Treatment Effects Using Stata

Enrique Pinzón

StataCorp LP

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México, D.F.
Motivation

We are interested in the outcomes of receiving a treatment in scenarios were researchers have observational data.

For instance:

- The impact on public education outcomes for schools that received a transfer and those that did not.
- Employment outcomes for individuals that participated in a job training program and those that did not.
- The effect on birth weight for babies of mothers that smoked relative to those of mothers that did not.
Observed Effect of Statin on Blood Pressure

Effect of Drug on Blood Pressure

- Untreated patients
- Treated patients
- Untreated mean = 160
- Treated mean = 160
Potential Outcomes of Statin on Blood Pressure

Effect of Drug on Blood Pressure

- Untreated mean = 185
- Treated mean = 141
How We Approach Treatment Effects

- We cannot observe individuals in both states simultaneously
  - Design a random experiment
  - We cannot do this because of technical or ethical concerns
- We need to account for covariates that are correlated with the treatment
- We will think of the problem in terms of models that govern the treatment result and the outcome
We cannot observe individuals in both states simultaneously

- Design a random experiment
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We need to account for covariates that are correlated with the treatment.

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Notation and Definitions

The potential outcome is denoted by the random variable $y_{\tau}$ with $\tau \in \{0, 1, \ldots, K\}$. The potential realizations will be denoted by:

- $y_{0i}$ is the outcome individual $i$ if they do not receive the treatment, where $i = 1 \ldots n$
- $y_{ki}$ is the potential outcome for individual $i$ if they receive different discrete levels of the treatment, where $k = 1 \ldots K$
- Usually people think about the binary case where there are only two levels $y_{0i}$ and $y_{1i}$

Potential outcome mean

$$POM = E (y_{\tau})$$

Average treatment effect

$$ATE = E (y_{ki} - y_{0i})$$

Average treatment effect on the treated

$$ATET = E (y_{ki} - y_{0i} | \tau = k)$$

From now on we will focus on binary treatments. All results are valid for multivariate treatments unless explicitly noted.
Assumptions

- We will be dealing with a cross-sectional random sample of $n$ individuals
- **Overlap:**

  \[ 0 < P(\tau_i = 1 | X_i = x) < 1 \]

- **Conditional Independence:** Conditional on the covariates, $X$, the potential outcomes, $y_0$, $y_1$, and the treatment, $\tau$, are independent
General Framework Illustrated with a Linear Example

OUTCOME MODEL:

\[
\begin{align*}
y_0 &= x \beta_0 + \varepsilon_0 \\
y_1 &= x \beta_1 + \varepsilon_1 \\
y &= \tau y_1 + (1 - \tau) y_0
\end{align*}
\]

TREATMENT MODEL:

\[
\tau = \begin{cases} 
1 & \text{if } w \gamma + \eta > 0 \\
0 & \text{otherwise}
\end{cases}
\]

- \( w \) refers to the covariates that determine the treatment
- \( y_0 \) and \( y_1 \) are not observed. Only \( y \), \( x \), \( w \), and \( \tau \) are observed
- The random disturbances \( \eta \), \( \varepsilon_0 \), and \( \varepsilon_1 \) are independent
- The functional forms for the outcome model do not need to be linear
- All the estimators we will see arise from combinations of the outcome model and the treatment model
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TREATMENT MODEL:

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Estimators Discussed Today

- Regression Adjustment (RA)
- Inverse Probability Weighting (IPW)
- Augmented Inverse Probability Weighting (AIPW)
- Inverse Probability Weighted Regression Adjustment (IPWRA)
- Nearest Neighbor Matching
- Propensity Score Matching
Effect of Smoking Mothers on Birthweight
Regression Adjustment (RA)

- We model the potential outcome and do not say anything about the treatment mechanism.
- A conditional expectation is estimated for the treatment and control groups.
- The results from the estimations are used to compute POMs and thereafter ATEs, and ATETs.
Graphical Representation of RA Estimation

Regression Lines for the Observations

Nonsmokers
Smokers

Birthweight

Mother's Age
Models for the Potential Outcome

| Outcome Model  | $E(y|x, z, \tau)$ |
|----------------|-------------------|
| linear         | $x\beta_\tau$    |
| logit          | $\exp(x\beta_\tau) / \{1 - \exp(x\beta_\tau)\}$ |
| probit         | $\Phi(x\beta_\tau)$ |
| poisson        | $\exp(x\beta_\tau)$ |
| hetprobit      | $\Phi(x\beta_\tau / z\alpha_\tau)$ |
Data from Cattaneo (2010) Journal of Econometrics

```
. describe bweight lbweight mbsmoke prenatal fbaby mmarried mage fage alcohol

<table>
<thead>
<tr>
<th>variable</th>
<th>storage</th>
<th>type</th>
<th>display</th>
<th>value label</th>
</tr>
</thead>
<tbody>
<tr>
<td>bweight</td>
<td>int</td>
<td>%9.0g</td>
<td></td>
<td>infant birth weight (grams)</td>
</tr>
<tr>
<td>lbweight</td>
<td>byte</td>
<td>%9.0g</td>
<td></td>
<td>1 if low birthweight baby</td>
</tr>
<tr>
<td>mbsmoke</td>
<td>byte</td>
<td>%9.0g</td>
<td></td>
<td>1 if mother smoked</td>
</tr>
<tr>
<td>prenatal</td>
<td>byte</td>
<td>%9.0g</td>
<td></td>
<td>trimester of first prenatal care visit</td>
</tr>
<tr>
<td>fbaby</td>
<td>float</td>
<td>%9.0g</td>
<td>YesNo</td>
<td>1 if first baby</td>
</tr>
<tr>
<td>mmarried</td>
<td>byte</td>
<td>%10.0g</td>
<td></td>
<td>1 if mother married</td>
</tr>
<tr>
<td>mage</td>
<td>byte</td>
<td>%9.0g</td>
<td></td>
<td>mother’s age</td>
</tr>
<tr>
<td>fage</td>
<td>byte</td>
<td>%9.0g</td>
<td></td>
<td>father’s age</td>
</tr>
<tr>
<td>alcohol</td>
<td>byte</td>
<td>%9.0g</td>
<td></td>
<td>1 if alcohol consumed during pregnancy</td>
</tr>
</tbody>
</table>
```

