Heterogeneous differences-in-differences in Stata

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Outline

Why heterogeneous treatment effects?

Classic differences-in-differences: Treatment effects are obtained by estimating

$$y_{it} = \beta_0 + \beta_1 D_{it} + \gamma_t + \gamma_g + \varepsilon_{it}$$

y_{it}: **Outcome** of interest.

D_{it}: Binary **treatment**.

γ_t: **Time** fixed effects

γ_g: **Group** fixed effects. Treatment happens at the **group level**.

$\beta_1 =$ ATT (average **treatment effect** on the treated)

Why heterogeneous treatment effects?

Classic differences-in-differences: Treatment effects are obtained by estimating

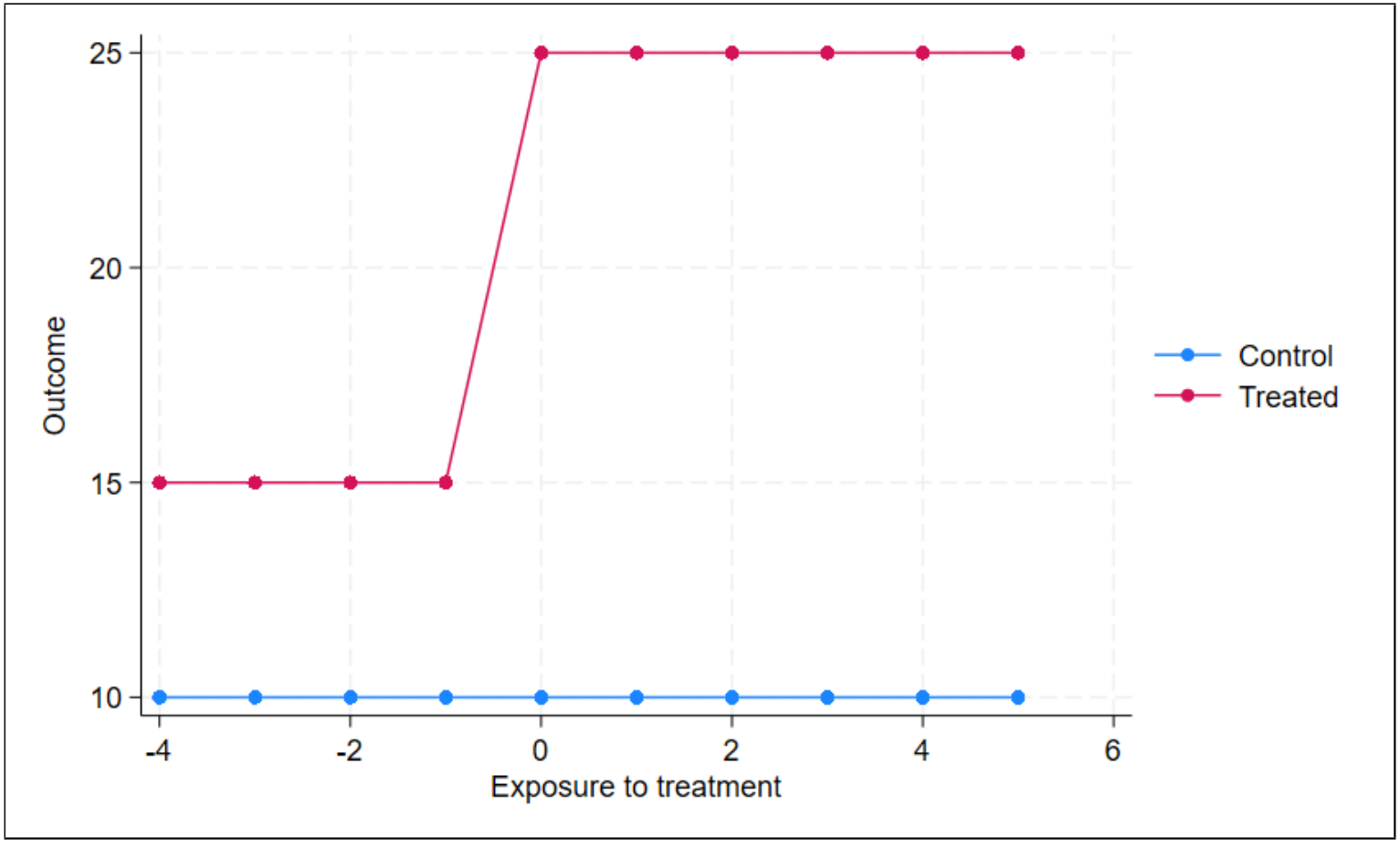
$$y_{it} = \beta_0 + \beta_1 D_{it} + \gamma_t + \gamma_g + \varepsilon_{it}$$

Implicit assumptions:

- ATT is the same **irrespective of when** unit is treated.
- ATT is **constant** after unit is treated.

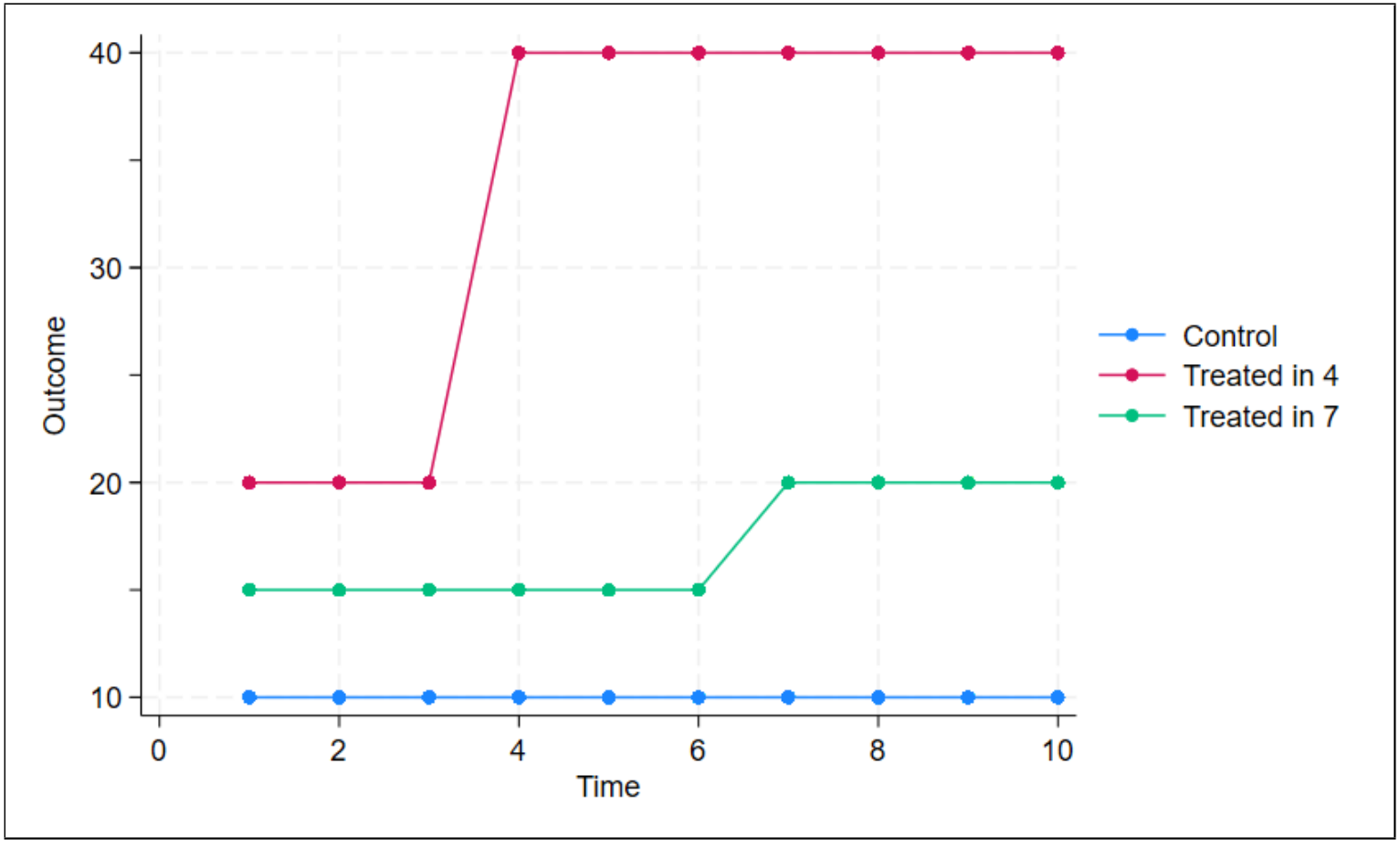
We are assuming **homogeneous treatment effects**.

