How to assess the fit of multilevel logit models with Stata? A project in progress

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"Models should not be true but it is important that they are applicable." (John W. Tukey)

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Contents

1. What is the problem ?

Current situation in applied research:

- An increasing number of people use multilevel logistic models for qualitative dependent variables with binary and ordinal outcome
- But users often complain that there are no fit measures for these models
- Neither Stata 16 nor SPSS 26 offer any fit measure for these models
- Let me demonstrate how to generalize the Pseudo R^2s for binary and ordinal logit model for the multilevel analysis

Which solutions does Stata provide?

- Indeed Stata estimates multilevel logit models for binary, ordinal and multinomial outcomes (melogit, meologit, gllamm) but it does not calculate any Pseudo R^2. It provides only the Akaike- (AIC) and Schwarz-Bayesian-Information Criteria (BIC)
- Stata provides a Wald test for the fixed effects and a Likelihood-Ratio-χ^2 test for the random effects of the exogenous variables
- Even special purpose programs like HLM, MlwiN, MPLUS or SuperMix do not calculate any Pseudo R^2

What can we learn from multilevel literature?

- Raudenbush & Bryk (2002), Heck & Thomas (2009) and Rabe-Hesketh & Skrondal (2013) do not mention Peudo R^2s at all
- Snijder & Bosker (2012) propose a variation of McKelvey & Zavoina Pseudo R^2 for random-intercept and intercept-as-outcome logit models. It is not implemented in any program
- Hox (2010) discusses the McFadden, Cox & Snell, Nagelkerke and McKelvey & Zavoina Pseudo R^2. He recommends the last one to assess the model fit

2. Summary of the econometric Monte-Carlo studies for testing Pseudo R^2s

- Econometricians made a lot of Monte-Carlo studies in the 1990s:
  - Hagle & Mitchell 1992
  - Veall & Zimmermann 1992, 1993, 1994
  - Windmeijer 1995
  - DeMaris 2002
- They systematically tested the most common Pseudo-R^2s for binary and ordinal probit / logit models

Which Pseudo R^2s were tested in these studies?

- Likelihood-based measures:
  - Maddala / Cox & Snell Pseudo R^2 (1983 / 1989)
  - Cragg & Uhler / Nagelkerke Pseudo R^2 (1970 / 1992)
- Log-Likelihood-based measures:
  - McFadden Pseudo R^2 (1974)
  - Aldrich & Nelson Pseudo R^2 (1984)
  - Aldrich & Nelson Pseudo R^2 with the Veall & Zimmer-mann correction (1992)
- Basing on the estimated probabilities:
  - Efron / Lave Pseudo R^2 (1970 / 1978)
- Basing on the variance decomposition of the estimated Probits / Logits:
  - McKelvey & Zavoina Pseudo R^2 (1975)

Results of the Monte-Carlo-Studies for binary and ordinal logits or probits

- The McKelvey & Zavoina Pseudo R^2 is the best estimator for the “true R^2” of the OLS regression
- The Aldrich & Nelson Pseudo R^2 with the Veall & Zimmermann correction is the best approximation of the McKelvey & Zavoina Pseudo R^2
- Lave / Efron, Aldrich & Nelson, McFadden and Cragg & Uhler Pseudo R^2 severely underestimate the “true R^2” of the OLS regression
- My personal advice:
  - Use the McKelvey & Zavoina Pseudo R^2 to assess the fit of binary and ordinal logit models

3. The generalization of the McKelvey & Zavoina Pseudo R^2 for the binary and ordinal multilevel logit model

- The multilevel logit model is a systematic extension of the classical binary and ordinal logit model for clustered subsamples (contextual units j)

  - The variance of the estimated logits is decomposed into ▸ Fixed effects, ▸ Random effects and ▸ Level-1 Error variance $\sigma^2(r_{ij})$
  - The variance of level 1 residua $\sigma^2(r_{ij})$ can not be estimated because of its own heteroscedasticity. It is replaced by the variance of the logistic density function ($\pi^2 / 3$) multiplied with the sample size

