Meta-analysis using Stata

Yulia Marchenko
Executive Director of Statistics
StataCorp LLC

2019 London Stata Conference
Acknowledgments

Brief introduction to meta-analysis

Stata’s meta-analysis suite

Meta-Analysis Control Panel

Motivating example: Effects of teacher expectancy on pupil IQ

Prepare data for meta-analysis

Meta-analysis summary: Forest plot

Heterogeneity: Subgroup analysis, meta-regression

Small-study effects and publication bias

Cumulative meta-analysis

Details: Meta-analysis models

Summary

Additional resources

References
Stata has a long history of meta-analysis methods contributed by Stata researchers, e.g. Palmer and Sterne (2016). We want to express our deep gratitude to Jonathan Sterne, Roger Harbord, Tom Palmer, David Fisher, Ian White, Ross Harris, Thomas Steichen, Mike Bradburn, Doug Altman (1948–2018), Ben Dwamena, and many more for their invaluable contributions. Their previous and still ongoing work on meta-analysis in Stata influenced the design and development of the official meta suite.
What is meta-analysis?

Meta-analysis (MA, Glass 1976) combines the results of multiple studies to provide a unified answer to a research question.

For instance,

- Does taking vitamin C prevent colds?
- Does exercise prolong life?
- Does lack of sleep increase the risk of cancer?
- Does daylight saving save energy?
- And more.
Does it make sense to combine different studies?

From Borenstein et al. (2009, chap. 40):

“In the early days of meta-analysis, Robert Rosenthal was asked whether it makes sense to perform a meta-analysis, given that the studies differ in various ways and that the analysis amounts to combining apples and oranges. Rosenthal answered that combining apples and oranges makes sense if your goal is to produce a fruit salad.”
Main goals of MA are:

- Provide an overall estimate of an effect, if sensible
- Explore between-study heterogeneity: studies often report different (and sometimes conflicting) results in terms of the magnitudes and even direction of the effects
- Evaluate the presence of publication bias—underreporting of nonsignificant results in the literature
Components of meta-analysis

- **Effect size**: standardized and raw mean differences, odds and risk ratios, risk difference, etc.
- **MA model**: common-effect, fixed-effects, random-effects
- **MA summary**—forest plot
- **Heterogeneity**—differences between effect-size estimates across studies in an MA
- **Small-study effects**—systematic differences between effect sizes reported by small versus large studies
- **Publication bias** or, more generally, **reporting bias**—systematic differences between studies included in an MA and all available relevant studies.
## Stata’s meta-analysis suite

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Declaration</strong></td>
<td></td>
</tr>
<tr>
<td>meta set</td>
<td>declare data using precalculated effect sizes</td>
</tr>
<tr>
<td>meta esize</td>
<td>calculate effect sizes and declare data</td>
</tr>
<tr>
<td>meta update</td>
<td>modify declaration of meta data</td>
</tr>
<tr>
<td>meta query</td>
<td>report how meta data are set</td>
</tr>
<tr>
<td><strong>Summary</strong></td>
<td></td>
</tr>
<tr>
<td>meta summarize</td>
<td>summarize MA results</td>
</tr>
<tr>
<td>meta forestplot</td>
<td>graph forest plots</td>
</tr>
</tbody>
</table>
**Heterogeneity**

- `meta summarize, subgroup()`
- `meta forestplot, subgroup()`
- `meta regress`
- `predict`
- `estat bubbleplot`
- `meta labbeplot`

**Small-study effects/ publication bias**

- `meta funnelplot`
- `meta bias`
- `meta trimfill`

**Cumulative analysis**

- `meta summarize, cumulative()`
- `meta forestplot, cumulative()`
You can work via commands or by using point-and-click: **Statistics > Meta-analysis.**

*(Continued on next page)*
Motivating example: Effects of teacher expectancy on pupil IQ

- Consider the famous meta-analysis study of Raudenbush (1984) that evaluated the effects of teacher expectancy on pupil IQ.
- The original study of Rosenthal and Jacobson (1968) discovered the so-called Pygmalion effect, in which expectations of teachers affected outcomes of their students.
- Later studies had trouble replicating the result.
- Raudenbush (1984) performed a meta-analysis of 19 studies to investigate the findings of multiple studies.
Data description

. webuse pupiliq
   (Effects of teacher expectancy on pupil IQ)
. describe studylbl stdmdiff se weeks week1

<table>
<thead>
<tr>
<th>variable name</th>
<th>storage type</th>
<th>display format</th>
<th>label</th>
<th>variable label</th>
</tr>
</thead>
<tbody>
<tr>
<td>studylbl</td>
<td>str26</td>
<td>%26s</td>
<td></td>
<td>Study label</td>
</tr>
<tr>
<td>stdmdiff</td>
<td>double</td>
<td>%9.0g</td>
<td></td>
<td>Standardized difference in means</td>
</tr>
<tr>
<td>se</td>
<td>double</td>
<td>%10.0g</td>
<td></td>
<td>Standard error of stdmdiff</td>
</tr>
<tr>
<td>weeks</td>
<td>byte</td>
<td>%9.0g</td>
<td></td>
<td>Weeks of prior teacher-student</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>contact</td>
</tr>
<tr>
<td>week1</td>
<td>byte</td>
<td>%9.0g</td>
<td>catweek1</td>
<td>Prior teacher-student contact &gt; 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>week</td>
</tr>
</tbody>
</table>
Meta-analysis using Stata