(StataCorp LP)
RA Linear Outcome Average Treatment Effect (ATE)

. teffects ra (bweight prenatall mmarried mage fbaby) (mbsmoke)
Iteration 0:   EE criterion = 7.734e-24
Iteration 1:   EE criterion = 1.196e-25
Treatment-effects estimation Number of obs = 4642
Estimator : regression adjustment
Outcome model : linear
Treatment model: none

|                | Coef.  | Std. Err. | z     | P>|z|   | [95% Conf. Interval] |
|----------------|--------|-----------|-------|-------|---------------------|
| bweight        |        |           |       |       |                     |
| ATE            |        |           |       |       |                     |
| mbsmoke        |        |           |       |       |                     |
| smoker vs nonsmoker | |        |       |       |                     |
|                 | -239.6392 | 23.82402  | -10.06 | 0.000 | -286.3334 -192.945  |
| POmean         |        |           |       |       |                     |
| mbsmoke        |        |           |       |       |                     |
| nonsmoker      |        |           |       |       |                     |
|                 | 3403.242 | 9.525207  | 357.29 | 0.000 | 3384.573 3421.911   |
RA Average Treatment Effect on the Treated (ATET)

```stata
. teffects ra (bweight prenatal1 mmarried mage fbaby) (mbsmoke), atet
Iteration 0: EE criterion = 7.629e-24
Iteration 1: EE criterion = 2.697e-26
Treatment-effects estimation
Number of obs = 4642
Estimator : regression adjustment
Outcome model : linear
Treatment model: none

| Coef. | Std. Err. | z    | P>|z|  | [95% Conf. Interval] |
|-------|-----------|------|------|---------------------|
| bweight ATET |         |      |      |                     |
|       mbsmoke smoker vs nonsmoker | -223.3017 | 22.7422 | -9.82 | 0.000 | -267.8755 -178.7278 |
| bweight POmean mbsmoke nonsmoker | 3360.961 | 12.75749 | 263.45 | 0.000 | 3335.957 3385.966 |
```

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```
. teffects ra (lbweight prenatal1 mmarried mage fbaby, probit) (mbsmoke)
Iteration 0:   EE criterion = 1.018e-18
Iteration 1:   EE criterion = 6.251e-34
Treatment-effects estimation                     Number of obs   =    4642
Estimator : regression adjustment
Outcome model : probit
Treatment model: none

|            | Coef.  | Std. Err. | z     | P>|z| | [95% Conf. Interval] |
|------------|--------|-----------|-------|-----|----------------------|
|            | lbweight |           |       |     |                      |
| ATE        |         |           |       |     |                      |
| mbsmoke    |         |           |       |     |                      |
| smoker vs  | .0500546 | .0118733  | 4.22  | 0.000 | .0267833 - .0733259  |
| nonsmoker  |         |           |       |     |                      |
| POmean     |         |           |       |     |                      |
| mbsmoke    |         |           |       |     |                      |
| nonsmoker  | .0517931 | .003734   | 13.87 | 0.000 | .0444745 - .0591116  |
```
RA Probit ATET

```
. teffects ra (lbweight prenatal1 mmarried mage fbaby, probit) (mbsmoke), atet
Iteration 0:   EE criterion = 1.018e-18
Iteration 1:   EE criterion = 2.165e-34
Treatment-effects estimation                          Number of obs       =       4642
Estimator : regression adjustment
Outcome model : probit
Treatment model: none

|                  | Coef.  | Std. Err. |    z    |    P>|z|    |      [95% Conf. Interval]     |
|------------------|--------|-----------|---------|--------|-------------------------------|
| lbweight         |        |           |         |        |                               |
| ATET             |        |           |         |        |                               |
| mbsmoke (smoker  |        |           |         |        |                               |
| vs nonsmoker)    | .0458142| .0119394  | 3.84    | 0.000  | .0224134 .0692149            |
| POmean mbsmoke   |        |           |         |        |                               |
| nonsmoker        | .0641478| .0054295  | 11.81   | 0.000  | .0535063 .0747894            |
```

(StataCorp LP)
Inverse Probability Weighting (IPW)

- In contrast to RA estimators, IPW estimate models for the treatment.
- We fit a model for the treatment and compute the probabilities of treatment.
- We then compute a weighted average, using the inverse of the probability of being in each group.
Inverse Probability Weight Calculation

```
.logistic mbsmoke mmarried alcohol mage fedu
Logistic regression
Number of obs = 60
LR chi2(4) = 46.50
Prob > chi2 = 0.0000
Log likelihood = -18.339432 Pseudo R2 = 0.5590

