Ensemble Learning Targeted Maximum Likelihood Estimation for Stata Users: 2018 Spanish Stata Conference

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https://github.com/migariane/SUGML

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Notation and definitions

Observed Data

- Treatment **A**.

  Often, A = 1 for treated and A = 0 for control.
- Confounders **W**.
- Outcome **Y**.

Potential Outcomes

- For patient i $\mathbf{Y_i(1)}$ and $\mathbf{Y_i(0)}$ set to A = a $Y^{(a)}$, namely A = 1 and A = 0.

Causal Effects

- Average Treatment Effect: **E[Y(1) - Y(0)]**.

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ATE estimators

Nonparametric

- G-formula plug-in estimator (generalization of standardization).

Parametric

- Regression adjustment (RA).
- Inverse probability treatment weighting (IPTW).
- Inverse-probability treatment weighting with regression adjustment (IPTW-RA) (Kang and Schafer, 2007).

Semi-parametric Double robust (DR) methods

- Augmented inverse-probability treatment weighting (Estimation Equations) (AIPTW) (Robins, 1994).
- Targeted maximum likelihood estimation (TMLE) **(van der Laan, 2006)**.

ATE estimators: drawbacks

Nonparametric

- Course of dimensionality (sparsity: zero empty cell)

Parametric

- Parametric models are misspecified (all models are wrong but some are useful, Box, 1976), and break down for high-dimensional data.
- (RA) Issue: extrapolation and biased if misspecification, no information about treatment mechanism.
- (IPTW) Issue: sensitive to course of dimensionality, inefficient in case of extreme weights and biased if misspecification. Non information about the outcome.

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Double-robust (DR) estimators

Pros: Semi-parametric Double-Robust Methods

- DR methods give **two chances at consistency** if any of two nuisance parameters is consistently estimated.
- DR methods are **less sensitive** to **course of dimensionality**.

Cons: Semi-parametric Double-Robust Methods

- DR methods are unstable and inefficient if the propensity score (PS) is small (**violation of positivity assumption**) (vand der Laan, 2007).
- AIPTW and IPTW-RA do not respect the **limits of the boundary space of Y**.
- **Poor performance** if **dual misspecification** (Benkeser, 2016).

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Targeted Maximum Likelihood Estimation (TMLE)

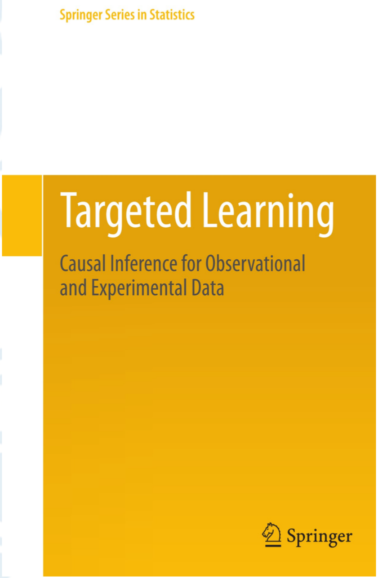
Pros: TMLE

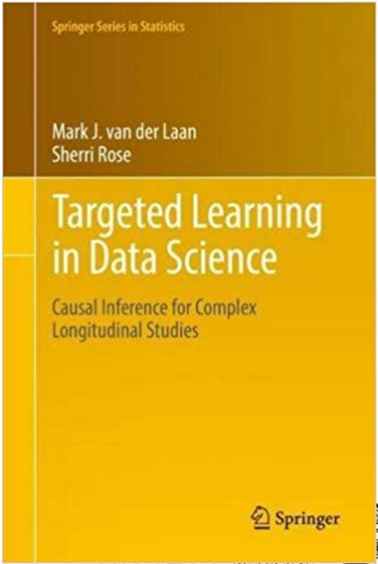
- (TMLE) is a general algorithm for the construction of **double-robust**, **semiparametric** MLE, efficient **substitution** estimator (Van der Laan, 2011)
- Better performance than competitors has been largely documented (Porter, et. al.,2011).
- (TMLE) **Respect bounds on Y**, **less sensitive** to **misspecification** and to **near-positivity** violations (Benkeser, 2016).
- (TMLE) **Reduces bias** through **ensemble learning** if misspecification, even dual misspecification.
- For the ATE, **Inference** is based on the **Efficient Influence Curve**. Hence, the **CLT** applies, making inference easier.