Classic differences-in-differences



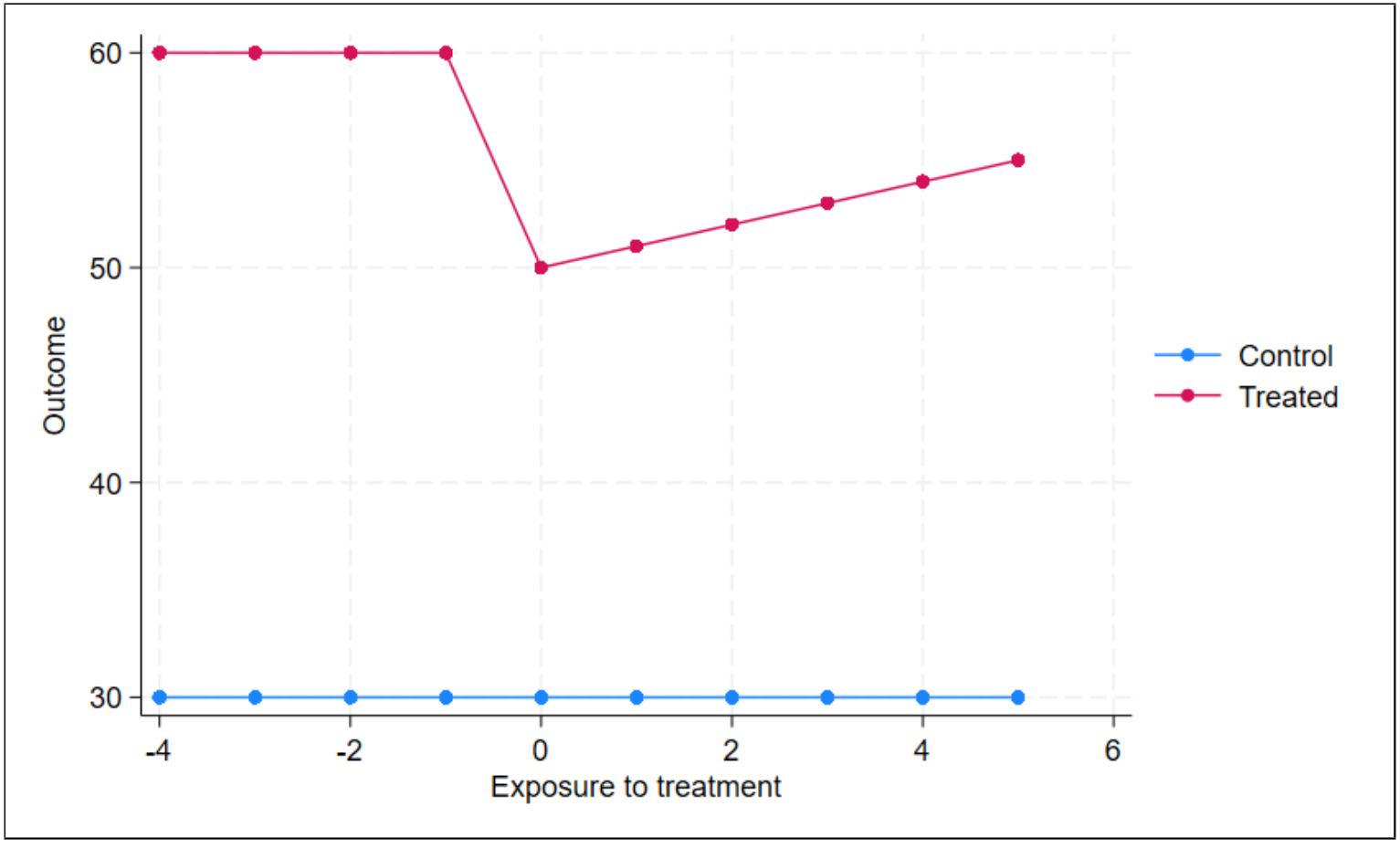
$ATT = DID =$ difference in treated$-$difference in control $= 10-0 = 10$

Cohort heterogeneity



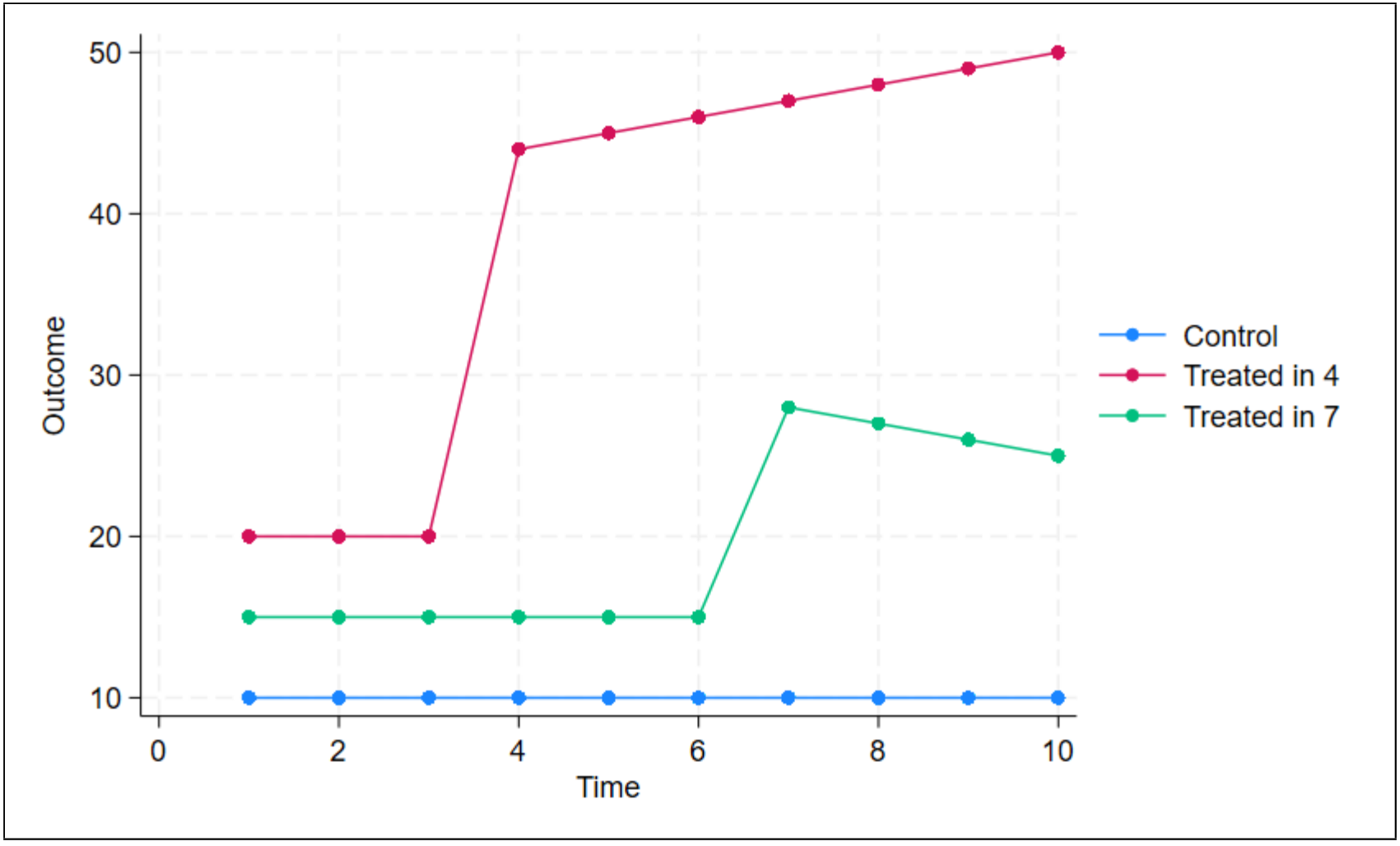
$ATT_{\text{red}} = 20$ $ATT_{\text{green}} = 5$

Time heterogeneity



$ATT_{t=0} = -10$ $ATT_{t=5} = -5$

Time-cohort heterogeneity



Heterogeneous DID

A **growing literature** has emerged to **estimate heterogeneous ATTs**:

- Callaway & Sant'Anna (2021), Wooldridge (2021), Chaisemartin and D'Haultfoeuille (2020)...