Motivating example: Effects of teacher expectancy on pupil IQ

Data description

```
. list studylbl stdmdiff se
```

<table>
<thead>
<tr>
<th></th>
<th>studylbl</th>
<th>stdmdiff</th>
<th>se</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Rosenthal et al., 1974</td>
<td>0.03</td>
<td>0.125</td>
</tr>
<tr>
<td>2</td>
<td>Conn et al., 1968</td>
<td>0.12</td>
<td>0.147</td>
</tr>
<tr>
<td>3</td>
<td>Jose &amp; Cody, 1971</td>
<td>-0.14</td>
<td>0.167</td>
</tr>
<tr>
<td>4</td>
<td>Pellegrini &amp; Hicks, 1972</td>
<td>1.18</td>
<td>0.373</td>
</tr>
<tr>
<td>5</td>
<td>Pellegrini &amp; Hicks, 1972</td>
<td>0.26</td>
<td>0.369</td>
</tr>
<tr>
<td>6</td>
<td>Evans &amp; Rosenthal, 1969</td>
<td>-0.06</td>
<td>0.103</td>
</tr>
<tr>
<td>7</td>
<td>Fielder et al., 1971</td>
<td>-0.02</td>
<td>0.103</td>
</tr>
<tr>
<td>8</td>
<td>Claiborn, 1969</td>
<td>-0.32</td>
<td>0.22</td>
</tr>
<tr>
<td>9</td>
<td>Kester, 1969</td>
<td>0.27</td>
<td>0.164</td>
</tr>
<tr>
<td>10</td>
<td>Maxwell, 1970</td>
<td>0.8</td>
<td>0.251</td>
</tr>
<tr>
<td>11</td>
<td>Carter, 1970</td>
<td>0.54</td>
<td>0.302</td>
</tr>
<tr>
<td>12</td>
<td>Flowers, 1966</td>
<td>0.18</td>
<td>0.223</td>
</tr>
<tr>
<td>13</td>
<td>Keshock, 1970</td>
<td>-0.02</td>
<td>0.289</td>
</tr>
<tr>
<td>14</td>
<td>Henrikson, 1970</td>
<td>0.23</td>
<td>0.29</td>
</tr>
<tr>
<td>15</td>
<td>Fine, 1972</td>
<td>-0.18</td>
<td>0.159</td>
</tr>
<tr>
<td>16</td>
<td>Grieger, 1970</td>
<td>-0.06</td>
<td>0.167</td>
</tr>
<tr>
<td>17</td>
<td>Rosenthal &amp; Jacobson, 1968</td>
<td>0.3</td>
<td>0.139</td>
</tr>
<tr>
<td>18</td>
<td>Fleming &amp; Anttonen, 1971</td>
<td>0.07</td>
<td>0.094</td>
</tr>
<tr>
<td>19</td>
<td>Ginsburg, 1970</td>
<td>-0.07</td>
<td>0.174</td>
</tr>
</tbody>
</table>
Declaration of your MA data is the first step of your MA in Stata.

- Use `meta set` to declare precomputed effect sizes.
- Use `meta esize` to compute (and declare) effect sizes from summary data.
• Declare precomputed effect sizes and their standard errors stored in variables `es` and `se`, respectively:

```
.meta set es se
```

• Or, compute, say, log odds-ratios from binary summary data stored in variables `n11`, `n12`, `n21`, and `n22`:

```
.meta esize n11 n12 n21 n22, esize(lnoratio)
```

• Or, compute, say, Hedges’s $g$ standardized mean differences from continuous summary data stored in variables `n1`, `m1`, `sd1`, `n2`, `m2`, `sd2`:

```
.meta esize n1 m1 sd1 n2 m2 sd2, esize(hedgesg)
```

• See [META] `meta data` for details.
Declaring pupil IQ dataset

Let’s use `meta set` to declare our pupil IQ data that contains precomputed effect sizes and their standard errors.

```
.meta set stdmdiff se
```

Meta-analysis setting information

Study information
- No. of studies: 19
- Study label: Generic
- Study size: N/A

Effect size
- Type: Generic
- Label: Effect Size
- Variable: stdmdiff

Precision
- Std. Err.: se
  - CI: [_meta_cil, _meta_ciu]
  - CI level: 95%

Model and method
- Model: Random-effects
- Method: REML
Declaring a meta-analysis model

In addition to effect sizes and their standard errors, one of the main components of your MA declaration is that of an MA model.

`meta` offers three models: random-effects (`random`), the default, common-effect (aka “fixed-effect”, common), and fixed-effects (`fixed`).

The selected MA model determines the availability of the MA methods and, more importantly, how you interpret the obtained results.