Log likelihood = -18.339432

mbsmoke       Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------    --------    --------    ------    ------     ------------------
 mmarried         .0785086   .0909212    -2.20    0.028     .0081122    .7597976
 alcohol        18.81727    27.98003     1.97    0.048     1.020649    346.9259
   mage            2.147569   .4593273     3.57    0.000     1.41218     3.265909
   fedu          .8189843   .1157528    -1.41    0.158     .6208252    1.080393
   _cons          4.46e-07   2.12e-06    -3.07    0.002     3.96e-11     .0050329

.predict ps
(option pr assumed; Pr(mbsmoke))
.replace ps = 1/ps if mbsmoke==1
(30 real changes made)
.replace ps = 1/(1-ps) if mbsmoke==0
(30 real changes made)
```
Inverse Propability Weighting Graphically
### Treatment Models

| Treatment Model | $P(\tau|w, z)$ |
|-----------------|----------------|
| logit           | $\exp(w\gamma_{\tau}) / \{1 - \exp(w\gamma_{\tau})\}$ |
| probit          | $\Phi(w\gamma_{\tau})$ |
| poisson         | $\exp(w\gamma_{\tau})$ |
| hetprobit       | $\Phi(w\gamma_{\tau} / z\theta_{\tau})$ |

- Only the logit model is available for multivalued treatments
. teffects ipw (bweight) (mbsmoke mmarried c.mage##c.mage fbaby medu)
Iteration 0:  EE criterion = 1.713e-21
Iteration 1:  EE criterion = 4.794e-27
Treatment-effects estimation  Number of obs  =  4642
Estimator       : inverse probability weighted
Outcome model   : weighted mean
Treatment model : logit

|                         | Coef.  | Std. Err. | z     | P>|z|  | [95% Conf. Interval] |
|-------------------------|--------|-----------|-------|------|---------------------|
| bweight                 |        |           |       |      |                     |
| ATE                     |        |           |       |      |                     |
| mbsmoke                 |        |           |       |      |                     |
| (smoker vs nonsmoker)   | -231.7203 | 25.17975 | -9.20 | 0.000 | -281.0717 -182.3689 |
| POmean                  |        |           |       |      |                     |
| mbsmoke                 |        |           |       |      |                     |
| nonsmoker               | 3403.527 | 9.576358 | 355.41 | 0.000 | 3384.757  3422.296  |
```
. teffects ipw (bweight) (mbsmoke mmarried c.mage##c.mage fbaby medu), atet
Iteration 0:   EE criterion =  1.714e-21
Iteration 1:   EE criterion =  3.735e-27
Treatment-effects estimation       Number of obs  =    4642
Estimator : inverse probability weighted
Outcome model : weighted mean
Treatment model: logit

<table>
<thead>
<tr>
<th>bweight</th>
<th>Robust</th>
<th></th>
<th></th>
<th></th>
<th>[95% Conf. Interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef.</td>
<td>Std. Err.</td>
<td>z</td>
<td>P&gt;</td>
<td>z</td>
</tr>
<tr>
<td>ATET mbsmoke</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>smoker vs</td>
<td>-225.6992</td>
<td>23.7133</td>
<td>-9.52</td>
<td>0.000</td>
<td>-272.1764</td>
</tr>
<tr>
<td>nonsmoker</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>POmean mbsmoke</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>nonsmoker</td>
<td>3363.359</td>
<td>14.28989</td>
<td>235.37</td>
<td>0.000</td>
<td>3335.351</td>
</tr>
</tbody>
</table>
```

The output shows the results of an inverse probability weighted (IPW) analysis using the `teffects ipw` command in Stata. The analysis estimates the average treatment effect on the treated (ATET) and the potential outcome means (POmean) for different smoking statuses. The table provides the coefficients, standard errors, z-scores, and p-values for these estimates, along with the 95% confidence intervals.
. teffects ipw (bweight) (mbsmoke mmarried c.mage##c.mage fbaby medu, probit)
Iteration 0:   EE criterion =  4.622e-21
Iteration 1:   EE criterion =  8.622e-26
Treatment-effects estimation  Number of obs   =  4642
Estimator : inverse probability weighted
Outcome model : weighted mean
Treatment model: probit

|                | Coef.  | Std. Err. |      z  |   P>|z|   |     [95% Conf. Interval]     |
|----------------|--------|-----------|--------|-------|-----------------------------|
| bweight        |        |           |        |       |                             |
| ATE            |        |           |        |       |                             |
| mbsmoke (smoker vs nonsmoker) | -230.6886 | 25.81524 | -8.94  | 0.000 | -281.2856 -180.0917         |
| POmean         |        |           |        |       |                             |
| mbsmoke nonsmoker | 3403.463  | 9.571369  | 355.59 | 0.000 | 3384.703 3422.222           |
. teffects ipw (bweight) (mbsmoke mmarried c.mage##c.mage fbaby medu, probit),
> ///
> atet
Iteration 0:  EE criterion =  4.621e-21
Iteration 1:  EE criterion =  7.103e-27
Treatment-effects estimation  Number of obs  =  4642
Estimator : inverse probability weighted
Outcome model : weighted mean
Treatment model: probit

| bweight     | Coef.  | Std. Err. |    z  | P>|z|  | [95% Conf. Interval] |
|-------------|--------|-----------|-------|------|----------------------|
| ATET        |        |           |       |      |                      |
| mbsmoke     | -225.1773 | 23.66458 | -9.52 | 0.000 | -271.559 --178.7955  |
| smoker (smoker vs nonsmoker) |        |           |       |      |                      |
| POmean      |        |           |       |      |                      |
| mbsmoke     | 3362.837 | 14.20149 | 236.79 | 0.000 | 3335.003 --3390.671  |
| nonsmoker   |        |           |       |      |                      |
Doubly Robust Estimators

- Doubly robust estimators model both the treatment and the outcome model.
- These models are interesting because they are consistent even if one of the models is misspecified.
- Augmented Inverse Probability Weighting (AIPW) and Inverse Probability Weighted Regression Adjustment (IPWRA) have this property.
Double Robust Estimators AIPW

- Estimate a treatment model and compute inverse-probability weights
- Estimate separate regression model of the outcome for each treatment level
  - We allow the outcome model to be estimated by nonlinear least squares or weighted nonlinear least squares
- Compute the weighted means of the treatment-specific predicted outcomes, where the weights are the inverse-probability weights computed in step.
. teffects aipw (bweight prenatal1 mmarried mage fbaby) ///
> (mbsmoke mmarried c.mage##c.mage fbaby medu)
Iteration 0:   EE criterion = 1.721e-21
Iteration 1:   EE criterion = 2.247e-26
Treatment-effects estimation                Number of obs  =  4642
Estimator : augmented IPW
Outcome model : linear by ML
Treatment model: logit

|                | bweight | Robust Coef. | Std. Err. | z     | P>|z|  | [95% Conf. Interval] |
|----------------|---------|--------------|-----------|-------|-----|----------------------|
| ATE            |         |              |           |       |     |                      |
| mbsmoke (smoker vs nonsmoker) |         | -232.0409    | 25.66973  | -9.04 | 0.000 | -282.3527   | -181.7292 |
| P0mean         |         | 3403.457     | 9.57043   | 355.64| 0.000 | 3384.7      | 3422.214  |

(StataCorp LP)
ATE for AIPW with Nonlinear Least Squares