... and to **diagnose/understand** treatment effect heterogeneity:

- Borusyak, Jaravel, and Spiess (2018), Goodman-Bacon (2021)...

Many of these features have been **incorporated into Stata 18**.

Outline

The heterogeneous DID model

Panel or **repeated cross-sectional** data with $\{1, \ldots, T\}$ periods:

t: a specific **time period**.

D_{it}: 1 if **unit is treated** in period t, 0 otherwise.

- **Irreversible treatment**: Once treated, unit remains treated.
- No unit is treated at $t = 1$.

G_i: **Group** of unit i. When did i **first receive** treatment?

- $G_i = 5$ if unit i **first received treatment** in $t = 5$.
- $G_i = \infty$ if unit i **never received treatment**.

X_i: Time-invariant **controls** for unit i.

Potential and observed outcomes

$Y_{i,t}(0)$: **potential outcome** of unit i at time t if it is **never treated**.

- If $G_i = \infty$, then $Y_{i,t}(0)$ is **observed**.
- If $G_i \neq \infty$, then $Y_{i,t}(0)$ is **unobserved**.

$Y_{i,t}(g)$: **potential outcome** of unit i at time t if it had been **first treated at time** g.

- If $G_i = g$, then $Y_{i,t}(g)$ is **observed**.
- If $G_i \neq g$, then $Y_{i,t}(g)$ is **unobserved**.

$Y_{i,t}$: **observed outcome** in the data.

- $Y_{i,t} = Y_{i,t}(0)$ when $G_i = \infty$.
- $Y_{i,t} = Y_{i,t}(g)$ when $G_i = g$.

Heterogeneous Treatment Effects

Group-time average treatment effects on the treated:

$$ATT(g,t) = \mathbb{E}\big[Y_{i,t}(g) - Y_{i,t}(0) | G_i = g\big]$$

In **group** g and **time** t, what was the **average effect of being treated?**

Up to $(T-1)^2$ different ATTs $\Rightarrow$ rich **heterogeneity!**

Problem: ATTs are based on **unobservables** $\Rightarrow$ **Assumptions**

Assumption 1: No anticipatory effect

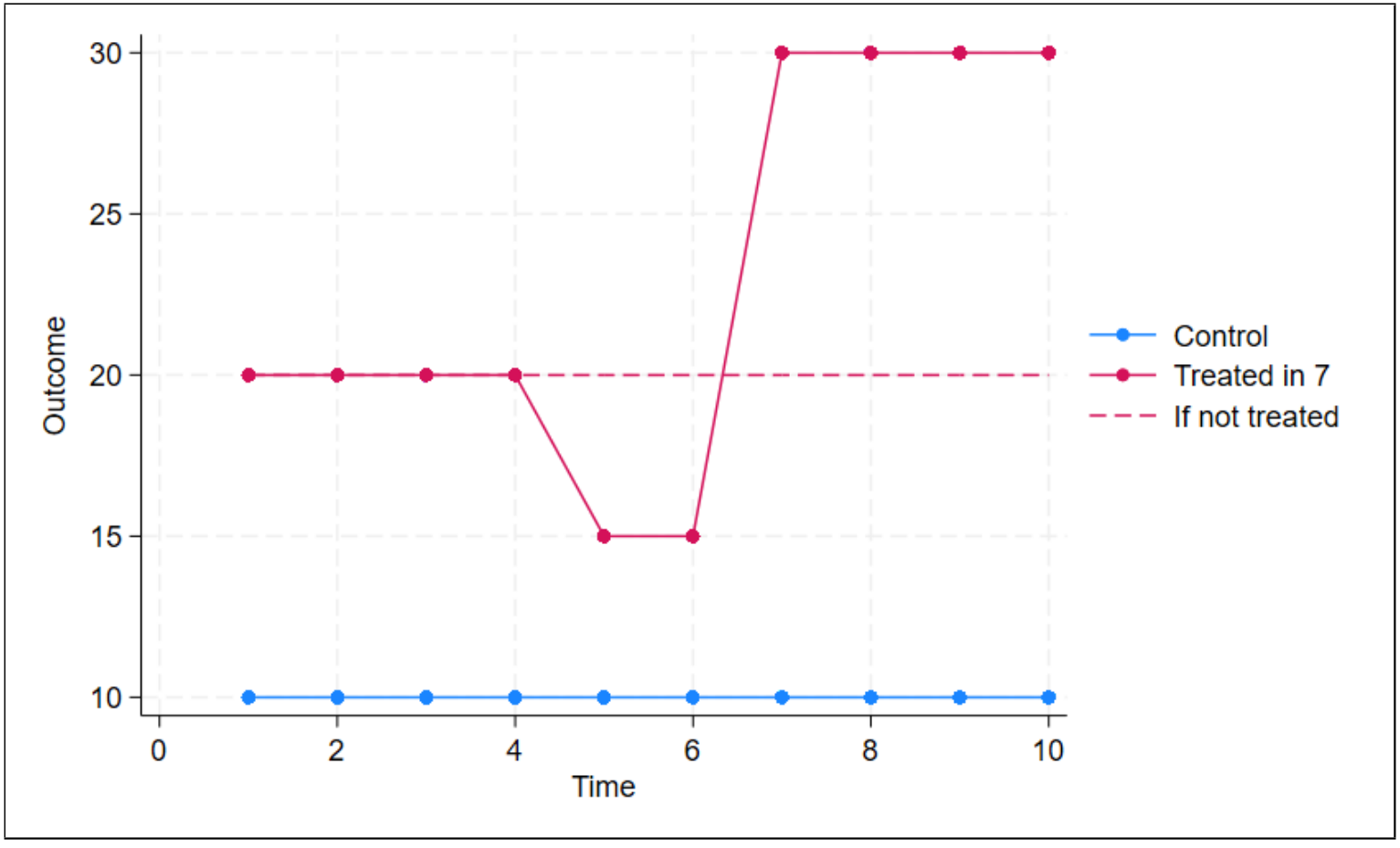
Before treatment happens (for $t < g$),

$$\mathbb{E}\big[Y_{i,t}(g)|X, G_i = g\big] = \mathbb{E}\big[Y_{i,t}(0)|X, G_i = g\big]$$

Outcome **doesn't respond in anticipation** to the treatment.

Anticipatory effects **bias** ATT estimation.

