Unbiased Instrumental Variables (IV) in Stata

Austin Nichols
2019 London Stata Conference

https://www.stata.com/meeting/uk19/
Magic Bullets

- Instrumental Variables (IV) methods are the only way to estimate causal effects in a variety of settings, including experiments (randomized control trials or RCTs) with imperfect compliance

- IV methods often exhibit poor performance
  - Bias & size distortion with many weak instruments
  - No finite moments when exactly identified

- Andrews and Armstrong (2017) offer a solution in Stata
Causal Diagram

• Conditioning on confounders does not in general solve the problem of endogenous participation in a treatment of interest.

• The receipt of a treatment (R=1) whose effect $\beta$ we want to measure may be randomly assigned (Z=1), but we still need IV to estimate impact.
Sign restriction allows unbiased IV

- IV has one fewer moments than overid restrictions, so exactly identified IV has no moments
  - Hirano and Porter (2015) show that mean, median, and quantile unbiased estimation are all impossible in the linear IV model with an unrestricted parameter space for the first stage

- This result no longer holds when the sign of the first stage is known (e.g. no defiers, some compliers):
  - In models with a single instrumental variable, Andrews and Armstrong (2017) show that there is a unique unbiased estimator based on the reduced form and first-stage regression estimates
  - This estimator is substantially less dispersed than the usual 2SLS estimator in finite samples

- In an RCT, we are very confident the first stage is positive
Model and Estimator

\[ Y = Z\pi \beta + u \] \( \leftarrow \) reduced form coef \( \xi_1 = (Z'Z)^{-1}(Z'Y) \)

\[ R = Z\pi + v \] \( \leftarrow \) first stage coef \( \xi_2 = (Z'Z)^{-1}(Z'R) \)

IV estimator constructs Wald ratio \( \xi_1 / \xi_2 \)

Assume \( u, v \) normal so \( (\xi_1, \xi_2) \sim N(\mu, \Sigma) \) w/variance \( \Sigma = (\sigma_1^2, \sigma_{12} \ \sigma_{12}, \sigma_2^2) \)

Let \( d = (\xi_1 - \xi_2 \sigma_{12}/\sigma_2^2). \) \( \text{E}[d] = \pi\beta - \pi\sigma_{12}/\sigma_2^2 \)

Voinov and Nikulin (1993) show that unbiased estimation of \( 1/\pi \) is possible if its sign is known:

Let \( t = \Phi(-\xi_2/\sigma_2)/\phi(\xi_2/\sigma_2)\sigma_2 \) then \( \text{E}[t] = 1/\pi \) and \( \text{E}[dt] = \text{E}[d]\text{E}[t] = \beta - \sigma_{12}/\sigma_2^2 \)

Estimator \( b_U = dt + s_{12}/v_2 \)
Further considerations

• $b_U$ is asymptotically equivalent to 2SLS when instruments are strong and thus $b_U$ can be used together with conventional 2SLS standard errors

• Optimal *estimation* and optimal *testing* are distinct questions in the context of weak instruments
  – $b_U$ is uniformly minimum risk unbiased for convex loss, but Moreira (2009) indicates that the Anderson–Rubin test is the *uniformly most powerful* unbiased two-sided test in the just-identified context (not a conditional t-test based on $b_U$)
  – more research needed on tests based on tests based on this unbiased IV estimator…
Small-Sample Properties

• Note this applies to bivariate normal errors with known variance, not the focal case of random assignment $Z=\{0,1\}$ and endogenous receipt of treatment $R=\{0,1\}$

  – Appendix B (Nonnormal errors and unknown reduced-form variance) “derives asymptotic results for the case with non-normal errors and an estimated reduced-form covariance matrix. Appendix B.1 shows asymptotic unbiasedness in the weak-instrument case. Appendix B.2 shows asymptotic equivalence with 2SLS in the strong-instrument case”

  – How does this approach perform in finite samples?
Stata command

- Estimator implemented as `aaniv` on SSC
- Download using `ssc install aaniv`
- So far, just one endogenous treatment and one excluded instrument (as of today), as is ideal for an RCT, but the command will be updated in future releases to a larger set of use cases
Small-Sample Properties

- Even with binary \( R \) and \( Z \), so non-normal errors by design, standard linear regression rejects the truth all the time, and unbiased IV outperforms standard IV/2SLS

  (this simulation has a high correlation between a normal variate that predicts \( R \) and the unobserved error that predicts the outcome \( Y \))
Distributions of Estimators by Sample Size and Correlation

Unbiased IV in Stata
Rejection rates about right for IV models, in large samples

Unbiased IV in Stata
Conclusion

• Unbiased IV performs as well as IV-2SLS in a setting that it is not explicitly designed for, with no bias and lower evident dispersion (but neither has a finite variance)
  – Report unbiased IV for an experiment, if only to enable meta-analysis; use `aaniv` in Stata (`ssc install aaniv`)

• Rejection rates for both Unbiased IV and IV 2SLS are approximately at the nominal rate when sample size is over a thousand
  – At smaller sample sizes, there is some under-rejection of a true null—using the deprecated t-tests, not the preferred AR test
Contact
Austin Nichols
Principal Scientist
austinnichols@gmail.com

Abt ASSOCIATES
BOLD THINKERS DRIVING REAL-WORLD IMPACT

abtassociates.com