```
. teffects aipw (bweight prenatall mmarried mage fbaby, poisson) ///
   > (mbsmoke mmarried c.mage##c.mage fbaby medu), nls
Iteration 0: EE criterion = .00018418
Iteration 1: EE criterion = 1.991e-17
Treatment-effects estimation                      Number of obs       =      4642
Estimator : augmented IPW
Outcome model : Poisson by NLS
Treatment model : logit

                   | Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
---------------------+-------------------------------------
   bweight            |                     
    ATE               |                     
      mbsmoke (smoker vs nonsmoker) | -232.1593   25.69692   -9.03 0.000  -282.5244   -181.7943
    POmean             |                     
      mbsmoke nonsmoker | 3403.444   9.570361   355.62 0.000  3384.687   3422.202
```
## Displaying Treatment and Outcome Equations

```
. teffects aipw (bweight prenatal1 mmarried mage fbaby, poisson) ///
> (mbsmoke mmarried c.mage##c.mage fbaby medu), aequations nolog
```

**Treatment-effects estimation**
- **Number of obs**: 4642
- **Estimator**: augmented IPW
- **Outcome model**: Poisson by ML
- **Treatment model**: logit

|                | Coef.   | Std. Err. | z      | P>|z| | [95% Conf. Interval] |
|----------------|---------|-----------|--------|------|---------------------|
| **ATE**        |         |           |        |      |                     |
| mbsmoke (smoker vs nonsmoker) | -232.1369 | 25.68896  | -9.04  | 0.000 | -282.4864 to -181.7875 |
| **POmean**     |         |           |        |      |                     |
| mbsmoke nonsmoker | 3403.444  | 9.570363  | 355.62 | 0.000 | 3384.686 to 3422.202 |
| **OME0**       |         |           |        |      |                     |
| prenatal1      | .0191803 | 0.0082502 | 2.32   | 0.020 | .0030102 to .0353503 |
| mmarried       | .0480049 | 0.0080048 | 6.00   | 0.000 | .0323158 to .0636939 |
| mage           | .0007522 | 0.006106  | 1.23   | 0.218 | -.004447 to .001949 |
| fbaby          | -.0209166| 0.0057619 | -3.63  | 0.000 | -.0322097 to -.0096235 |
| _cons          | 8.072261 | 0.159896  | 504.84 | 0.000 | 8.040922 to 8.1036  |
| **OME1**       |         |           |        |      |                     |
| prenatal1      | .0080848 | 0.012943  | 0.62   | 0.532 | -.0172831 to .0334526 |
| mmarried       | .0426096 | 0.0130351 | 3.27   | 0.001 | .0170612 to .0681579 |
| mage           | -.0023601| 0.0013552 | -1.74  | 0.082 | -.0050163 to .0002961 |
| fbaby          | .0131662 | 0.0126163 | 1.04   | 0.297 | -.0115613 to .0378937 |
| _cons          | 8.07972  | 0.034184  | 241.77 | 0.000 | 8.014221 to 8.145219 |
| **TME1**       |         |           |        |      |                     |
| mmarried       | -1.145706 | .0975846  | -11.74 | 0.000 | -1.336969 to -.9544439 |
| mage           | .321518  | .0657363  | 4.89   | 0.000 | .1926773 to .4503588 |
| c.mage#c.mage  | -.0060368| .0012234  | -4.93  | 0.000 | -.0084346 to -.0036389 |
| fbaby          | -.3864258| .0894428  | -4.32  | 0.000 | -.5617305 to -.2111211 |
| medu           | -.1420833| .0179132  | -7.93  | 0.000 | -.1771926 to -.106974 |
| _cons          | -2.950915| .8302955  | -3.55  | 0.000 | -4.578264 to -1.323565 |

(StataCorp LP)
Double Robust Estimators Inverse Probability Weighted Regression Adjustment (IPWRA)

- Estimate a treatment model and compute inverse-probability weights
- Use the estimated inverse-probability weights and fit weighted regression models of the outcome for each treatment level
- Compute the means of the treatment-specific predicted outcomes
ATET for Inverse Probability Weighted Regression Adjustment