Anticipatory effects bias DID



$$DID = (30 - 15) - 0 = 30 - 15 = 15 \neq 10 = ATT$$

Assumption 2: Parallel trends with never-treated

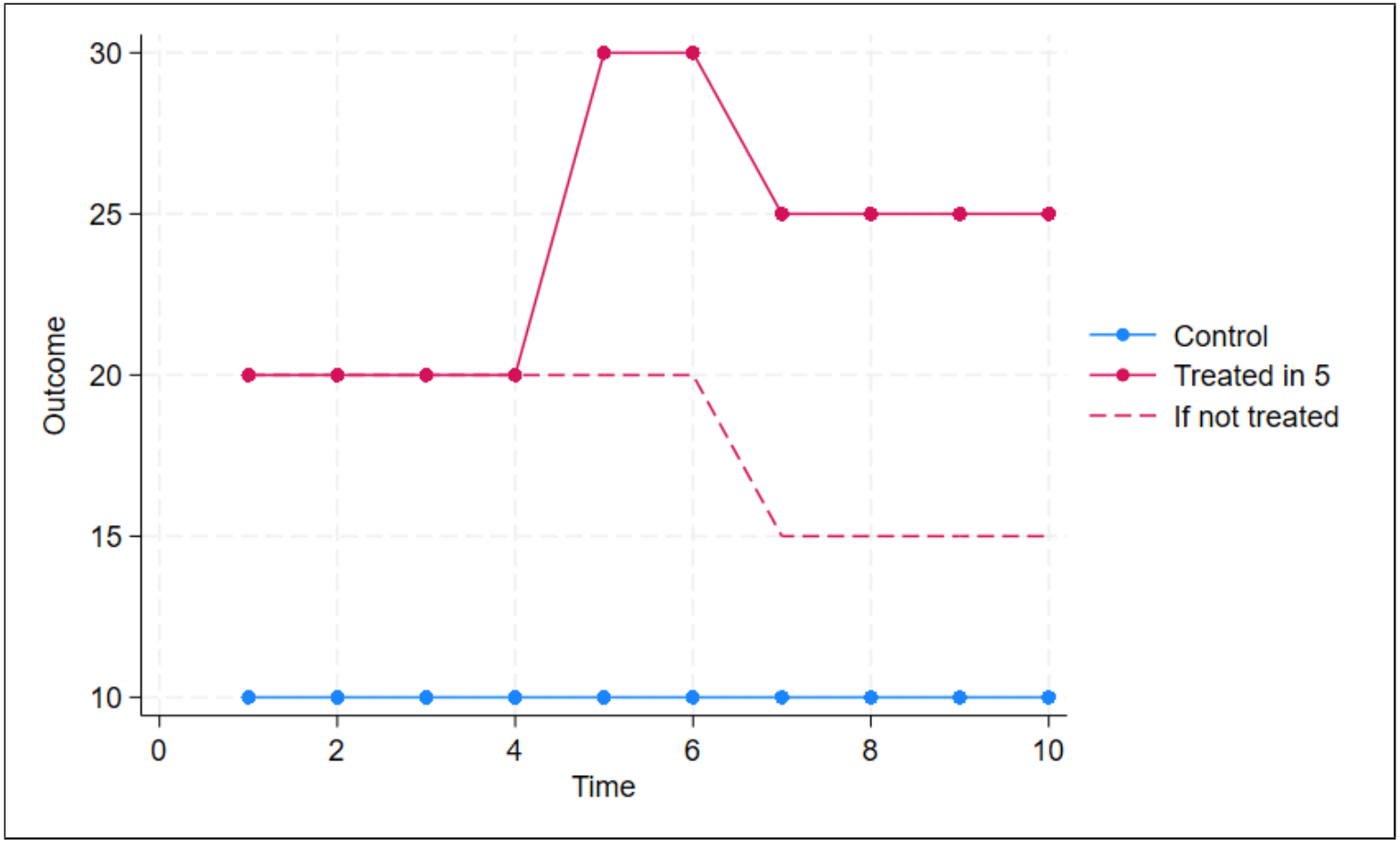
After treatment happens $(t \geq g)$,

$$\mathbb{E}\left[Y_{i,t}(0) - Y_{i,t-1}(0) | X, G_i = g\right] = \mathbb{E}\left[Y_{i,t}(0) - Y_{i,t-1}(0) | X, G_i = \infty\right]$$

If group **had not been treated**, outcome would move as in the **never treated group**.

Violations of this assumption **bias** ATT estimation.

Non-parallel trends effects bias DID



$$DID = (25 - 20) - 0 = 5 - 0 = 5 \neq 10 = ATT$$

Identification – Callaway, Sant'Anna (2021)

Theorem: Given some **technical conditions**, if **assumptions 1 and 2** hold

$$\Rightarrow ATT(g,t) \text{ can be } \textbf{estimated from the data}.$$

Result also holds if **parallel trends** with **not-yet treated groups**.

Option `controlgroup` lets you **choose the control group**

Outline

Commands `hdidregress` and `xthdidregress`

Estimate ATTs that vary over group/cohort and over time:

- `hdidregress` for **cross-sectional data**
- `xthdidregress` for **panel data**.

Both commands come with **four estimators**:

- Callaway, Sant'Anna (2021):
  - **Regression adjusted**
  - Inverse-**probability weighting**
  - **Augmented** inverse-probability weighting
- Wooldridge (2021):
  - **Extended** two-way fixed effects

Regression adjusted estimator

Syntax:

`xthdidregress ra` (*ovar* [*omvarlist*]) (*tvar*) [*if*] [*in*] [*weight*], `group`(*groupvar*) [*options*]

ovar: **continuous outcome** of interest

omvarlist: **covariates** in the **outcome model**

tvar: binary **treatment**

groupvar: categorical variable indicating **group level at which treatment occurs. Required.**

The RA estimator

$$ATT(g,t) = \mathbb{E}\left[\frac{G_g}{\mathbb{E}\left[G_g\right]}\left(Y_t - Y_{g-1} - m^{nev}_{g,t}(X)\right)\right]$$

$m^{nev}_{g,t}(X)$: **Difference in the control group** conditional on X.

- $m^{nev}_{g,t}(X) = \mathbb{E}\left[Y_t - Y_{g-1} | X, G_i = \infty\right]$

The term in orange is the **difference in the differences** between group g and the control group.

Heuristically

Algorithm:

1. `keep` if time is t or $g-1$
2. `keep` if cohort is g or C
3. `generate` $\Delta Y = Y_t - Y_{g-1}$
4. `regress` ΔY on X for the group C and predict $\hat{m}^{nev}_{g,t}(X)$
5. `generate` $\widehat{TE} = \Delta Y - \hat{m}^{nev}_{g,t}(X)$
6. `summarize` $\widehat{TE}$ if cohort is g
7. Repeat for each g and t.

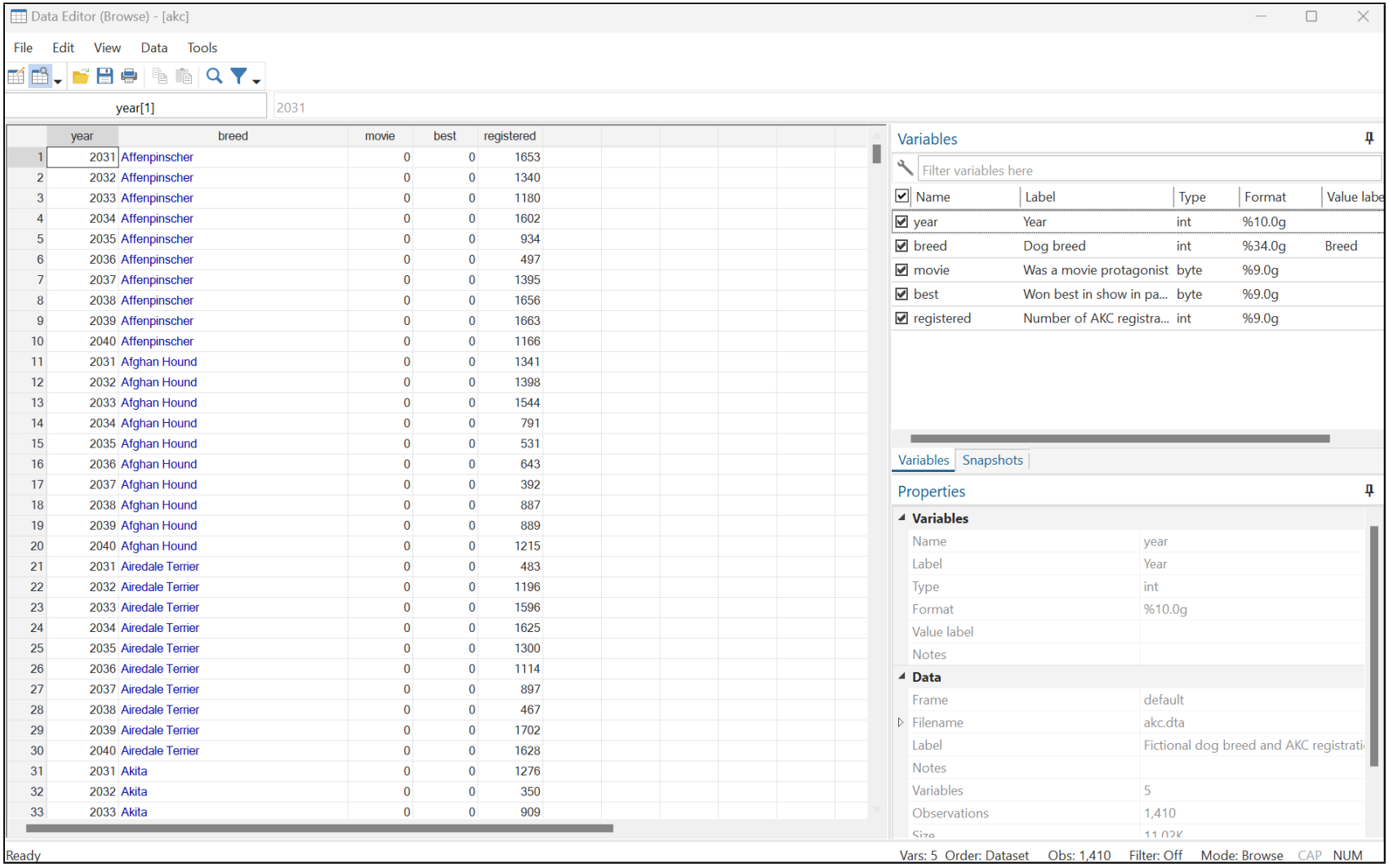
Example: the RA estimator in Stata

Question: How is the **number of registrations of a dog breed** in the American Kennel Club affected by that **dog breed being the protagonist** in a movie?