Cons: TMLE

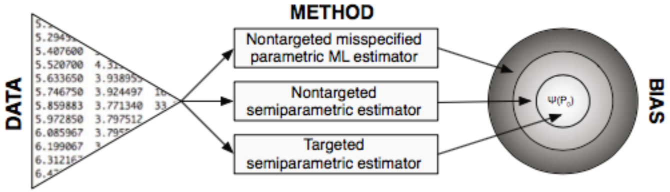
- The procedure is only available in R: **tmle** package (Gruber, 2011).

Targeted learning





Why Targeted learning?



Source: Mark van der Laan and Sherri Rose. Targeted learning: causal inference for observational and experimental data. Springer Series in Statistics, 2011.

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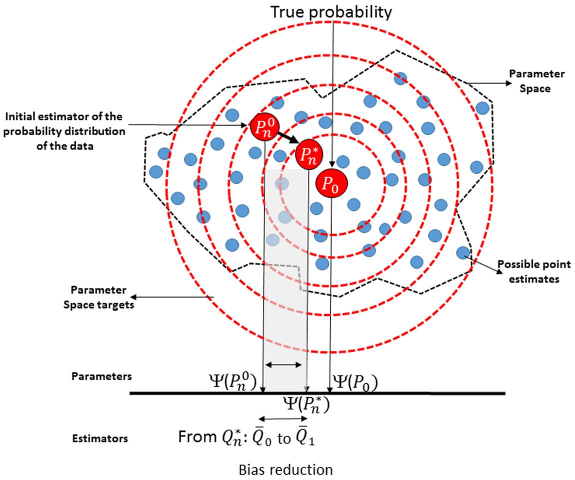
MC simulations: Luque-Fernandez et al, 2017 (in press, American Journal of Epidemiology)

	ATE		BIAS (%)		RMSE		95%CI coverage (%)	
	N=1,000	N=10,000	N=1,000	N=10,000	N=1,000	N=10,000	N=1,000	N=10,000
First scenario* (correctly specified models)								
True ATE	**-0.1813**							
Naïve	-0.2234	-0.2218	23.2	22.3	0.0575	0.0423	77	89
AIPTW	-0.1843	-0.1848	1.6	1.9	0.0534	0.0180	93	94
IPTW-RA	-0.1831	-0.1838	1.0	1.4	0.0500	0.0174	91	95
TMLE	-0.1832	-0.1821	1.0	**0.4**	0.0482	0.0158	**95**	**95**
Second scenario ** (misspecified models)								
True ATE	**-0.1172**							
Naïve	-0.0127	-0.0121	89.2	89.7	0.1470	0.1100	0	0
BFit AIPTW	-0.1155	-0.0920	1.5	**11.7**	0.0928	0.0773	65	65
BFit IPTW-RA	-0.1268	-0.1192	8.2	**1.7**	0.0442	0.0305	52	73
TMLE	-0.1181	-0.1177	**0.8**	**0.4**	0.0281	0.0107	**93**	**95**

*First scenario : correctly specified models and near-positivity violation

**Second scenario: misspecification, near-positivity violation and adaptive model selection

TMLE ROAD MAP



TMLE STEPS

Substitution estimation: $\hat{E}(Y \mid A, W)$

- First compute the outcome regression $\mathbf{E}(\mathbf{Y} \mid \mathbf{A}, \mathbf{W})$ using the **Super-Learner** to then derive the Potential Outcomes and compute $\psi^{(0)} = \mathbf{E}(Y(1) \mid A = 1, W) - \mathbf{E}(Y(0) \mid A = 0, W)$.

- Estimate the exposure mechanism P(A=1|,W) using the **Super-Learner** to predict the values of the propensity score.