```
teffects ipwra (bweight prenatal1 mmarried mage fbaby) ///
> (mbsmoke mmarried c.mage##c.mage fbaby medu), atet
Iteration 0:   EE criterion =  4.620e-21
Iteration 1:   EE criterion =  1.345e-26
Treatment-effects estimation Number of obs   =    4642
Estimator : IPW regression adjustment
Outcome model : linear
Treatment model: logit

|                | Coef.  | Std. Err. | z     | P>|z|   | [95% Conf. Interval] |
|----------------|--------|-----------|-------|-------|----------------------|
| bweight        |        |           |       |       |                      |
| ATET           |        |           |       |       |                      |
| mbsmoke        |        |           |       |       |                      |
| (smoker vs     |        |           |       |       |                      |
| nonsmoker)     |        |           |       |       |                      |
|                | -224.0108 | 23.846   | -9.39 | 0.000 | -270.7481       -177.2735 |
| POmean         |        |           |       |       |                      |
| mbsmoke        |        |           |       |       |                      |
| nonsmoker      |        |           |       |       |                      |
|                | 3361.671 | 14.54939 | 231.05| 0.000 | 3333.154        3390.187  |
```

(StataCorp LP)
Displaying Treatment and Outcome Equations

```stata
. teffects ipwra (bweight prenatal1 mmarried mage fbaby) ///
> (mbsmoke mmarried c.mage##c.mage fbaby medu), atet aequations
Iteration 0:   EE criterion =  4.620e-21
Iteration 1:   EE criterion =  1.345e-26
Treatment-effects estimation
Estimator : IPW regression adjustment
Outcome model : linear
Treatment model: logit

|                | Coef.  | Std. Err. | z     | P>|z|   | [95% Conf. Interval] |
|----------------|--------|-----------|-------|-------|---------------------|
|                | bweight|           |       |       |                     |
| ATET           |        |           |       |       |                     |
| mbsmoke (smoker vs nonsmoker) | -224.0108 | 23.846 | -9.39 | 0.000 | -270.7481 -177.2735 |
| POmean         |        |           |       |       |                     |
| mbsmoke nonsmoker | 3361.671 | 14.54939 | 231.05 | 0.000 | 3333.154 3390.187  |
| OME0           |        |           |       |       |                     |
| prenatal1      | 77.07926 | 40.4633 | 1.90  | 0.057 | -2.227341 156.3859  |
| mmarried       | 138.9961 | 29.48776 | 4.71  | 0.000 | 81.20114 196.791   |
| mage           | 4.482273 | 3.033008 | 1.48  | 0.139 | -1.462313 10.42686  |
| fbaby          | -73.85266 | 32.55461 | -2.27 | 0.023 | -137.6585 -10.0468  |
| _cons          | 3157.337 | 72.75786 | 43.40 | 0.000 | 3014.734 3299.939  |
| OME1           |        |           |       |       |                     |
| prenatal1      | 25.11133 | 40.37541 | 0.62  | 0.534 | -54.02302 104.2457  |
| mmarried       | 133.6617 | 40.86443 | 3.27  | 0.001 | 53.5689 213.7545   |
| mage           | -7.370881 | 4.21817 | -1.75 | 0.081 | -15.63834 .8965804 |
| fbaby          | 41.43991 | 39.70712 | 1.04  | 0.297 | -36.38461 119.2644  |
| _cons          | 3227.169 | 104.4059 | 30.91 | 0.000 | 3022.537 3431.801  |
| TME1           |        |           |       |       |                     |
| mmarried       | -1.145706 | 0.0975846 | -11.74 | 0.000 | -1.336969 -0.9544439 |
| mage           | .321518 | 0.0657363 | 4.89  | 0.000 | 0.1926773 .4503588 |
| c.mage##c.mage | -.0060368 | .0012234 | -4.93 | 0.000 | -.0084346 -.0036389 |
| fbaby          | -.3864258 | .0894428 | -4.32 | 0.000 | -0.5617305 -0.211121 |
| medu           | -.1420833 | .0179132 | -7.93 | 0.000 | -0.1771926 -0.106974 |
| _cons          | -2.950915 | .8302955 | -3.55 | 0.000 | -4.578264 -1.323565 |
```

(StataCorp LP)
Nearest Neighbor Matching

- Can be understood as an outcome model within our framework
- Matches the closest individuals in terms of covariates
- Is a nonparametric estimate with an asymptotic bias. We provide a bias correction option.
- These estimators are nondifferentiable therefore the bootstrap is not allowed
- These estimators do not allow for multivalued treatments
Nearest Neighbor Matching

- Can be understood as an outcome model within our framework
- Matches the closest individuals in terms of covariates
- Is a nonparametric estimate with an asymptotic bias. We provide a bias correction option.
- These estimators are nondifferentiable therefore the bootstrap is not allowed
- These estimators do not allow for multivalued treatments
ATE with `nnmatch`

