Cointegrating VAR and Probability Forecasting: An application for small open economies

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StataCorp LP

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International Monetary Fund

Barcelona, Spain
Outline

- Why Cointegrating VAR models
- Why Probability Forecasting
- VEC and Cointegrating VAR Models
  - Theoretical comments
  - Case study: Uruguay
- Probability Forecasting
  - Some theoretical comments
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Why Cointegrating VAR

- vec- command: The default (Johansen normalization)

. webuse lutkepohl
(Quarterly SA West German macro data, Bil DM, from Lutkepohl 1993 Table E.1)
. vec linvest lconsumption lincome, lags(4) rank(2) noetable
Vector error-correction model
Sample: 1961q1 - 1982q4 No. of obs = 88
Identification: beta is exactly identified
Johansen normalization restrictions imposed

| beta      | Coef. | Std. Err. | z     | P>|z|  | [95% Conf. Interval] |
|-----------|-------|-----------|-------|-------|---------------------|
| _ce1      |       |           |       |       |                     |
| linvestment| 1     |           | .     | .     | .                   |
| lconsumption| 0   | (omitted)| .     |       |                     |
| lincome    | -.863381 | .0468879  | -18.41 | 0.000 | -.9552796 -.7714825 |
| _cons     | .034525 |           | .     | .     | .                   |
| _ce2      |       |           |       |       |                     |
| linvestment| 8.67e-19 |           | .     | .     | .                   |
| lconsumption| 1   |           | .     |       | .                   |
| lincome    | -.9670451 | .0045147  | -214.20 | 0.000 | -.9758938 -.9581964 |
| _cons     | -.1447284 |           | .     | .     | .                   |
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- **-vec- command:** *Theoretical long-run relationship*

```
constraint define 1 [_ce1]linvest =1
constraint define 2 [_ce1]lincome =-.75
constraint define 3 [_ce2]lconsumption=1
constraint define 4 [_ce2]lincome =-.9
vec linvest lconsumption lincome, lags(4) rank(2) noetable ///
>   bconstraints(1/4) nolog nocnsreport
```

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|--------------|----------|-----------|-------|-----|---------------------|
| _ce1         |          |           |       |     |                     |
| linvestment  | 1        |           | .     | .   |                     |
| lconsumption | -.1172    | .0480998  | -2.44 | 0.015 | -.2115187 -.022971 |
| lincome      | -.75     |           | .     | .   |                     |
| _cons        | .0514938 |           | .     | .   |                     |
| _ce2         |          |           |       |     |                     |
| linvestment  | -.0776    | .0068539  | -11.33| 0.000 | -.0910876 -.0642206 |
| lconsumption | 1        |           | .     | .   |                     |
| lincome      | -.9      |           | .     | .   |                     |
| _cons        | -.1474    |           | .     | .   |                     |
```
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| linvestment    | 1      |           |        |      |                      |
| lconsumption   | -.1172448 | .0480998 | -2.44  | 0.015| -.2115187 - -.022971 |
| lincome        | -.75   |           |        |      |                      |
| _cons          | .0514938 |           |        |      |                      |
| \_ce2          |        |           |        |      |                      |
| linvestment    | -.0776541 | .0068539 | -11.33 | 0.000| -.0910876 - -.0642206|
| lconsumption   | 1      |           |        |      |                      |
| lincome        | -.9    |           |        |      |                      |
| _cons          | -.1474094 |           |        |      |                      |

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### Why Cointegrating VAR

```
. vec linvest lconsumption lincome, ...

