Teaching consumer theory with maximum likelihood estimation of demand systems

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1 The course
Outline

1. The course
2. The template
The course

The template

The application

The outcomes
Applied economics – connecting theory to data
PhD program, course, and students

- Applied economics – connecting theory to data
- Indirect utility, $V(p, w)$, and expenditure, $e(p, u)$ – empirical content of theory
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- Beyond Mas-Collel, Whinston, and Green to weak separability, and demographic scaling
Applied economics – connecting theory to data

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First year PhD students simultaneously take first PhD econometrics course
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- First year PhD students simultaneously take first PhD econometrics course
- Provide introduction to SUR, maximum likelihood, bootstrapping, and nonparametric regression
Estimation problem

- Equation system

\[ y_1 = f(X_1; \beta_1) + \varepsilon_1 \]
\[ y_2 = f(X_2; \beta_2) + \varepsilon_2 \]
\[ \vdots \]
\[ y_k = f(X_k; \beta_k) + \varepsilon_k \]

\[ \varepsilon \sim \mathcal{N}(0, \Sigma) \]
Estimation problem

**Equation system**

\[
\begin{align*}
    y_1 &= f(X_1; \beta_1) + \varepsilon_1 \\
    y_2 &= f(X_2; \beta_2) + \varepsilon_2 \\
    & \vdots \\
    y_k &= f(X_k; \beta_k) + \varepsilon_k \\
    \varepsilon &\sim N(0, \Sigma)
\end{align*}
\]

**Log likelihood function**

\[
\begin{align*}
    \ln L &= -\frac{n}{2} \ln(2\pi) - \frac{1}{2} \ln |\Sigma| \\
           & - \frac{1}{2} (y - f(X; \beta))' \Sigma^{-1} (y - f(X; \beta))
\end{align*}
\]
Concentrated log likelihood function

\[
\ln L = -\frac{n}{2} \ln(2\pi) \\
- \frac{1}{2} \ln \left| (y - f(X; \beta)) (y - f(X; \beta))' \right| \\
- \frac{1}{2} I \\
\Rightarrow \hat{\beta}_{ML} = \min_{\beta} \frac{1}{2} \ln \left| (y - f(X; \beta)) (y - f(X; \beta))' \right|
\]
File structure of the template

- findit demand system estimation → st0029 from Stata journal 2-4
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Summary

File structure of the template

- `findit demand system estimation → st0029` from Stata journal 2-4
- `ado files: ln1_quaids, quaids_delta, quaids_params, quaids_vec, vec_sum`
File structure of the template

- findit demand system estimation → st0029 from Stata journal 2-4
- ado files: lnl_quaids, quaids_delta, quaids_params, quaids_vec, vec_sum
- ancillary files: quaids.do, food.dta
The template

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- ado files: lnl_quaids, quaids_delta, quaids_params, quaids_vec, vec_sum
- ancillary files: quaids.do, food.dta
- quaids share equations:

\[ s_i = \frac{\partial \ln a(p)}{\partial \ln p_i} + \frac{\partial b(p)}{\partial \ln p_i} \ln w + \frac{\partial \lambda(p)}{\partial \ln p_i} \frac{1}{b(p)} (\ln w)^2 \]
The template do file

```
ml model d0 lnl_quaids () /a2 /a3 /b1 /b2 /b3 /*
*/ /g11 /g21 /g31 /g22 /g32 /g33 /l1 /l2 /l3
ml search
ml maximize, noclear nooutput

mat b = e(b)
quaid_vec b beta
mat v = e(V)
quaid_delta r
mat var = r*v*r'
glo anames ""
glo bnames ""
glo lnames ""
glo gnames ""
```
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```bash
forv i = 1/$NEQN {
    glo anames "$anames alpha:‘i’"
    glo bnames "$bnames beta:‘i’"
    glo lnames "$lnames lambda:‘i’"
    forv j = ‘i’/$NEQN {
        glo gnames "$gnames gamma:‘j’‘i’"
    }
    glo names "$anames $bnames $gnames $lnames"
    mat colnames beta = $names
    mat colnames var = $names
    mat rownames var = $names
    estimates post beta var
    estimates display
```
Call to recover parameters:
```
tempname alpha beta gamma lambda