See Details: Meta-analysis models below as well as Meta-analysis models in [META] Intro and Declaring a meta-analysis model in [META] meta data.
Meta-analysis summary

- Use `meta summarize` to obtain MA summary in a table.
- Use `meta forestplot` to summarize MA data graphically—produce forest plot.
- See [META] `meta summarize` and [META] `meta forestplot` for details.
. meta summarize

**Effect-size label:** Effect Size  
**Effect size:** stdmdiff  
**Std. Err.:** se

---

**Meta-analysis summary**  
Random-effects model  
Method: REML  
Heterogeneity:  
\( \tau^2 = 0.0188 \)  
\( I^2 (\%) = 41.84 \)  
\( H^2 = 1.72 \)

---

<table>
<thead>
<tr>
<th>Study</th>
<th>Effect Size</th>
<th>[95% Conf. Interval]</th>
<th>% Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Study 1</td>
<td>0.030</td>
<td>-0.215 to 0.275</td>
<td>7.74</td>
</tr>
<tr>
<td>Study 2</td>
<td>0.120</td>
<td>-0.168 to 0.408</td>
<td>6.60</td>
</tr>
<tr>
<td>Study 3</td>
<td>-0.140</td>
<td>-0.467 to 0.187</td>
<td>5.71</td>
</tr>
<tr>
<td>Study 4</td>
<td>1.180</td>
<td>0.449 to 1.911</td>
<td>1.69</td>
</tr>
<tr>
<td>Study 5</td>
<td>0.260</td>
<td>-0.463 to 0.983</td>
<td>1.72</td>
</tr>
<tr>
<td>Study 6</td>
<td>-0.060</td>
<td>-0.262 to 0.142</td>
<td>9.06</td>
</tr>
<tr>
<td>Study 7</td>
<td>-0.020</td>
<td>-0.222 to 0.182</td>
<td>9.06</td>
</tr>
<tr>
<td>Study 8</td>
<td>-0.320</td>
<td>-0.751 to 0.111</td>
<td>3.97</td>
</tr>
<tr>
<td>Study 9</td>
<td>0.270</td>
<td>-0.051 to 0.591</td>
<td>5.84</td>
</tr>
<tr>
<td>Study 10</td>
<td>0.800</td>
<td>0.308 to 1.292</td>
<td>3.26</td>
</tr>
<tr>
<td>Study 11</td>
<td>0.540</td>
<td>-0.052 to 1.132</td>
<td>2.42</td>
</tr>
<tr>
<td>Study 12</td>
<td>0.180</td>
<td>-0.257 to 0.617</td>
<td>3.89</td>
</tr>
<tr>
<td>Study 13</td>
<td>-0.020</td>
<td>-0.586 to 0.546</td>
<td>2.61</td>
</tr>
<tr>
<td>Study 14</td>
<td>0.230</td>
<td>-0.338 to 0.798</td>
<td>2.59</td>
</tr>
<tr>
<td>Study 15</td>
<td>-0.180</td>
<td>-0.492 to 0.132</td>
<td>6.05</td>
</tr>
<tr>
<td>Study 16</td>
<td>-0.060</td>
<td>-0.387 to 0.267</td>
<td>5.71</td>
</tr>
<tr>
<td>Study 17</td>
<td>0.300</td>
<td>0.028 to 0.572</td>
<td>6.99</td>
</tr>
<tr>
<td>Study 18</td>
<td>0.070</td>
<td>-0.114 to 0.254</td>
<td>9.64</td>
</tr>
<tr>
<td>Study 19</td>
<td>-0.070</td>
<td>-0.411 to 0.271</td>
<td>5.43</td>
</tr>
</tbody>
</table>

**\( \theta \)**  
0.084 | -0.018 | 0.185

---

Test of \( \theta = 0 \):  
\( z = 1.62 \)  
\( \text{Prob} > |z| = 0.1052 \)

Test of homogeneity:  
\( Q = \chi^2(18) = 35.83 \)  
\( \text{Prob} > Q = 0.0074 \)

---

Yulia Marchenko (StataCorp)
Use meta update to modify your MA settings.

.meta update, studylabel(studylbl) eslabel(Std. Mean Diff.)
-> meta set stdmdiff se , random(reml) studylabel(studylbl) eslabel(Std. Mean Diff.)

Meta-analysis setting information from meta set

Study information
  No. of studies: 19
  Study label: studylbl
  Study size: N/A

Effect size
  Type: Generic
  Label: Std. Mean Diff.
  Variable: stdmdiff

Precision
  Std. Err.: se
  CI: [_meta_cil, _meta_ciu]
  CI level: 95%

Model and method
  Model: Random-effects
  Method: REML
Forest plot

- Use `meta forestplot` to produce forest plots.
- Specify options or use the **Graph Editor** to modify the default look.

```
.meta forestplot
    Effect-size label: Std. Mean Diff.
    Effect size: stdmdiff
    Std. Err.: se
    Study label: studylbl
```

*(Continued on next page)*
Random–effects REML model
Between-study heterogeneity

- The previous forest plot reveals noticeable between-study variation.
- Raudenbush (1984) suspected that the amount of time that the teachers spent with students prior to the experiment may influence the teachers’ susceptibility to researchers’ categorization of students.
- One solution is to incorporate moderators (study-level covariates) into an MA.
- Subgroup analysis for categorical moderators.
- Meta-regression for continuous and a mixture of moderators.
Binary variable `week1` divides the studies into high-contact (`week1=1`) and low-contact (`week1=0`) groups.

```
.meta forestplot, subgroup(week1)
```

