5th Nordic and Baltic Stata Users Group meeting
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Linear Quantile Mixed-Effects Models

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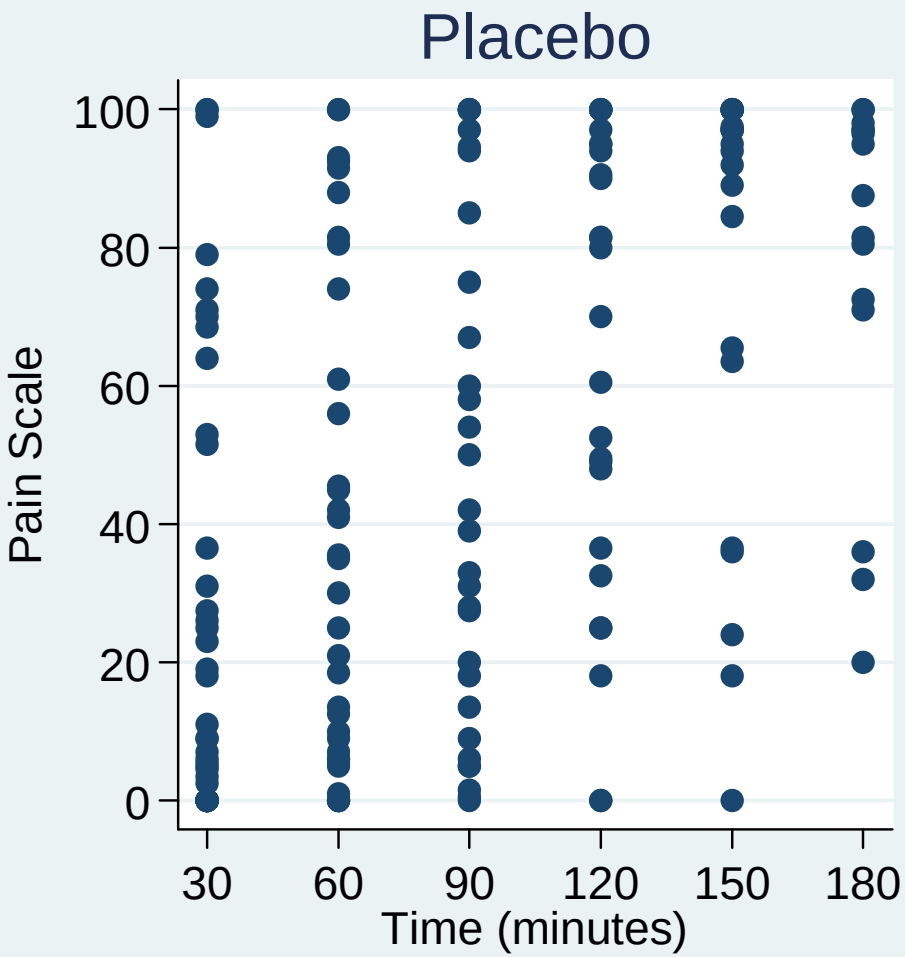
Joint work with Nicola Orsini

