Introduction to Markov-switching regression models using the `mswitch` command

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October 22, 2015
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Outline

1. When we use Markov-Switching Regression Models
2. Introductory concepts
3. Markov-Switching Dynamic Regression
   - Predictions
     - State probabilities predictions
     - Level predictions
   - State expected durations
   - Transition probabilities
4. Markov-Switching AR Models

When we use Markov-Switching Regression Models

- The parameters of the data generating process (DGP) vary over a set of different unobserved states.
- We do not know the current state of the DGP, but we can estimate the probability of each possible state.

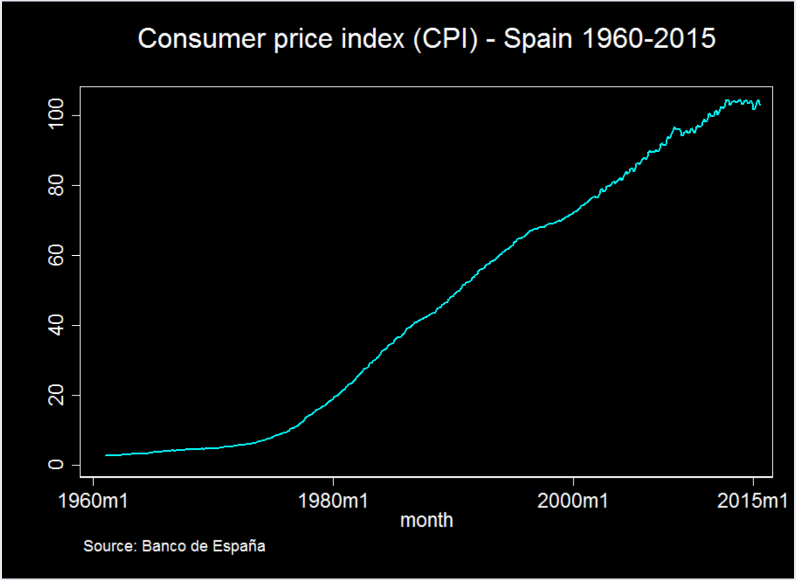
When we use Markov-Switching Regression Models

- Some examples:
  - In economics
    - Asymmetrical behavior over GDP expansions and recessions (Hamilton 1989).
    - Exchange rates (Engel and Hamilton 1990).
    - Interest rates (García and Perron 1996).
    - Stock returns (Kim et al. 1998).
  - In epidemiology: Incidence rates of infectious disease in epidemic and nonepidemic states (Lu et al. 2010).
  - In psychology: manic depressive states (Hamaker et al. 2010).

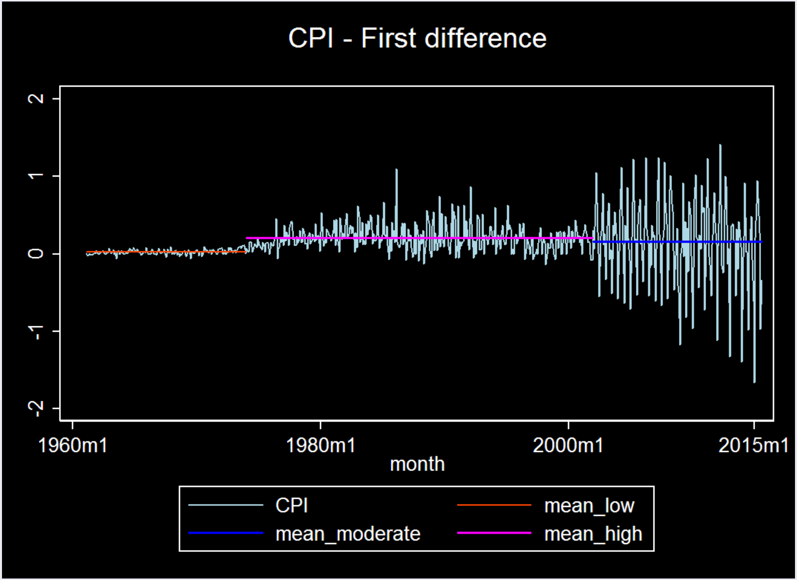
When we use Markov-Switching Regression Models

- The time series in all those examples are characterized by DGPs with dynamics that are state dependent.
  - States may be recessions and expansions, high/low volatility, depressive/non-depressive, epidemic/non-epidemic states, etc.
  - Any of the parameters (beta estimates, sigma, AR components) may be different for each state.