- Effect-size label: Std. Mean Diff.
- Effect size: stdmdiff
- Std. Err.: se
- Study label: studylbl

(Continued on next page)
Study | Std. Mean Diff. with 95% CI | Weight (%)
--- | --- | ---

**<= 1 week**
Pellegrini & Hicks, 1972 | 1.18 [ 0.45, 1.91] | 1.69
Pellegrini & Hicks, 1972 | 0.26 [ -0.46, 0.98] | 1.72
Kester, 1969 | 0.27 [ -0.05, 0.59] | 5.84
Maxwell, 1970 | 0.80 [ 0.31, 1.29] | 3.26
Carter, 1970 | 0.54 [ -0.05, 1.13] | 2.42
Flowers, 1966 | 0.18 [ -0.26, 0.62] | 3.89
Keshock, 1970 | -0.02 [ -0.59, 0.55] | 2.61
Rosenthal & Jacobson, 1968 | 0.30 [ 0.03, 0.57] | 6.99
Heterogeneity: $\tau^2 = 0.02$, $\hat{I}^2 = 22.40\%$, $H^2 = 1.29$ | 0.37 [ 0.19, 0.56] |
Test of $\theta_i = \theta_j$; $Q(7) = 11.20$, $p = 0.13$

**> 1 week**
Rosenthal et al., 1974 | 0.03 [ -0.21, 0.27] | 7.74
Conn et al., 1968 | 0.12 [ -0.17, 0.41] | 6.60
Jose & Cody, 1971 | -0.14 [ -0.47, 0.19] | 5.71
Evans & Rosenthal, 1969 | -0.06 [ -0.26, 0.14] | 9.06
Fielder et al., 1971 | -0.02 [ -0.22, 0.18] | 9.06
Claiborn, 1969 | -0.32 [ -0.75, 0.11] | 3.97
Henrikson, 1970 | 0.23 [ -0.34, 0.80] | 2.59
Fine, 1972 | -0.18 [ -0.49, 0.13] | 6.05
Grieger, 1970 | -0.06 [ -0.39, 0.27] | 5.71
Fleming & Anttonen, 1971 | 0.07 [ -0.11, 0.25] | 9.64
Ginsburg, 1970 | -0.07 [ -0.41, 0.27] | 5.43
Heterogeneity: $\tau^2 = 0.00$, $\hat{I}^2 = 0.00\%$, $H^2 = 1.00$ | -0.02 [ -0.10, 0.06] |
Test of $\theta_i = \theta_j$; $Q(10) = 6.40$, $p = 0.78$

**Overall**
Heterogeneity: $\tau^2 = 0.02$, $\hat{I}^2 = 41.84\%$, $H^2 = 1.72$
Test of $\theta = \theta_j$; $Q(18) = 35.83$, $p = 0.01$
Test of group differences: $Q_a(1) = 14.77$, $p = 0.00$

Random-effects REML model
Heterogeneity: Meta-regression

- Perform meta-regression using a continuous variable, weeks.

```
.meta regress weeks
Effect-size label: Std. Mean Diff.
Effect size: stdmdiff
Std. Err.: se
Random-effects meta-regression
Method: REML
```

Number of obs = 19

Residual heterogeneity:
- tau2 = .01117
- I2 (%) = 29.36
- H2 = 1.42
- R-squared (%) = 40.70
- Wald chi2(1) = 7.51
- Prob > chi2 = 0.0061

|        | Coef.   | Std. Err. | z     | P>|z|   | [95% Conf. Interval] |
|--------|---------|-----------|-------|-------|----------------------|
| weeks  | -.0157453 | .0057447  | -2.74 | 0.006 | -.0270046 -.0044859 |
| _cons  | .1941774  | .0633563  | 3.06  | 0.002 | .0700013 .3183535   |

Test of residual homogeneity: Q_res = chi2(17) = 27.66
Prob > Q_res = 0.0490
Meta-regression: Bubble plot

- Explore the relationship between effect sizes and weeks.
  
  . estat bubbleplot

- Negative relationship; some of the more precise studies are outlying studies.
Funnel plot

- Explore funnel-plot asymmetry visually.

```
.meta funnelplot
   Effect-size label: Std. Mean Diff.
   Effect size: stdmdiff
   Std. Err.: se
   Model: Common-effect
   Method: Inverse-variance
```

![Funnel plot image]
Test for funnel-plot asymmetry

Explore funnel-plot asymmetry more formally.