- Compute **HAW** = $\left(\frac{\mathbb{I}(A_i=1)}{P(A_i=1|W_i)} - \frac{\mathbb{I}(A_i=0)}{P(A_i=0|W_i)}\right)$ for each individual, named the **clever covariate H.**

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Fluctuation step: Epsilon

Fluctuation step ($\hat{\epsilon}_0$, $\hat{\epsilon}_1$)

- Update $\Psi^{(0)}$ through a fluctuation step incorporating the information from the exposure mechanism:

$$\mathbf{H(1)W} = \frac{\mathbb{I}(A_i{=}1)}{\hat{P}(A_i{=}1|W_i)} \text{ and, } \mathbf{H(0)W} = -\frac{\mathbb{I}(A_i{=}0)}{\hat{P}(A_i{=}0|W_i)}.$$

- This step aims to **reduce bias** minimising the mean squared error (MSE) for (Ψ) and considering the **bounds of the limits of Y.**
- The fluctuation parameters ($\hat{\epsilon}_0$, $\hat{\epsilon}_1$) are estimated using maximum likelihood procedures (in Stata):

. glm Y HAW, fam(binomial) nocons offset(E(Y| A, W))

. mat e = e(b),

. gen double $\epsilon = e[1,1]$,

Targeted estimate of the ATE ($\widehat{\Psi}$)

$\Psi^{(0)}$ update using ϵ (epsilon)

$$\mathbf{E}^*(Y \mid A = 1, W) = \text{expit}\left[\textbf{logit}\left[E(Y \mid A = 1, W)\right] + \hat{\epsilon}_{\mathbf{1}} H_1(1, W)\right]$$

$$\mathbf{E}^*(Y \mid A = 0, W) = \text{expit}\left[\textbf{logit}\left[E(Y \mid A = 0, W)\right] + \hat{\epsilon}_{\mathbf{0}} H_0(0, W)\right]$$

Targeted estimate of the ATE from $\Psi^{(0)}$ to $\Psi^{(1)}$: ($\widehat{\Psi}$)

$$\Psi^{(1)} : \hat{\Psi} = \left[\mathbf{E}^*(Y(1) \mid A = 1, W) - \mathbf{E}^*(Y(0) \mid A = 0, W)\right]$$

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TMLE inference: Influence curve

TMLE inference

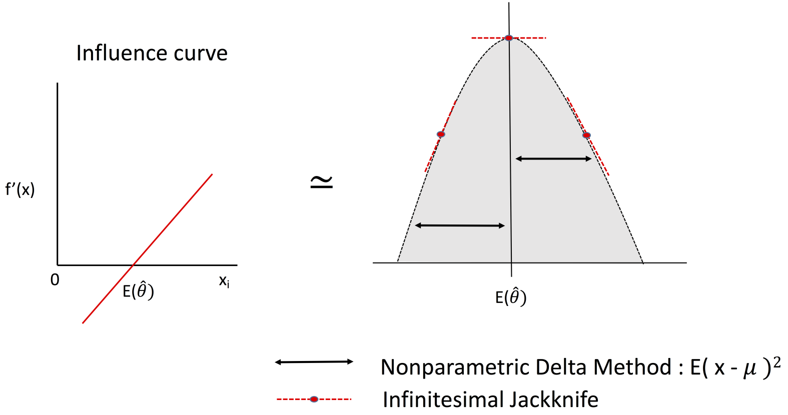
$$\textbf{IC} = \left(\frac{(A_i = 1)}{P(A_i = 1 \mid W_i)} - \frac{(A_i = 0)}{P(A_i = 0 \mid W_i)} \right) [Y_i - E_1(Y \mid A_i, W_i)] +$$
$$[E_1(Y(1) \mid A_i = 1, W_i) - E_1(Y(0) \mid A_i = 0, W_i)] - \psi$$

$$\textbf{Standard Error} : \sigma(\psi_0) = \frac{SD(IC_n)}{\sqrt{n}}$$

TMLE inference

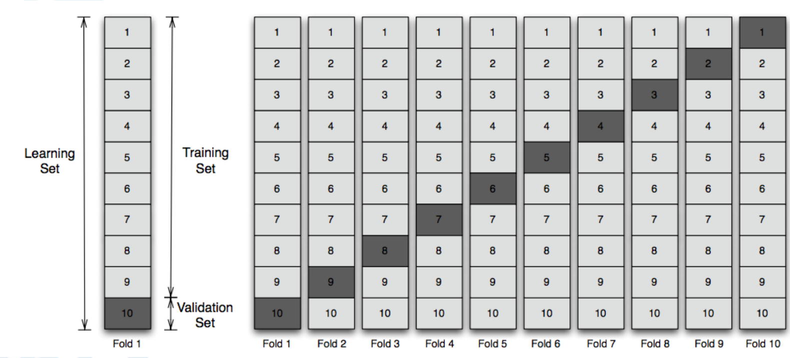
- The **Efficient IC**, first introduced by Hampel (1974), is used to apply readily the **CLT** for statistical inference using TMLE.
- The **Efficient IC** is the same as the infinitesimal jackknife and the **nonparametric delta method**. Also named the "**canonical gradient**" of the pathwise derivative of the target parameter ψ or "**approximation by averages**"(Efron, 1982).