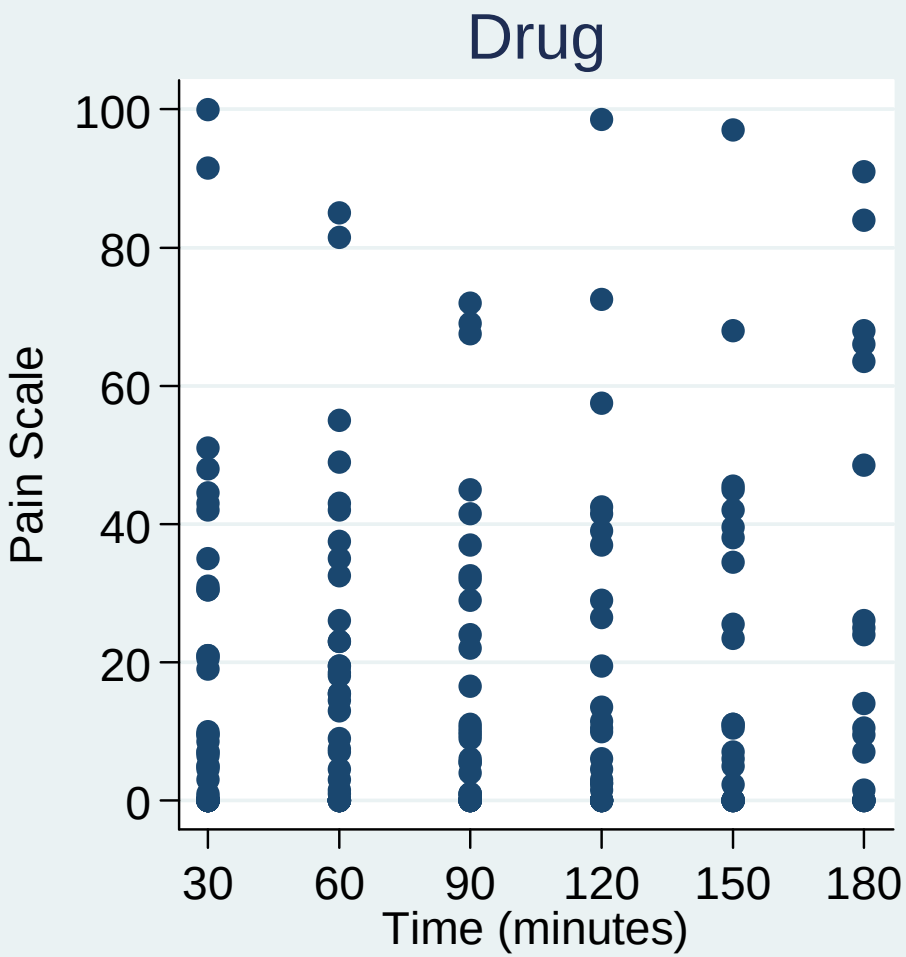
Longitudinal Data on Labor Pain

83 women in labor were randomized to drug or placebo (Davis, *Stat Med* 1991). They self-reported pain every 30 minutes on a 100-mm visual analogue scale.

