

Estimating the ATE of an endogenously assigned treatment from a sample with endogenous selection by regression adjustment using an extended regression models

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- Fictional data on wellness program from large company

```
. use wprogram2
```

```
. describe
```

```
Contains data from wprogram2.dta
```

```
obs:      3,000
```

```
vars:      8
```

```
size:     96,000
```

```
28 Jul 2017 07:13
```

variable name	storage type	display format	value label	variable label
wchange	float	%9.0g	changel	Weight change level
age	float	%9.0g		Years over 50
over	float	%9.0g		Overweight (tens of pounds)
phealth	float	%9.0g		Prior health score
prog	float	%9.0g	yesno	Participate in wellness program
wtprog	float	%9.0g	yesno	Offered work time to participate in program
wtsamp	float	%9.0g		Offered work time to participate in sample
insamp	float	%9.0g		In sample: attended initial and final weigh in

```
Sorted by:
```

- Three levels of wchange

```
. tabulate wchange prog
```

Weight change level	Participate in wellness program		Total
	No	Yes	
Loss	154	960	1,114
No change	251	299	550
Gain	184	36	220
Total	589	1,295	1,884

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- Data are observational

Dealing with observational data

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- Table does not account for
 - how observed covariates that affect program participation also affect the potential outcome variables
 - Assume the treatment is as good as random after conditioning on covariates
 - Conditional mean independence
 - Exogenous treatment assignment
 - teffects

Dealing with observational data

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- Table does not account for
 - how observed unobserved error that affect program participation also affect the potential outcome variables
 - Endogenous treatment assignment
 - `eteffects` and `etregress` for continuous outcomes
 - `etpoisson` for count outcomes
 - Need Stata command for ordinal outcome

Dealing with observational data

```
. tabulate wchange prog
```

Weight change level	Participate in wellness program		Total
	No	Yes	
Loss	154	960	1,114
No change	251	299	550
Gain	184	36	220
Total	589	1,295	1,884

- Table does not account for
 - the possibility that unobserved errors in the process that caused some of 3,000 individuals not to show for the final weigh in may also affect the potential outcome variables
 - Endogenous loss to follow up
 - Endogenous sample selection

Ordinal Potential outcomes

- Because the outcome `wchange` is ordinal, there are really three binary outcomes
 - `wchange==“Loss”`,
 - `wchange==“No Change”`, and
 - `wchange==“Gain”`

Ordinal Potential outcomes

- In the potential outcome framework, there is an outcome for each person when they participate and when they do not participate

Ordinal Potential outcomes

- In the potential outcome framework, there is an outcome for each person when they participate and when they do not participate
- Thus, there are really three binary outcomes for each potential outcome

Participate		Not participate	
$wchange_p$	== "Loss"	$wchange_{np}$	== "Loss"
$wchange_p$	== "No change"	$wchange_{np}$	== "No change"
$wchange_p$	== "Gain"	$wchange_{np}$	== "Gain"

Potential outcome framework

- For each outcome (Loss, No change, and Gain), we only observe one of these two potential outcomes for each individual

Potential outcome framework

- For each outcome (Loss, No change, and Gain), we only observe one of these two potential outcomes for each individual
- We estimate the parameters of a model and use the estimated parameters to predict what each person does in the unobserved potential outcome
 - Regression adjustment

Average treatment effects

- In the case of one outcome, the average treatment effect (ATE) is

$$\mathbf{E}[y_p - y_{np}]$$

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Average treatment effects

- In the case of one outcome, the average treatment effect (ATE) is

$$\mathbf{E}[y_p - y_{np}]$$

- As there are three outcomes, there are three ATEs
 - one for “Loss”, one for “No Change”, and one for “Gain”

$$ATE_{Loss} = \mathbf{E}[(wchange_p == \text{“Loss”}) - (wchange_{np} == \text{“Loss”})]$$

$$ATE_{Nochange} = \mathbf{E}[(wchange_p == \text{“No change”}) - (wchange_{np} == \text{“No change”})]$$

$$ATE_{Gain} = \mathbf{E}[(wchange_p == \text{“Gain”}) - (wchange_{np} == \text{“Gain”})]$$

Average treatment effects

- I will provide some details about the average treatment effect for “Loss”
- The details for the outcomes of “No change” and “Gain” are analogous