IC: Geometric interpretation



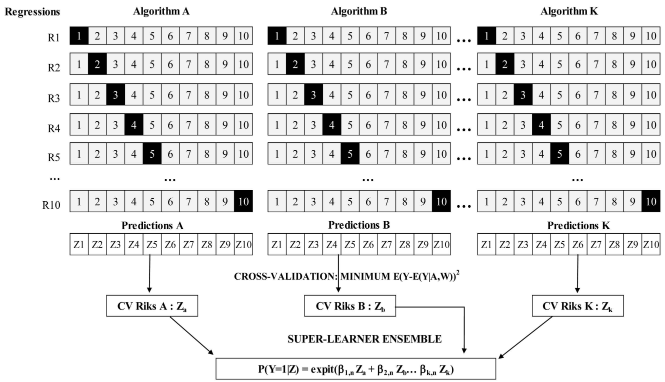
Estimate of the ψ Standard Error using the efficient Influence Curve.
Image credit: Miguel Angel Luque-Fernandez

Targeted learning



Source: Mark van der Laan and Sherri Rose. Targeted learning: causal inference for observational and experimental data. Springer Series in Statistics, 2011.

Super-Learner: Ensemble learning



To apply the **EIC** we need data-adaptive estimation for both, the model of the outcome, and the model of the treatment.
Asymptotically, the final weighted combination of algorithms (Super Learner) performs as well as or better than the best-fitting algorithm (van der Laan, 2007).

Stata **ELTMLE**

Ensemble Learning Targeted Maximum Likelihood Estimation

- **eltmle** is a Stata program implementing R-TMLE for the ATE for a binary or continuous outcome and binary treatment.
- **eltmle** includes the use of a **super-learner**(Polley E., et al. 2011).
- I used the default Super-Learner algorithms implemented in the base installation of the tmle-R package v.1.2.0-5 (Susan G. and Van der Laan M., 2007).
- i) stepwise selection, ii) GLM, iii) a GLM interaction.
- Additionally, **eltmle** users will have the option to include Bayes GLM and GAM.

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Stata ELTMLE

Syntax eltmle Stata command

eltmle Y A W [, **tmle tmlebgam tmleglsrf**]

Y: Outcome: numeric binary or continuous variable.

A: Treatment or exposure: numeric binary variable.

W: Covariates: vector of numeric and categorical variables.

Stata Implementation: overall structure

eltmle.ado

```
capture program drop eltmle
program define eltmle
     syntax varlist(min=3) [if] [pw] [, tmle tmlebgam tmleglsrf]
     version 13.2
     marksample touse
     local var `varlist' if `touse'
     tokenize `var'
     local yvar = "`1'"
     global flag = cond(`yvar'<=1,1,0)
     qui sum `yvar'
     global b = `r(max)'
     global a = `r(min)'
     qui replace `yvar' = (`yvar' - `r(min)') / (`r(max)' - `r(min)') if `yvar'>1
     local dir `c(pwd)'
     cd "`dir'"
     qui export delimited `var' using "data.csv", nolabel replace
     if "`tmlebgam'" == "" & "`tmleglsrf'" == "" {
        tmle `varlist'
        }
     else if "`tmlebgam'" == "tmlebgam" {
        tmlebgam `varlist'
        }
     else if "`tmleglsrf'" == "tmleglsrf" {
        tmleglsrf `varlist'
        }
end
```