Data

```
. webuse akc, clear
(Fictional dog breed and AKC registration data)

.
. describe

Contains data from https://www.stata-press.com/data/r18/akc.dta
 Observations:         1,410                  Fictional dog breed and AKC registration data
    Variables:             5                  1 Feb 2023 14:23
-------------------------------------------------------------------------------
Variable      Storage   Display    Value
    name         type    format    label      Variable label
-------------------------------------------------------------------------------
year            int     %10.0g                Year
breed           int     %34.0g     Breed      Dog breed
movie           byte    %9.0g                 Was a movie protagonist
best            byte    %9.0g                 Won best in show in past 10 years
registered      int     %9.0g                 Number of AKC registrations
-------------------------------------------------------------------------------
Sorted by: breed
```

Data



	year	breed	movie	best	registered
1	2031	Affenpinscher	0	0	1653
2	2032	Affenpinscher	0	0	1340
3	2033	Affenpinscher	0	0	1180
4	2034	Affenpinscher	0	0	1602
5	2035	Affenpinscher	0	0	934
6	2036	Affenpinscher	0	0	497
7	2037	Affenpinscher	0	0	1395
8	2038	Affenpinscher	0	0	1656
9	2039	Affenpinscher	0	0	1663
10	2040	Affenpinscher	0	0	1166
11	2031	Afghan Hound	0	0	1341
12	2032	Afghan Hound	0	0	1398
13	2033	Afghan Hound	0	0	1544
14	2034	Afghan Hound	0	0	791
15	2035	Afghan Hound	0	0	531
16	2036	Afghan Hound	0	0	643
17	2037	Afghan Hound	0	0	392
18	2038	Afghan Hound	0	0	887
19	2039	Afghan Hound	0	0	889
20	2040	Afghan Hound	0	0	1215
21	2031	Airedale Terrier	0	0	483
22	2032	Airedale Terrier	0	0	1196
23	2033	Airedale Terrier	0	0	1596
24	2034	Airedale Terrier	0	0	1625
25	2035	Airedale Terrier	0	0	1300
26	2036	Airedale Terrier	0	0	1114
27	2037	Airedale Terrier	0	0	897
28	2038	Airedale Terrier	0	0	467
29	2039	Airedale Terrier	0	0	1702
30	2040	Airedale Terrier	0	0	1628
31	2031	Akita	0	0	1276
32	2032	Akita	0	0	350
33	2033	Akita	0	0	909

Staggered treatment

```
. tabulate year movie
```

Year	Was a movie protagonist 0	1	Total
2031	141	0	141
2032	141	0	141
2033	141	0	141
2034	137	4	141
2035	137	4	141
2036	134	7	141
2037	119	22	141
2038	119	22	141
2039	119	22	141
2040	119	22	141
Total	1,307	103	1,410

Output 1

```
. xtset breed year

Panel variable: breed (strongly balanced)
 Time variable: year, 2031 to 2040
         Delta: 1 unit

.
. xthdidregress ra (registered best) (movie), group(breed)
note: variable _did_cohort, containing cohort indicators formed by treatment variable movie and group variable breed, was added to the dataset.

Computing ATET for each cohort and time:
Cohort 2034 (9): ......... done
Cohort 2036 (9): ......... done
Cohort 2037 (9): ......... done

Treatment and time information

Time variable: year
Time interval: 2031 to 2040
Control:       _did_cohort = 0
Treatment:     _did_cohort > 0
```

	_did_cohort
Number of cohorts	4
Number of obs	
Never treated	1190
2034	40
2036	30
2037	150

Output 2

Heterogeneous-treatment-effects regression — Number of obs = 1,410; Number of panels = 141

Estimator: Regression adjustment
Panel variable: **breed**
Treatment level: **breed**
Control group: Never treated

(Std. err. adjusted for **141** clusters in **breed**)

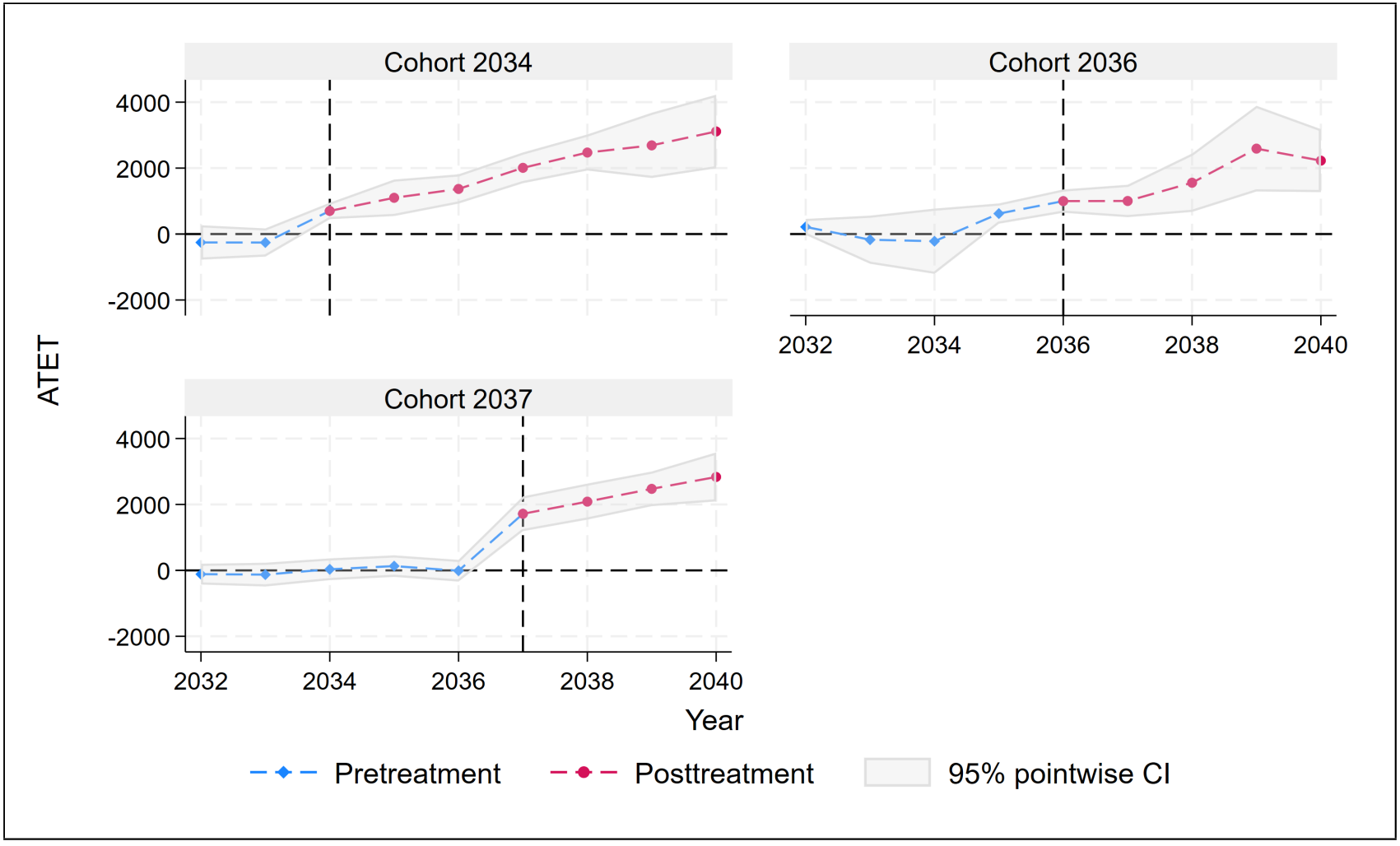
Cohort	ATET	Robust std. err.	z	P>|z|	[95% conf.	interval]
2034						
year						
2032	-254.8927	266.1024	-0.96	0.338	-776.4439	266.6584
2033	-257.5329	217.9389	-1.18	0.237	-684.6852	169.6194
2034	701.1318	127.0935	5.52	0.000	452.0331	950.2304
2035	1099.044	282.0704	3.90	0.000	546.196	1651.892
2036	1367.632	225.8702	6.05	0.000	924.9343	1810.329
2037	2008.294	237.2396	8.47	0.000	1543.313	2473.275
2038	2472.624	278.2949	8.88	0.000	1927.176	3018.072
2039	2689.615	504.3324	5.33	0.000	1701.142	3678.088
2040	3110.97	568.916	5.47	0.000	1995.915	4226.025
2036						
year						
2032	216.0259	122.9107	1.76	0.079	-24.87472	456.9265
2033	-172.5154	372.0776	-0.46	0.643	-901.7741	556.7433
2034	-218.0495	504.5267	-0.43	0.666	-1206.904	770.8045
2035	621.033	156.1306	3.98	0.000	315.0227	927.0434
2036	999.0781	180.1055	5.55	0.000	646.0779	1352.078
2037	1003.333	250.5916	4.00	0.000	512.1829	1494.484
2038	1556.669	451.6914	3.45	0.001	671.3697	2441.967
2039	2590.674	662.6979	3.91	0.000	1291.81	3889.538
2040	2225.712	486.9917	4.57	0.000	1271.225	3180.198