- the average treatment effect (ATE) of the program on the Loss outcome ATE_{Loss}

$$ATE_{Loss} = \mathbf{E}[(wchange_p == "Loss") - (wchange_{np} == "Loss")]$$

- The first line says that ATE_{Loss} is the mean difference in the outcomes when everyone participates instead of no one participates

- the average treatment effect (ATE) of the program on the Loss outcome ATE_{Loss}

$$\begin{aligned}ATE_{Loss} &= \mathbf{E}[(wchange_p == "Loss") - (wchange_{np} == "Loss")] \\ &= \mathbf{E}[wchange_p == "Loss"] - \mathbf{E}[wchange_{np} == "Loss"]\end{aligned}$$

- The second line says that the mean of the differences is the difference in the means

- the average treatment effect (ATE) of the program on the Loss outcome ATE_{Loss}

$$\begin{aligned}ATE_{Loss} &= \mathbf{E}[(wchange_p == "Loss") - (wchange_{np} == "Loss")] \\ &= \mathbf{E}[wchange_p == "Loss"] - \mathbf{E}[wchange_{np} == "Loss"] \\ &= \Pr[wchange_p == "Loss"] - \Pr[wchange_{np} == "Loss"]\end{aligned}$$

- The third line says that because the mean of binary outcome is the probability that the event is true, the ATE_{Loss} is the difference in the probability an individual is in the state of "Loss" when everyone participates instead of no one participates

- I am going to use the ERM comand eoprobit to estimate the parameters of $\Pr[\text{wchange}_p == \text{"Loss"} | \mathbf{x}]$ and $\Pr[\text{wchange}_{np} == \text{"Loss"} | \mathbf{x}]$ and

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- Then I use margins or estat teffects to estimate

$$\begin{aligned}
 & \mathbf{E}[\Pr[\text{wchange}_p == \text{"Loss"} | \mathbf{x}]] - \mathbf{E}[\Pr[\text{wchange}_{np} == \text{"Loss"} | \mathbf{x}]] \\
 &= \Pr[\text{wchange}_p == \text{"Loss"}] - \Pr[\text{wchange}_{np} == \text{"Loss"}] \\
 &= ATE_{\text{Loss}}
 \end{aligned}$$

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$$\begin{aligned} & \mathbf{E}[\Pr[\text{wchange}_p == \text{"Loss"} | \mathbf{x}]] - \mathbf{E}[\Pr[\text{wchange}_{np} == \text{"Loss"} | \mathbf{x}]] \\ &= \Pr[\text{wchange}_p == \text{"Loss"}] - \Pr[\text{wchange}_{np} == \text{"Loss"}] \\ &= ATE_{Loss} \end{aligned}$$

- The ATE_{Loss} is the mean difference in the probability an individual is in the state of "Loss" when everyone participates instead of no one participates

Models for the ordinal outcome

- For exogenous treatment, we do a one-step equivalent to fitting two separate ordinal probit models
 - One fit to participants
 - Another fit to non participants

Model for participants

$$wchange = \begin{cases} \text{"Loss"} & \text{if } \mathbf{x}\beta_0 + \epsilon_0 \leq cut1_0 \\ \text{"No change"} & \text{if } cut1_0 < \mathbf{x}\beta_0 + \epsilon_0 \leq cut2_0 \\ \text{"Gain"} & \text{if } cut2_0 < \mathbf{x}\beta_0 + \epsilon_0 \end{cases}$$

$$\mathbf{x}\beta_0 = \beta_{1,0}age + \beta_{2,0}over + \beta_{3,0}phealth$$

for the observations at which prog=0, and

ϵ_0 , is standard normal

Model for nonparticipants

$$wchange = \begin{cases} \text{"Loss"} & \text{if } \mathbf{x}\beta_1 + \epsilon_1 \leq cut1_1 \\ \text{"No change"} & \text{if } cut1_1 < \mathbf{x}\beta_1 + \epsilon_1 \leq cut2_1 \\ \text{"Gain"} & \text{if } cut2_1 < \mathbf{x}\beta_1 + \epsilon_1 \end{cases}$$

$$\mathbf{x}\beta_1 = \beta_{1,1}age + \beta_{2,1}over + \beta_{3,1}phealth$$

for the observations at which prog=1

ϵ_1 is standard normal

$$wchange = \begin{cases} \text{"Loss"} & \text{if } \mathbf{x}\beta_0 + \epsilon_0 \leq cut1_0 \\ \text{"No change"} & \text{if } cut1_0 < \mathbf{x}\beta_0 + \epsilon_0 \leq cut2_0 \\ \text{"Gain"} & \text{if } cut2_0 < \mathbf{x}\beta_0 + \epsilon_0 \end{cases}$$