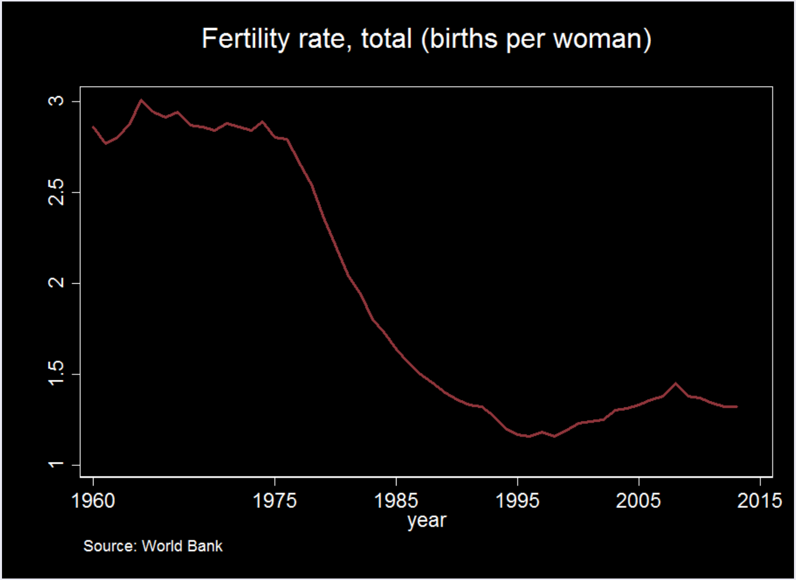
Different volatilities - IPC for Spain



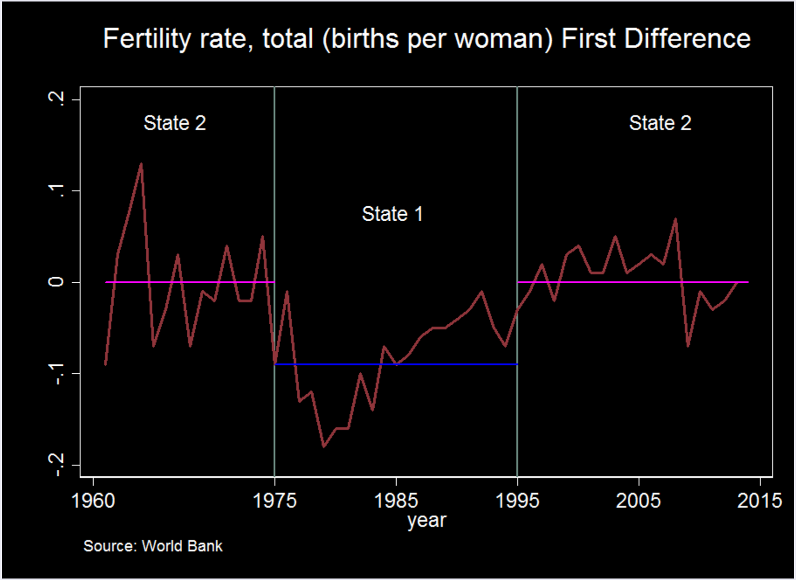
Different volatilities - IPC for Spain



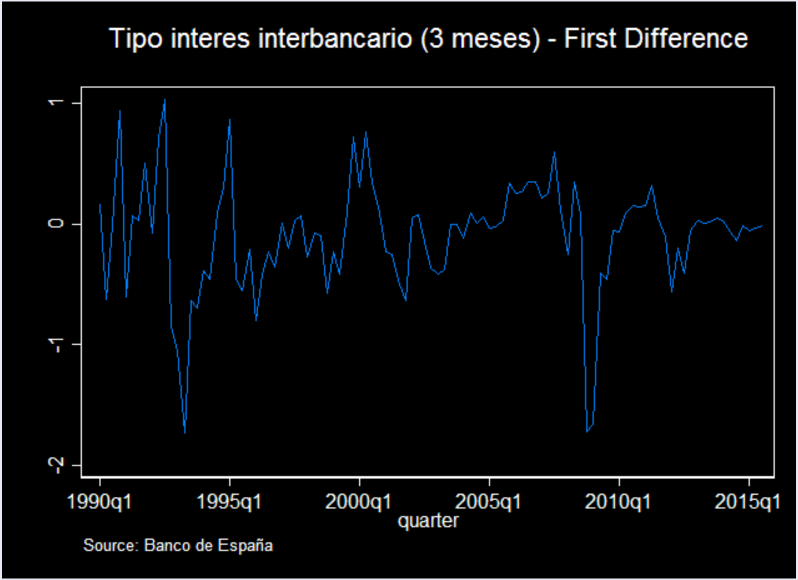
Different State Levels? - Fertility rate Spain 1960-2014



Different State Levels? - Fertility rate Spain 1960-2014



Different AR structure Levels? - Interbank interest rate UEM



Introductory Concepts

Markov-Switching Regression Models

- Models for time series that transition over a set of finite states.
- States are unobserved and the process can switch among states throughout the sample.
- The time of transition between states and the duration in a particular state are both random.
- The transitions follow a Markov process.
- We can estimate state-dependent and state-independent parameters.

