Teaching consumer theory with maximum likelihood estimation of demand systems

Carl H Nelson

Ag. and Con. Economics
Univ. of Illinois Urbana-Champaign

SNASUG08
7/24/08

Outline

1 The course

Outline

Outline

Outline

PhD program, course, and students

- Applied economics – connecting theory to data

PhD program, course, and students

- Applied economics – connecting theory to data
- Indirect utility, $V(p, w)$, and expenditure, $e(p, u)$ – empirical content of theory

PhD program, course, and students

- Applied economics – connecting theory to data
- Indirect utility, $V(p, w)$, and expenditure, $e(p, u)$ – empirical content of theory
- Beyond Gorman polar form, $V(p, w) = a(p) + b(p)w$ to Muellbauer, Lewbel and rank

PhD program, course, and students

- Applied economics – connecting theory to data
- Indirect utility, $V(p, w)$, and expenditure, $e(p, u)$ – empirical content of theory
- Beyond Gorman polar form, $V(p, w) = a(p) + b(p)w$ to Muellbauer, Lewbel and rank
- Beyond Mas-Collel, Whinston, and Green to weak separability, and demographic scaling

PhD program, course, and students

- Applied economics – connecting theory to data
- Indirect utility, $V(p, w)$, and expenditure, $e(p, u)$ – empirical content of theory
- Beyond Gorman polar form, $V(p, w) = a(p) + b(p)w$ to Muellbauer, Lewbel and rank
- Beyond Mas-Collel, Whinston, and Green to weak separability, and demographic scaling
- First year PhD students simultaneously take first PhD econometrics course

PhD program, course, and students

- Applied economics – connecting theory to data
- Indirect utility, $V(p, w)$, and expenditure, $e(p, u)$ – empirical content of theory
- Beyond Gorman polar form, $V(p, w) = a(p) + b(p)w$ to Muellbauer, Lewbel and rank
- Beyond Mas-Collel, Whinston, and Green to weak separability, and demographic scaling
- First year PhD students simultaneously take first PhD econometrics course
- Provide introduction to SUR, maximum likelihood, bootstrapping, and nonparametric regression

Estimation problem

- Equation system

$$
\begin{aligned}
y_1 &= f(X_1; \beta_1) + \varepsilon_1 \\
y_2 &= f(X_2; \beta_2) + \varepsilon_2 \\
&\vdots \\
y_k &= f(X_k; \beta_k) + \varepsilon_k
\end{aligned}
\qquad \varepsilon \sim N(0, \Sigma)
$$

Estimation problem

- Equation system

$$\begin{aligned} y_1 &= f(X_1; \beta_1) + \varepsilon_1 \\ y_2 &= f(X_2; \beta_2) + \varepsilon_2 \\ &\vdots \\ y_k &= f(X_k; \beta_k) + \varepsilon_k \end{aligned} \qquad \varepsilon \sim N(0, \Sigma)$$

- Log likelihood function

$$\ln L = -\frac{n}{2} \ln(2\pi) - \frac{1}{2} \ln |\Sigma| \\ -\frac{1}{2} (y - f(X;\beta))' \Sigma^{-1} (y - f(X;\beta))$$

Estimating equation

- Concentrated log likelihood function

$$\ln L = -\frac{n}{2}\ln(2\pi)$$

$$-\frac{1}{2}\ln\left|(y - f(X;\beta))\,(y - f(X;\beta))'\right|$$

$$-\frac{1}{2}I$$

$$\Rightarrow \widehat{\beta}_{ML} = \min_{\beta} \frac{1}{2}\ln\left|(y - f(X;\beta))\,(y - f(X;\beta))'\right|$$

File structure of the template

- findit demand system estimation $\longrightarrow$ st0029 from Stata journal 2-4

File structure of the template

- findit demand system estimation $\rightarrow$ st0029 from Stata journal 2-4
- ado files: lnl_quaids, quaids_delta, quaids_params, quaids_vec, vec_sum

File structure of the template

- findit demand system estimation $\rightarrow$ st0029 from Stata journal 2-4
- ado files: lnl_quaids, quaids_delta, quaids_params, quaids_vec, vec_sum
- ancillary files: quaids.do, food.dta