Let's have a short look at the lucky winner

- McKelvey & Zavoina Pseudo R^2 (M & Z Pseudo R^2)

$$M\ \&\ Z\ Pseudo\ R^2 = \frac{\sum_{i=1}^{n}\left(\hat{y}_i^* - \overline{\hat{y}^*}\right)^2}{\sum_{i=1}^{n}\left(\hat{y}_i^* - \overline{\hat{y}^*}\right)^2 + n \times \frac{\pi^2}{3}}$$

Range: $0 \leq$ M & Z-Pseudo $R^2 \leq 1$

Legend:

$\sum_{i=1}^{n}\left(\hat{y}_i^* - \overline{\hat{y}^*}\right)^2$: Sum of squares of the estimated logits (latent variable Y^*)

n : Sample size

$\frac{\pi^2}{3}$: Variance of the logistic density function

Generalization to the 2-level logit model 2

- Prediction of the latent variable Y* (estimated binary or cumulative logit) in two ways
  - 1.Population-Average Prediction with the fixed effects of the exogenous variables (all random effects hold at zero)
    - Stata-command: predict newvar1 if e(sample), xb
  - 2. Unit-Specific Prediction of the fixed and random effects of the exogenous variable
    - Stata-command: predict newvar2 if e(sample), eta
- Therefore, the variation of the estimated logits (Y*) can be calculated in two different ways
  - 1. Only for the fixed effects of the exogenous variables
  - 2. For the fixed and random effects of the exogenous variables

Generalization to the 2-level logit model 3

- Therefore we get two different McKelvey & Zavoina Pseudo R^2s
  - 1. "Population-Average" M & Z Pseudo R^2 (fixed effects)
  - 2. "Unit-Specific" M & Z Pseudo R^2 (fixed & random effects)

- The "Unit-Specific" M & Z Pseudo R^2 uses all estimated fixed and random effects for prediction. Therefore it assesses the fit more realistically as its "Population-Average" counterpart

Let’s have a short look at the lucky loser

- McFadden Pseudo R^2 (1974)

$$McFadden\ Pseudo\ R^2\left(\rho^2\right) = 1 - \left[\frac{\ln L_A}{\ln L_0}\right]$$

Range: $0 \leq$ McFadden Pseudo $R^2 < 1$

but ρ^2 does not reach the maximum of 1.0

Rule of thumb: $0.20 \leq$ McFadden Pseudo $R^2 \leq 0.40$ marks an excellent fit (McFadden 1979: 307)

Legend: $\ln L_A$: Log-Likelihood of the actual model
$\ln L_0$: Log-Likelihood of the zero model

Generalization to the 2-level logit model 4

- Conditions of application
  - Maximum-Likelihood estimation of the fixed and random effects of the exogenous variables
  - Actual and zero model have to use the same sample
  - Choice of the "appropriate zero model" (M_0) depends on our knowlege to which contextual unit the respondent belongs
    - Membership known: Random-Intercept-Only Logit model estimates the proportion of Y* which can be maximally explained by the contextual units (= ANOVA model)
    - Membership unkown: Fixed-Intercept-Only Logit model estimates only the marginal distribution of Y* (= true zero model)