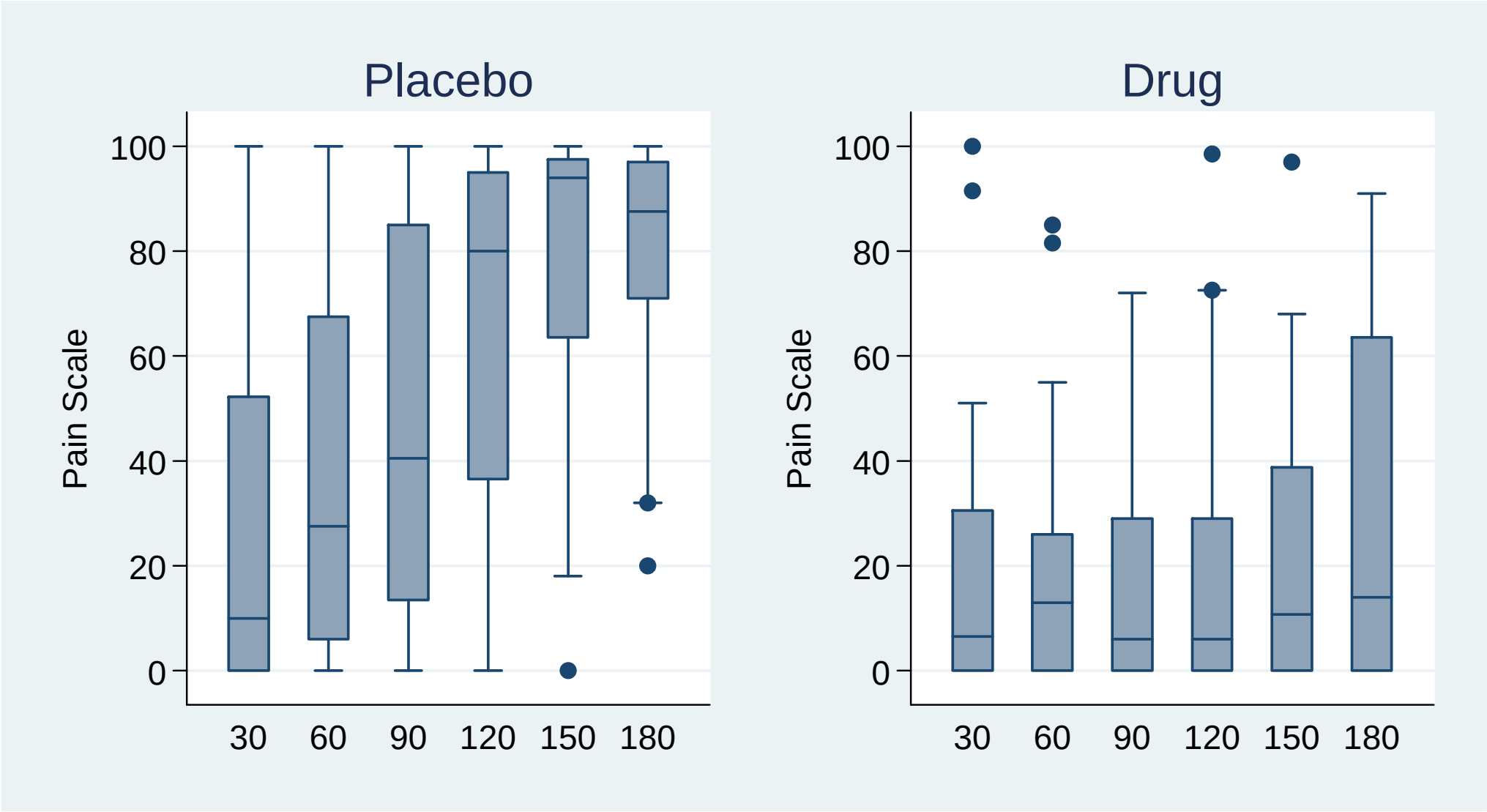
treatment	subject	time	pain
1	1	30	0
1	1	60	0
1	1	90	0
1	2	30	0
1	2	60	0
1	2	90	0
1	2	120	2.5
1	2	150	2.3
1	2	180	14