Stata Implementation: R code for calling the SL

```
program tmle
// Write R Code dependencies: foreign Surperlearner
set more off
qui: file close _all
qui: file open rcode using SLS.R, write replace
qui: file write rcode ///
        `"set.seed(123)"' _newline ///
        `"list.of.packages <- c("foreign","SuperLearner")"' _newline ///
        `"new.packages <- list.of.packages[!(list.of.packages %in% installed.packages()[,"Package"])]"' _newline ///
        `"if(length(new.packages)) install.packages(new.packages, repos='http://cran.us.r-project.org')"' _newline ///
        `"library(SuperLearner)"' _newline ///
        `"library(foreign)"' _newline ///
        `"data <- read.csv("data.csv", sep=",")"' _newline ///
        `"attach(data)"' _newline ///
        `"SL.library <- c("SL.glm","SL.step","SL.glm.interaction")"' _newline ///
        `"n <- nrow(data)"' _newline ///
        `"nvar <- dim(data)[[2]]"' _newline ///
        `"Y <- data[,1]"' _newline ///
        `"A <- data[,2]"' _newline ///
        `"X <- data[,2:nvar]"' _newline ///
        `"W <- data[,3:nvar]"' _newline ///
        `"X1 <- X0 <- X"' _newline ///
        `"X1[,1] <- 1"' _newline ///
        `"X0[,1] <- 0"' _newline ///
        `"newdata <- rbind(X,X1,X0)"' _newline ///
        `"Q <- try(SuperLearner(Y = data[,1] ,X = X, SL.library=SL.library, family=binomial(), newX=newdata, method="meth
        `"Q <- as.data.frame(Q[[4]])"' _newline ///
        `"QAW <- Q[1:n,]"' _newline ///
        `"Q1W <- Q[((n+1):(2*n)),]"' _newline ///
        `"Q0W <- Q[((2*n+1):(3*n)),]"' _newline ///
        `"g <- suppressWarnings(SuperLearner(Y = data[,2], X = W, SL.library = SL.library, family = binomial(), method =
        `"ps <- g[[4]]"' _newline ///
        `"ps[ps<0.025] <- 0.025"' _newline ///
        `"ps[ps>0.975] <- 0.975"' _newline ///
        `"data <- cbind(data,QAW,Q1W,Q0W,ps,Y,A)"' _newline ///
        `"write.dta(data, "data2.dta")"'
qui: file close rcode
```

Stata Implementation: Batch file executing R

```
qui: file close rcode

// Write bacth file to find R.exe path and R version
set more off
qui: file close _all
qui: file open bat using setup.bat, write replace
qui: file write bat ///
`"@echo off"' _newline ///
`"SET PATHROOT=C:\Program Files\R\"' _newline ///
`"echo Locating path of R..."' _newline ///
`"echo."' _newline ///
`"if not exist "%PATHROOT%" goto:NO_R"' _newline ///
`"for /f "delims=" %%r in (' dir /b "%PATHROOT%R*" ') do ("' _newline ///
        `"echo Found %%r"' _newline ///
        `"echo shell "%PATHROOT%%%r\bin\x64\R.exe" CMD BATCH SLS.R > runr.do"' _newline ///
        `"echo All set!"' _newline ///
        `"goto:DONE"' _newline ///
`")"' _newline ///
`":NO_R"' _newline ///
`"echo R is not installed in your system."' _newline ///
`"echo."' _newline ///
`"echo Download it from https://cran.r-project.org/bin/windows/base/"' _newline ///
`"echo Install it and re-run this script"' _newline ///
`":DONE"' _newline ///
`"echo."' _newline ///
`"pause"'
qui: file close bat

//Run batch
shell setup.bat
//Run R
do runr.do

// Read Revised Data Back to Stata
clear
quietly: use "data2.dta", clear

// Q to logit scale
gen logQAW = log(QAW / (1 - QAW))
gen logQ1W = log(Q1W / (1 - Q1W))
gen logQ0W = log(Q0W / (1 - Q0W))

// Clever covariate HAW
```

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Output for continuous outcome

```
. use http://www.stata-press.com/data/r14/cattaneo2.dta
(Excerpt from Cattaneo (2010) Journal of Econometrics 155: 138-154)

. eltmle bweight mbsmoke mage medu prenatal mmarried, tmle
/Users/MALF/Dropbox/CAUSALITY/TARGETED-MACHINE-LEARNING/STATA-ELTMLE/Githu


    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
        POM1 |      4,642    2833.081    74.84581   2580.186   2958.981
        POM0 |      4,642    3062.785    89.55875   2867.102   3166.985
          PS |      4,642    .1861267     .110755   .0372202   .8494988
________________________________
TMLE: Additive Causal Effect
________________________________
Risk Differences:-229.70; EST VAR:600.9; 95%CI:(-277.75,-181.66);
p-value: 0.0000
________________________________
TMLE: Causal Risk Ratio (CRR)
________________________________
CRR:      0.93; 95%CI:(0.91,0.94)
________________________________
TMLE: Marginal Odds Ratio (MOR)
________________________________
MOR:      0.83; 95%CI:(0.80,0.87)
```