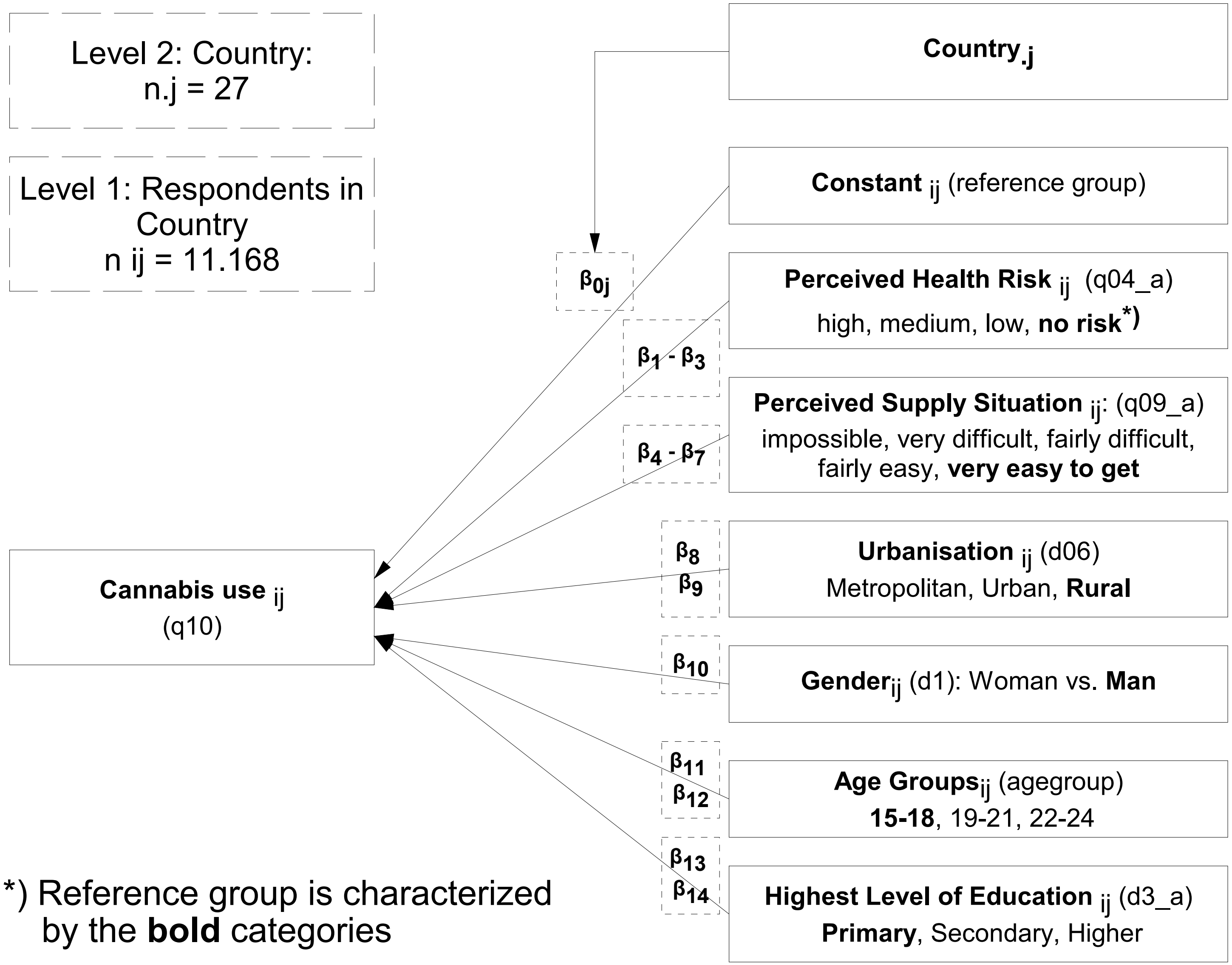
Generalization to the 2-level logit model 5

- Calculation of McFadden Pseudo R^2 is possible in two different ways using the following ones as zero model
  - 1. Random-Intercept-Only Logit-Model (RIOM)
    - It measures the proportional reduction of the log likelihood of the actual model in comparison with the RIOM caused by the fixed effects of the exogenous variables
    - Its Likelihood-Ratio χ^2 test refers to all fixed effects of the exogenous level 1 and level 2 variables
  - 2. Fixed-Intercept-Only Logit-Model (FIOM)
    - It measures the proportional reduction of the log likelihood of the actual model in comparison with the FIOM caused by fixed and random effects of all exogenous variables
    - Its Likelihood-Ratio χ^2 test refers to all fixed and random effects of the exogenous level 1 and level 2 variables