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$$\mathbf{x}\beta_1 = \beta_{1,1}age + \beta_{2,1}over + \beta_{3,1}phealth$$

for the observations at which prog=1

ϵ_0 , and ϵ_1 are normal

$corr(\epsilon_0, \epsilon_1)$ is not identified or estimated


```
. estat teffects
```

```
Predictive margins
```

```
Number of obs      =      3,000
```

```
ATE_Pr0      : Pr(wchange=0=Loss)  
ATE_Pr1      : Pr(wchange=1=No change)  
ATE_Pr2      : Pr(wchange=2=Gain)
```

	Unconditional				[95% Conf. Interval]	
	Margin	Std. Err.	z	P> z		
ATE_Pr0 prog (Yes vs No)	.4374574	.0238647	18.33	0.000	.3906834	.4842314
ATE_Pr1 prog (Yes vs No)	-.1688022	.0244607	-6.90	0.000	-.2167443	-.1208601
ATE_Pr2 prog (Yes vs No)	-.2686552	.0198483	-13.54	0.000	-.3075572	-.2297532

- When everyone joins the program instead of when no one participants in the program,

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- When everyone joins the program instead of when no one participants in the program,
 - On average, the probability of “Loss” goes up by .44

```
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Predictive margins
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```
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```

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	Unconditional				[95% Conf. Interval]	
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- When everyone joins the program instead of when no one participants in the program,
 - On average, the probability of “Loss” goes up by .44
 - On average, the probability of “No change” goes down by .17

```
. estat teffects
```

```
Predictive margins
```

```
Number of obs = 3,000
```

```
ATE_Pr0      : Pr(wchange=0=Loss)  
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```

	Unconditional Margin	Std. Err.	z	P> z	[95% Conf. Interval]	
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- When everyone joins the program instead of when no one participants in the program,
 - On average, the probability of “Loss” goes up by .44
 - On average, the probability of “No change” goes down by .17
 - On average, the probability of “Gain” goes down .27

None lost to follow up

- Some observations on `wchange` are missing
- No observations on covariates are missing
- Can do predictions for all cases

ATE: How (2)

```
. estimates restore oprobit
(results oprobit are active now)

. margins prog,          ///
>   predict(outlevel("Loss"))    ///
>   predict(outlevel("No change"))  ///
>   predict(outlevel("Gain")) noesample

Predictive margins                    Number of obs      =      3,000
Model VCE      : OIM

1._predict    : Pr(wchange==Loss), predict(outlevel("Loss"))
2._predict    : Pr(wchange==No change), predict(outlevel("No change"))
3._predict    : Pr(wchange==Gain), predict(outlevel("Gain"))
```

	Delta-method				
	Margin	Std. Err.	z	P> z	[95% Conf. Interval]
__predict#prog					
1#No	.2721432	.0191116	14.24	0.000	.2346853 .3096012
1#Yes	.7096007	.0142655	49.74	0.000	.6816407 .7375606
2#No	.4260522	.0203869	20.90	0.000	.3860947 .4660097
2#Yes	.25725	.0133175	19.32	0.000	.2311483 .2833518
3#No	.3018046	.0191367	15.77	0.000	.2642973 .3393118
3#Yes	.0331493	.0055184	6.01	0.000	.0223334 .0439652

ATE: How (3)

```
. margins r.prog,          ///
>   predict(outlevel("Loss"))    ///
>   predict(outlevel("No change"))  ///
>   predict(outlevel("Gain"))    ///
>   contrast(nowald)            ///
>   noesample

Contrasts of predictive margins
Model VCE      : OIM
1._predict    : Pr(wchange==Loss), predict(outlevel("Loss"))
2._predict    : Pr(wchange==No change), predict(outlevel("No change"))
3._predict    : Pr(wchange==Gain), predict(outlevel("Gain"))
```