Pain over Time: a Scatter Plot

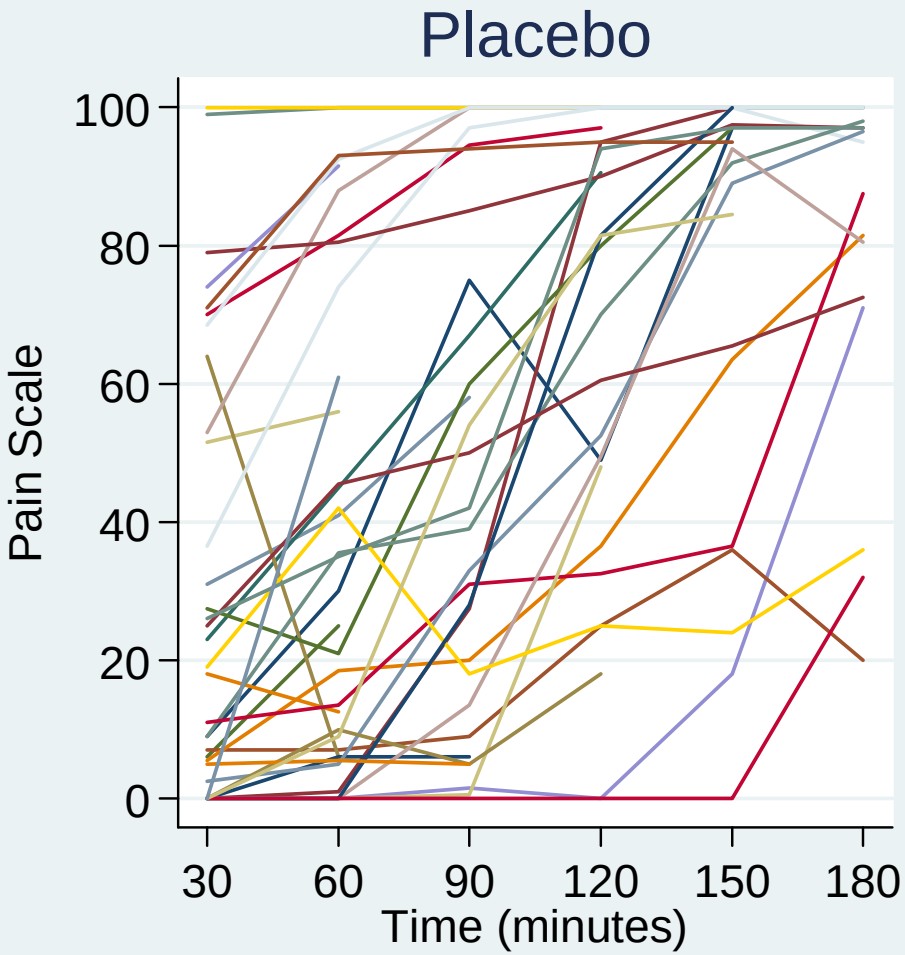


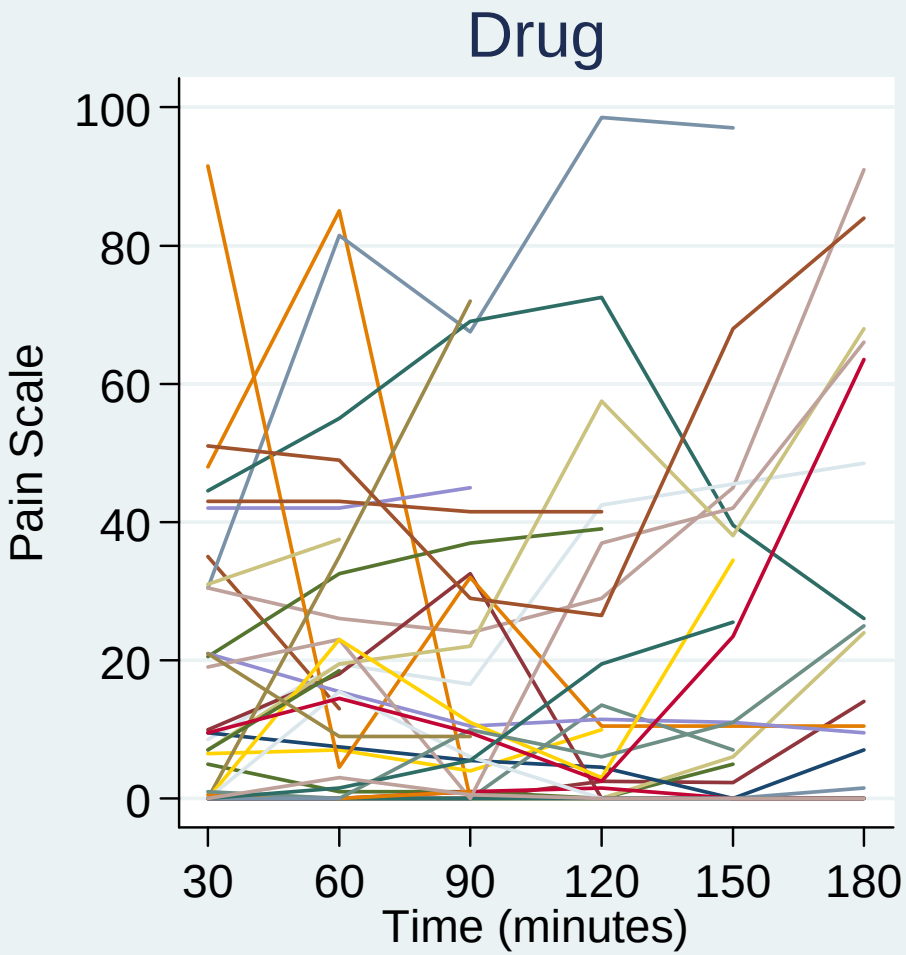


Pain over Time: a Box Plot



Pain over Time: a Spaghetti Plot





Logistic Quantile Mixed Effects Model

We consider a logistic quantile random-intercept regression model

$$logit(pain_{ij}) = \beta_0 + u_i + \beta_1 time_{ij} + \beta_2 trt_{ij} + \beta_2 time_{ij} \times trt_{ij} + \lambda\varepsilon_{ij}$$