4. Example of application

- Flash Eurobarometer No 330 about youth attitudes on drugs (2011)
  - WebCATI-Survey of n_{ij} = 12.313 respondents (aged 15 -24) in $n_{.j}$ = 27 EU member states (contextual units j)
  - My focus:
    - prevalence of cannabis use by juveniles and young adults (q10): Have you used cannabis by yourself?
      - **1) never**
      - **2) more than 12 months ago**
      - **3) less than 12 months ago**
      - **4) in the last 30 days**
  - Let us have a look at the exogenous variables in the following diagram

Theoretical 2-level-model: RIM



*) Reference group is characterized by the **bold** categories

Stata-Output Version16

- Fixed effects
- Thresholds
- Random effect

```
Mixed-effects ologit regression                 Number of obs     =     11,168
Group variable:         country                 Number of groups  =         27

                                                Obs per group:
                                                              min =        211
                                                              avg =      413.6
                                                              max =        490

Integration method: mvaghermite                 Integration pts.  =          7

                                                Wald chi2(14)     =    3363.39
Log pseudolikelihood = -7424.2751               Prob > chi2       =     0.0000
                                     (Std. Err. adjusted for 27 clusters in country)
------------------------------------------------------------------------------------
                         |               Robust
                  q10ord |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------------------+----------------------------------------------------------
                    q4_a |
               high risk |  -2.656545   .1498799   -17.72   0.000    -2.950304   -2.362785
             medium risk |  -1.668222   .1015971   -16.42   0.000    -1.867348   -1.469095
                low risk |  -.7527713   .0463199   -16.25   0.000    -.8435567   -.6619859
                         |
                    q9_a |
              impossible |  -2.976197   .1910924   -15.57   0.000    -3.350731   -2.601663
          very difficult |  -2.132899   .1642871   -12.98   0.000    -2.454896   -1.810903
        fairly difficult |  -1.527717   .1004747   -15.21   0.000    -1.724644   -1.330791
             fairly easy |  -.6241175   .0843823    -7.40   0.000    -.7895038   -.4587312
                         |
                      d6 |
       metropolitan zone |   .3950294   .1038006     3.81   0.000      .191584    .5984749
other town/urban centre  |   .2082467   .0751678     2.77   0.006     .0609206    .3555729
                         |
                      d1 |
                  female |  -.4777186   .0525238    -9.10   0.000    -.5806634   -.3747738
                         |
                agegroup |
                 19 - 21 |   .4967552   .0607537     8.18   0.000     .3776801    .6158303
                 22 - 24 |   .6850326   .0788467     8.69   0.000     .5304958    .8395694
                         |
                    d3_a |
     secondary education |  -.0122673   .0702398    -0.17   0.861    -.1499348    .1254003
        higher education |  -.0268133   .1227001    -0.22   0.827    -.2673011    .2136745
-------------------------+----------------------------------------------------------
                   /cut1 |  -.3882657   .1392505                     -.6611918   -.1153396
                   /cut2 |   .7153171   .1438259                      .4334236    .9972106
                   /cut3 |   1.902703   .1602572                      1.588605    2.216802
-------------------------+----------------------------------------------------------
country                  |
               var(_cons)|   .2617043   .0850279                      .1384379    .4947284
------------------------------------------------------------------------------------
```

What does Stata offer to assess the fit?

- Akaike (AIC) and Schwarz Bayesian Information Criterion (BIC)
  - Decision rule: Choose the model with the lowest AIC or BIC

Akaike's information criterion and Bayesian information criterion

Model	N	ll(null)	ll(model)	df	AIC	BIC
fiom_w4	11,168	.	-9343.799	3	18693.60	18715.56
riom_w4	11,168	.	-9036.540	4	18081.08	18110.36
rim_w4	11,168	.	-7424.275	18	14884.55	15016.32

Note: BIC uses N = number of observations. See [R] BIC note.