File structure of the template

- findit demand system estimation → st0029 from Stata journal 2-4
- ado files: lnl_quaids, quaids_delta, quaids_params, quaids_vec, vec_sum
- ancillary files: quaids.do, food.dta
- quaids share equations:

$$s_i = \frac{\partial \ln a(p)}{\partial \ln p_i} + \frac{\partial b(p)}{\partial \ln p_i} \ln w + \frac{\partial \lambda(p)}{\partial \ln p_i} \frac{1}{b(p)} (\ln w)^2$$

The template do file


```
ml model d0 lnl_quaids () /a2 /a3 /b1 /b2 /b3 /*
*/ /g11 /g21 /g31 /g22 /g32 /g33 /l1 /l2 /l3
ml search
ml maximize, noclear nooutput

mat b = e(b)
quaids_vec b beta
mat v = e(V)
quaids_delta r
mat var = r*v*r'

glo anames ""
glo bnames ""
glo lnames ""
glo gnames ""
```

template do file – continued

```
forv i = 1/$NEQN {
glo anames "$anames alpha:`i'"
glo bnames "$bnames beta:`i'"
glo lnames "$lnames lambda:`i'"
forv j = `i'/$NEQN {
glo gnames "$gnames gamma:`j'`i'"
}
}
glo names "$anames $bnames $gnames $lnames"
mat colnames beta = $names
mat colnames var = $names
mat rownames var = $names
estimates post beta var
estimates display
```

Inside lnl_quaids – using restrictions

- Call to recover parameters:
tempname alpha beta gamma lambda
tempvar deflexp lnpindex bofp
local nm1 = $NEQN-1
/* Get the parameters out of b. */
quaids_params ‘b’ ‘alpha’ ‘beta’ ‘gamma’ ‘lambda’

Inside lnl_quaids – using restrictions

- Call to recover parameters:
  tempname alpha beta gamma lambda
  tempvar deflexp lnpindex bofp
  local nm1 = $NEQN-1
  /* Get the parameters out of b. */
  quaids_params ‘b’ ‘alpha’ ‘beta’ ‘gamma’ ‘lambda’
- Quaids_params does the work:
  /* Gamma will take more work. */
  matrix ‘gamma’ = J($NEQN, $NEQN, 0)
  /* First get the (k-1) by (k-1) symmetric matrix. */
  local k = 1
  forvalues j = 1/‘nm1’ {
  forvalues i = ‘j’/‘nm1’ {
  matrix ‘gamma’[‘i’, ‘j’] = ‘b’[1, (2*$NEQN - 2 + ‘k’)]
  if (‘i’ = ‘j’) {
  matrix ‘gamma’[‘j’, ‘i’] = ‘gamma’[‘i’, ‘j’]

Almost ideal demand system with full demographics

- share equations

$$s_i^h(p,z,w) = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \left(\beta_i + \theta_{1i} z_1^h + \theta_{2i} z_2^h\right)$$
$$ln\left(w - \ln(1 + \rho_1 z_1^h + \rho_2 z_2^h) - \ln a(p)\right)$$

Almost ideal demand system with full demographics

- share equations

$$
\begin{aligned}
s_i^h(p,z,w) = & \alpha_i + \sum_j \gamma_{ij} \ln p_j + \left(\beta_i + \theta_{1i} z_1^h + \theta_{2i} z_2^h\right) \\
& ln \left(w - \ln(1 + \rho_1 z_1^h + \rho_2 z_2^h) - \ln a(p) \right)
\end{aligned}
$$

- Reference household contains 2 adults

Almost ideal demand system with full demographics

- share equations

$$s_i^h(p,z,w) = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \left(\beta_i + \theta_{1i} z_1^h + \theta_{2i} z_2^h\right)$$
$$ln\left(w - \ln(1 + \rho_1 z_1^h + \rho_2 z_2^h) - \ln a(p)\right)$$

- Reference household contains 2 adults
- z_1 – children 10 and under; z_2 – children 11–18.