Markov-Switching Regression Models

- Let's then define a Markov Chain:
  - Assume the states are defined by a random variable S_t that takes the integer values 1, 2, ..., N.
  - Then, the probability of the current state, j, only depends on the previous state:

$$P(S_t = j | S_{t-1} = i, S_{t-2} = k, S_{t-3} = w...) = P(S_t = j | S_{t-1} = i) = p_{ij}$$

Markov-Switching Regression Models

- Let's define a simple constant only model with three states:

$$y_t = \mu_s + \varepsilon_t$$

Where:

$$\begin{aligned} \mu_s &= \mu_1 \quad \text{if} \quad s = 1 \\ \mu_s &= \mu_2 \quad \text{if} \quad s = 2 \\ \mu_s &= \mu_3 \quad \text{if} \quad s = 3 \end{aligned}$$

- We do not know with certainty the current state, but we can estimate the probability.
- We can also estimate the transition probabilities:
  - p_{ij} : probability of being in state j in the current period given that the process was in state i in the previous period.

Transition probabilities, expected duration, tests

- We will then be interested in obtaining the matrix with the transition probabilities:

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}$$

Where:

$$p_{11} + p_{12} + p_{13} = 1$$
$$p_{21} + p_{22} + p_{23} = 1$$
$$p_{31} + p_{32} + p_{33} = 1$$

- We will also be interested in the expected duration for each state.
- We can perform tests for comparing parameters across states

Markov-switching dynamic regression

Markov-switching dynamic regression

- Allow states to switch according to a Markov process
- Allow for quick adjustments after a change of state.
- Often applied to high frequency data (monthly,weekly,etc.)

Markov-switching dynamic regression

- The model can be written as:

$$y_t = \mu_s + x_t\alpha + z_t\beta_s + \epsilon_{s,t}$$

Where:

y_t: Dependent variable

μ_s: State-dependent intercept

x_t: Vector of exog. variables with state invariant coefficients α

z_t: Vector of exog. variables with state-dependent coefficients β_s

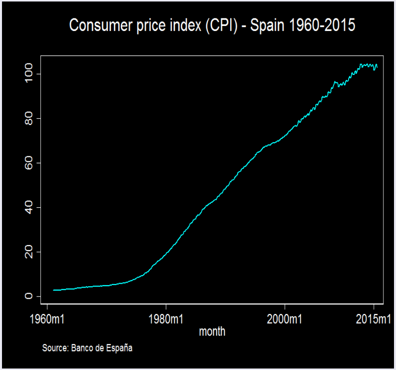
$\epsilon_{s,t}$ ~iid $N(0, \sigma_s^2)$

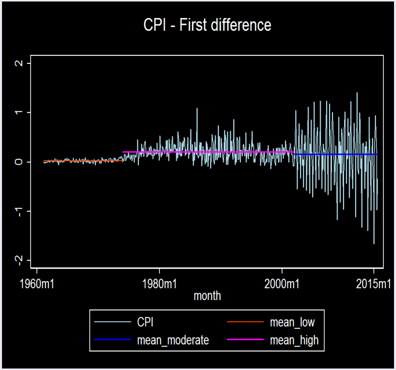
Markov-switching dynamic regression

```
mswitch dr depvar [nonswitch_varlist] [if] [in] [, options]
```

Markov switching dynamic regression

- Example 1:
  - Consumer price index for Spain 2011=100
  - Period: 1961m1 - 2015m8
  - Source: Banco de España





