Estimating (S,s) rule regression models

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2019 London Stata Conference
5 September 2019
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There are many economic variables such as product prices, or firm employment levels that exhibit infrequent adjustments.

These outcomes can occur when there are costs associated with making changes (e.g. menu costs), which lead agents to adopt an \((S,s)\) decision rule.

Such rules are characterized by a band of inaction, where agents tolerate some deviation from an optimal frictionless outcome, provided the deviation is not too large.

This presentation describes a new command \texttt{xtss} for estimating state dependent \((S,s)\) models, for panel data applications. This is based on Fougere et al. (2010) and Dhyne et al. (2011).
Fougere et al. (2010) estimate a price rigidity model to assess the impacts of the minimum wage on prices in French restaurants. This is based on a flexible (S,s) model where the thresholds vary over time and across restaurants.

Dhyne et al. (2011) use price observations on various goods in Belgium and France to study the importance of real and nominal rigidities in price adjustments. They find that asymmetry in price adjustments are caused by trends in cost and not asymmetry in the (S,s) bounds.

Gautier and Saout (2015) use a similar specification to Fougere et al. (2010) to examine the speed at which refined oil prices are passed-through to gasoline prices.
The model

Consider the case where the dependent variable is the price of product \( i = 1, \ldots, N \) in period \( t = 1, \ldots, T \) denoted \( p_{it} \).

Letting \( p_{it}^* \) denote the latent desired or frictionless price, the price decision rule is given by:

\[
p_{it} = \begin{cases} 
  p_{it}^* & \text{if } p_{it}^* - p_{it-1} > c_{it}^u \\
  p_{it}^* & \text{if } p_{it}^* - p_{it-1} < -c_{it}^d \\
  p_{it-1} & \text{if } -c_{it}^d \leq p_{it}^* - p_{it-1} \leq c_{it}^u.
\end{cases} \tag{1}
\]

where \( c_{it}^u \) and \( -c_{it}^d \) are stochastic (S,s) bounds which differ across \( i \) and \( t \) and allow for asymmetric menu costs.

Price equals the frictionless value when the difference \( p_{it}^* - p_{it-1} \) is above \( c_{it}^u \) (price rise) or below \( -c_{it}^d \) (price fall).
In the analysis of prices, the thresholds measure the extent to which changes are costly and represent nominal rigidity.

Dhyne et al. (2011) assumes these are normality distributed with time-invariant means; however this leads to a non-zero probability of a price rise and a price fall.

For example if $p_{it}^* - p_{it-1} = 0.1$, $c_{it}^u = -0.2$ and $-c_{it}^d = 0.2$, the gap is above the upper bound but also below the negative of the lower bound.

Such outcomes are inconsistent with the price decision rule where the thresholds must be non-negative.
The model

To avoid this issue, I modify their model and allow the thresholds to have a normal distribution truncated at zero:

\[ c_{it}^u \sim N^+ (\mu_{it}^u, \sigma_c^2) \]
\[ c_{it}^d \sim N^+ (\mu_{it}^d, \sigma_c^2) \]

Following Gautier and Saout (2015) this also allows the means to depend on explanatory variables \( z_{it} \) that modify the timing of the price adjustment:

\[ \mu_{it}^u = z_{it}' \lambda_u \]
\[ \mu_{it}^d = z_{it}' \lambda_d \]
The final equation is for the frictionless price, which is observed when prices are amended.

Letting $x_{it}$ denote a vector of exogenous explanatory variables:

$$p^*_t = x_{it}'\beta + u_i + \epsilon_{it}, \quad \epsilon_{it} \sim iidN(0, \sigma^2_{\epsilon})$$ (4)

where $u_i \sim iidN(0, \sigma^2_u)$ is an individual specific random effect that contains unobserved product heterogeneity.