*** Output Omitted ***
```

|                         | Coef.   | Std. Err. | z      | P>|z|  | [95% Conf. Interval] |
|-------------------------|---------|-----------|--------|------|---------------------|
| D_linvestment           |         |           |        |      |                     |
| _ce1                    |         |           |        |      |                     |
| L1.                     | -0.1040128 | 0.0790621 | -1.32  | 0.188 | -0.2589717 0.0509461 |
| _ce2                    |         |           |        |      |                     |
| L1.                     | 0.6979509  | 0.4424682 | 1.58  | 0.115 | -0.169271 1.565173  |
| linvestment             |         |           |        |      |                     |
| LD.                     | -0.2246769 | 0.1290886 | -1.74  | 0.082 | -0.4776859 0.028332 |
| L2D.                    | -0.1279608 | 0.1286637 | -0.99  | 0.320 | -0.3801371 0.1242155 |
| L3D.                    | 0.0304474  | 0.126218  | 0.24  | 0.809 | -0.2169353 0.2778301 |
| lconsumption            |         |           |        |      |                     |
| LD.                     | 0.0546931  | 0.6438309 | 0.08  | 0.932 | -1.207192 1.316578  |
| L2D.                    | 0.3857924  | 0.6440095 | 0.60  | 0.549 | -0.8764429 1.648028  |
| L3D.                    | -0.0152393 | 0.5789952 | -0.03 | 0.979 | -1.150049 1.11957   |
| lincome                 |         |           |        |      |                     |
| LD.                     | 1.059818  | 0.6446678 | 1.64  | 0.100 | -0.2037081 2.323343  |
| L2D.                    | 0.7647925  | 0.5900074 | 1.30  | 0.195 | -0.3916007 1.921186  |
| L3D.                    | 0.7128608  | 0.5460636 | 1.31  | 0.192 | -0.3574042 1.783126  |
| _cons                   | 0.0007756  | 0.0211156 | 0.04  | 0.971 | -0.0406103 0.0421615 |

*** Output Omitted ***
Why Probability Forecasting

- Uncertainty expressed in terms of confidence intervals
Why Probability Forecasting

- Uncertainty expressed in terms of simulations for single and composed events

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VECM and Cointegrating VAR Models
Cointegrating VAR Models - Theoretical comments

- Based on the vector error correction (VEC) model specification.
  - The specification assumes that economic theory characterizes the long-run behavior.
  - The short-run fluctuations represent deviations from the equilibrium.
  - The short-run and long-run economic concepts are linked to the statistical concept of stationarity.


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Reduced form for a VEC model

\[ \Delta y_t = \gamma_0 + \gamma_1 t - \alpha \beta y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \mu_t \]  

(1)

Where:

- \( y_t \): I(1) Endogenous variables
- \( \alpha \beta \): Matrices containing the long-run adjustment coefficients and cointegrating relationships
- \( \Gamma_i \): Matrix with coefficients associated to short-run dynamic effects
- \( \gamma_0, \gamma_1 \): Vectors with coefficients associated to the intercepts and trends
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  - Use economic theory to impose restrictions to identify $\alpha$ and $\beta$.
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- Small open economy with soy, livestock, leather, and rice as the main export products.

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Uruguay: Some economic indicators

- Exports
- Imports
- Exports price index
- GDP

Source: International Monetary Fund
Variables in the cointegrating VAR

- **m1**: Currency and demand deposits
- **pib**: Gross domestic product (GDP)
- **tipp906bn**: Interest rate.
- **tcpn**: Exchange rate.
- **ipcp97**: Consumer price index (1997 = 100):
- **mt**: Imports
- **xt**: Exports
- **ipex**: Exports price index.
**Long-run cointegrating relationships**

\[ ce1: \text{lm1} = \beta_{11} \times \text{lpib} + \beta_{12} \times \text{ltipp906bn} + \beta_{10} \]