	Delta-method			
	Contrast	Std. Err.	[95% Conf. Interval]	
prog@_predict				
(Yes vs No) 1	.4374574	.0238486	.390715	.4841999
(Yes vs No) 2	-.1688022	.0243512	-.2165296	-.1210748
(Yes vs No) 3	-.2686552	.0199165	-.3076908	-.2296196

Endogenous Treatment model

The potential-outcome model for an endogenous treatment

- Allows the coefficients to differ for the treated and not-treated state
- Allows the cut offs to differ for the treated and not-treated state
- Allows for distinct (nonzero) correlations between the errors driving treatment assignment and the errors driving the ordinal outcomes for the treated and not-treated states

$$prog = (\mathbf{x}\gamma + \gamma_1 w_{tprog} + \eta > 0)$$

$$prog = (\mathbf{x}\gamma + \gamma_1 wtprog + \eta > 0)$$

$$wchange = \begin{cases} \text{"Loss"} & \text{if } \mathbf{x}\beta_0 + \epsilon_0 \leq cut1_0 \\ \text{"No change"} & \text{if } cut1_0 < \mathbf{x}\beta_0 + \epsilon_0 \leq cut2_0 \\ \text{"Gain"} & \text{if } cut2_0 < \mathbf{x}\beta_0 + \epsilon_0 \end{cases}$$

$$\mathbf{x}\beta_0 = \beta_{1,0}age + \beta_{2,0}over + \beta_{3,0}phealth$$

for the observations at which $prog=0$, and

$$prog = (\mathbf{x}\gamma + \gamma_1 wtprog + \eta > 0)$$

$$wchange = \begin{cases} \text{"Loss"} & \text{if } \mathbf{x}\beta_0 + \epsilon_0 \leq cut1_0 \\ \text{"No change"} & \text{if } cut1_0 < \mathbf{x}\beta_0 + \epsilon_0 \leq cut2_0 \\ \text{"Gain"} & \text{if } cut2_0 < \mathbf{x}\beta_0 + \epsilon_0 \end{cases}$$

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$$wchange = \begin{cases} \text{"Loss"} & \text{if } \mathbf{x}\beta_1 + \epsilon_1 \leq cut1_1 \\ \text{"No change"} & \text{if } cut1_1 < \mathbf{x}\beta_1 + \epsilon_1 \leq cut2_1 \\ \text{"Gain"} & \text{if } cut2_1 < \mathbf{x}\beta_1 + \epsilon_1 \end{cases}$$

$$\mathbf{x}\beta_1 = \beta_{1,1}age + \beta_{2,1}over + \beta_{3,1}phealth$$

for the observations at which $prog=1$

$$prog = (\mathbf{x}\gamma + \gamma_1 wtprog + \eta > 0)$$

$$wchange = \begin{cases} \text{"Loss"} & \text{if } \mathbf{x}\beta_0 + \epsilon_0 \leq cut1_0 \\ \text{"No change"} & \text{if } cut1_0 < \mathbf{x}\beta_0 + \epsilon_0 \leq cut2_0 \\ \text{"Gain"} & \text{if } cut2_0 < \mathbf{x}\beta_0 + \epsilon_0 \end{cases}$$

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for the observations at which $prog=1$

ϵ_0 , ϵ_1 , and η are correlated and joint normal

ρ_0 correlation between ϵ_0 and η

ρ_1 correlation between ϵ_1 and η

Endogenous treatment model

```

. eoprobit wchange age over phealth , ///
>     entreat(prog = age over phealth wtprog, pocorr ) ///
>     vce(robust) vsquish nolog
Extended ordered probit regression           Number of obs   =       1,884
                                              Wald chi2(6)     =       137.27
Log pseudolikelihood = -2335.2213          Prob > chi2      =       0.0000
    
```

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
wchange						
prog#c.age						
No	.4919782	.1357859	3.62	0.000	.2258427	.7581137
Yes	-.1111304	.1183412	-0.94	0.348	-.3430749	.1208142
prog#c.over						
No	.4659558	.0789709	5.90	0.000	.3111757	.6207359
Yes	.0458895	.0794788	0.58	0.564	-.109886	.2016651
prog# c.phealth						
No	-.3162974	.0872579	-3.62	0.000	-.4873198	-.145275
Yes	-.6880971	.0713535	-9.64	0.000	-.8279474	-.5482467
prog						
age	-.9224146	.1057226	-8.72	0.000	-1.129627	-.7152021
over	-.9957274	.0675412	-14.74	0.000	-1.128106	-.863349
phealth	.7483889	.0604543	12.38	0.000	.6299007	.8668771
wtprog	1.718043	.1160706	14.80	0.000	1.490549	1.945537