In the context of price modeling, this process would arise as a log-linear expression under isoelastic demand and constant marginal costs where $x_{it}$ contains factor prices.
Estimation

- Letting $I_{it}^u = 1$ indicate a price rise and $I_{it}^d = 1$ a price fall, the observed price in (1) is:

$$p_{it} = p_{it-1} + (I_{it}^u + I_{it}^d)(p_{it}^* - p_{it-1})$$

- Letting $d_{it} = x_{it}'\beta + u_i - p_{it-1}$, from (4) the above becomes:

$$\Delta p_{it} = (I_{it}^u + I_{it}^d)(d_{it} + \epsilon_{it}) \tag{5}$$

where the indicators

$$I_{it}^u = \begin{cases} 1 & \text{if } d_{it} + \epsilon_{it} > c_{it}^u \\ 0 & \text{if } d_{it} + \epsilon_{it} \leq c_{it}^u \end{cases} \tag{6}$$

$$I_{it}^d = \begin{cases} 1 & \text{if } d_{it} + \epsilon_{it} < -c_{it}^d \\ 0 & \text{if } d_{it} + \epsilon_{it} \geq -c_{it}^d \end{cases} \tag{7}$$
Estimation

- Analogous to a Tobit II model, OLS based on the amended prices will be inconsistent as $E[\epsilon_{it} \mid I_{it}^u = 1 \cup I_{it}^d = 1] \neq 0$. Instead the model is estimated by maximum likelihood.

- Given the first-order Markovian property of the model, the contribution of product $i$ to the likelihood given $p_{i0}$ is:

$$L_i = \int_{-\infty}^{\infty} \prod_{t=2}^{T} f(\Delta p_{it} \mid p_{it-1}, u_i, x_{it}, z_{it}) f(u_i) du_i$$

- The integral in the above is approximated by Monte Carlo integration using Halton draws.

- To derive $f(\Delta p_{it} \mid p_{it-1}, u_i, x_{it}, z_{it})$ we distinguish between the cases where prices rise, prices fall and prices remain constant.
Price rise

- Suppressing the dependence on $u_i$, $x_{it}$ and $z_{it}$ to simply the exposition, the contribution to the likelihood of a price rise is:

$$f(\Delta p_{it}, I_{it}^u = 1 \mid p_{it-1}) = Pr(c_{it}^u < \Delta p_{it} \mid \Delta p_{it}) f(\Delta p_{it} \mid p_{it-1})$$

- From (2) and (5), the components in the above are:

$$f(\Delta p_{it} \mid p_{it-1}) = \frac{1}{\sigma_\epsilon} \phi \left( \frac{\Delta p_{it} - d_{it}}{\sigma_\epsilon} \right)$$

$$Pr(c_{it}^u < \Delta p_{it} \mid \Delta p_{it}) = \frac{\Phi\left( \frac{\Delta p_{it} - \mu_{it}^u}{\sigma_c} \right) - \Phi\left( \frac{-\mu_{it}^u}{\sigma_c} \right)}{\Phi\left( \frac{\mu_{it}^u}{\sigma_c} \right)}$$
Price fall

- The contribution to the likelihood of a price fall is:

\[
f(\Delta p_{it}, l_{it}^d = 1 \mid p_{it-1}) = Pr(c_{it}^d < -\Delta p_{it} \mid \Delta p_{it})f(\Delta p_{it} \mid p_{it-1})
\]

- The first component in the above is:

\[
Pr(c_{it}^d < -\Delta p_{it} \mid \Delta p_{it}) = \frac{\Phi\left(-\Delta p_{it} - \frac{\mu_{it}^d}{\sigma_c}\right) - \Phi\left(-\frac{\mu_{it}^d}{\sigma_c}\right)}{\Phi\left(\frac{\mu_{it}^d}{\sigma_c}\right)}
\]
Price constancy

- The contribution from no change in price occurs when both $p^*_it - p_{it-1} < c^u_{it}$ and $p^*_it - p_{it-1} > -c^d_{it}$; from (6)-(7) this is:

$$Pr(I^u_{it} = 0, I^d_{it} = 0 \mid p_{it-1}) = Pr(\epsilon_{it} - c^u_{it} < -d_{it}, \epsilon_{it} + c^d_{it} > -d_{it} \mid p_{it-1})$$

- As $\epsilon_{1it} \leq \epsilon_{2it}$, the above simplifies to:

$$= Pr(\epsilon_{1it} < -d_{it} \mid p_{it-1}) - Pr(\epsilon_{2it} < -d_{it} \mid p_{it-1})$$

(8)