Simulations comparing Stata ELTMLE vs R-TMLE

```
. mean psi aipw eltmle
  Mean estimation
  Number of obs    =   1,000
  -------------------------
               |    Mean
  -------------+-----------
          True |    .173
          aipw |    .170
        eltmle |    .170
  -------------------------
        R-TMLE |    .170
  -------------------------
```

SIM and online open-source tutorials

Link to the tutorials

MA Luque-Fernandez et al. Targeted maximum likelihood estimation for a binary treatment: A tutorial. SIM. 2018.
https://onlinelibrary.wiley.com/doi/full/10.1002/sim.7628

https://migariane.github.io/TMLE.nb.html

Stata Implementation: source code

https://github.com/migariane/eltmle

Stata installation and step by step commented syntax

github install migariane/eltmle
which eltmle
viewsource eltmle.ado

One sample simulation: TMLE reduces bias

https://github.com/migariane/SUGML

Next steps for ELTMLE

Next steps

- Stata Journal manuscript.
- Improving the user interface for **eltmle**.
- Include more machine learning algorithms.
- Implementation of Ensemble Learning in Stata (Super-Learner).
- Recently, we have implemented the **cross-validated AUC**: https://github.com/migariane/cvAUROC. Also available at **ssc**: ssc install cvAUROC

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Thank you

THANK YOU FOR YOUR TIME

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Background: Potential Outcomes framework

Rubin and Heckman

- This framework was developed first by statisticians (Rubin, 1983) and econometricians (Heckman, 1978) as a new approach for the estimation of **causal effects** from observational data.
- We will keep separate the **causal framework** (a conceptual issue briefly introduce here) and the **"how to estimate causal effects"** (an statistical issue also introduced here)

Causal effects with OBSERVATIONAL data

ASSUMPTIONS for Identification

- Rosebaum & Rubin, 1983: **The Ignorable Treatment Assignment** (A.K.A Ignorability, Unconfoundeness or Conditional Mean Independence).
- **POSITIVITY**.
- **SUTVA**.

Causal effect with OBSERVATIONAL data

IGNORABILITY

$$(\mathbf{Y_i(1)}, \mathbf{Y_i(0)}) \perp \mathbf{A_i} \mid \mathbf{W_i}$$

POSITIVITY

POSITIVITY: $\mathbf{P(A = a \mid W) > 0}$ for all a, W

SUTVA

- We have assumed that there is only on version of the treatment (**consistency**) **Y(1)** if **A = 1** and **Y(0)** if **A = 0**.
- The assignment to the treatment to one unit doesn't affect the outcome of another unit (**no interference**) or **IID** random variables.
- The model used to estimate the assignment probability has to **be correctly specified**.

Causal effect

Potential Outcomes

We only observe:

$$Y_i(1) = Y_i(A = 1) \text{ and } Y_i(0) = Y_i(A = 0)$$

However we would like to know what would have happened if:

Treated $\mathbf{Y_i(1)}$ would have been non-treated $Y_i(A = 0) = Y_i(0)$.

Controls $Y_i(0)$ would have been treated $\mathbf{Y_i(A = 1) = Y_i(1)}$.

Identifiability

- How we can identify the effect of the potential outcomes $\mathbf{Y}^a$ if they are not observed?
- How we can estimate the expected difference between the potential outcomes **E[Y(1) - Y(0)]**, namely the **ATE**.