Almost ideal demand system with full demographics

- share equations

$$s_i^h(p,z,w) = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \left(\beta_i + \theta_{1i} z_1^h + \theta_{2i} z_2^h\right)$$
$$ln \left(w - \ln(1 + \rho_1 z_1^h + \rho_2 z_2^h) - \ln a(p)\right)$$

- Reference household contains 2 adults
- z_1 – children 10 and under; z_2 – children 11–18.
- Brian Poi provided templates to construct data set from raw data.

Construction of alternative ado files

- First attempts frustrating due to ignorance of syntax

Construction of alternative ado files

- First attempts frustrating due to ignorance of syntax
- Macros and dereferencing were the most puzzling

Construction of alternative ado files

- First attempts frustrating due to ignorance of syntax
- Macros and dereferencing were the most puzzling
- Baum's "A little bit of Stata programming goes a long way . . . "

Construction of alternative ado files

- First attempts frustrating due to ignorance of syntax
- Macros and dereferencing were the most puzzling
- Baum's "A little bit of Stata programming goes a long way . . . "
- Example has helped students improve efficiency of post-estimation code

Econometric problems using demand system

- Nonparametric regression suggests rank 2 demand

Econometric problems using demand system

- Nonparametric regression suggests rank 2 demand
- Likelihood ratio tests of full and partial demographics

Econometric problems using demand system

- Nonparametric regression suggests rank 2 demand
- Likelihood ratio tests of full and partial demographics
- Recovery of expenditure function, $e(p, u)$, to calculate welfare measures

Econometric problems using demand system

- Nonparametric regression suggests rank 2 demand
- Likelihood ratio tests of full and partial demographics
- Recovery of expenditure function, $e(p, u)$, to calculate welfare measures
- Jorgenson, Christensen translog function also coded.

Saving and restoring coefficients

A helpful command from Mata matters mata

```
b = st_matrix("e(b)")
fh = fopen("demcoeff.mat", "w")
fputmatrix(fh,b)
fclose(fh)
end
mata
fh = fopen("demcoeff.mat", "r")
X = fgetmatrix(fh)
fclose(fh)
st_matrix("b", X)
end
```

Student learning outcomes

- Linear aids with Stone's price index produces controversies about price derivatives.

Student learning outcomes

- Linear aids with Stone's price index produces controversies about price derivatives.
- Nonlinear demand focuses student attention on elasticity calculation. The strong empirical content of preference based demand theory is evident.

Student learning outcomes

- Linear aids with Stone's price index produces controversies about price derivatives.
- Nonlinear demand focuses student attention on elasticity calculation. The strong empirical content of preference based demand theory is evident.
- Without estimation, theory is just algebra without economic interpretation.

Student learning outcomes

- Linear aids with Stone's price index produces controversies about price derivatives.
- Nonlinear demand focuses student attention on elasticity calculation. The strong empirical content of preference based demand theory is evident.
- Without estimation, theory is just algebra without economic interpretation.
- Demographics in estimation emphasizes the significant effect of household composition.

Student learning outcomes

- Linear aids with Stone's price index produces controversies about price derivatives.
- Nonlinear demand focuses student attention on elasticity calculation. The strong empirical content of preference based demand theory is evident.
- Without estimation, theory is just algebra without economic interpretation.
- Demographics in estimation emphasizes the significant effect of household composition.
- Estimation of theoretically consistent demand systems provides good experience calculating compensating and equivalent variation.

Summary

- McCullough and Vinod *AER* 93(2003):873–892 caused considerable discussion on Statalist

Summary

- McCullough and Vinod *AER* 93(2003):873–892 caused considerable discussion on Statalist
- The behavior of probit estimation on Madalla's death penalty data

Summary

- McCullough and Vinod *AER* 93(2003):873–892 caused considerable discussion on Statalist
- The behavior of probit estimation on Madalla's death penalty data
- Many PhD students do not learn details of nonlinear estimation in econometrics courses.

Summary

- McCullough and Vinod *AER* 93(2003):873–892 caused considerable discussion on Statalist
- The behavior of probit estimation on Madalla's death penalty data
- Many PhD students do not learn details of nonlinear estimation in econometrics courses.
- The practical experience of nonlinear demand estimation makes students aware of the need for care in even the simplest nonlinear models.