- Evaluating the above requires the CDF of the sum of a normal and truncated normal random variable.
Price constancy

- The pdf’s of $\epsilon_1$ and $\epsilon_2$ can be derived using the convolution formula. Focusing on $\epsilon_1$, after considerable algebra this yields:

$$f(\epsilon_1) = \frac{1}{s \Phi(\mu_u / \sigma_c)} \Phi \left( a + b \frac{\epsilon_1 + \mu_u}{s} \right) \phi \left( \frac{\epsilon_1 + \mu_u}{s} \right)$$  \hspace{1cm} (9)

- The components in the above expression are:

$$s = \sqrt{\sigma_\epsilon^2 + \sigma_c^2}$$

$$a = \mu_u \frac{s}{\sigma_\epsilon \sigma_c}$$

$$b = -\frac{\sigma_c}{\sigma_\epsilon}$$  \hspace{1cm} (10)
Making the substitution $z = (\epsilon_1 + \mu^u)/s$ in (9) and integrating yields CDF:

$$Pr(\epsilon_1 \leq m) = \frac{1}{\Phi(\mu^u/s)} \int_{-\infty}^{m+\mu^u/s} \Phi(a + bz) \phi(z) dz$$

Based on in Owen (1980), the above is:

$$Pr(\epsilon_1 \leq m) = \frac{1}{\Phi(\mu^u/s)} \Phi_2 \left[ \frac{a}{\sqrt{1 + b^2}}, \frac{m + \mu^u}{s}, \rho = \frac{-b}{\sqrt{1 + b^2}} \right]$$

where $\Phi_2(x, y, \rho)$ is the bivariate normal CDF.
For $\epsilon_2$ the result is based on $\mu^d$, the term $z = (\epsilon_2 - \mu_d)/s$ and correlation coefficient is negative.

Substituting the expressions for $s, a, b$ in (10), the probability of no change in price in (8) is given by:

$$f(\Delta p_{it} | p_{it-1}, u_i) \quad \Phi_2 \left[ \frac{\mu_{it}^u}{\sigma_c}, \frac{\mu_{it}^u - d_{it}}{\sqrt{\sigma_e^2 + \sigma_c^2}}, \frac{\sigma_c}{\sqrt{\sigma_e^2 + \sigma_c^2}} \right] - \Phi \left( \frac{\mu_{it}^u}{\sigma_c} \right)$$

This completes the derivation of $f(\Delta p_{it} | p_{it-1}, u_i)$. The resulting MLE will be consistent if $N$ or $T \to \infty$. 
Stata Command: \texttt{xtss}

\begin{verbatim}
\texttt{xtss} \texttt{depvar} [\texttt{indepvars}] [\texttt{if}] [\texttt{in}] [\texttt{,} \texttt{thold(varlist)} \texttt{diff} \texttt{re} \texttt{hdraws(#)} \texttt{burn(#)} \texttt{level(#)} \texttt{noconstant}]
\end{verbatim}

\texttt{thold(varlist)} identifies the variables that appear in the mean equations of the upper and lower thresholds.

\texttt{diff} allows the coefficients in the mean equations of the upper and lower thresholds to differ; the default is they are the same.

\texttt{re} specifies that the model includes a random effect.

\texttt{hdraws(#)} specifies the number of halton draws for the simulation in the random effects model; the default is 50.

\texttt{burn(#)} specifies the initial sequences to drop; default is 15.
Numerical example

In this example, data is simulated on product prices supplied by various firms from the following DGP:

$$\log p_{it}^* = \alpha_j + \log \text{material}_{it} + 0.2 \text{cartel}_t + u_i + N(0, 0.1)$$

$$u_i \sim N(0, 0.1)$$

where $\alpha_j$ is a firm fixed effect, $\text{material}_{it}$ are material costs and $\text{cartel}_t = 1$ indicates collusion between firms.

Collusion also impacts the number of price changes, reducing the threshold for a rise and increasing the threshold for a fall:

$$c_{it}^u \sim N^+(0.2 - 0.1 \text{cartel}_t, 0.1)$$

$$c_{it}^d \sim N^+(0.2 + 0.1 \text{cartel}_t, 0.1)$$
Data is simulated for 150 products and 4 firms between 2005Q1 and 2009Q4 and the cartel operates from 2007Q1.