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
wchange						
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phealth	.7483889	.0604543	12.38	0.000	.6299007	.8668771
wtprog	1.718043	.1160706	14.80	0.000	1.490549	1.945537
_cons	.3398047	.0690413	4.92	0.000	.2044863	.475123
/wchange						
prog#c.cut1						
No	.1953761	.1544741			-.1073875	.4981397
Yes	-.133868	.0985578			-.3270377	.0593017
prog#c.cut2						
No	1.193014	.111908			.9736779	1.412349
Yes	1.170747	.1289195			.9180695	1.423425
corr(e. prog, wchange)						
prog						

	Yes	-.1111304	.1183412	-0.94	0.348	-.3430749	.1208142
prog#c.over	No	.4659558	.0789709	5.90	0.000	.3111757	.6207359
	Yes	.0458895	.0794788	0.58	0.564	-.109886	.2016651
prog#c.phealth	No	-.3162974	.0872579	-3.62	0.000	-.4873198	-.145275
	Yes	-.6880971	.0713535	-9.64	0.000	-.8279474	-.5482467
prog	age	-.9224146	.1057226	-8.72	0.000	-1.129627	-.7152021
	over	-.9957274	.0675412	-14.74	0.000	-1.128106	-.863349
	phealth	.7483889	.0604543	12.38	0.000	.6299007	.8668771
	wtprog	1.718043	.1160706	14.80	0.000	1.490549	1.945537
	_cons	.3398047	.0690413	4.92	0.000	.2044863	.475123
/wchange	prog#c.cut1						
	No	.1953761	.1544741			-.1073875	.4981397
	Yes	-.133868	.0985578			-.3270377	.0593017
	prog#c.cut2						
	No	1.193014	.111908			.9736779	1.412349
	Yes	1.170747	.1289195			.9180695	1.423425
corr(e.prog, e.wchange)	prog						
	No	-.6325687	.1073524	-5.89	0.000	-.7992197	-.3755982
	Yes	-.4199058	.1042067	-4.03	0.000	-.6015292	-.1970056

```
. estat teffects
```

```
Predictive margins
```

```
Number of obs = 3,000
```

```
ATE_Pr0 : Pr(wchange=0=Loss)  
ATE_Pr1 : Pr(wchange=1=No change)  
ATE_Pr2 : Pr(wchange=2=Gain)
```

	Unconditional Margin	Std. Err.	z	P> z	[95% Conf. Interval]	
ATE_Pr0 prog (Yes vs No)	.1082033	.0606482	1.78	0.074	-.0106649	.2270715
ATE_Pr1 prog (Yes vs No)	-.0066579	.0439074	-0.15	0.879	-.0927147	.079399
ATE_Pr2 prog (Yes vs No)	-.1015455	.0233349	-4.35	0.000	-.147281	-.0558099

- When everyone joins the program instead of when no one participants in the program,

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Predictive margins
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```
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ATE_Pr0      : Pr(wchange=0=Loss)  
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	Unconditional Margin	Std. Err.	z	P> z	[95% Conf. Interval]	
ATE_Pr0 prog (Yes vs No)	.1082033	.0606482	1.78	0.074	-.0106649	.2270715
ATE_Pr1 prog (Yes vs No)	-.0066579	.0439074	-0.15	0.879	-.0927147	.079399
ATE_Pr2 prog (Yes vs No)	-.1015455	.0233349	-4.35	0.000	-.147281	-.0558099

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```

. margins r.prog,          ///
>   predict(fix(prog) outlevel("Loss"))    ///
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```

Contrasts of predictive margins

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1._predict : Pr(wchange==Loss), predict(fix(prog) outlevel("Loss"))
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```

	Unconditional			
	Contrast	Std. Err.	[95% Conf. Interval]	
prog@_predict				
(Yes vs No) 1	.1082033	.0606482	-.0106649	.2270715
(Yes vs No) 2	-.0066579	.0439074	-.0927147	.0793999
(Yes vs No) 3	-.1015455	.0233349	-.147281	-.0558099