```
. teffects nnmatch (bweight mage prenatal1 mmarried fbaby) (mbsmoke)
Treatment-effects estimation
Estimator : nearest-neighbor matching
Outcome model : matching
Distance metric: Mahalanobis

|            | Coef.  | Std. Err. | z     | P>|z|   | [95% Conf. Interval] |
|------------|--------|-----------|-------|-------|---------------------|
| bweight    |        |           |       |       |                     |
| ATE        |        |           |       |       |                     |
| mbsmoke    |        |           |       |       |                     |
| smoker vs  |        |           |       |       |                     |
| nonsmoker  |        |           |       |       |                     |
```

- ATE with `nnmatch` (bweight mage prenatal1 mmarried fbaby) (mbsmoke)
- Treatment-effects estimation
- Estimator: nearest-neighbor matching
- Outcome model: matching
- Distance metric: Mahalanobis

|            | Coef.  | Std. Err. | z     | P>|z|   | [95% Conf. Interval] |
|------------|--------|-----------|-------|-------|---------------------|
| ATE        |        |           |       |       |                     |
| mbsmoke    |        |           |       |       |                     |
| smoker vs  |        |           |       |       |                     |
| nonsmoker  |        |           |       |       |                     |

- **ATE**: 
  - `mbsmoke` (smoker vs nonsmoker)
  - Coef: -240.3306
  - Std. Err: 28.43006
  - z: -8.45
  - P>|z|: 0.000
  - [95% Conf. Interval]: -296.0525 to -184.6087
Exact Matching and Different Distance

```
. teffects nnmatch (bweight mage) (mbsmoke), ///
> ematch(prenatal mmarried fbaby) metric(euclidean)
```

Treatment-effects estimation
Number of obs = 4642
Estimator : nearest-neighbor matching
Matches: requested = 1
Outcome model : matching
Distance metric: Euclidean

|                | Coef.    | Std. Err. | z     | P>|z|    | [95% Conf. Interval] |
|----------------|----------|-----------|-------|--------|---------------------|
| bweight        |          |           |       |        |                     |
| ATE            |          |           |       |        |                     |
| mbsmoke        |          |           |       |        |                     |
| smoker vs      |          |           |       |        |                     |
| nonsmoker      | -240.3306| 28.43006  | -8.45| 0.000  | [-296.0525, -184.6087]|
```
. teffects nnmatch (bweight mage fage) (mbsmoke), ///
>  ematch(prenatal1 mmarried fbaby) biasadj(mage fage)

Treatment-effects estimation  Number of obs = 4642
Estimator : nearest-neighbor matching  Matches: requested = 1
Outcome model : matching  min = 1
Distance metric: Mahalanobis  max = 25

|         | Coef.  | Std. Err. | z   | P>|z|  | [95% Conf. Interval] |
|---------|--------|-----------|-----|------|---------------------|
| ATE     |        |           |     |      |                     |
| mbsmoke (smoker vs nonsmoker) | -223.8389 | 26.19973  | -8.54 | 0.000 | -275.1894 -172.4883 |
Can be classified within the class of treatment models

Estimate the treatment probabilities (propensity scores)

Assign values to unobserved outcomes based on observed ones with similar propensity scores

Estimate ATE

These estimators are nondifferentiable therefore the bootstrap is not allowed

These estimators do not allow for multivalued treatments
Propensity Score Matching

- Can be classified within the class of treatment models
- Estimate the treatment probabilities (propensity scores)
- Assign values to unobserved outcomes based on observed ones with similar propensity scores
- Estimate ATE
- These estimators are nondifferentiable therefore the bootstrap is not allowed
- These estimators do not allow for multivalued treatments
Propensity Score Matching controlling matches

```
. teffects psmatch (bweight) (mbsmoke mmmarried c.mage#c.mage fbaby medu), ///
> nneighbor(2)
Treatment-effects estimation          Number of obs =  4642
Estimator : propensity-score matching   Matches: requested =  2
Outcome model : matching                min =  2
Treatment model: logit                 max =  74

| Coef.  | Std. Err. | z    | P>|z|  | [95% Conf. Interval] |
|--------|-----------|------|------|----------------------|
| ATE mbsmoke (smoker vs nonsmoker)    | -214.2469 | 27.47783 | -7.80 | 0.000 | -268.1025  | -160.3914 |
```
Conclusion

- We have presented a host of treatment effects estimators within a unified framework.
- The estimators are parametric and nonparametric and in the parametric cases can be consistent under misspecification of the potential outcome or treatment models.
- The estimators provide estimates and inference for quantities of interest for researchers, POM, ATE, ATET.
Double Robustness I

- Let \( P(\tau|x, z, \hat{\gamma}) =: M_P(\hat{\gamma}) \) be our estimated conditional treatment probabilities.
- Let \( E(y|x, z, \tau, \hat{\beta}) =: M_E(\hat{\beta}_\tau) \) define our estimated conditional means.
- We define the following estimators for the POMs:

\[
\hat{E}(y_1) = \frac{1}{n} \sum_{i=1}^{n} \left[ \frac{\tau_i y_i}{M_P(\hat{\gamma})} - \frac{\{\tau_i - M_P(\hat{\gamma})\}}{M_P(\hat{\gamma})} M_E(\hat{\beta}_1) \right]
\]

\[
\hat{E}(y_0) = \frac{1}{n} \sum_{i=1}^{n} \left[ \frac{(1 - \tau_i) y_i}{1 - M_P(\hat{\gamma})} - \frac{\{\tau_i - M_P(\hat{\gamma})\}}{1 - M_P(\hat{\gamma})} M_E(\hat{\beta}_0) \right]
\]
Double Robustness I