for the jth observation on the ith woman.
We assume

$$u_i \sim N(0, \sigma^2)$$
$$\varepsilon_{ij} \sim AL(p): f(\varepsilon \mid p) = p(1-p)\exp(I_{\varepsilon \leq 0}\varepsilon - p\varepsilon)$$

The conditional p-quantile of pain given covariates and random effects is

$$Q_{pain|time,trt,u}(p) = \ logit^{-1}(\beta_0 + u + \beta_1 time + \beta_2 trt + \beta_2 time \times trt)$$

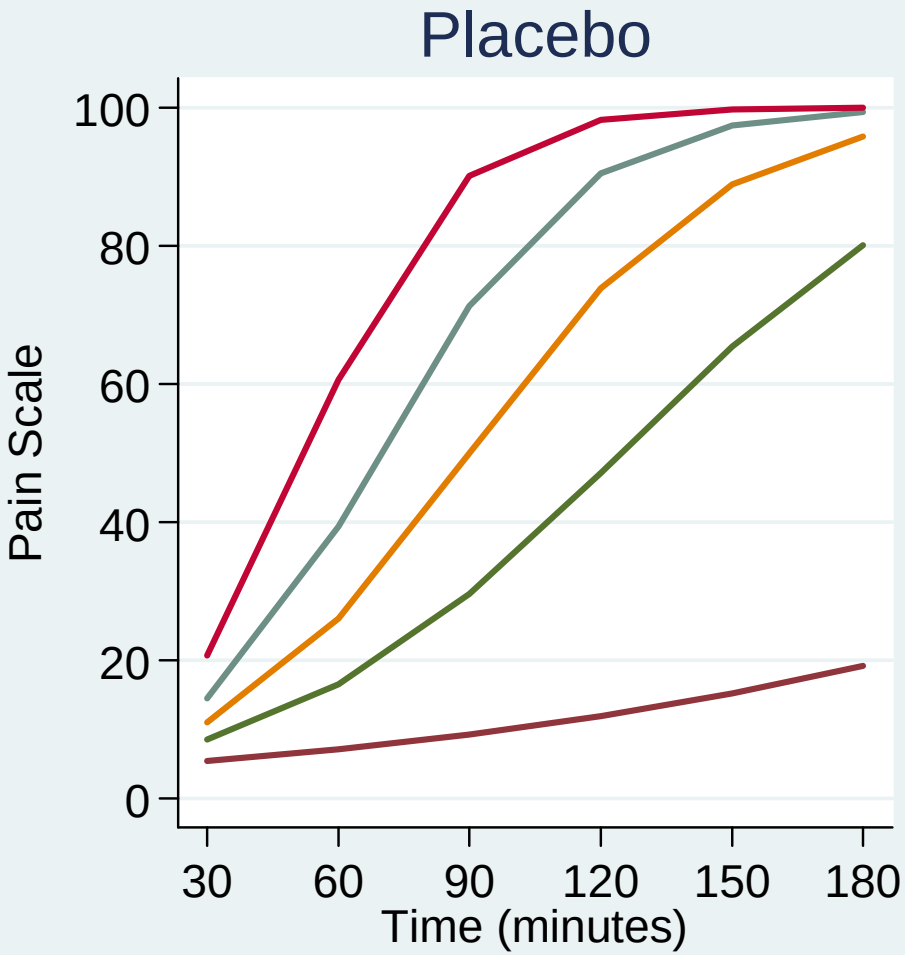
The "logit" link constrains inference on the p-quantile within 0 and 1.

