The `grreg` command for geometric rate regression

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The `grreg` command was developed with Nicola Orsini

Survival in Cancer Patients

```
.strate
[... output omitted ...]
+------------------------------+
| D   Y Rate Lower Upper      |
+------------------------------|
| 152 136.9425 1.10996 0.94681 1.30121 |
+------------------------------+
```

A patient died on average 1.11 times a year.

A Trivial Example

A cohort of 100 subjects is followed up for 2 days.

<table>
<thead>
<tr>
<th>Day</th>
<th>At risk</th>
<th>Deaths</th>
<th>Survivors</th>
<th>Geometric Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>80</td>
<td>20</td>
<td>80/100 = 0.80</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>4</td>
<td>16</td>
<td>4/20 = 0.20</td>
</tr>
</tbody>
</table>

Average geometric rate = 1 – (16/100)^(1/2) = 0.60

The average daily risk is 60%.
If 60% died every day, 16 would survive 2 days.
The incidence rate is 84/120 = 0.70 deaths/person-day.
Geometric Rate Regression

Poisson regression models incidence rates

Geometric rate regression models geometric rates

Geometric rate regression was introduced in Bottai (2015)

Geometric rates over Proportions

Let $T$ be a time variable and $S(t) = P(T > t)$.

The geometric rate over the time interval $(0, t)$ is

$$g(0, t) = 1 - S(t)^{1/t}$$

The geometric rate over the proportion interval $(0, p)$ is

$$g(0, p) = 1 - (1 - p)^{1/Q(p)}$$

where $p = P(T \leq t)$ and $Q(p)$ is the quantile function.

The Primal Idea

**Proposition (Bottai, 2015).** The geometric rate

$$g(0, p) = 1 - (1 - p)^{1/Q(p)}$$

is the $(1 - p)$-quantile of the transformed time variable

$$T^* = 1 - (1 - p)^{1/T}$$

That is $P[T^* \leq g(0, p)] = 1 - p$.

The above follows directly from the fact that for a fixed $p$ the function $1 - (1 - p)^{1/t}$ is monotonically decreasing in $t$ for $t > 0$.

Geometric Rate Regression

A geometric rate regression model with linear link

$$g(0, p \mid x) = x'\gamma$$

A geometric rate regression model with log link

$$\log g(0, p \mid x) = x'\gamma$$
Eighty-nine percent of the patients died during follow up. I estimate the following geometric regression model

$$\log g(0, 0.89) = \gamma_0$$

```
. grreg years, fail(died) p(.89)
```

Geometric rate regression

<table>
<thead>
<tr>
<th>Proportion: .89</th>
<th>No. of subjects = 175</th>
</tr>
</thead>
<tbody>
<tr>
<td>Link: log</td>
<td>No. of failures = 152</td>
</tr>
</tbody>
</table>

| years | Rate Ratio | Std. Err. | z   | P>|z| | [95% Conf. Interval] |
|-------|------------|-----------|-----|-------|----------------------|
| _cons | .6641147   | .0148485  | -18.31 | 0.000 | .6356407 .6938642 |

The risk of dying in a year was 66.4% (0.3% in a day).
Survival by Treatment

![Survival by Treatment graph](image)

**Geometric Rates by Treatment**

```
. grreg years trt, f(died)
```

Geometric rate regression

Proportion: .5  
No. of subjects = 347

Link: log  
No. of failures = 286

| Robust | Rate Ratio | Std. Err. | z   | P>|z| | [95% Conf. Interval] |
|--------|------------|-----------|-----|------|---------------------|
| years  |            |           |     |      |                     |
| trt    | .802882    | .0724971  | -2.43 | 0.015 | .6726537 .9583229   |
| _cons  | .7054235   | .0450773  | -5.46 | 0.000 | .6223824 .7995444   |

Half of the patients on MPA died at a yearly rate of 0.71. The risk on IFN was 20% smaller.

Survival in a Cohort of Men from Sweden

![Survival in a Cohort of Men from Sweden graph](image)

**Mortality Rates and Physical Activity**

I estimated the following geometric rate regression model

\[
\log g(0.25 | x) = \gamma_0 + \gamma_1 \text{PA2} + \gamma_2 \text{PA3} + \gamma_3 \text{PA4} + \text{covariates}
\]

The PA’s were indicators of physical activity level.

This example is presented in Bottai (2015). The data are described in Zheng (2015).
### Mortality Rate Ratios

The table shows the estimated rates for the first 25% of deaths.

<table>
<thead>
<tr>
<th>Physical Activity</th>
<th>Quantile (years)</th>
<th>Annual Rate (%)</th>
<th>Rate Ratio</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Crude</td>
<td>Adjusted</td>
</tr>
<tr>
<td>Very low</td>
<td>6.3 (5.5-7.0)</td>
<td>4.5 (4.1-4.9)</td>
<td>1.00 (referent)</td>
<td>1.00 (referent)</td>
</tr>
<tr>
<td>Low</td>
<td>8.7 (7.9-9.4)</td>
<td>3.3 (3.0-3.6)</td>
<td>0.73 (0.64,0.82)</td>
<td>0.74 (0.65,0.84)</td>
</tr>
<tr>
<td>High</td>
<td>9.6 (8.9-10.4)</td>
<td>2.9 (2.7-3.2)</td>
<td>0.66 (0.58,0.75)</td>
<td>0.74 (0.65,0.83)</td>
</tr>
<tr>
<td>Very high</td>
<td>9.8 (9.1-10.6)</td>
<td>2.9 (2.6-3.2)</td>
<td>0.64 (0.57,0.73)</td>
<td>0.73 (0.64,0.82)</td>
</tr>
</tbody>
</table>

The mortality rate decreased over levels of physical activity. In the most active it was 36% smaller than in the least active.

### Final Remarks

The `greg` command can estimate geometric rate regression.

Geometric rates have not been used in medical sciences, yet.

They have long been used in demography and finance.

Incidence rates are different from geometric rates.

Assumptions can improve efficiency; for example

\[ S(t) = \exp[-(\lambda t)^{\theta}] \iff g(0, p) = 1 - (1 - p) \lambda \left[ -\log(1-p) \right]^{-1/\theta} \]

### References