Output 3

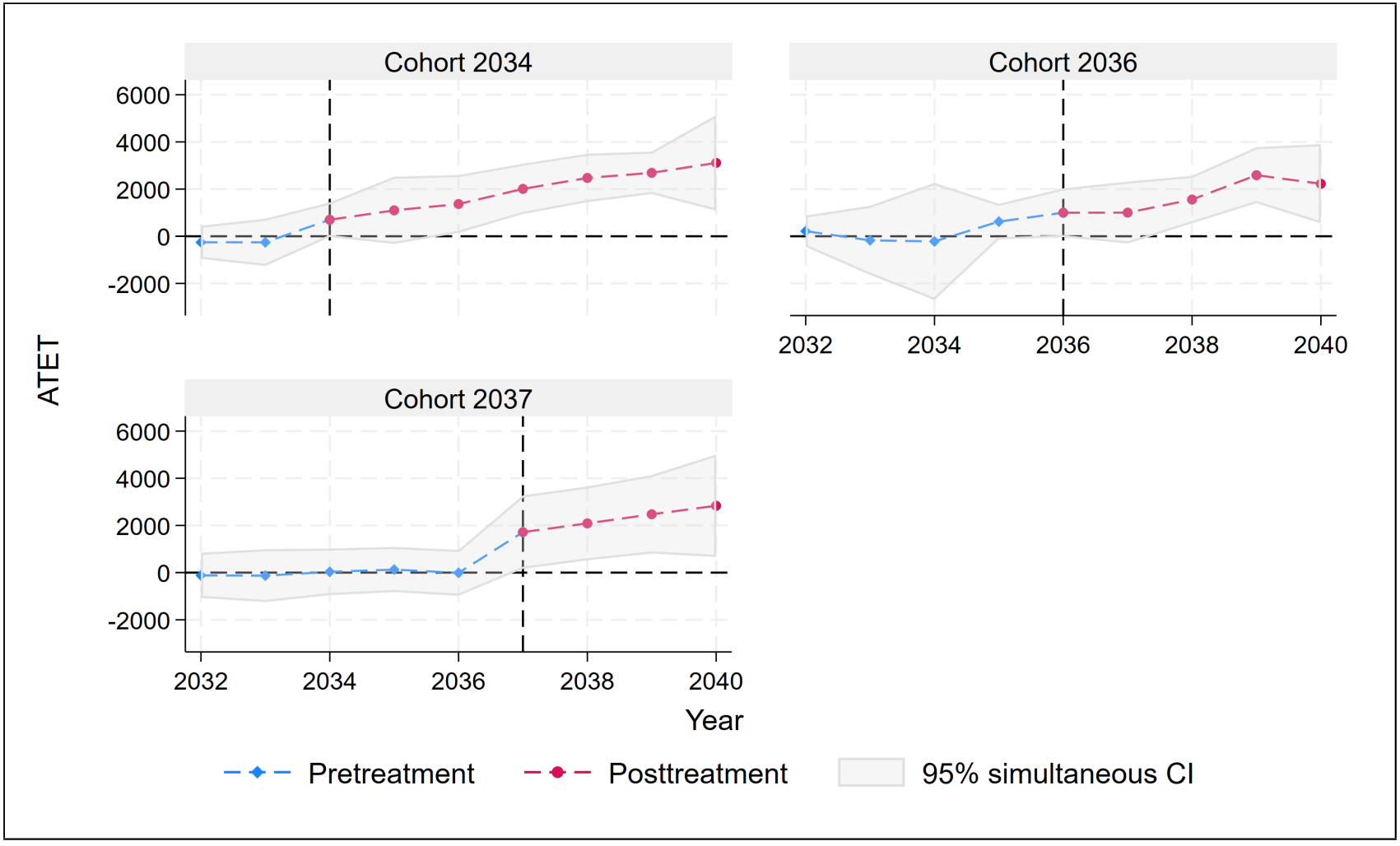
2037						
year						
2032	-114.582	160.0972	-0.72	0.474	-428.3668	199.2028
2033	-127.9856	183.3941	-0.70	0.485	-487.4315	231.4603
2034	33.40901	168.0312	0.20	0.842	-295.9262	362.7442
2035	130.3495	166.2261	0.78	0.433	-195.4477	456.1468
2036	-10.48288	167.5059	-0.06	0.950	-338.7884	317.8226
2037	1717.016	268.5592	6.39	0.000	1190.65	2243.383
2038	2086.798	278.0215	7.51	0.000	1541.886	2631.71
2039	2473.611	268.186	9.22	0.000	1947.976	2999.246
2040	2835.117	378.6699	7.49	0.000	2092.938	3577.296

Note: ATET computed using covariates.

Graphical representation: `estat atetplot`



Simultaneous confidence intervals: `estat atetplot, sci`



No anticipatory effects test – `estat ptrends`

```
Parallel-trends test (pretreatment time period)
H0: Treatment effects in all the pretreatment periods are zero

   chi2(11) =  57.68
Prob > chi2 = 0.0000
```

Change the control group – `controlgroup()`

Sometimes all units are eventually treated. We need to use as controls the not yet treated.

```
xthdidregress ra (registered best) (movie), group(breed)
controlgroup(notyet)
```

```
Heterogeneous-treatment-effects regression          Number of obs     = 1,410
                                                    Number of panels =   141
Estimator:       Regression adjustment
Panel variable:  breed
Treatment level: breed
Control group:   Not yet treated
```

```
output omitted
```

Inverse-probability weighting estimator

Syntax:

`xthdidregress ipw` (*ovar*) (*tvar* [*tmvarlist*]) [*if*] [*in*] [*weight*], `group`(*groupvar*) [*options*]

ovar: **continuous outcome** of interest

tmvarlist: **covariates** in the **treatment model**

tvar: binary **treatment**

groupvar: categorical variable indicating **group level at which treatment occurs. Required.**

The IPW estimator

$$ATT(g,t) = \mathbb{E}\left[\left(\frac{G_g}{\mathbb{E}\left[G_g\right]} - \frac{\frac{p_g(X)}{1-p_g(X)}}{\mathbb{E}\left[\frac{p_g(X)}{1-p_g(X)}\right]}\right)\left(Y_t - Y_{g-1}\right)\right]$$

$p_g(X)$: **Probability of being in group g** given X and given that observation is either in g or C.

- Generalized **propensity score**.

The term in orange is the **inverse-probability weight**.

Augmented inverse-probability weighting estimator

Syntax:

`xthdidregress aipw` (*ovar* [*omvarlist*]) (*tvar* [*tmvarlist*]) [*if*] [*in*] [*weight*], `group`(*groupvar*) [*options*]

ovar: **continuous outcome** of interest

omvarlist: **covariates** in the **outcome model**

tmvarlist: **covariates** in the **treatment model**

tvar: binary **treatment**

groupvar: categorical variable indicating **group level at which treatment occurs. Required.**

The AIPW estimator

$$ATT(g,t) = \mathbb{E}\left[\left(\frac{G_g}{\mathbb{E}\left[G_g\right]} - \frac{\frac{p_g(X)}{1-p_g(X)}}{\mathbb{E}\left[\frac{p_g(X)}{1-p_g(X)}\right]}\right)\left(Y_t - Y_{g-1} - m^{nev}_{g,t}(X)\right)\right]$$

$p_g(X)$: **Probability of being in group g** given X and given that observation is either in g or C.