- Let \( P(\tau|x, z, \hat{\gamma}) =: M_P(\hat{\gamma}) \) be our estimated conditional treatment probabilities
- Let \( E(y|x, z, \tau, \hat{\beta}) =: M_E(\hat{\beta}_\tau) \) define our estimated conditional means
- We define the following estimators for the POMs

\[
\hat{E}(y_1) = \frac{1}{n} \sum_{i=1}^{n} \left[ \frac{\tau_i y_i}{M_P(\hat{\gamma})} - \frac{\tau_i - M_P(\hat{\gamma})}{M_P(\hat{\gamma})} M_E(\hat{\beta}_1) \right]
\]

\[
\hat{E}(y_0) = \frac{1}{n} \sum_{i=1}^{n} \left[ \frac{(1 - \tau_i) y_i}{1 - M_P(\hat{\gamma})} - \frac{\tau_i - M_P(\hat{\gamma})}{1 - M_P(\hat{\gamma})} M_E(\hat{\beta}_0) \right]
\]
We will focus on $\hat{E}(y_1)$ (a similar argument follows for $\hat{E}(y_0)$).

By the law of large numbers it follows that $\hat{E}(y_1)$ has the following probability limit:

$$\hat{E}(y_1) \xrightarrow{p} E \left[ \frac{\tau y}{M_P(\gamma)} - \frac{\{\tau - M_P(\gamma)\}}{M_P(\gamma)} M_E(\beta_1) \right]$$

$$= E \left[ \frac{\tau y_1}{M_P(\gamma)} - \frac{\{\tau - M_P(\gamma)\}}{M_P(\gamma)} M_E(\beta_1) + y_1 - y_1 \right]$$

$$= E \left[ \frac{\tau y_1}{M_P(\gamma)} - \frac{\{\tau - M_P(\gamma)\}}{M_P(\gamma)} M_E(\beta_1) + y_1 - y_1 \frac{M_P(\gamma)}{M_P(\gamma)} \right]$$

$$= E \left[ \frac{y_1 (\tau - M_P(\gamma))}{M_P(\gamma)} - \frac{\{\tau - M_P(\gamma)\}}{M_P(\gamma)} M_E(\beta_1) + y_1 \right]$$

$$= E \left[ \frac{\{\tau - M_P(\gamma)\}}{M_P(\gamma)} (y_1 - M_E(\beta_1)) + y_1 \right]$$

$$= E (y_1) + E \left[ \frac{\{\tau - M_P(\gamma)\}}{M_P(\gamma)} (y_1 - M_E(\beta_1)) \right]$$
Intuition Behind Double Robustness II

- We will focus on $\hat{E}(y_1)$ (a similar argument follows for $\hat{E}(y_0)$).
- By the law of large numbers it follows that $\hat{E}(y_1)$ has the following probability limit:

$$
\hat{E}(y_1) \xrightarrow{p} E \left[ \frac{\tau y}{M_P(\gamma)} - \frac{\{\tau - M_P(\gamma)\}}{M_P(\gamma)} M_E(\beta_1) \right]
$$

$$
= E \left[ \frac{\tau y_1}{M_P(\gamma)} - \frac{\{\tau - M_P(\gamma)\}}{M_P(\gamma)} M_E(\beta_1) + y_1 - y_1 \right]
$$

$$
= E \left[ \frac{\tau y_1}{M_P(\gamma)} - \frac{\{\tau - M_P(\gamma)\}}{M_P(\gamma)} M_E(\beta_1) + y_1 - y_1 \frac{M_P(\gamma)}{M_P(\gamma)} \right]
$$

$$
= E \left[ \frac{y_1 (\tau - M_P(\gamma))}{M_P(\gamma)} - \frac{\{\tau - M_P(\gamma)\}}{M_P(\gamma)} M_E(\beta_1) + y_1 \right]
$$

$$
= E \left[ \frac{\{\tau - M_P(\gamma)\}}{M_P(\gamma)} (y_1 - M_E(\beta_1)) + y_1 \right]
$$

$$
= E(y_1) + E \left[ \frac{\{\tau - M_P(\gamma)\}}{M_P(\gamma)} (y_1 - M_E(\beta_1)) \right]
$$
Intuition Behind Double Robustness III

\[
\hat{E}(y_1) \overset{p}{\to} E(y_1) + E \left[ \frac{\{\tau - M_P(\gamma)\}}{M_P(\gamma)} (y_1 - M_E(\beta_1)) \right]
\]

Given conditional independence of treatment and outcome conditional on the regressors by the law of iterated expectations:

- If the outcome model is correctly specified \( E[y_1 - M_E(\beta_1)] = 0 \). This implies that even if the treatment model is incorrectly specified, \( \hat{E}(y_1) \overset{p}{\to} E(y_1) \)

- Similarly if the treatment model is correctly specified \( E[\tau - M_P(\gamma)] = 0 \). Thus, even if \( E[y_1 - M_E(\beta_1)] \neq 0 \) we have that \( \hat{E}(y_1) \overset{p}{\to} E(y_1) \)
\( \hat{E}(y_1) \overset{p}{\rightarrow} E(y_1) + E\left[ \frac{\{\tau - M_P(\gamma)\}}{M_P(\gamma)} (y_1 - M_E(\beta_1)) \right] \)