G-Formula, (Robins, 1986)

G-Formula for the **identification** of the ATE with observational data

$$
\begin{aligned}
E(Y^a) &= \sum_y E(Y^a \mid W = w) P(W = w) \\
&= \sum_y E(Y^a \mid A = a, W = w) P(W = w) \text{ by consistency} \\
&= \sum_y E(Y = y \mid A = a, W = w) P(W = w) \text{ by ignorability}
\end{aligned}
$$

The **ATE**=

$$
\sum_{\mathbf{w}} \left[\sum_{\mathbf{y}} \mathbf{P}(\mathbf{Y} = \mathbf{y} \mid \mathbf{A} = \mathbf{1}, \mathbf{W} = \mathbf{w}) - \sum_{\mathbf{y}} \mathbf{P}(\mathbf{Y} = \mathbf{y} \mid \mathbf{A} = \mathbf{0}, \mathbf{W} = \mathbf{w}) \right] \mathbf{P}(\mathbf{W} = \mathbf{w})
$$

$$
P(W = w) = \sum_{y,a} P(W = w, A = a, Y = y)
$$

G-Formula, (Robins, 1986)

G-Formula for the identification of the ATE with observational data

The **ATE**=

$$\sum_{\mathbf{w}} \left[\sum_{\mathbf{y}} \mathbf{P}(\mathbf{Y}=\mathbf{y} \mid \mathbf{A}=\mathbf{1}, \mathbf{W}=\mathbf{w}) - \sum_{\mathbf{y}} \mathbf{P}(\mathbf{Y}=\mathbf{y} \mid \mathbf{A}=\mathbf{0}, \mathbf{W}=\mathbf{w}) \right] \mathbf{P}(\mathbf{W}=\mathbf{w})$$

$$P(W=w) = \sum_{y,a} P(W=w, A=a, Y=y)$$

G-Formula

- The sums is generic notation. In reality, likely involves sums and integrals (we are just integrating out the W's).
- The **g-formula** is a **generalization of standardization** and allow to estimate unbiased treatment effect estimates.

Regression-adjustment

$$\widehat{ATE}_{RA} = N^{-1} \sum_{i=1}^{N} [E(Y_i \mid A = 1, W_i) - E(Y_i \mid A = 0, W_i)]$$

$$m_A(w_i) = E(Y_i \mid A_i = A, W_i)$$

$$\widehat{ATE}_{RA} = N^{-1} \sum_{i=1}^{N} [\hat{m}_1(w_i) - \hat{m}_0(w_i)]$$

IPTW (Inverse probability treatment weighting)

Survey theory (Horvitz-Thompson)

$$\hat{P}_i = E(A_i \mid W_i) \text{ ; So , } \frac{\mathbf{1}}{\hat{\mathbf{p}}_\mathbf{i}} \text{ , if A = 1 and , } \frac{\mathbf{1}}{(\mathbf{1} - \hat{\mathbf{p}}_\mathbf{i})} \text{ , if A = 0}$$

Average over the total number of individuals

$$\widehat{ATE}_{IPTW} = N^{-1} \sum_{i=1}^{N} \frac{A_i Y_i}{\hat{p}_i} - N^{-1} \sum_{i=1}^{N} \frac{(1 - A_i) Y_i}{(1 - \hat{p}_i)}$$

AIPTW

AIPTW (Augmented Inverse probability treatment weighting)

Solving Estimating Equations

$$\widehat{ATE}_{AIPTW} =$$

$$N^{-1}\sum_{i=1}^{N}[(Y(1) \mid A_i = 1, W_i) - (Y(0) \mid A_i = 0, W_i)] +$$

$$N^{-1}\sum_{i=1}^{N}\left(\frac{(A_i = 1)}{P(A_i = 1 \mid W_i)} - \frac{(A_i = 0)}{P(A_i = 0 \mid W_i)}\right)[Y_i - E(Y \mid A_i, W_i)]$$

TMLE inference: INFLUENCE CURVE

M-ESTIMATORS: Semi-parametric and Empirical processes theory

An estimator is **asymptotically linear** with **influence function** φ **(IC)** if the estimator can be **approximate by an empirical average** in the sense that

$$(\hat{\theta} - \theta_0) = \frac{1}{n}\sum_{i=1}^{n}(IC) + Op(1/\sqrt{n})$$

(Bickel, 1997).

TMLE inference: Bickel (1993); Tsiatis (2007); Van der Laan (2011); Kennedy (2016)

- The **IC** estimation is a more general approach than M-estimation.
- The **Efficient IC** has mean zero $E(IC_{\hat{\psi}}(y_i, \psi_0)) = 0$ and **finite variance.**
- By the Weak Law of the Large Numbers, the **Op** converges to zero in a rate $1/\sqrt{n}$ as n $\rightarrow\infty$ (Bickel, 1993).
- The **Efficient IC** requires **asymptotically linear** estimators.

Thank you

THANK YOU FOR YOUR TIME

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