. use stickyprices.dta,clear
describe
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  vars: 9
  size: 108,000

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Sorted by: id quarter
Note: Dataset has changed since last saved.
Path of material costs

- Collusion leads to a 20% rise in potential prices. The formation of the cartel is triggered by a large rise in material costs over the same period.
The price paths are plotted for products 1 to 4. Prices are sticky with infrequent changes.
Prices remains constant for 65% of the sample, increases for 22% and falls for 13%.

The frequency of price rises increases during the cartel as threshold for a rise is reduced.

```
. tab direction cartel, nofreq col
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Histogram of % price changes

Graphs by cartel indicator
### Maximum likelihood estimates

- **xtss ln_price i.firm ln_materials cartel, thold(cartel) diff re**

(output omitted)

**ML random effects regression**

- Number of obs = 2,850
- Wald chi2(4) = 59062.83
- Prob > chi2 = 0.0000
- Log likelihood = -446.50523

| ln_price     | Coef.  | Std. Err. | z     | P>|z| | [95% Conf. Interval] |
|--------------|--------|-----------|-------|------|----------------------|
| **Model**    |        |           |       |      |                      |
| firm         | 1.015957 | .022717   | 44.72 | 0.000 | .9714329 - 1.060482 |
| firm2        | 1.029548 | .020269   | 50.79 | 0.000 | .9898214 - 1.069274 |
| firm3        | 1.029548 | .020269   | 50.79 | 0.000 | .9898214 - 1.069274 |
| ln_materials | .986478  | .0252836  | 39.02 | 0.000 | .936923 - 1.036033  |
| cartel       | .2121362 | .0261593  | 8.11  | 0.000 | .160865 - .2634074  |
| _cons        | .9832395 | .0166212  | 59.16 | 0.000 | .9506626 - 1.015816 |
| **Lower_threshold** | | | | | |
| cartel       | .0957907 | .0106661  | 8.98  | 0.000 | .0748855 - .1166959 |
| _cons        | .1920669 | .0082463  | 23.29 | 0.000 | .1759045 - .2082294 |
| **Upper_threshold** | | | | | |
| cartel       | -.0795848 | .0122898 | -6.48 | 0.000 | -.1036724 - -.0554972 |
| _cons        | .1918862 | .0082121  | 23.37 | 0.000 | .1757908 - .2079816 |
| /lnsigma_c   | -2.436115 | .0622327  | -39.15| 0.000 | -2.558089 - -2.314142 |
| /lnsigma_e   | -2.318582 | .018449   | -125.67| 0.000 | -2.354741 - -2.282422 |
| /lnsigma_u   | -2.214386 | .0571133  | -38.77| 0.000 | -2.326326 - -2.102446 |

| sigma_c      | .0875001 | .0054454  |       |      | .0774526 - .098851   |
| sigma_e      | .0984131 | .0018156  |       |      | .0949181 - .1020368  |
| sigma_u      | .1092206 | .0062379  |       |      | .0976539 - .1221573  |
Comparison with other estimators

- The frictionless price model (4) is estimated by OLS/RE and FE using amended prices (direction!="constant").

- The estimates are biased with no significant cartel overcharge.

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t statistics in parentheses
* p<0.1, ** p<0.05, *** p<0.01
Model predictions

- Predicted prices are the average of 2000 simulated trajectories using draws of $\epsilon_{it}$, $u_i$, $c_{it}^u$ and $c_{it}^d$ at the parameter estimates.
Price overcharge due to collusion

- The following plots the difference with and without the cartel. The maximum is above 20% as the frequency of rises increase.
Pass through of material costs

- The following shows the dynamic response across all products from a permanent 1% rise in material costs.
Conclusion

- This presentation has described a new Stata command `xtss` for estimating state dependent \((S,s)\) models based on Fougere et al. (2010) and Dhyne et al. (2011).

- This estimator is appropriate when the decision to amend a variable occurs when the deviation from a frictionless outcome is outside the stochastic \((S,s)\) thresholds.

- As per sample selection models, usual estimators of the outcome equation (4) fitted to the amendments will be inconsistent as the amendment decision is endogenous.