Markov switching dynamic regression with three states

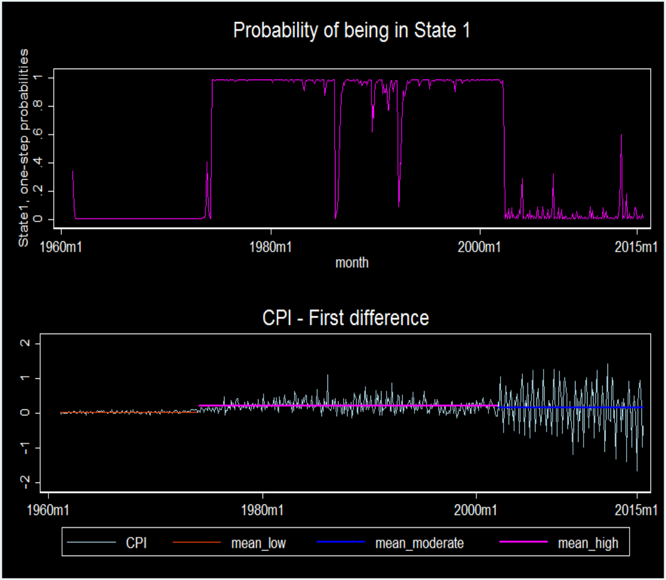
- Code (D. indicates first difference)
  - Fit the model
    - **mswitch dr D.ipc, states(3) varswitch nolog**
  - Predict probabilities of being at each state
    - **predict pr_state1 pr_state2 pr_state3, pr**

Markov switching dynamic regression with three states

```
. mswitch dr D.ipc,states(3) varswitch nolog
Markov-switching dynamic regression
Sample:  1961m2 -  2015m8                    No. of obs        =       655
Number of states =    3                      AIC               =   -0.8669
Unconditional probabilities: transition      HQIC              =   -0.8350
                                             SBIC              =   -0.7847
Log likelihood = 295.91091
```

D.ipc	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
State1						
_cons	.2060383	.0089781	22.95	0.000	.1884416	.223635
State2						
_cons	.0262835	.0027906	9.42	0.000	.020814	.0317529
State3						
_cons	.1668645	.0430856	3.87	0.000	.0824182	.2513108
sigma1	.1594772	.0065446			.1471524	.1728342
sigma2	.0321699	.0020182			.0284478	.0363791
sigma3	.5534114	.0306889			.4964158	.6169509

MSDR - Example 1: Probability of being in State 1



Markov switching dynamic regression with three states

```
. mswitch dr D.ipc,states(3) varswitch nolog
Markov-switching dynamic regression
Sample:  1961m2 -  2015m8                        No. of obs      =       655
Number of states =     3                         AIC             =   -0.8669
Unconditional probabilities: transition          HQIC            =   -0.8350
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```

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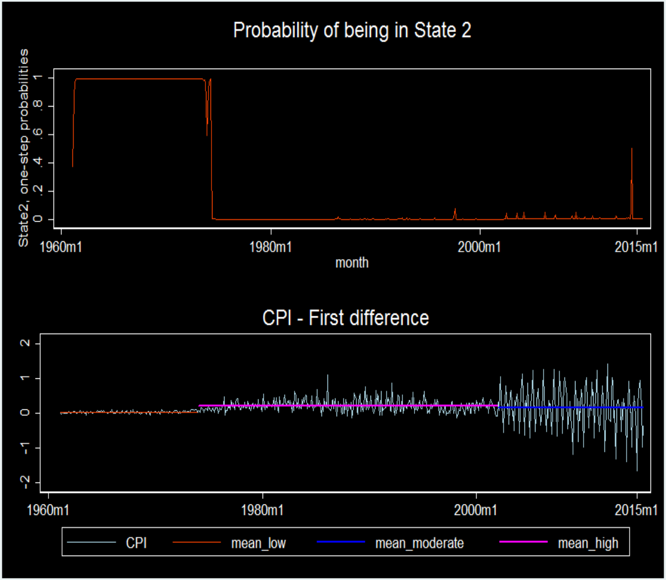
sigma3	.5534114	.0306889			.4964158	.6169509

MSDR - Example 1: Probability of being in State 2

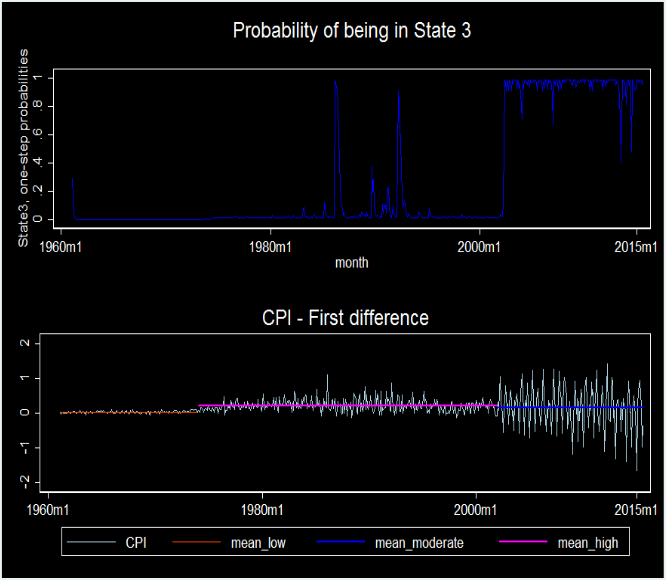


Markov switching dynamic regression with three states

