xsmle - A Command to Estimate Spatial Panel Data Models in Stata

Federico Belotti[⋆],
Gordon Hughes[∘], Andrea Piano Mortari[⋆]

[⋆] CEIS, University of Rome Tor Vergata
[∘] School of Economics, University of Edinburg.

2013 Italian Stata Users Group Meeting
Florence, November 14 2013

Outline

1. Motivation

2. The `xsmle` command
   - Options common to all spatial models
   - The weighting matrix
   - Models' specific options
   - `xsmle` postestimation command

3. Examples
   - Estimation
   - Postestimation
   - Testing

A general specification for Spatial Panel models:

$$
\begin{aligned}
y_{it} &= \alpha + \tau y_{it-1} + \rho \sum_{j=1}^{n} w_{ij} y_{jt} + \sum_{k=1}^{K} x_{itk} \beta_{\mathbf{k}} + \sum_{k=1}^{K} \sum_{j=1}^{n} w_{ij} x_{jtk} \theta_{\mathbf{k}} + \\
&+ \mu_i + \gamma_t + \nu_{it} \qquad (1) \\
\nu_{it} &= \lambda \sum_{j=1}^{n} m_{ij} \nu_{it} + \epsilon_{it} \qquad i = 1, ..., n \qquad t = 1, ..., T \qquad (2)
\end{aligned}
$$

Static Models ($\tau = 0$) and Dynamic Models ($\tau \neq 0$, Yu et al. (2008))

- if $\theta = 0$ and $\rightarrow$ Spatial Autoregressive Model with Auto Regressive disturbances (SAC)
- if $\lambda = 0 \rightarrow$ Spatial Durbin Model (SDM)
- if $\lambda = 0$ and $\theta = 0 \rightarrow$ Spatial Autoregressive Model (SAR)
- if $\rho = 0$ and $\theta = 0 \rightarrow$ Spatial Error Model (SEM)
- if $\rho = 0$, $\theta = 0$, and $\mu_i = \phi \sum_{j=1}^{n} w_{ij} \mu_i + \eta_i \rightarrow$ Generalised Spatial Panel Random Effects model (GSPRE)

A number of spatial-related routines have been written by users and available through SSC. A non-comprehensive list includes:

Data management and visualization

- shp2dta by K. Crow
- spmat by D.M. Drukker *et al*
- spwmatrix by P.W. Jeanty
- spmap by M. Pisati
- geocode3 by S. Bernhard

Cross sectional data

- spreg: SAR, SEM, SAC *via* ML or GS2SLS by D. M. Drukker *et al*
- spivreg: SAC *via* GS2SLS by D. M. Drukker *et al*
- spmlreg: SAR, SEM, SDM, SAC *via* ML by P.W. Jeanty
- spatreg: SAR, SEM *via* ML by M. Pisati
- spautoreg: SAR, SEM, SDM, SAC *via* ML or GS2SLS by E.A. Shehata

Panel data

- spreg*xt suite SAR, SEM, SDM, SAC *via* LS, GLS, GMM or GS2SLS by E.A. Shehata

DGP - 250 replications

$$y_{it} = \rho \sum_{j=1}^{n} w_{ij} y_{jt} + 0.3 x_{1it} + 0.7 x_{2it} + \mu_i + \gamma_t + \epsilon_{it} \quad n = 1, \ldots, 188 \quad t = 1, \ldots, 5$$

where the nuisance parameters μ_i $(i = 1, \ldots, n)$ are drawn from an iid standard Gaussian random variable. To allow for dependence between the unit-specific effects and the regressors, we generate the latter as follows

$$x_{kit} = 0.4\mu_i + (1 - 0.4^2)^{1/2} z_{kit},$$

where $k = 1, 2$ and the z_{kit} is an iid standard Gaussian random variable.