- `fix(prog)` gets us the effect of the program that is not contaminated by the selection effect/correlation between ϵ and η that increases the participation among people more likely to lose weight

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- `fix(prog)` causes the value of `prog` not to affect ϵ , even though they are correlated
 - `fix(prog)` specifies that the part of ϵ that is correlated with `prog` be integrated out

- This type of prediction is sometimes called the structural prediction or an average structural function; see Blundell and Powell (2003), Blundell and Powell (2004), Wooldridge (2005), Wooldridge (2010), and Wooldridge (2014),
- The difference between the mean of the average of the structural predictions when $prog=1$ and the mean of the average of the structural predictions when $prog=0$ is an average treatment effect (Blundell and Powell (2003) and Wooldridge (2014))

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- Reconsider our fictional weight-loss program

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 - are independent of the unobservables that affect program participation
 - and they are independent of the unobservables that affect the outcomes with and without the program,
 - the previously discussed estimator consistently estimates the effects
- Any dependence among the unobservables must be modeled

$$insamp = (\mathbf{x}\alpha + \alpha_1 wtsamp + \xi > 0)$$

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$$prog = (\mathbf{x}\gamma + \gamma_1 wtprog + \eta > 0)$$

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$$wchange = \begin{cases} \text{"Loss"} & \text{if } \mathbf{x}\beta_0 + \epsilon_0 \leq cut1_0 \\ \text{"No change"} & \text{if } cut1_0 < \mathbf{x}\beta_0 + \epsilon_0 \leq cut2_0 \\ \text{"Gain"} & \text{if } cut2_0 < \mathbf{x}\beta_0 + \epsilon_0 \end{cases}$$

$$\mathbf{x}\beta_0 = \beta_{1,0}age + \beta_{2,0}over + \beta_{3,0}phealth$$

for the observations at which $prog=0$, and

$$insamp = (\mathbf{x}\alpha + \alpha_1 wtsamp + \xi > 0)$$

$$prog = (\mathbf{x}\gamma + \gamma_1 wtprog + \eta > 0)$$

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for the observations at which prog=0, and

$$wchange = \begin{cases} \text{"Loss"} & \text{if } \mathbf{x}\beta_1 + \epsilon_1 \leq cut1_1 \\ \text{"No change"} & \text{if } cut1_1 < \mathbf{x}\beta_1 + \epsilon_1 \leq cut2_1 \\ \text{"Gain"} & \text{if } cut2_1 < \mathbf{x}\beta_1 + \epsilon_1 \end{cases}$$

$$\mathbf{x}\beta_1 = \beta_{1,1}age + \beta_{2,1}over + \beta_{3,1}phealth$$

for the observations at which prog=1

$$insamp = (\mathbf{x}\alpha + \alpha_1 wtsamp + \xi > 0)$$

$$prog = (\mathbf{x}\gamma + \gamma_1 wtprog + \eta > 0)$$

$$wchange = \begin{cases} \text{"Loss"} & \text{if } \mathbf{x}\beta_0 + \epsilon_0 \leq cut1_0 \\ \text{"No change"} & \text{if } cut1_0 < \mathbf{x}\beta_0 + \epsilon_0 \leq cut2_0 \\ \text{"Gain"} & \text{if } cut2_0 < \mathbf{x}\beta_0 + \epsilon_0 \end{cases}$$

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$$\mathbf{x}\beta_1 = \beta_{1,1}age + \beta_{2,1}over + \beta_{3,1}phealth$$

for the observations at which prog=1

ξ , ϵ_0 , ϵ_1 , and η are correlated and joint normal

distinct correlations between each treatment error and others

```

. eoprobit wchange age over phealth , ///
>     entreat(prog = age over phealth wtprog, pocorr ) ///
>     select(insamp = age over phealth wtsamp ) ///
>     vce(robust) vsquish nolog

```

```

Extended ordered probit regression           Number of obs   =       3,000
                                           Selected        =       1,884
                                           Nonselected     =       1,116
                                           Wald chi2(6)    =       163.70
                                           Prob > chi2     =       0.0000
Log pseudolikelihood = -4483.9683