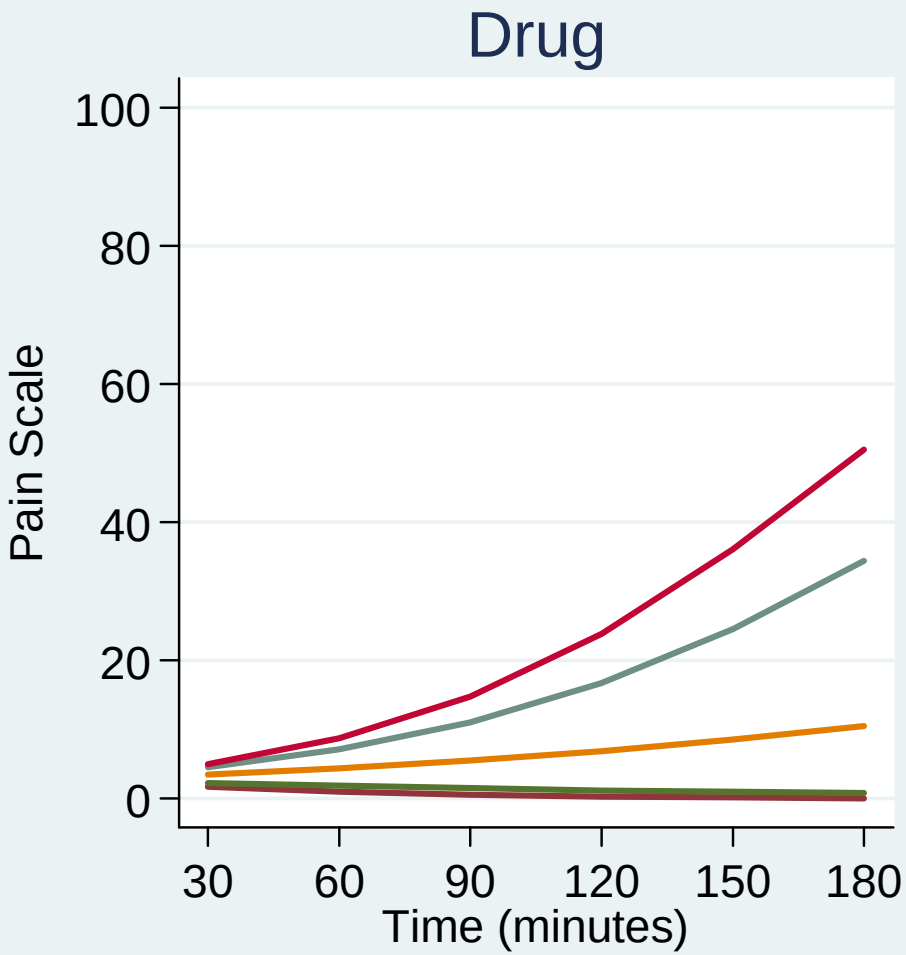
A Prototype for the “xtqreg” Command

The prototype calls the R package “lqmm” developed by Marco Geraci.