. meta bias, egger
   Effect-size label: Std. Mean Diff.
   Effect size: stdmdiff
   Std. Err.: se

Regression-based Egger test for small-study effects
Random-effects model
Method: REML
H0: beta1 = 0; no small-study effects
   beta1 = 1.83
   SE of beta1 = 0.724
   z = 2.53
   Prob > |z| = 0.0115

Beware of the presence of heterogeneity! See **Small-study effects** below.
Contour-enhanced funnel plot

- Add 1%, 5%, and 10% significance contours

```stata
.meta funnelplot, contours(1 5 10)

Effect-size label: Std. Mean Diff.
Effect size: stdmdiff
Std. Err.: se
Model: Common-effect
Method: Inverse-variance
```
Small-study effects

Keeping in mind the presence of heterogeneity in these data, let's produce funnel plots separately for each group of week1.

```
.meta funnelplot, by(week1)
Effect-size label: Std. Mean Diff.
Effect size: stdmdiff
Std. Err.: se
Model: Common-effect
Method: Inverse-variance
```
Or, more formally,

```
. meta bias i.week1, egger
    Effect-size label:  Std. Mean Diff.
    Effect size:       stdmdiff
    Std. Err.:        se

Regression-based Egger test for small-study effects
Random-effects model
Method: REML
Moderators: week1

H0: beta1 = 0; no small-study effects
    beta1 =     0.30
    SE of beta1 = 0.729
    z =    0.41
    Prob > |z| = 0.6839
```
When publication bias is suspect, you can use the trim-and-fill method to assess the impact of publication bias on the MA results.

In our example, the asymmetry of the funnel plot is likely due to heterogeneity, not publication bias.

But, for the purpose of demonstration, let’s go ahead and apply the trim-and-fill method to these data.
Meta-analysis using Stata

Small-study effects and publication bias

Assess publication bias

. `meta trimfill, funnel`

Effect-size label: Std. Mean Diff.

Effect size: stdmdiff

Std. Err.: se

Nonparametric trim-and-fill analysis of publication bias

Linear estimator, imputing on the left

Iteration

Number of studies = 22

Model: Random-effects observed = 19

Method: REML imputed = 3

Pooling

Model: Random-effects

Method: REML

<table>
<thead>
<tr>
<th>Studies</th>
<th>Std. Mean Diff.</th>
<th>[95% Conf. Interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed</td>
<td>0.084</td>
<td>-0.018 0.185</td>
</tr>
<tr>
<td>Observed + Imputed</td>
<td>0.028</td>
<td>-0.117 0.173</td>
</tr>
</tbody>
</table>

(Continued on next page)
Meta-analysis using Stata

Assess publication bias

Funnel plot

- Standard error
- Observed studies
- Imputed studies
- Pseudo 95% CI

Yulia Marchenko (StataCorp)
Cumulative meta-analysis

- Cumulative MA performs multiple MAs by accumulating studies one at a time after ordering them with respect to the variable of interest.
- Cumulative MA is useful for monitoring the trends in effect-size estimates with respect to the ordering variable.
- Use option cumulative() with meta summarize or meta forestplot to perform cumulative MA.

```
. meta forestplot, cumulative(weeks)
Effect-size label: Std. Mean Diff.
  Effect size: stdmdiff
  Std. Err.: se
  Study label: studylbl
```

(Continued on next page)
<table>
<thead>
<tr>
<th>Study</th>
<th>Std. Mean Diff. with 95% CI</th>
<th>P-value</th>
<th>weeks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pellegrini &amp; Hicks, 1972</td>
<td>1.18 [0.45, 1.91]</td>
<td>0.002</td>
<td>0</td>
</tr>
<tr>
<td>Pellegrini &amp; Hicks, 1972</td>
<td>0.72 [−0.18, 1.62]</td>
<td>0.118</td>
<td>0</td>
</tr>
<tr>
<td>Kester, 1969</td>
<td>0.52 [−0.03, 1.06]</td>
<td>0.064</td>
<td>0</td>
</tr>
<tr>
<td>Carter, 1970</td>
<td>0.49 [0.13, 0.86]</td>
<td>0.008</td>
<td>0</td>
</tr>
<tr>
<td>Flowers, 1966</td>
<td>0.39 [0.13, 0.64]</td>
<td>0.003</td>
<td>0</td>
</tr>
<tr>
<td>Maxwell, 1970</td>
<td>0.48 [0.20, 0.76]</td>
<td>0.001</td>
<td>1</td>
</tr>
<tr>
<td>Keshock, 1970</td>
<td>0.42 [0.15, 0.68]</td>
<td>0.002</td>
<td>1</td>
</tr>
<tr>
<td>Rosenthal &amp; Jacobson, 1968</td>
<td>0.37 [0.19, 0.56]</td>
<td>0.000</td>
<td>1</td>
</tr>
<tr>
<td>Rosenthal et al., 1974</td>
<td>0.32 [0.12, 0.52]</td>
<td>0.002</td>
<td>2</td>
</tr>
<tr>
<td>Henrikson, 1970</td>
<td>0.31 [0.13, 0.49]</td>
<td>0.001</td>
<td>2</td>
</tr>
<tr>
<td>Fleming &amp; Anttonen, 1971</td>
<td>0.26 [0.10, 0.42]</td>
<td>0.001</td>
<td>2</td>
</tr>
<tr>
<td>Evans &amp; Rosenthal, 1969</td>
<td>0.23 [0.07, 0.38]</td>
<td>0.005</td>
<td>3</td>
</tr>
<tr>
<td>Grieger, 1970</td>
<td>0.20 [0.05, 0.34]</td>
<td>0.008</td>
<td>5</td>
</tr>
<tr>
<td>Ginsburg, 1970</td>
<td>0.17 [0.04, 0.31]</td>
<td>0.013</td>
<td>7</td>
</tr>
<tr>
<td>Fielder et al., 1971</td>
<td>0.14 [0.02, 0.26]</td>
<td>0.019</td>
<td>17</td>
</tr>
<tr>
<td>Fine, 1972</td>
<td>0.12 [0.00, 0.24]</td>
<td>0.043</td>
<td>17</td>
</tr>
<tr>
<td>Jose &amp; Cody, 1971</td>
<td>0.10 [−0.01, 0.21]</td>
<td>0.071</td>
<td>19</td>
</tr>
<tr>
<td>Conn et al., 1968</td>
<td>0.10 [−0.00, 0.20]</td>
<td>0.056</td>
<td>21</td>
</tr>
<tr>
<td>Claiborn, 1969</td>
<td>0.08 [−0.02, 0.18]</td>
<td>0.105</td>
<td>24</td>
</tr>
</tbody>
</table>