local nm1 = $NEQN-1
/* Get the parameters out of b. */
quaisds_params 'b' 'alpha' 'beta' 'gamma' 'lambda'
```
Call to recover parameters:
\[
\text{tempname alpha beta gamma lambda}
\]
\[
\text{tempvar deflexp Inpindex bofp}
\]
\[
\text{local nm1 = $NEQN-1}
\]
/* Get the parameters out of b. */
\[
\text{quaids_params ‘b’ ‘alpha’ ‘beta’ ‘gamma’ ‘lambda’}
\]

Quaids_params does the work:
/* Gamma will take more work. */
\[
\text{matrix ‘gamma’ = J($NEQN, $NEQN, 0)}
\]
/* First get the (k-1) by (k-1) symmetric matrix. */
\[
\text{local k = 1}
\]
\[
\text{forvalues j = 1/'nm1' {}
\]
\[
\text{forvalues i = ‘j’/’nm1’ {}
\]
\[
\text{matrix ‘gamma’[‘i’, ‘j’] = ‘b’[1, (2*$NEQN - 2 + ‘k’)]}
\]
if (‘i’ = ‘j’) {
\[
\text{matrix ‘gamma’[‘j’, ‘i’] = ‘gamma’[‘i’, ‘j’]}
\]
Almost ideal demand system with full demographics

- share equations

\[
s_i^h(p, z, w) = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \left( \beta_i + \theta_1 i z_1^h + \theta_2 i z_2^h \right) \ln \left( w - \ln \left( 1 + \rho_1 z_1^h + \rho_2 z_2^h \right) - \ln a(p) \right)
\]
Almost ideal demand system with full demographics

- **Share equations**

  \[
  s^h_i(p, z, w) = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \left( \beta_i + \theta_1 z^h_1 + \theta_2 z^h_2 \right) \ln \left( w - \ln \left( 1 + \rho_1 z^h_1 + \rho_2 z^h_2 \right) - \ln a(p) \right)
  \]

- Reference household contains 2 adults
Almost ideal demand system with full demographics

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- Reference household contains 2 adults
- **z_1** – children 10 and under; **z_2** – children 11–18.
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- share equations

\[ s_i^h(p, z, w) = \alpha_i + \sum_j \gamma_{ij} \ln p_j + (\beta_i + \theta_1 z_1^h + \theta_2 z_2^h) \ln \left( w - \ln (1 + \rho_1 z_1^h + \rho_2 z_2^h) - \ln a(p) \right) \]

- Reference household contains 2 adults
- \( z_1 \) – children 10 and under; \( z_2 \) – children 11–18.
- Brian Poi provided templates to construct data set from raw data.
First attempts frustrating due to ignorance of syntax
Construction of alternative ado files

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Construction of alternative ado files

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Construction of alternative ado files

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- Example has helped students improve efficiency of post-estimation code
Econometric problems using demand system

- Nonparametric regression suggests rank 2 demand
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- Jorgenson, Christensen translog function also coded.
A helpful command from Mata matters mata

```
b = st_matrix("e(b)"
fh = fopen("demcoeff.mat", "w")
fputmatrix(fh,b)
fclose(fh)
end
mata
fh = fopen("demcoeff.mat", "r")
X = fgetmatrix(fh)
fclose(fh)
st_matrix("b", X)
end
```
Student learning outcomes

- Linear aids with Stone’s price index produces controversies about price derivatives.
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- Without estimation, theory is just algebra without economic interpretation.
- Demographics in estimation emphasizes the significant effect of household composition.
- Estimation of theoretically consistent demand systems provides good experience calculating compensating and equivalent variation.
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- McCullough and Vinod *AER* 93(2003):873–892 caused considerable discussion on Statalist
- The behavior of probit estimation on Madalla’s death penalty data
- Many PhD students do not learn details of nonlinear estimation in econometrics courses.
- The practical experience of nonlinear demand estimation makes students aware of the need for care in even the simplest nonlinear models.