\[ ce2: \text{lmt} = \beta_{21} \times \text{lpib} + \beta_{22} \times \text{ltcpn} + \beta_{20} \]

\[ ce3: \text{lxt} = \beta_{31} \times \text{lipcp97} + \beta_{32} \times \text{ltcpn} + \beta_{33} \times \text{lipex} + \beta_{30} \]

**Long-run adjustment restrictions**

\[ [D \text{lipex}] \alpha_{ce1} = 0 \]

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** Restrictions on long-run cointegrating equations (bconstraints) **

* Restriction on long run equation for lm1 *

constraint 1 \([ _{ce1}lm1=1 \]
constraint 2 \([ _{ce1}lipcp97=0 \]
constraint 3 \([ _{ce1}ltcpn=0 \]
constraint 4 \([ _{ce1}lmt=0 \]
constraint 5 \([ _{ce1}lxt=0 \]
constraint 6 \([ _{ce1}lipex=0 \]

* Restrictions on long run equation for lmt *

constraint 7 \([ _{ce2}lm1=0 \]
constraint 8 \([ _{ce2}ltipp906bn=0 \]
constraint 9 \([ _{ce2}lipcp97=0 \]
constraint 10 \([ _{ce2}lmt=1 \]
constraint 11 \([ _{ce2}lxt=0 \]
constraint 12 \([ _{ce2}lipex=0 \]

* Restrictions on long run equation for lxt *

constraint 13 \([ _{ce3}lm1=0 \]
constraint 14 \([ _{ce3}lpib=0 \]
constraint 15 \([ _{ce3}ltipp906bn=0 \]
constraint 16 \([ _{ce3}lmt=0 \]
constraint 17 \([ _{ce3}lxt=1 \]

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constraint 4 \[ ce1 \]lmt=0  
constraint 5 \[ ce1 \]lxt=0  
constraint 6 \[ ce1 \]lipex=0

* Restrictions on long run equation for lmt *

constraint 7 \[ ce2 \]lm1=0  
constraint 8 \[ ce2 \]ltipp906bn=0  
constraint 9 \[ ce2 \]lipcp97=0  
constraint 10 \[ ce2 \]lmt=1  
constraint 11 \[ ce2 \]lxt=0  
constraint 12 \[ ce2 \]lipex=0

* Restrictions on long run equation for lxt *

constraint 13 \[ ce3 \]lm1=0  
constraint 14 \[ ce3 \]lpib=0  
constraint 15 \[ ce3 \]ltipp906bn=0  
constraint 16 \[ ce3 \]lmt=0  
constraint 17 \[ ce3 \]lxt=1
** Restrictions on long-run cointegrating equations (bconstraints) **

* Restriction on long run equation for lm1 *

constraint 1 \(_{ce1}lm1=1\)
constraint 2 \(_{ce1}lipcp97=0\)
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constraint 4 \(_{ce1}lmt=0\)
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constraint 17 \(_{ce3}lxt=1\)
* Restrictions on long-run adjustment coefficients (aconstraints) *

constraint 18 [D_lipex]l._ce1=0
constraint 19 [D_lipex]l._ce2=0
constraint 20 [D_lipex]l._ce3=0

*** VEC model specification ***

vec lm1 lpib ltip906bn lipcp97 ltcpn lmt lxt lipex, ///
   bconstraints(1/17) ///
   aconstraints(18/20) ///
   lags(4) rank(3) noetable ///
   ltolerance(1e-7) tolerance(1e-4)
* Restrictions on long-run adjustment coefficients (aconstraints) *

constraint 18 [D_lipex]1._ce1=0
constraint 19 [D_lipex]1._ce2=0
constraint 20 [D_lipex]1._ce3=0

*** VEC model specification ***

vec lm1 lpib ltip906bn lipcp97 ltcpn lmt lxt lipex, ///
   bconstraints(1/17) ///
   aconstraints(18/20) ///
   lags(4) rank(3) notable ///
   ltolerance(1e-7) tolerance(1e-4)
## Cointegrating Equations

Sample: 1990q1 - 2011q2  
No. of obs = 86

AIC = -27.58711  
HQIC = -24.96839  
SBIC = -21.08023

### Cointegrating equations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Parms</th>
<th>chi2</th>
<th>P&gt;chi2</th>
</tr>
</thead>
<tbody>
<tr>
<td>_ce1</td>
<td>2</td>
<td>170.89</td>
<td>0.0000</td>
</tr>
<tr>
<td>_ce2</td>
<td>2</td>
<td>115.2276</td>
<td>0.0000</td>
</tr>
<tr>
<td>_ce3</td>
<td>3</td>
<td>111.1596</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Identification: beta is overidentified

| beta     | Coef.  | Std. Err. | z     | P>|z| | [95% Conf. Interval] |
|----------|--------|-----------|-------|------|---------------------|
| _ce1     |        |           |       |      |                     |
| lm1      | 1      |           | .     | .    | .                   |
| lpib     | -2.388095 | .2667403 | -8.95 | 0.000 | -2.910897 -1.865294 |
| ltipp906bn | .2394962 | .0401545 | 5.96  | 0.000 | .1607948 .3181976  |
| lipcp97  | 0      | (omitted) | .     | .    | .                   |