```
. mswitch dr D.ipc,states(3) varswitch nolog
Markov-switching dynamic regression
Sample:  1961m2 -  2015m8                      No. of obs       =       655
Number of states =    3                        AIC              =   -0.8669
Unconditional probabilities: transition        HQIC             =   -0.8350
                                               SBIC             =   -0.7847
Log likelihood = 295.91091
```

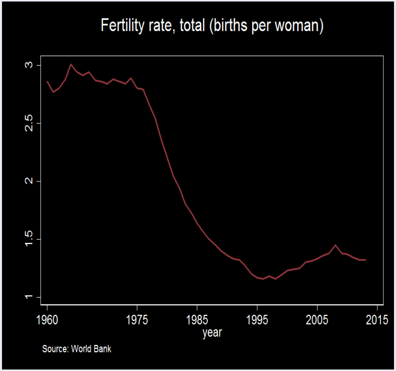
D.ipc	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
State1						
_cons	.2060383	.0089781	22.95	0.000	.1884416	.223635
State2						
_cons	.0262835	.0027906	9.42	0.000	.020814	.0317529
State3						
_cons	.1668645	.0430856	3.87	0.000	.0824182	.2513108
sigma1	.1594772	.0065446			.1471524	.1728342
sigma2	.0321699	.0020182			.0284478	.0363791
sigma3	.5534114	.0306889			.4964158	.6169509

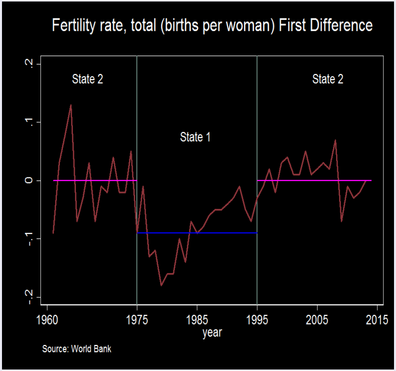
MSDR - Example 1: Probability of being in State 3



Markov switching dynamic regression

- Example 2:
  - Fertility rate (total births per woman) for Spain
  - Period: 1960 - 2013
  - Source: World Bank





Markov switching dynamic regression

- Example 2: Fertility rate in Spain (1960-2013)

Variables:

fertility: Fertility rate (total births per woman) for Spain

ch_mortality: Mortality rate, under 5 (per 1,000 live births)

gni_pcapita: GNI per capita (thousands - 2005 US$)

school_access: Primary and secondary school enrollment, (gross), gender parity index_(GPI)

Markov switching dynamic regression

- Fit the model

```
mswitch dr D.fertility D.ch_mortality, states(2) varswitch ///
        switch(D.gni_pcapita D.school_access) vsquish
```

- Test on equality of coefficients across states

```
test _b[State1:D.school_access]=_b[State2:D.school_access],notest
test _b[State1:D.gni_pcapita]=_b[State2:D.gni_pcapita],accumulate
```

- Transition probabilities and expected duration

```
estat transition
estat duration
```

Markov switching dynamic regression for fertility rate

```
. mswitch dr D.fertility D.ch_mortality, states(2) varswitch ///
>    switch(D.gni_pcapita D.school_access) vsquish

Markov-switching dynamic regression
```

D.fertility	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
D.fertility						
ch_mortality						
D1.	.0632344	.0107531	5.88	0.000	.0421587	.08431
State1						
gni_pcapita						
D1.	.0434944	.0115159	3.78	0.000	.0209236	.0660652
school_access						
D1.	-1.042254	.1833868	-5.68	0.000	-1.401686	-.6828228
_cons	-.058854	.0117087	-5.03	0.000	-.0818027	-.0359054
State2						
gni_pcapita						
D1.	.0647241	.0149331	4.33	0.000	.0354557	.0939925
school_access						
D1.	-1.15316	.557033	-2.07	0.038	-2.244925	-.0613957
_cons	-.0078558	.0089525	-0.88	0.380	-.0254023	.0096907
sigma1	.0087495	.0020577			.0055183	.0138728
sigma2	.0372011	.0046423			.0291296	.0475091

Markov switching dynamic regression for fertility rate

- Test on the equality of coefficients across states

```
. test _b[State1:D.gni_pcapita]=_b[State2:D.gni_pcapita],notest
( 1)  [State1]D.gni_pcapita - [State2]D.gni_pcapita = 0