- Given conditional independence of treatment and outcome conditional on the regressors by the law of iterated expectations:
  - If the outcome model is correctly specified \( E[y_1 - M_E(\beta_1)] = 0 \). This implies that even if the treatment model is incorrectly specified, \( \hat{E}(y_1) \overset{p}{\rightarrow} E(y_1) \)
  - Similarly if the treatment model is correctly specified \( E[\tau - M_P(\gamma)] = 0 \). Thus, even if \( E[y_1 - M_E(\beta_1)] \neq 0 \) we have that \( \hat{E}(y_1) \overset{p}{\rightarrow} E(y_1) \)
Estimation I: An Example of Moment Based Estimation

We define the projection model by:

\[ y = X\beta + \varepsilon \]

\[ E(X'\varepsilon) = 0 \]

\( \beta \) is then given by:

\[ 0 = E(X'\varepsilon) \]

\[ 0 = E(X'\{y - X\beta\}) \]

\[ \beta = E(X'X)^{-1}E(X'y) \]

A consistent estimator of \( \beta \) is:

\[ \hat{\beta} = \left(\frac{X'X}{n}\right)^{-1} \left(\frac{X'y}{n}\right) \]
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We define the projection model by:

\[
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E(X'\epsilon) = 0
\]

\(\beta\) is then given by:

\[
0 = E(X'\epsilon) \\
0 = E(X'\{y - X\beta\}) \\
\beta = E(X'X)^{-1}E(X'y)
\]

A consistent estimator of \(\beta\) is:

\[
\hat{\beta} = \left(\frac{X'X}{n}\right)^{-1}\left(\frac{X'y}{n}\right)
\]
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\[ \beta = E (X'X)^{-1} E (X'y) \]

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\[ 0 = E(X'\epsilon) \]

\[ 0 = E(X'\{y - X\beta\}) \]

\[ \beta = E(X'X)^{-1} E(X'y) \]

A consistent estimator of \( \beta \) is:

\[ \hat{\beta} = \left( \frac{X'X}{n} \right)^{-1} \left( \frac{X'y}{n} \right) \]
Estimation II: Methodology We Employ

- The different specifications for the outcome generate moment conditions
- We can then use GMM to estimate the parameters of interest
- For the linear model:

$$0 = E \left[ \tau (y - x\beta_1)'x + (1 - \tau) (y - x\beta_0)'x \right]$$

- For the probit and logit models

$$0 = E \left( \tau \left[ \frac{g(x\beta_1)}{G(x\beta_1)} \{ y - G(x\beta_1) \} \right] + (1 - \tau) \left[ \frac{g(x\beta_1)}{G(x\beta_0)} \{ y - G(x\beta_0) \} \right] \right)$$

- $G(.)$ is either the standard normal CDF or the logistic function
- $g(.)$ is the derivative of $G(.)$
Estimation II: Methodology We Employ

- The different specifications for the outcome generate moment conditions
- We can then use GMM to estimate the parameters of interest
- For the linear model:

  \[ 0 = E \left[ \tau (y - x\beta_1)'x + (1 - \tau) (y - x\beta_0)'x \right] \]

- For the probit and logit models

  \[ 0 = E \left( \tau \left[ \frac{g(x\beta_1) \{y - G(x\beta_1)\}}{G(x\beta_1) \{1 - G(x\beta_1)\}} \right] + (1 - \tau) \left[ \frac{g(x\beta_1) \{y - G(x\beta_0)\}}{G(x\beta_0) \{1 - G(x\beta_0)\}} \right] \right) \]

  - \( G(.) \) is either the standard normal CDF or the logistic function
  - \( g(.) \) is the derivative of \( G(.) \)
Distance

The distance function is given by:

$$\|x_i - x_j\|_S = \left\{ (x_i - x_j)' S^{-1} (x_i - x_j) \right\}^{1/2}$$

where $S$ can be:

$$S = \begin{cases} 
\frac{(X-x1_n)'W(X-x1_n)}{\sum_{i=1}^{n} w_i^{-1}} & \text{if metric is mahalanobis} \\
\text{diagonal } \left\{ \frac{(X-x1_n)'W(X-x1_n)}{\sum_{i=1}^{n} w_i^{-1}} \right\} & \text{if metric is ivariance} \\
I_k & \text{if metric is euclidean}
\end{cases}$$

Above $1_n$ is an $n$ vector of ones, $W$ is a matrix of frequency weights.
ATE for Propensity Score Matching

```
. teffects psmatch (bweight) (mbsmoke mmarried c.mage##c.mage fbaby medu), ///
  > generate(ps)
```

Treatment-effects estimation               Number of obs = 4642
Estimator : propensity-score matching       Matches: requested = 1
Outcome model : matching                    min = 1
Treatment model: logit                      max = 74

|         | Coef.  | Std. Err. | z     | P>|z|  | [95% Conf. Interval] |
|---------|--------|-----------|-------|------|----------------------|
| bweight |        |           |       |      |                      |
| ATE     |        |           |       |      |                      |
| mbsmoke |        |           |       |      |                      |
| smoker  |        |           |       |      |                      |
| vs      |        |           |       |      |                      |
| nonsmoker |      |           |       |      |                      |

-210.9683  32.021  -6.59  0.000  -273.7284 -148.2083
Matches Generated by the Estimator

Observed (solid) and Pscores (hollow)

Birthweight

Mother's age

Smokers
Nonsmokers
Imbens and Wooldridge (2009) JEL for a recent survey
Regression Discontinuity. Lee and Lemieux (2010) JEL
Stata also offers estimation in the presence of endogeneity