| ltcpn    | 0      | (omitted) | .     | .    | .                   |
| lmt      | 0      | (omitted) | .     | .    | .                   |
| lxt      | 0      | (omitted) | .     | .    | .                   |
| lipex    | 0      | (ommitted) | .     | .    | .                   |
| _cons    | 34.28762 | .       | .     | .    | .                   |
## Cointegrating Equations

| beta      | Coef.       | Std. Err. | z        | P>|z|   | [95% Conf. Interval] |
|-----------|-------------|-----------|----------|-------|----------------------|
| _ce2      |             |           |          |       |                      |
| lm1       | 0 (omitted) |           |          |       |                      |
| lpt9097   |-1.190284    | .1176426  | -10.12   | 0.000 | -1.420859 - .9597084 |
| ltip9096bn| 0 (omitted) |           |          |       |                      |
| ltpcp97   | 0 (omitted) |           |          |       |                      |
| ltcpn     | .1906142    | .0365307  | 5.22     | 0.000 | .1190154 .262213     |
| lmt       | 1           |           | .        | .     |                      |
| lxt       | 0 (omitted) |           |          |       |                      |
| lipex     | 0 (omitted) |           |          |       |                      |
| _cons     | 4.087814    |           |          | .     |                      |
| _ce3      |             |           |          |       |                      |
| lm1       | 0 (omitted) |           |          |       |                      |
| lpt9097   | 268.7157    | 28.66662  | 9.37     | 0.000 | 212.5301 324.9012    |
| ltip9096bn| 0 (omitted) |           |          |       |                      |
| ltcpn     | -130.17     | 27.2982   | -4.77    | 0.000 | -183.6735 -76.66648  |
| lmt       | 0 (omitted) |           |          |       |                      |
| lxt       | 1           |           | .        | .     |                      |
| lipex     | -337.4693   | 45.183    | -7.47    | 0.000 | -426.0264 -248.9123  |
| _cons     | 470.4702    |           |          | .     |                      |
Long-run cointegrating relationships

\[
l_{m1} = 2.39 \times l_{pib} - 0.24 \times l_{tipp906bn} - 34.29
\]

\[
l_{mt} = 1.19 \times l_{pib} - 0.19 \times l_{tcpn} - 4.09
\]

\[
l_{xt} = -268.7 \times l_{ipcp97} + 130.2 \times l_{tcpn} + 337.5 \times l_{ipex} - 470.5
\]
Probability Forecasting

- Obtain the probability that a single or joint event occurs, conditional on the information contained in the estimation sample.
- We could define the event in terms of the levels of one or more variables, for one or more future time periods.
- It is associated to the uncertainty inherent to the predictions produced by regression models.
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Probability Forecasting

- This methodology can be applied to a wide spectrum of models. Our focus here is on the predictions from a cointegrating VAR model.

- In general, forecasting based on econometric models are subject to:
  - Future uncertainty
  - Parameters uncertainty
  - Model uncertainty
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- **Future uncertainty with parametric approach**
  - Let’s consider a standard linear regression model:
    \[ y_t = x_{t-1} \cdot \beta + \mu_t \; ; \; \mu \sim N(0, \sigma^2) \]
  - For \( \sigma^2 \) known we could simulate \( y_{T+1}^{(s)} \):
    \[ y_{T+1}^{(s)} = x_t \cdot \hat{\beta} + \mu_{T+1}^{(s)} \; ; \; s = 1, 2, \ldots, S \]

Where:
\( \mu_{T+1}^{(s)} \) is the s-th random draw from \( N(0, \sigma^2) \)
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- Computations for VAR cointegrating models

  - Let’s consider the VECM model:

    \[ \Delta y_t = \gamma_0 + \gamma_1 \times t - \alpha \beta y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \mu_t \]

Non-Parametric Approach