. test _b[State1:D.school_access]=_b[State2:D.school_access],accumulate

( 1)  [State1]D.gni_pcapita - [State2]D.gni_pcapita = 0
( 2)  [State1]D.school_access - [State2]D.school_access = 0
           chi2(  2) =    1.23
         Prob > chi2 =    0.5411
```

- Test on the equality of sigma across states

```
. test _b[lnsigma1:_cons]=_b[lnsigma2:_cons]
( 1)  [lnsigma1]_cons - [lnsigma2]_cons = 0
           chi2(  1) =   29.23
         Prob > chi2 =    0.0000
```

Markov switching dynamic regression for fertility rate

```
. mswitch dr D.fertility D.(ch_mortality gni_pcapita school_access),      ///
>         states(2) varswitch vsquish
```

Markov-switching dynamic regression

D.fertility	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
D.fertility						
ch_mortality						
D1.	.048953	.0072042	6.80	0.000	.034833	.0630729
gni_pcapita						
D1.	.0505516	.0091887	5.50	0.000	.0325422	.068561
school_access						
D1.	-.6381832	.2571355	-2.48	0.013	-1.142159	-.134207
State1						
_cons	-.0704097	.0082148	-8.57	0.000	-.0865104	-.054309
State2						
_cons	.0013952	.0071861	0.19	0.846	-.0126893	.0154798
sigma1	.019568	.0037427			.0134505	.0284678
sigma2	.0278482	.0041421			.0208062	.0372737

Markov switching dynamic regression for fertility rate

- Transition probabilities

```
. estat transition
```

Number of obs = 42

Transition Probabilities	Estimate	Std. Err.	[95% Conf.	Interval]
p11	.9178466	.06678	.6632132	.9844686
p12	.0821534	.06678	.0155314	.3367868
p21	.0311448	.0327426	.003818	.2123665
p22	.9688552	.0327426	.7876335	.996182

Markov switching dynamic regression for fertility rate

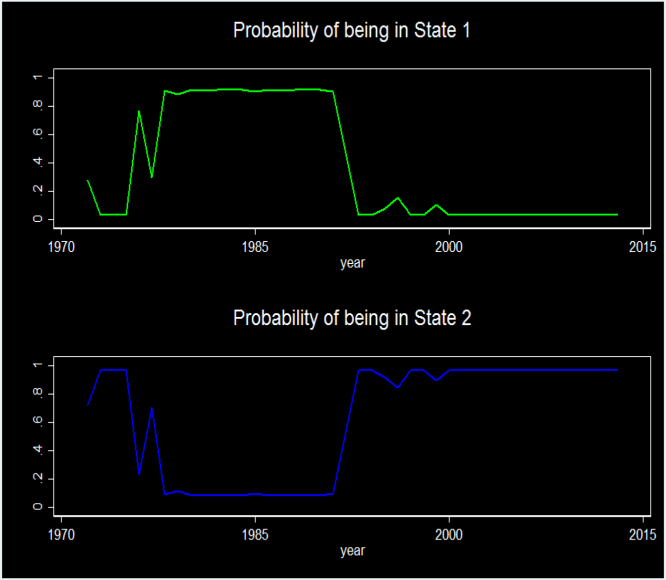
- Expected duration