  - Looking at AIC and BIC, the RIM fits best of all bad models
  - But we do not know how well the RIM really fits !

Output of my fit_meologit_2lev.ado 1

- Assessing the fit by the McKelvey & Zavoina-, McFadden-Pseudo R^2s for the fixed & random effects

```
. fit_meologit_2lev
Fit-measures for the MELOGIT/MEOLOGIT Model:

McKelvey&Zavoina-Pseudo R2 (fixed & random effects)=   0.5097

McKelvey&Zavoina-Pseudo R2 (fixed effects only)=   0.4728


Just estimating the Fixed-/ Random-Intercept-Only-Logit Model


McFadden Pseudo R2 (fixed effects only) = 0.1784

McFadden Pseudo R2 (fixed & random effects)  = 0.2054
```

Output of my fit_meologit_2lev.ado 2

- Intra-Class Correlation and corresponding Likelihood-Ratio-χ^2 tests for fixed & random effects

```
ICC of Random-Intercept-Only-Logit Model (Sample M(A))
Intra-Class-Correlation (Level 2) =    0.1431

H0: ICC of Level 2 is zero in the population
LR-chi2 test statistic (  1) =   614.52    Prob > chi2 = 0.0000

LR-chi2 test: H0: all fixed effects are zero in the population

LR-chi2 test statistic ( 14) =  3224.53    Prob > chi2 = 0.0000

LR-chi2 test: H0: all fixed & random effects are zero in the population
LR-chi2 test statistic ( 15) =  3839.05    Prob > chi2 = 0.0000
```

What you get afterwards

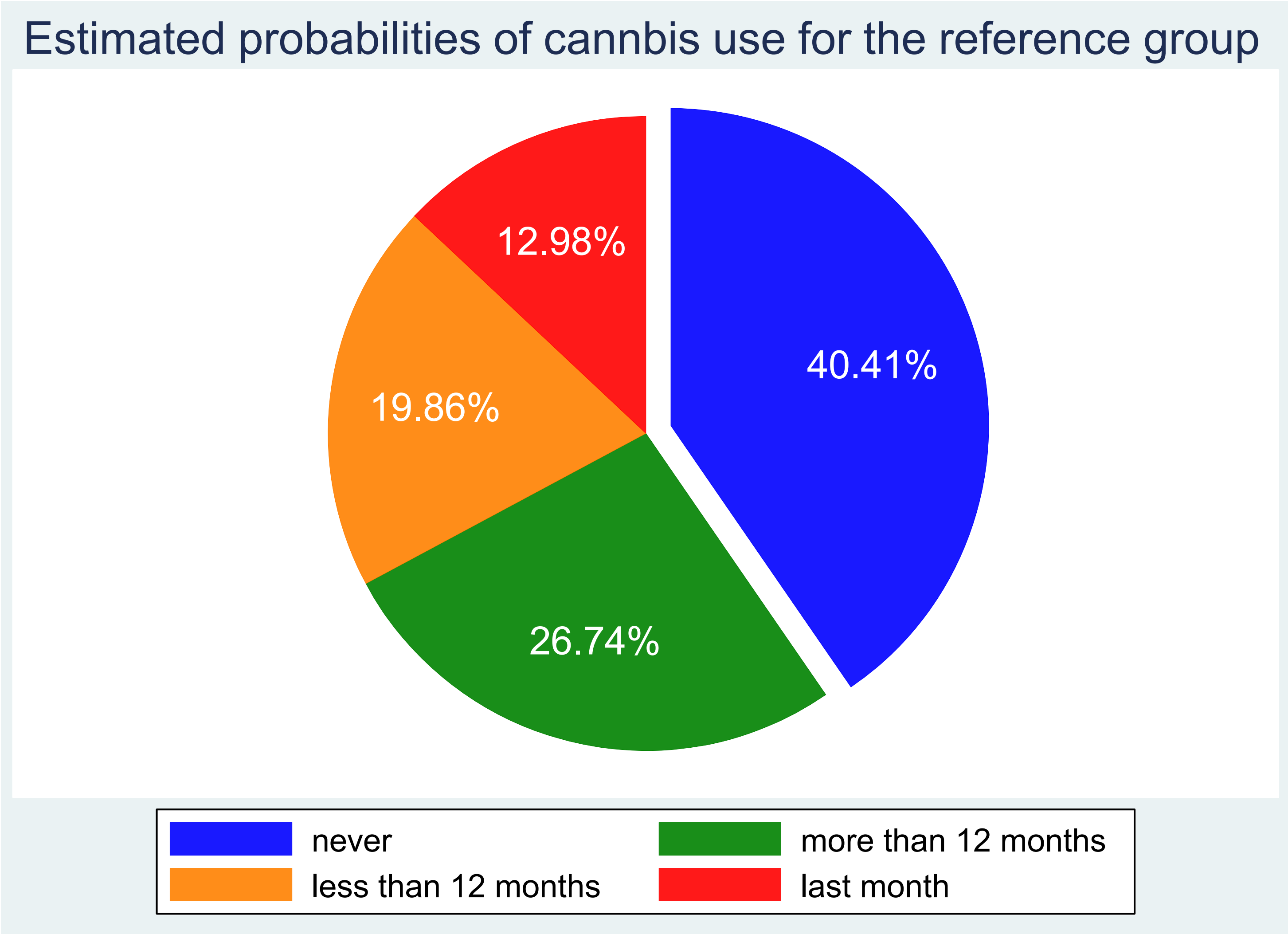
- r-containers of fit_meologit_2lev.ado

```
. return list

scalars:
          r(p_chi2_fr) =  0
              r(df_fr) =  15
            r(chi2_fr) =  3839.047042018567
           r(p_chi2_f) =  0
               r(df_f) =  14
             r(chi2_f) =  3224.530363760103
              r(p_icc) =  0
             r(df_icc) =  1
           r(chi2_icc) =  614.5166782584638
           r(icc_riom) =  .1431007329841504
          r(mcr2_fiom) =  .2054328872219874
          r(mcr2_riom) =  .1784162011068596
              r(mzr2f) =  .4728218379947466
             r(mzr2fr) =  .5097289342586476
```