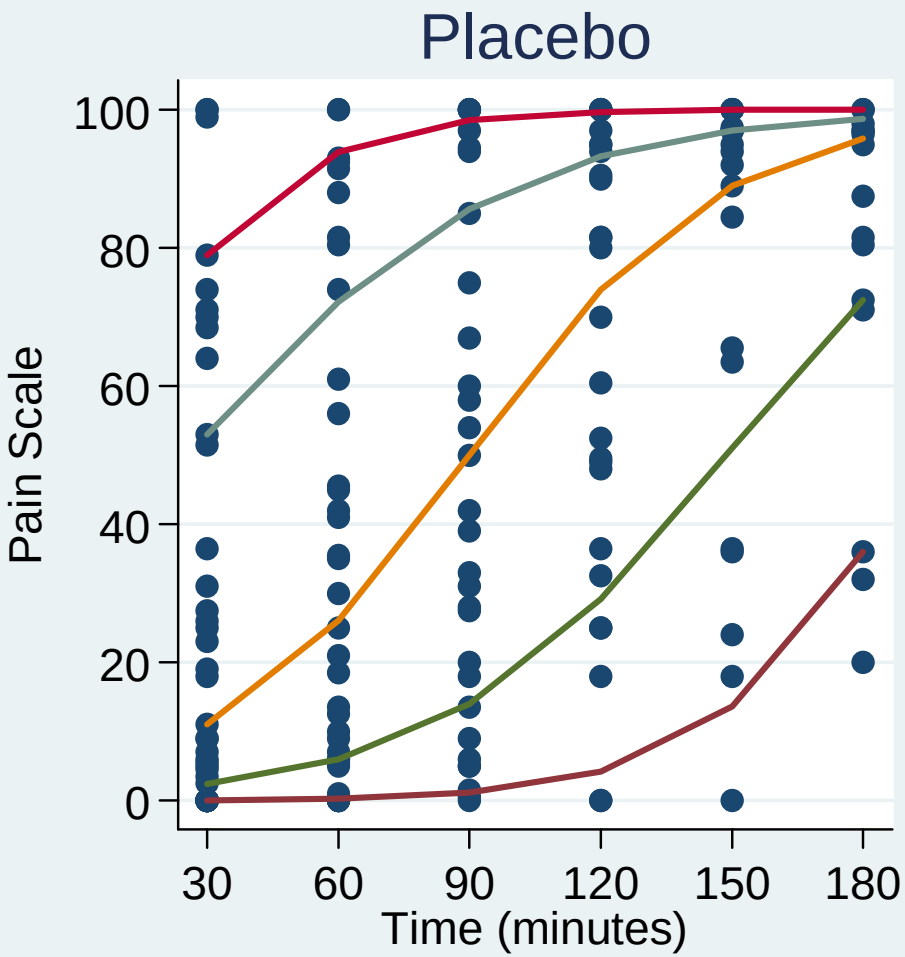
```
. xtqreg logit_pain time treatment treatment_time, quantile(.1 .25 .5 .75 .9)

Mixed-effects Quantile regression                    No. of obs    =        358
Optimization: Gradient-Search                        No. of groups =         83
Group variable: subject                            No. of bootstrap =         20
Covariance random-effects: Independent
------------------------------------------------------------------------------
             |              Bootstrap
  logit_pain |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
                          [... output omitted ...]
q50          |
        time |   .0345962   .0043228     8.00   0.000     .0261237    .0430687
   treatment |  -.4051901   .8642464    -0.47   0.639    -2.099082    1.288702
treatment_time |  -.0268845    .005356    -5.02   0.000     -.037382    -.016387
       _cons |  -3.115767     .60243    -5.17   0.000    -4.296509   -1.935026

                          [... output omitted ...]
------------------------------------------------------------------------------
```