```
. estat duration
```

Number of obs = 42

Expected Duration	Estimate	Std. Err.	[95% Conf.	Interval]
State1	12.17235	9.894526	2.969237	64.38565
State2	32.10806	33.75521	4.70884	261.9202

Markov switching dynamic regression for fertility rate



Markov-switching AR model

Markov-switching AR model

- Allow states to switch according to a Markov process
- Allow a gradual adjustment after a change of state.
- Often applied to lower frequency data (quarterly, yearly, etc.)

Markov-switching AR model

- The model can be written as:

$$y_t = \mu_{s_t} + x_t\alpha + z_t\beta_{s_t} + \sum_{i=1}^{P} \phi_{i,s_t}(y_{t-i} - \mu_{s_{t-i}} - x_{t-i}\alpha + z_{t-i}\beta_{s_{t-i}}) + \epsilon_{t,s}$$

Where:

y_t: Dependent variable

μ_{s_t}: State-dependent intercept

x_t: Vector of exog. variables with state invariant coefficients α

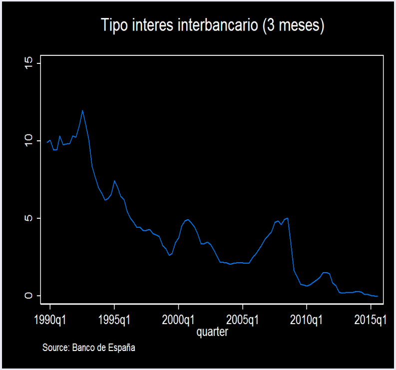
z_t: Vector of exog. variables with state-dependent coefficients β_{s_t}

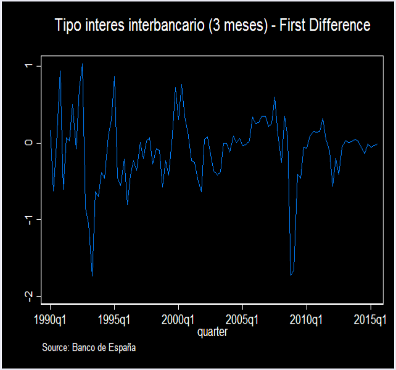
ϕ_{i,s_t}: i^{th} AR term in state s_t

$\epsilon_{s,t}$ ~iid $N(0, \sigma_s^2)$

Markov switching AR model

- Example 3:
  - Interbank interest rate for Spain
  - Period: 1989Q4 - 2015Q3
  - Source: Banco de España





Markov switching AR model

- Example 3: Interest rate in Spain (1990Q1-2015Q3)

  Variables:

  r_interbank: Three months interbank rate

  ipc: Consumer price index

Markov switching AR model

- Fit the model

```
mswitch ar D.r_interbank D.ipc if tin(1990q2,2012q4), ///
        states(2) ar(1) arswitch varswitch constant     ///
        switch(,noconstant) nolog
```

- Transition probabilities and expected duration

```
        estat transition
        estat duration
```

Markov switching AR model

```
. mswitch ar D.r_interbank D.ipc if tin(1990q2,2012q4),states(2) ar(1) ///
>      arswitch  nolog switch(,noconstant) constant varswitch vsquish

Markov-switching autoregression
Sample: 1990q2 - 2012q4                        No. of obs       =        91
Number of states =     2                       AIC              =    1.1681
Unconditional probabilities: transition        HQIC             =    1.2572

------------------------------------------------------------------------------
D.           |
 r_interbank |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
D.r_interb~k |
         ipc |
         D1. |   .1345492   .0430415     3.13   0.002     .0501895     .218909
       _cons |  -.1287786   .0299325    -4.30   0.000    -.1874453   -.0701119
-------------+----------------------------------------------------------------
State1       |
          ar |
          L1. |  -.5821326   .0868487    -6.70   0.000    -.7523529   -.4119122
-------------+----------------------------------------------------------------
State2       |
          ar |
         L1. |    .600846   .1133802     5.30   0.000     .3786249    .8230671
-------------+----------------------------------------------------------------
      sigma1 |     .10039    .021533                      .0659346    .1528509
-------------+----------------------------------------------------------------
      sigma2 |   .4279839   .0404373                      .3556339    .5150526
------------------------------------------------------------------------------
```

Markov switching AR model

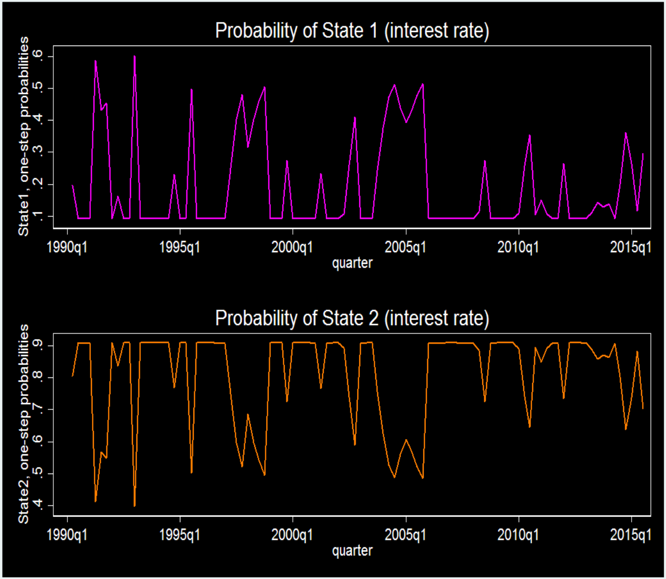
```
. estat transition
Number of obs = 91
```

Transition Probabilities	Estimate	Std. Err.	[95% Conf.	Interval]
p11	.6238106	.1906249	.2523082	.8906938
p12	.3761894	.1906249	.1093062	.7476918
p21	.0917497	.0529781	.0282364	.2599153
p22	.9082503	.0529781	.7400847	.9717636