- Simulated errors are drawn from in sample residuals.
- The Choleski decomposition for the estimated Var-Cov matrix of the error term is used in a two-stage procedure combined with the simulated errors.
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- Computations for VAR cointegrating models

  - Let’s consider the VECM model:

    \[ \Delta y_t = \gamma_0 + \gamma_1 * t - \alpha \beta y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \mu_t \]

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Sample code for Probability forecasting after VEC

** VEC model **
```
vec lm1 lpib ltipp906bn lipcp97 ltcpn lmt lxt lipex, ///
bconstraints(1/17) aconstraint(18/20) lags(4) rank(3) ...
```

** Residuals for VEC ecuations **
```
foreach x of varlist lm1 lmt lxt lpib ... lipex {
    predict double res_`x´ if e(sample),resid equat(D_`x´)
}
```

** Transform residuals for simulation - Garrat et al. **
```
matrix sigma=e(omega)
matrix P=cholesky(sigma)
mkmat res_lm1 res_lmt res_lxt res_lpib ... res_lipex ///
    if tin(1990q1,2011q2),matrix(res)
matrix invP_res=inv(P)*res´
matrix invP_rs1=invP_res´ /* Matrix in last line on p.167 */
```
Sample code for Probability forecasting after VEC

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matrix P=cholesky(sigma)
mkmat res_lm1 res_lmt res_lxt res_lpib ... res_lipex ///
   if tin(1990q1,2011q2),matrix(res)
matrix invP_res=inv(P)*res´
matrix invP_rs1=invP_res´ /* Matrix in last line on p.167 */
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** VEC model **
vec lm1 lpib ltip906bn lpcp97 ltcpn lmt lxt lipex, ///
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- **Simulation**
  - Generate dynamic predictions (from Cointegrating VAR) for the forecasting period.
  - Replications
    - Draw sample of transformed residuals and add them to point dynamic forecasts.
  - Define events and obtain proportions with simulated dynamic predictions.
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Case study: Uruguay
**Case study: Uruguay**

- Scenarios for Export Prices

### Interannual Change in Exports Prices

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Quarter</th>
<th>Inertial</th>
<th>Moderate Impact</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011Q3</td>
<td>17.26</td>
<td>16.40</td>
<td></td>
</tr>
<tr>
<td>2011Q4</td>
<td>13.63</td>
<td>9.90</td>
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<tr>
<td>2012Q1</td>
<td>10.28</td>
<td>2.70</td>
<td></td>
</tr>
<tr>
<td>2012Q2</td>
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<td>-2.00</td>
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</tr>
<tr>
<td>2012Q3</td>
<td>6.04</td>
<td>-3.60</td>
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<tr>
<td>2012Q4</td>
<td>7.41</td>
<td>-2.40</td>
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</tr>
</tbody>
</table>
Case study: Uruguay

Probability forecasting for GDP

Scenarios for Change in GDP

<table>
<thead>
<tr>
<th>Event</th>
<th>Year</th>
<th>Proportion</th>
<th>Std. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in GDP &lt; 6</td>
<td>2011</td>
<td>.72</td>
<td>.04512609</td>
</tr>
<tr>
<td>Change in GDP &gt; 7.5</td>
<td>2011</td>
<td>.06</td>
<td>.02386833</td>
</tr>
<tr>
<td>Change in GDP &gt; 5.5 and &lt; 7.5</td>
<td>2011</td>
<td>.33</td>
<td>.04725816</td>
</tr>
<tr>
<td>Change in GDP &lt; 4.2</td>
<td>2012</td>
<td>.41</td>
<td>.04943111</td>
</tr>
<tr>
<td>Change in GDP &gt; 5.9</td>
<td>2012</td>
<td>.33</td>
<td>.04725816</td>
</tr>
<tr>
<td>Change in GDP &gt; 4.2 and &lt; 5.9</td>
<td>2012</td>
<td>.26</td>
<td>.0440844</td>
</tr>
</tbody>
</table>
Case study: Uruguay

Kernel density estimate for Change in GDP 2012

- Kernel: epanechnikov
- Bandwidth: 0.7219

![Graph showing the kernel density estimate for Change in GDP 2012. The graph displays a peak around the change value of 5, with decreasing density towards 0 and 10.]
References


Questions - Comments