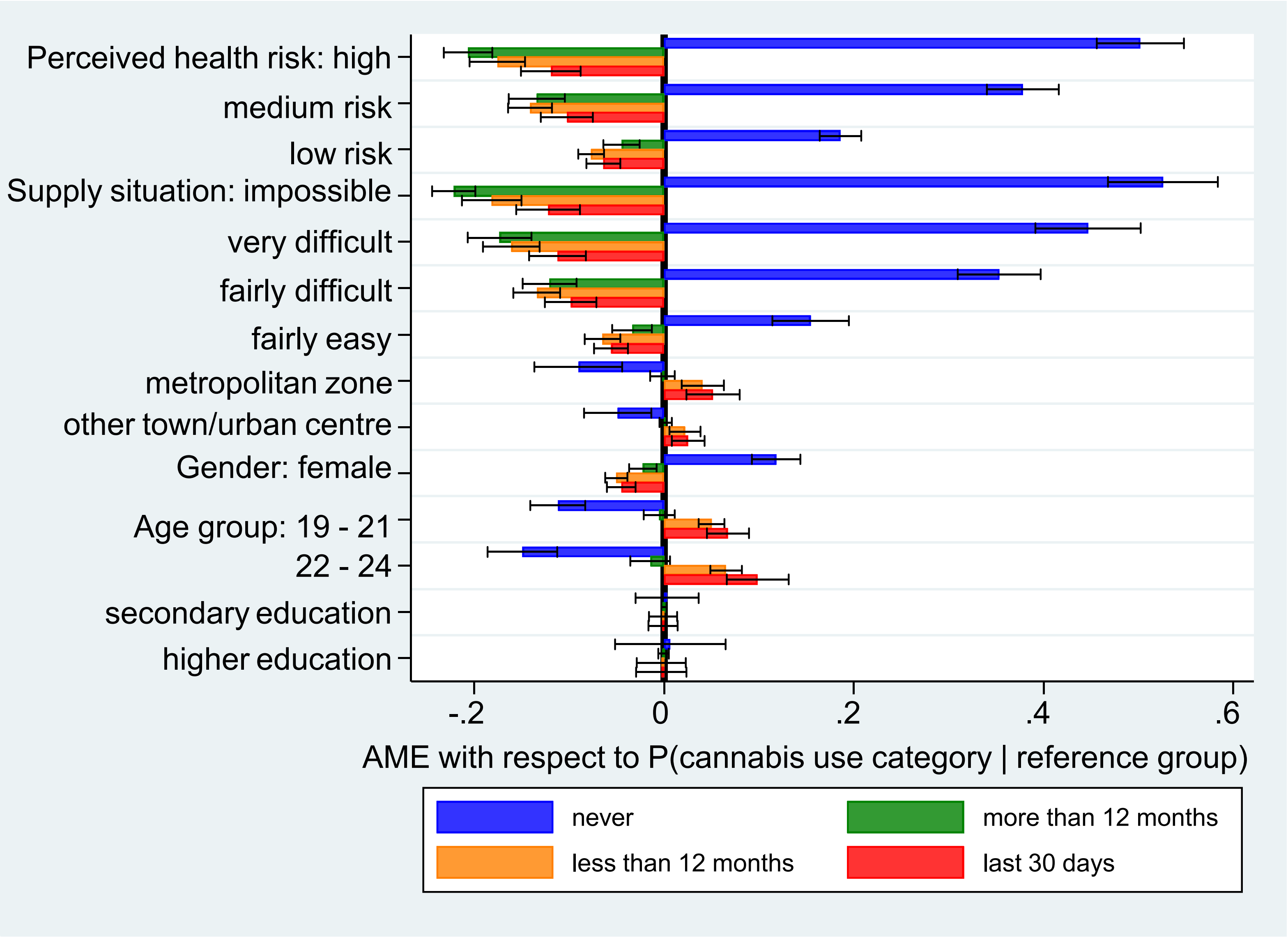
How do the effects look like?