Random-effects REML model
Details: Meta-analysis models

- **Common-effect (CE) model** (aka fixed-effect model, notice singular “fixed”):

  \[ \hat{\theta}_j = \theta + \epsilon_j \]

  \( \theta \) is the true common effect, \( \hat{\theta}_j \)'s are \( K \) previously estimated study-specific effects with their standard errors \( \hat{\sigma}^2_j \)'s, and \( \epsilon_j \sim N(0, \hat{\sigma}^2_j) \).

- **Fixed-effects (FE) model**:

  \[ \hat{\theta}_j = \theta_j + \epsilon_j \]

  \( \theta_j \)'s are unknown, “fixed” study-specific effects.

- **Random-effects (RE) model**:

  \[ \hat{\theta}_j = \theta_j + \epsilon_j = \theta + u_j + \epsilon_j \]

  \( \theta_j \sim N(\theta, \tau^2) \) or \( u_j \sim N(0, \tau^2) \).
Estimator of the overall effect

- The three models differ in the population parameter, $\theta_{\text{pop}}$, they estimate:
  - CE model: $\theta_{\text{pop}} = \theta$ is a common effect;
  - FE model: $\theta_{\text{pop}}$ is a weighted average of the $K$ true study effects (Rice, Higgins, and Lumley 2018); and
  - RE model: $\theta_{\text{pop}} = \theta$ is the mean of the distribution of the study effects.

- But they all use the weighted average as the estimator of $\theta_{\text{pop}}$:
  \[
  \hat{\theta}_{\text{pop}} = \frac{\sum_{j=1}^{K} w_j \hat{\theta}_j}{\sum_{j=1}^{K} w_j}
  \]
  where $w_j$ depends on the model.
Random-effects model: Stata’s default

- Study-specific effects may vary between studies.
- They are viewed as a random sample from a larger population of studies.
- RE model adjusts for unexplained between-study variability.
- RE model is Stata’s default for MA.
. quietly meta update, nometashow
. meta summarize

Meta-analysis summary
Number of studies = 19

Random-effects model
Heterogeneity:
Method: REML
tau2 = 0.0188
I2 (%) = 41.84
H2 = 1.72

Effect Size: Std. Mean Diff.

<table>
<thead>
<tr>
<th>Study</th>
<th>Effect Size</th>
<th>[95% Conf. Interval]</th>
<th>% Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rosenthal et al., 1974</td>
<td>0.030</td>
<td>-0.215</td>
<td>0.275 7.74</td>
</tr>
<tr>
<td>Conn et al., 1968</td>
<td>0.120</td>
<td>-0.168</td>
<td>0.408 6.60</td>
</tr>
<tr>
<td>Jose &amp; Cody, 1971</td>
<td>-0.140</td>
<td>-0.467</td>
<td>0.187 5.71</td>
</tr>
<tr>
<td>Pellegrini &amp; Hicks, 1972</td>
<td>1.180</td>
<td>0.449</td>
<td>1.911 1.69</td>
</tr>
<tr>
<td>Pellegrini &amp; Hicks, 1972</td>
<td>0.260</td>
<td>-0.463</td>
<td>0.983 1.72</td>
</tr>
<tr>
<td>Evans &amp; Rosenthal, 1969</td>
<td>-0.060</td>
<td>-0.262</td>
<td>0.142 9.06</td>
</tr>
<tr>
<td>Fielder et al., 1971</td>
<td>-0.020</td>
<td>-0.222</td>
<td>0.182 9.06</td>
</tr>
<tr>
<td>Claiborn, 1969</td>
<td>-0.320</td>
<td>-0.751</td>
<td>0.111 3.97</td>
</tr>
<tr>
<td>Kester, 1969</td>
<td>0.270</td>
<td>-0.051</td>
<td>0.591 5.84</td>
</tr>
<tr>
<td>Maxwell, 1970</td>
<td>0.800</td>
<td>0.308</td>
<td>1.292 3.26</td>
</tr>
<tr>
<td>Carter, 1970</td>
<td>0.540</td>
<td>-0.052</td>
<td>1.132 2.42</td>
</tr>
<tr>
<td>Flowers, 1966</td>
<td>0.180</td>
<td>-0.257</td>
<td>0.617 3.89</td>
</tr>
<tr>
<td>Keshock, 1970</td>
<td>-0.020</td>
<td>-0.586</td>
<td>0.546 2.61</td>
</tr>
<tr>
<td>Henrikson, 1970</td>
<td>0.230</td>
<td>-0.338</td>
<td>0.798 2.59</td>
</tr>
<tr>
<td>Fine, 1972</td>
<td>-0.180</td>
<td>-0.492</td>
<td>0.132 6.05</td>
</tr>
<tr>
<td>Grieger, 1970</td>
<td>-0.060</td>
<td>-0.387</td>
<td>0.267 5.71</td>
</tr>
<tr>
<td>Rosenthal &amp; Jacobson, 1968</td>
<td>0.300</td>
<td>0.028</td>
<td>0.572 6.99</td>
</tr>
<tr>
<td>Fleming &amp; Anttonen, 1971</td>
<td>0.070</td>
<td>-0.114</td>
<td>0.254 9.64</td>
</tr>
<tr>
<td>Ginsburg, 1970</td>
<td>-0.070</td>
<td>-0.411</td>
<td>0.271 5.43</td>
</tr>
</tbody>
</table>