```
. estat duration
Number of obs = 91
```

Expected Duration	Estimate	Std. Err.	[95% Conf.	Interval]
State1	2.658235	1.346997	1.33745	9.148609
State2	10.89922	6.293423	3.847408	35.41533

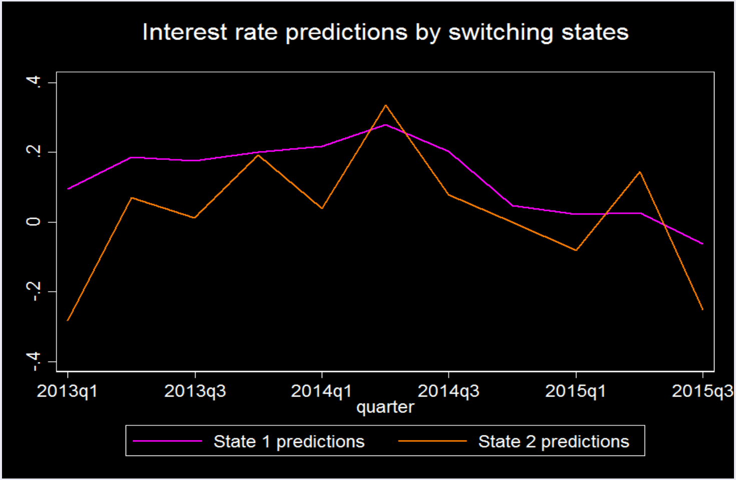
MSAR - Example 3: Probability of being in each State



Markov switching AR model

```
. predict state*,yhat dynamic(tq(2012q4))

. forvalues i=1/2 {
  2.    generate y_st`i´=state`i´+L.r_interbank
  3. }
```



Markov switching AR model

- Comparing results from a different dynamic model (ARCH)

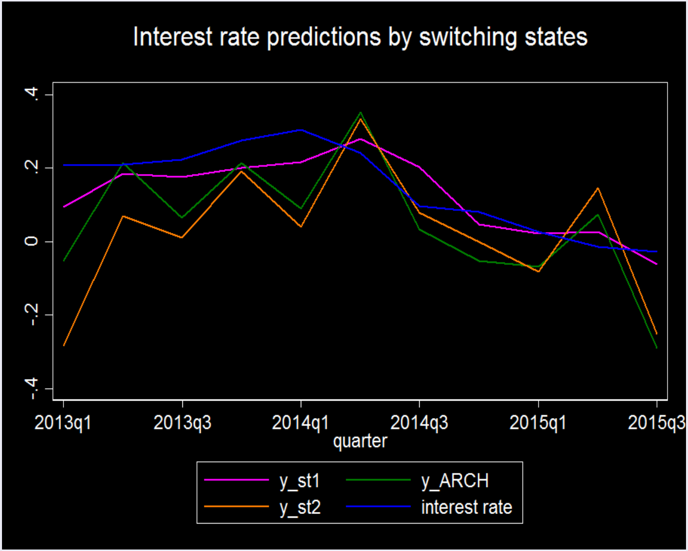
```
. arch D.r_interbank D.ipc if tin(1990q2,2012q4),arch(1) ///
>          garch(1) ar(1) nolog vsquish
ARCH family regression -- AR disturbances

Sample: 1990q2 - 2012q4                         Number of obs   =         91
Distribution: Gaussian                          Wald chi2(2)    =      18.56
Log likelihood = -45.09683                      Prob > chi2     =     0.0001

------------------------------------------------------------------------------
D.           |                 OPG
 r_interbank |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
r_interbank  |
         ipc |
         D1. |   .0765736   .0491625     1.56   0.119    -.0197832    .1729305
       _cons |  -.1778991    .081251    -2.19   0.029    -.3371482   -.0186501
-------------+----------------------------------------------------------------
ARMA         |
          ar |
         L1. |   .5016758    .126954     3.95   0.000     .2528505    .7505011
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   .3809789   .2505937     1.52   0.128    -.1101757    .8721334
       garch |
         L1. |   .5630487   .2146123     2.62   0.009     .1424163     .983681
       _cons |   .0227445   .0202104     1.13   0.260    -.0168671    .0623561
------------------------------------------------------------------------------
```

Markov switching AR model

- Comparing results from a different dynamic model (ARCH)



Summary

1. When we use Markov-Switching Regression Models
2. Introductory concepts
3. Markov-Switching Dynamic Regression
    - Predictions
        - State probabilities predictions
        - Level predictions
    - State expected durations
    - Transition probabilities
4. Markov-Switching AR Models

References

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