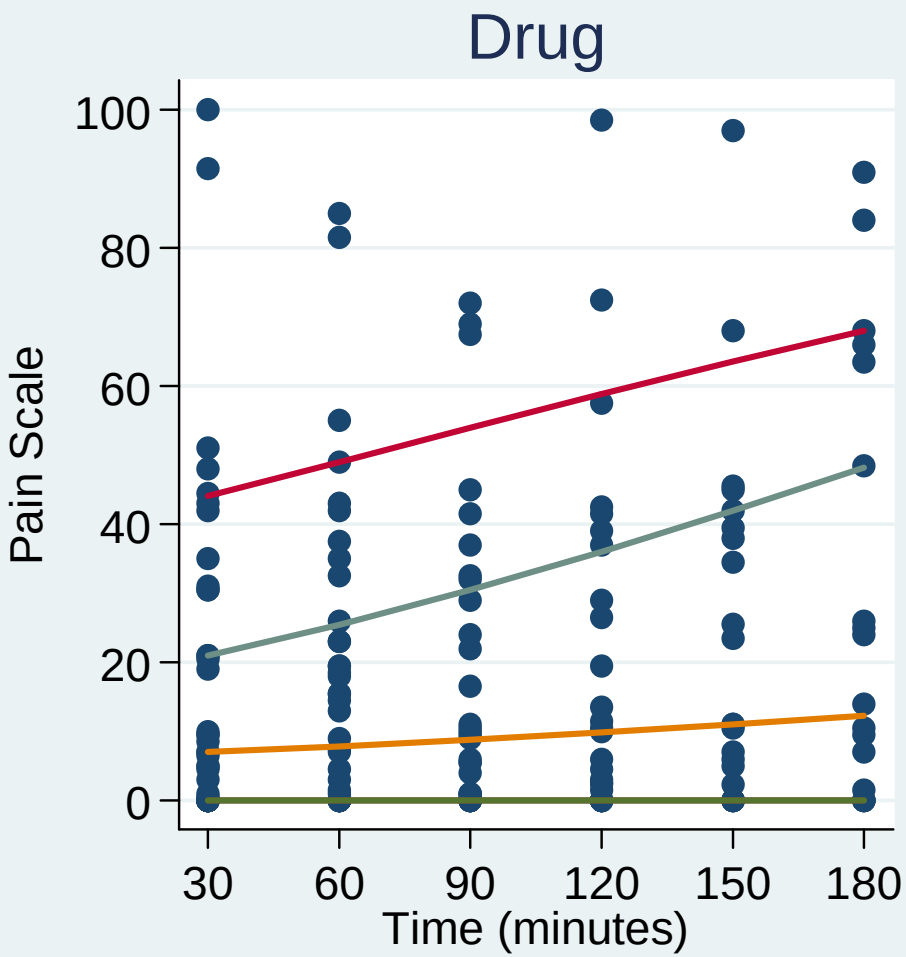
Conditional 10th, 25th, 50th, 75th, and 90th Percentiles





Marginal 10th, 25th, 50th, 75th, and 90th Percentiles





Possible Applications

Quantile mixed effects models can be applied to

- ✓ Longitudinal data
- ✓ Matched case-control data
- ✓ Twins cohort data
- ✓ Clustered data
- ✓ Spatially dependent data
- ✓ Multivariate outcomes
- ✓ Latent variable analysis

To do-list

- ✘ Implement the xtqreg command in Mata
- ✘ Improve computational stability and speed

xtqreg: Syntax

```
    xtqreg depvar [indepvars] [if] [in] [, xtqreg_options]

xtqreg_options              Description
-------------------------------------------------------------------------
Model

  quantiles(numlist)        specifies quantiles; default is quantiles(.5)

  reps(#)                   generates # bootstrap samples; default is
                              reps(20)

  method(string)            specifies the optimization algorithm

  random(varlist)           specifies the random effects

  covariance(vartype)       specifies the variance-covariance of the random
                              effects

  noconstant                suppresses constant term

Reporting

  level(#)                  set confidence level; default is level(95)
-------------------------------------------------------------------------
```

xtqreg: Random Effects Covariance Structures

vartype	Description
independent	one variance parameter per random effect, all covariances zero; the default unless a factor variable is specified
exchangeable	equal variances for random effects, and one common pairwise covariance
identity	equal variances for random effects, all covariances zero; the default for factor variables
unstructured	all variances and covariances distinctly estimated

See [R] sqreg postestimation for features available after estimation.

References

Linear Quantile Mixed Effects Models

Geraci and Bottai. Quantile regression for longitudinal data using the asymmetric Laplace distribution. *Biostatistics* 2007

Liu and Bottai. Mixed-Effects Models for Conditional Quantiles with Longitudinal Data. *International Journal of Biostatistics* 2009

Geraci and Bottai. Linear quantile mixed models. *Statistics and Computing* 2013

Logistic quantile regression

Bottai, Cai, and McKeown. Logistic quantile regression for bounded outcomes. *Statistics in Medicine* 2010

Orsini and Bottai. Logistic quantile regression in Stata. *Stata Journal* 2011

www.imm.ki.se/biostatistics