- The baseline



- Reference Group
  - Men
  - Age 15-18
  - Location: rural
  - Education: primary
  - Perceived health risk: no risk
  - Perceived supply situation: very easy to get

Ben Jann's Coefplot for the 4 categories



5. Conclusions 1

- What's known
  - The Monte-Carlo-simulation studies show that the McKelvey & Zavoina Pseudo R^2 is the best fit measure for binary and ordinal logit models
- What's new
  - Generalization of the M & Z-Pseudo R^2 to binary and ordinal multilevel logit models. The prediction of estimated logits bases upon the fixed effects only or upon fixed and random effects of exogenous variables
  - The McFadden-Pseudo R^2 bases upon the fixed effects only or upon fixed and random effects of the exogenous variables using a context-independent zero model

5. Conclusions 2

- What's new
  - Simultaneous Likelihood-Ratio-χ^2 test for the estimated fixed effects using the Random-Intercept-Only Model (RIOM) as the zero model
  - Simultaneous Likelihood-Ratio-χ^2 test for the estimated fixed and random effects using the Fixed-Intercept-Only Model (FIOM) as the zero model
  - Use of probability weights for each level j
  - You get all Pseudo-R2s and tests in r-containers
- That's why
  - I suggest to use my fit_meologit_2lev.ado and fit_meologit_3lev.ado to assess the fit of 2- and 3-level logit models with binary and ordinal outcome

Closing words

- Thank you for your attention
- Do you have some questions?

