>
</table>