$m^{nev}_{g,t}(X)$: **Difference in the control group** conditional on X.

Inverse-probability weight in orange. **Augmented** term in violet

Doubly robust

Extended two-way fixed effects estimator

Syntax:

`xthdidregress twfe` (*ovar* [*omvarlist*]) (*tvar*) [*if*] [*in*] [*weight*], `group`(*groupvar*) [*options*]

ovar: **continuous outcome** of interest

omvarlist: **covariates** in the **outcome model**

tvar: binary **treatment**

groupvar: categorical variable indicating **group level at which treatment occurs. Required.**

The TWFE estimator

Consider the **extended two-way fixed effects regression**:

$$Y_{it'} = \eta + \sum_{g=q}^{T} \alpha_g G_{ig} + \sum_{t=q}^{T} \gamma_t f_t + \sum_{g=q}^{T} \sum_{t=q}^{T} \tau_{g,t} D_{it} G_{ig} f_t + \varepsilon_{it'}$$

q: **first treatment** period

f_t: 1 if $t' = t$, 0 otherwise.

$\tau_{g,t} =$ $ATT(g,t)$

Remarks:

- **Covariates** would enter **fully interacted** in the model.

Outline

Aggregating treatment effects

You might be interested in exploring **heterogeneity just by**:

- **Cohort**
- **Time**
- **Exposure** to treatment (event studies)
- Even **no heterogeneity** at all

Some **post-estimation tools** come handy in this case.

Suppose you've just fitted **a heterogeneous DID model**:

```
xthdidregress ra (registered best) (movie), group(breed)
```

Overall aggregation – `estat aggregation, overall`

Overall ATET — Number of obs = **1,410**

(Std. err. adjusted for **141** clusters in **breed**)

registered	ATET	Robust std. err.	z	P>|z|	[95% conf.	interval]
movie (1 vs 0)	**2093.318**	**122.5752**	**17.08**	**0.000**	**1853.075**	**2333.561**

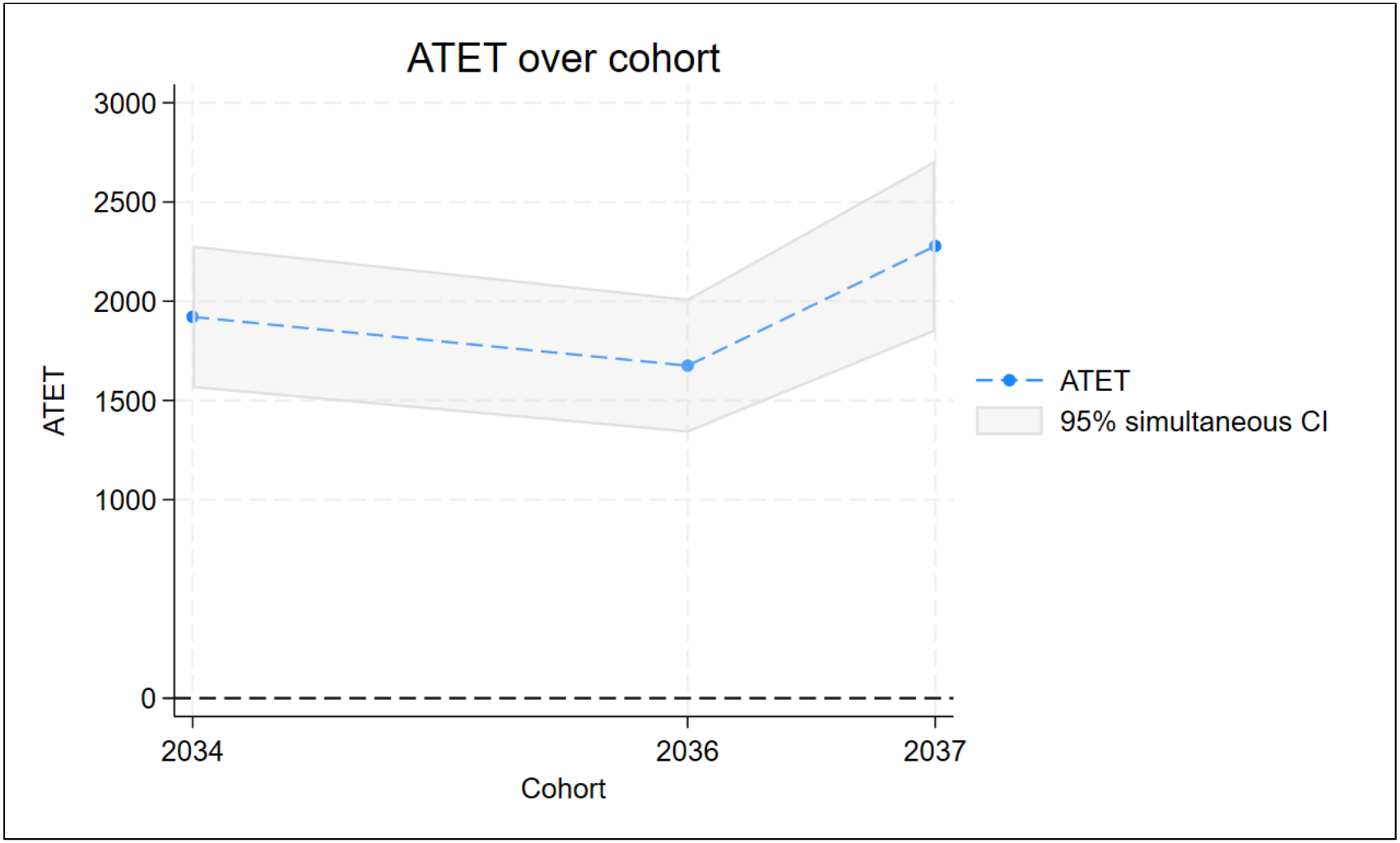
Aggregation by cohort

```
. estat aggregation, cohort graph sci

ATET over cohort                                Number of obs = 1,410
                                                Replications  =   999

                (Std. err. adjusted for 141 clusters in breed)
```

Cohort	Observed ATET	Bootstrap std. err.	Simultaneous [95% conf.	interval]
2034	1921.33	135.3652	1561.16	2281.5
2036	1675.093	120.9415	1353.3	1996.886
2037	2278.136	175.554	1811.034	2745.238

```
Note: Simultaneous confidence intervals provide inference
      across all aggregations simultaneously.
```