theta 0.084 -0.018 0.185

Test of theta = 0: z = 1.62
Prob > |z| = 0.1052
Test of homogeneity: Q = chi2(18) = 35.83
Prob > Q = 0.0074
Common-effect model

- Historically known as a “fixed-effect model” (singular “fixed”)
- New terminology due to Rice, Higgins, and Lumley (2018)
- One common effect: $\theta_1 = \theta_2 = \ldots = \theta_K = \theta$
- Should not be used in the presence of study heterogeneity
- For demonstration purposes only here, ...
. meta summarize, common

Meta-analysis summary
Common-effect model
Method: Inverse-variance

Effect Size: Std. Mean Diff.

<table>
<thead>
<tr>
<th>Study</th>
<th>Effect Size</th>
<th>[95% Conf. Interval]</th>
<th>% Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rosenthal et al., 1974</td>
<td>0.030</td>
<td>-0.215 0.275</td>
<td>8.52</td>
</tr>
<tr>
<td>Conn et al., 1968</td>
<td>0.120</td>
<td>-0.168 0.408</td>
<td>6.16</td>
</tr>
<tr>
<td>Jose &amp; Cody, 1971</td>
<td>-0.140</td>
<td>-0.467 0.187</td>
<td>4.77</td>
</tr>
<tr>
<td>Pellegrini &amp; Hicks, 1972</td>
<td>1.180</td>
<td>0.449 1.911</td>
<td>0.96</td>
</tr>
<tr>
<td>Pellegrini &amp; Hicks, 1972</td>
<td>0.260</td>
<td>-0.463 0.983</td>
<td>0.98</td>
</tr>
<tr>
<td>Evans &amp; Rosenthal, 1969</td>
<td>-0.060</td>
<td>-0.262 0.142</td>
<td>12.55</td>
</tr>
<tr>
<td>Fielder et al., 1971</td>
<td>-0.020</td>
<td>-0.222 0.182</td>
<td>12.55</td>
</tr>
<tr>
<td>Claiborn, 1969</td>
<td>-0.320</td>
<td>-0.751 0.111</td>
<td>2.75</td>
</tr>
<tr>
<td>Kester, 1969</td>
<td>0.270</td>
<td>-0.051 0.591</td>
<td>4.95</td>
</tr>
<tr>
<td>Maxwell, 1970</td>
<td>0.800</td>
<td>0.308 1.292</td>
<td>2.11</td>
</tr>
<tr>
<td>Carter, 1970</td>
<td>0.540</td>
<td>-0.052 1.132</td>
<td>1.46</td>
</tr>
<tr>
<td>Flowers, 1966</td>
<td>0.180</td>
<td>-0.257 0.617</td>
<td>2.68</td>
</tr>
<tr>
<td>Keshock, 1970</td>
<td>-0.020</td>
<td>-0.586 0.546</td>
<td>1.59</td>
</tr>
<tr>
<td>Henrikson, 1970</td>
<td>0.230</td>
<td>-0.338 0.798</td>
<td>1.58</td>
</tr>
<tr>
<td>Fine, 1972</td>
<td>-0.180</td>
<td>-0.492 0.132</td>
<td>5.27</td>
</tr>
<tr>
<td>Grieger, 1970</td>
<td>-0.060</td>
<td>-0.387 0.267</td>
<td>4.77</td>
</tr>
<tr>
<td>Rosenthal &amp; Jacobson, 1968</td>
<td>0.300</td>
<td>0.028 0.572</td>
<td>6.89</td>
</tr>
<tr>
<td>Fleming &amp; Anttonen, 1971</td>
<td>0.070</td>
<td>-0.114 0.254</td>
<td>15.07</td>
</tr>
<tr>
<td>Ginsburg, 1970</td>
<td>-0.070</td>
<td>-0.411 0.271</td>
<td>4.40</td>
</tr>
</tbody>
</table>

| theta                  | 0.060       | -0.011 0.132         |

Test of theta = 0: z = 1.65    Prob > |z| = 0.0981
Fixed-effects model

- Study-specific effects may vary between studies.
- They are considered “fixed”.
- FE model produces the same estimates as the CE model but their interpretation is different!
- Two different options, common and fixed, are provided to emphasize the conceptual differences between the two models.
Meta-analysis using Stata