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Appendix

Multilevel ordered logit model 1

- Equations of the 2-level-ordered logit model

Level 2: Between-Context Regression

2a) Logistic Intercept-as-Outcome Model:

$$\beta_{0j} = 0 + \gamma_{01} \times Z_{.j} + u_{0j}$$

2b) Logistic Slope-as-Outcome Model:

$$\beta_{1j} = \gamma_{10} + \gamma_{11} \times Z_{.j} + u_{1j}$$

Level 1: Within-Context Regression

1) $$\ln\left[\frac{\mathrm{P}(\mathrm{Y} > k)}{\mathrm{P}(\mathrm{Y} \leq k)}\right] = \beta_{0j} + \beta_{1j} \times X_{ij} - \sum_{K=1}^{k-1} \delta_k \{+r_{ij}\}$$

Notation of Raudenbush & Bryk (2002):

γ: fixed-effect estimator
Z: exogenous level 2 variable
β: random-effect estimator
X: exogenous level 1 variable
u_{0j}: residuum random-intercept
u_{1j}: residuum random-slope
r_{ij}: residuum of within-context-logistic regression
δ_k: threshold for category k of Y

Single equation notation: 2a) and 2b) in 1)

$$\ln\left[\frac{\mathrm{P}(\mathrm{Y} > k)}{\mathrm{P}(\mathrm{Y} \leq k)}\right] = \left(0 + \gamma_{01} \times Z_{.j} + u_{0j}\right) + \left(\gamma_{10} \times X_{ij} + \gamma_{11} \times X_{ij} \times Z_{.j} + u_{1j} \times X_{ij}\right) - \sum_{k=1}^{K-1} \delta_k \{+r_{ij}\}$$

Multilevel ordered logit model 2

- Interpretation of the residua of the Between-Context Regression

$$3a)\ u_{0j} = \beta_{0j} - \left[\gamma_{00} + \gamma_{01} \times Z_{.j}\right] = \beta_{0j} - \widehat{\beta_{0j}}$$

$$3b)\ u_{1j} = \beta_{1j} - \left[\gamma_{10} + \gamma_{11} \times Z_{.j}\right] = \beta_{1j} - \widehat{\beta_{1j}}$$

- Assumptions for the residua of the logistic 2-level logit model

Level 1:

1.1) r_{ij} is binomial distributed with an expected value of zero

and a variance $\sigma^2_{r_{ij}} = \widehat{P_{ij}}(Y=1) \times \left(1 - \widehat{P_{ij}}(Y=1)\right)$

1.2) Heteroscedasticity of r_{ij} in all contextual units j

Multilevel ordered logit model 3

- Implication for the level 1 residuum r_{ij}
  - The variance $\sigma^2(r_{ij})$ can not be estimated because of its own heteroscedasticity. It is replaced by the variance of the logistic density function ($\pi^2 / 3$)

- Residua of level 2

2.1) u_{kj} is normal distributed with an expected value of zero and a covariance matrix T of the residua

$$E\begin{bmatrix} u_{0j} \\ u_{1j} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad T = \begin{bmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{bmatrix} \qquad \sigma^2_{u_{0j}} = \tau_{00} \qquad \sigma^2_{u_{1j}} = \tau_{11}$$

$$\sigma_{u_{0j},u_{1j}} = \tau_{10} = \tau_{01}$$

2.2) The residua of level 1 and level 2 are not correlated:

$$\sigma_{u_{0j},r_{ij}} = \sigma_{u_{1j},r_{ij}} = 0$$

Alternative in Stata: Information criteria

- Calculation of Akaike- (AIC) and Schwarz Bayesian-Information-Criteria (BIC)

$$AIC = -2 \times \ln L_{M_A} + 2 \times k$$

$$BIC = -2 \times \ln L_{M_A} + \ln N \times k$$

deviance ($-2 \times \ln L_{M_A}$)   complexity of the model (k)

Range: $0 < AIC \leq +\infty$

$0 < BIC \leq +\infty$

Legend:

$\ln L_{M_A}$: Log Likelihood of actual model

k : Number of estimated parameters

N : *Sample size*

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