Aggregation by cohort – Graph



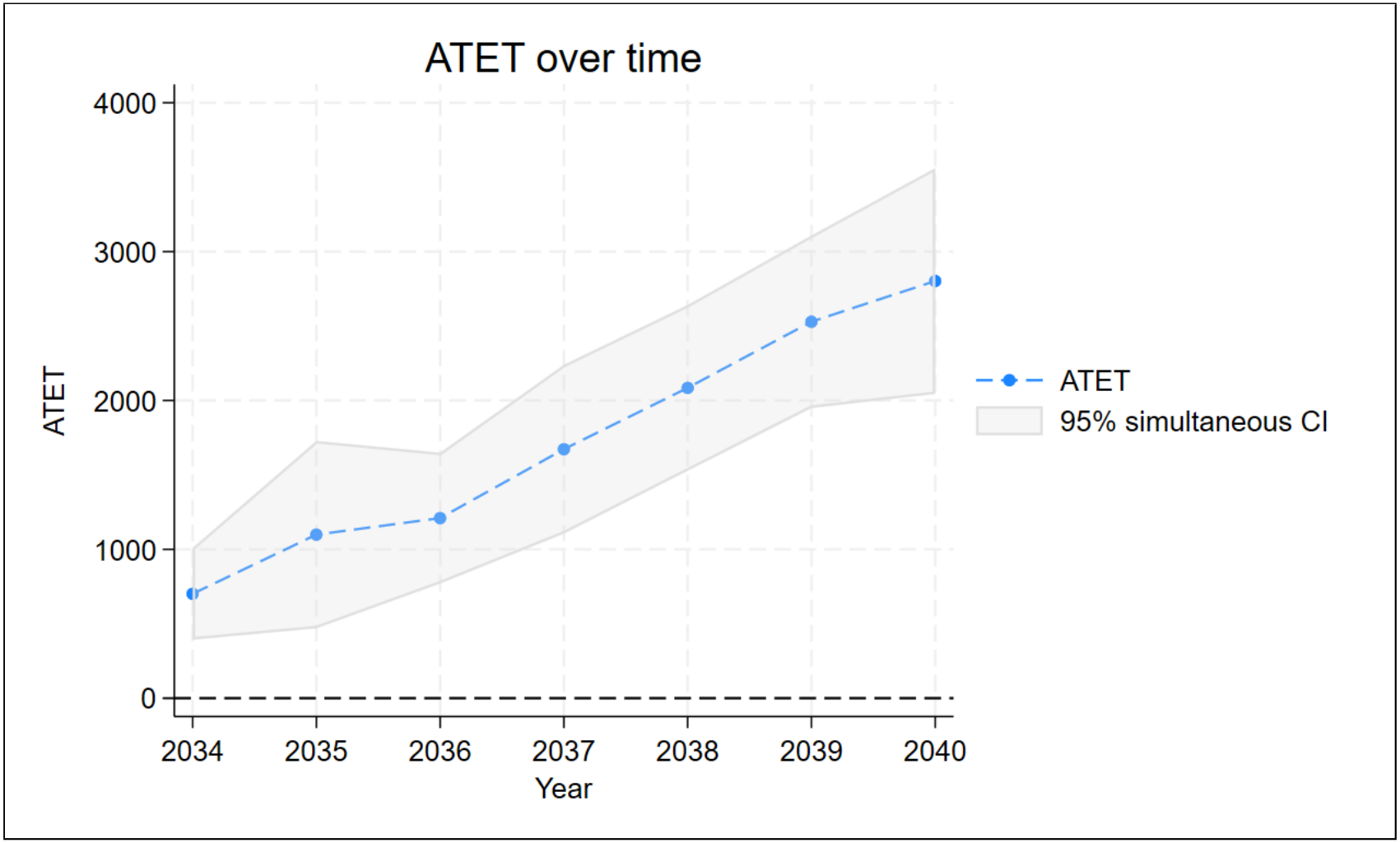
Aggregation by time

```
. estat aggregation, time graph sci

ATET over time                                  Number of obs = 1,410
                                                Replications  =   999

                    (Std. err. adjusted for 141 clusters in breed)
```

Time	Observed ATET	Bootstrap std. err.	Simultaneous [95% conf.	interval]
2034	701.1318	120.8096	388.4994	1013.764
2035	1099.044	263.6189	416.8482	1781.24
2036	1209.68	172.2839	763.8421	1655.518
2037	1672.655	205.9913	1139.589	2205.722
2038	2084.658	216.9237	1523.301	2646.015
2039	2528.847	219.2507	1961.468	3096.227
2040	2802.171	287.8548	2057.258	3547.085

```
Note: Simultaneous confidence intervals provide inference
      across all aggregations simultaneously.
```

Aggregation by time – Graph



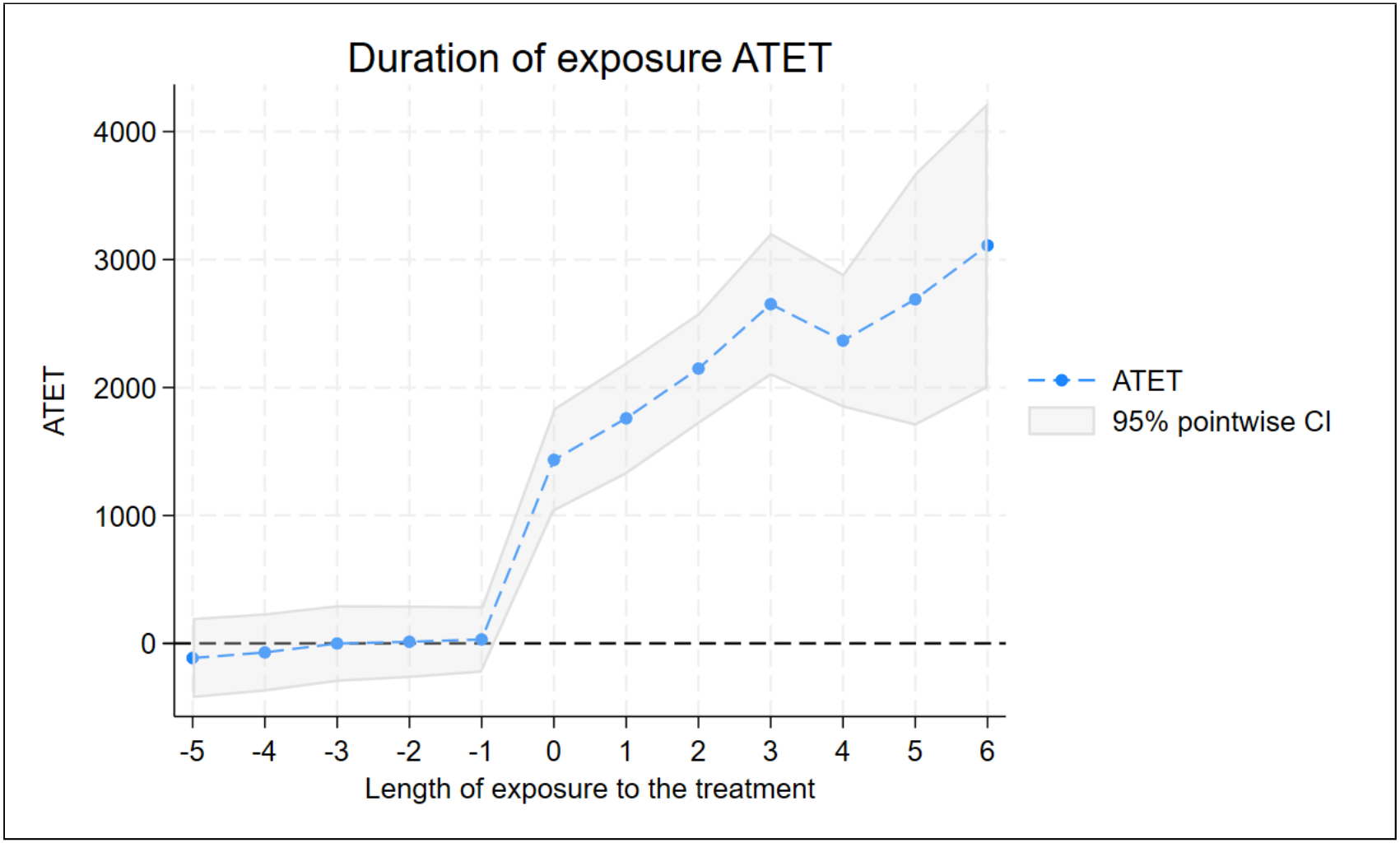
Aggregation by exposure

```
. estat aggregation, dynamic graph

Duration of exposure ATET                          Number of obs = 1,410

                                (Std. err. adjusted for 141 clusters in breed)
```

Exposure	ATET	Robust std. err.	z	P>|z|	[95% conf.	interval]
-5	-114.582	160.0972	-0.72	0.474	-428.3668	199.2028
-4	-70.65034	156.3185	-0.45	0.651	-377.029	235.7283
-3	-.9117242	153.0999	-0.01	0.995	-300.982	299.1585
-2	12.79653	144.8216	0.09	0.930	-271.0486	296.6417
-1	30.71473	132.8508	0.23	0.817	-229.668	291.0975
0	1434.409	206.3277	6.95	0.000	1030.014	1838.804
1	1759.461	224.0229	7.85	0.000	1320.385	2198.538
2	2147.486	221.903	9.68	0.000	1712.564	2582.408
3	2651.452	284.8928	9.31	0.000	2093.073	3209.832
4	2366.805	267.4253	8.85	0.000	1842.661	2890.949
5	2689.615	504.3324	5.33	0.000	1701.142	3678.088
6	3110.97	568.916	5.47	0.000	1995.915	4226.025

Note: Exposure is the number of periods since the first treatment time.

Aggregation by exposure – Graph



Outline

Conclusion

1. **Heterogeneous DID is a powerful tool** to better understand treatment effects.
2. **Easy to implement** in Stata 18:
   - `xthdidregress` for **panel data**
   - `hdidregress` for **repeated cross section**
   - Results displayed as **tables or graphs**.
3. **Treatment effects** can be **aggregated by**:
   - Cohort,
   - Time
   - Exposure to treatment
   - Overall

Thank you!

Questions?