```

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
wchange						
prog#c.age						
No	.4174575	.1335097	3.13	0.002	.1557832	.6791318
Yes	-.0779536	.1120819	-0.70	0.487	-.2976301	.141723
prog#c.over						
No	.5046857	.0836683	6.03	0.000	.3406989	.6686725
Yes	.1930521	.0973183	1.98	0.047	.0023118	.3837924
prog#						
c.phealth						
No	-.4250361	.091857	-4.63	0.000	-.6050726	-.2449996
Yes	-.8098627	.0753678	-10.75	0.000	-.9575809	-.6621444
insamp						
age	-.0231005	.0805424	-0.29	0.774	-.1809607	.1347597
over	-.7639994	.0450909	-16.94	0.000	-.852376	-.6756229
phealth	.7765721	.0467569	16.61	0.000	.6849303	.8682139
wtsamp	2.611108	.2660121	9.82	0.000	2.089734	3.132483
cons	.2832551	.0516926	5.48	0.000	.1819395	.3845707

c.phealth							
	No	-.4250361	.091857	-4.63	0.000	-.6050726	-.2449996
	Yes	-.8098627	.0753678	-10.75	0.000	-.9575809	-.6621444
insamp							
	age	-.0231005	.0805424	-0.29	0.774	-.1809607	.1347597
	over	-.7639994	.0450909	-16.94	0.000	-.852376	-.6756229
	phealth	.7765721	.0467569	16.61	0.000	.6849303	.8682139
	wtsamp	2.611108	.2660121	9.82	0.000	2.089734	3.132483
	_cons	.2832551	.0516926	5.48	0.000	.1819395	.3845707
prog							
	age	-.9371024	.0818803	-11.44	0.000	-1.097585	-.7766199
	over	-1.060975	.0492229	-21.55	0.000	-1.15745	-.9645
	phealth	.890558	.0494954	17.99	0.000	.7935487	.9875673
	wtprog	1.644504	.0731516	22.48	0.000	1.501129	1.787878
	_cons	.0153225	.0527572	0.29	0.771	-.0880796	.1187247
/wchange							
	prog#c.cut1						
	No	-.2754667	.1708586			-.6103433	.05941
	Yes	-.4323606	.1401249			-.7070003	-.1577208
	prog#c.cut2						
	No	.6797857	.1534354			.3790578	.9805137
	Yes	.7803365	.2260056			.3373737	1.223299
corr(e.ins~p, e.wchange)							
	prog						
	No	-.5779184	.1004465	-5.75	0.000	-.7420068	-.3484981
	Yes	-.5355424	.1948537	-2.75	0.006	-.81217	-.0623165
corr(e.prog, e.wchange)							

prog#c.cut1						
No	-.2754667	.1708586			-.6103433	.05941
Yes	-.4323606	.1401249			-.7070003	-.1577208
prog#c.cut2						
No	.6797857	.1534354			.3790578	.9805137
Yes	.7803365	.2260056			.3373737	1.223299
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No	-.5779184	.1004465	-5.75	0.000	-.7420068	-.3484981
Yes	-.5355424	.1948537	-2.75	0.006	-.81217	-.0623165
corr(e.prog, e.wchange)						
prog						
No	-.6031412	.1119322	-5.39	0.000	-.7790275	-.3392526
Yes	-.4940044	.0934446	-5.29	0.000	-.6547774	-.2904625
corr(e.prog, e.insamp)						
	.4745668	.0298397	15.90	0.000	.4140283	.5309257

- Nonzero correlations between e.insamp and e.wchange imply endogenous sample selection for outcomes
- Nonzero correlations between e.prog and e.wchange imply endogenous treatment assignment

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Predictive margins
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```
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	Unconditional Margin	Std. Err.	z	P> z	[95% Conf. Interval]	
ATE_Pr0 prog (Yes vs No)	.1406344	.0785061	1.79	0.073	-.0132346	.2945035
ATE_Pr1 prog (Yes vs No)	.0210902	.0369635	0.57	0.568	-.0513569	.0935372
ATE_Pr2 prog (Yes vs No)	-.1617246	.0642328	-2.52	0.012	-.2876187	-.0358305

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```

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Contrasts of predictive margins

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More about ERM commands

- The commands `eregress`, `eprobit`, and `eintreg` fit ERMs handle continuous-and-unbounded, binary, and censored/corner outcomes
- Look at

<http://www.stata.com/manuals/erm.pdf>

for more examples and a wealth of details

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