Meta-analysis models

Fixed-effects model

```
.meta summarize, fixed

Meta-analysis summary
Number of studies = 19

Fixed-effects model
Heterogeneity:
Method: Inverse-variance
I2 (%) = 49.76
H2 = 1.99

Effect Size: Std. Mean Diff.

<table>
<thead>
<tr>
<th>Study</th>
<th>Effect Size</th>
<th>[95% Conf. Interval]</th>
<th>% Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rosenthal et al., 1974</td>
<td>0.030</td>
<td>-0.215</td>
<td>0.275</td>
</tr>
<tr>
<td>Conn et al., 1968</td>
<td>0.120</td>
<td>-0.168</td>
<td>0.408</td>
</tr>
<tr>
<td>Jose &amp; Cody, 1971</td>
<td>-0.140</td>
<td>-0.467</td>
<td>0.187</td>
</tr>
<tr>
<td>Pellegrini &amp; Hicks, 1972</td>
<td>1.180</td>
<td>0.449</td>
<td>1.911</td>
</tr>
<tr>
<td>Pellegrini &amp; Hicks, 1972</td>
<td>0.260</td>
<td>-0.463</td>
<td>0.983</td>
</tr>
<tr>
<td>Evans &amp; Rosenthal, 1969</td>
<td>-0.060</td>
<td>-0.262</td>
<td>0.142</td>
</tr>
<tr>
<td>Fielder et al., 1971</td>
<td>-0.020</td>
<td>-0.222</td>
<td>0.182</td>
</tr>
<tr>
<td>Claiborn, 1969</td>
<td>-0.320</td>
<td>-0.751</td>
<td>0.111</td>
</tr>
<tr>
<td>Kester, 1969</td>
<td>0.270</td>
<td>-0.051</td>
<td>0.591</td>
</tr>
<tr>
<td>Maxwell, 1970</td>
<td>0.800</td>
<td>0.308</td>
<td>1.292</td>
</tr>
<tr>
<td>Carter, 1970</td>
<td>0.540</td>
<td>-0.052</td>
<td>1.132</td>
</tr>
<tr>
<td>Flowers, 1966</td>
<td>0.180</td>
<td>-0.257</td>
<td>0.617</td>
</tr>
<tr>
<td>Keshock, 1970</td>
<td>-0.020</td>
<td>-0.586</td>
<td>0.546</td>
</tr>
<tr>
<td>Henrikson, 1970</td>
<td>0.230</td>
<td>-0.338</td>
<td>0.798</td>
</tr>
<tr>
<td>Fine, 1972</td>
<td>-0.180</td>
<td>-0.492</td>
<td>0.132</td>
</tr>
<tr>
<td>Grieger, 1970</td>
<td>-0.060</td>
<td>-0.387</td>
<td>0.267</td>
</tr>
<tr>
<td>Rosenthal &amp; Jacobson, 1968</td>
<td>0.300</td>
<td>0.028</td>
<td>0.572</td>
</tr>
<tr>
<td>Fleming &amp; Anttonen, 1971</td>
<td>0.070</td>
<td>-0.114</td>
<td>0.254</td>
</tr>
<tr>
<td>Ginsburg, 1970</td>
<td>-0.070</td>
<td>-0.411</td>
<td>0.271</td>
</tr>
</tbody>
</table>

theta | 0.060 | -0.011 | 0.132 |
```

Test of theta = 0: z = 1.65
Prob > |z| = 0.0981

Test of homogeneity: Q = chi2(18) = 35.83
Prob > Q = 0.0074
Summary

- `meta` is a new suite of commands available in Stata 16 to perform MA.
- Three MA models are supported: random-effects (default, `random`), common-effect (aka “fixed-effect”, `common`), and fixed-effects (`fixed`).
- Various estimation methods are supported including DerSimonian–Laird and Mantel–Haenszel.
- Declare and compute your effect sizes and standard errors upfront using `meta set` or `meta esize`. Declare other information for your entire MA session. Use `meta update` to update any meta settings during your MA session.
Compute basic MA summary using `meta summarize` and produce forest plots using `meta forestplot`.

Explore heterogeneity via subgroup analysis (e.g., `meta forestplot`, `subgroup()`) or meta-regression (`meta regress`).

Explore small-study effects and publication bias by producing funnel plots (`meta funnelplot`, `meta funnelplot`, `contours()`) and by testing for funnel-plot asymmetry (`meta bias`).

Assess the impact of publication bias, when it is suspected, by using `meta trimfill`.

Perform cumulative MA by using `meta forestplot`, `cumulative()` and `meta summarize`, `cumulative()`.
Additional resources

- Full list of MA features: https://www.stata.com/features/meta-analysis/
- YouTube: Meta-analysis in Stata—https://youtu.be/8zzZojXnXJg


Raudenbush, S. W. 1984. Magnitude of teacher expectancy effects on pupil IQ as a function of the credibility of expectancy induction:
A synthesis of findings from 18 experiments. *Journal of Educational Psychology* 76: 85–97.