	$\rho = 0.3$		$\rho = 0.5$		$\rho = 0.7$	
	bias	MSE	bias	MSE	bias	MSE
xsmle	-0.0013	0.0020	-0.0016	0.0014	-0.0016	0.0007
spregfext	0.1473	0.0255	0.1972	0.0408	0.1859	0.0352
xtivreg2	0.0174	0.0091	0.0153	0.0063	0.0112	0.0033

xsmle fits (balanced) Spatial Panel data models *via* maximum likelihood (ML)

Requirements:

- (At least) Stata Version 10
- The $n \times n$ matrix of spatial weights. xsmle will deal with the longitudinal dimension automatically
- Data must be tsset or xtset

The basic xsmle syntax is the following

xsmle *depvar* [*indepvars*] [*if*] [*in*] [*weight*] [, options]

- The default model is the random-effects SAR model
- Only aweight are allowed but the declared weight variable must be constant within each panel unit
- The mi prefix is allowed
- Factor variables are allowed

Options common to all spatial models

- model(*name*) specifies the spatial model to be estimated. May be *sar* for the Spatial-AutoRegressive model, *sdm* for the Spatial Durbin Model, *sem* for the Spatial-Error Model, *sac* for the Spatial-Autoregressive with Spatially Autocorrelated Errors Model, *gspre* for the Generalised Spatial Random Effects Model.
- re use the random effects estimator; the default. This option cannot be specified when model(*sac*).
- fe use the fixed effects estimator. This option cannot be specified when model(*gspre*).
- type(*type_options* [, leeyu]) specifies fixed-effects type; only for fe estimators. May be ind for spatial fixed effects, time for time fixed effects or both for both spatial and time fixed effects. Suboption leeyu allows to transform the data according to Lee and Yu (2010) approach and can be used only when type(ind).

- noconstant suppresses the constant term in the model. Only for re estimators.
- noeffects suppresses the computation of direct, indirect and total effects.
- nsim(#) sets the number of simulations for the LeSage and Pace (2009) procedure to compute the standard errors of the direct, indirect and total effects.
- constraints(*constraints_list*) applies specified linear constraints.
- from(*init_specs*) specifies initial values for the coefficients.
- level(#) sets confidence level for confidence intervals; default is level(95).
- postscore save observation-by-observation scores in the estimation results list.
- posthessian save the Hessian corresponding to the full set of coefficients in the estimation results list.
- hausman performs the Hausman test.

Variance estimation

This section describes the arguments of the `vce(`*vcetype*`)` option.

- *oim* observed information matrix.
- *opg* outer product of the gradient vectors.
- *robust* clustered sandwich estimator where *clustvar* is the *panelvar*.
- *cluster clustvar* clustered sandwich estimator.
- *dkraay(#)* Driscoll-Kraay robust estimator. Where # is the maximum lag used in the calculation.

In `xsmle` the spatial weighting matrix can be

- a Stata matrix
- a `spmat` object

In both cases the matrix can be standardized or not.

e.g.

- a Stata matrix can be created using `matrix` define, imported from Mata using st_matrix("*string scalar name*", *real matrix*) or imported from GIS softwares like GeoDa using

  spwmatrix gal using *path_to_gal_file*, wname(*name_of_the_matrix*)

- `spmat objects` are created by `spmat`

  spmat import *name_of_the_object* using *path_to_file*

SAR model

- `wmatrix`(*name*) specifies the weight matrix for the spatial-autoregressive term.
- `dlag` includes (time) lagged dependent variable in the model.

SDM model

- `wmatrix`(*name*) specifies the weight matrix for the spatial-autoregressive term.
- `dmatrix`(*name*) specifies the weight matrix for the spatially lagged regressors; default is to use the matrix specified in `wmat(`*name*`)`.
- `durbin`(*dvarlist*) specifies the regressors that have to be spatially lagged; default is to lag all independent variables specified in *varlist*.
- `dlag` includes (time) lagged dependent variable in the model.

SEM model

- `ematrix(name)` specifies the weight matrix for the spatial-autocorrelated error term.

SAC model

- `wmatrix(name)` specifies the weight matrix for the spatial-autoregressive term.
- `ematrix(name)` specifies the weight matrix for the spatial-autocorrelated error term.

GSPRE model

- `wmatrix(name)` specifies the weight matrix for the spatial-autocorrelated random-effects.
- `ematrix(name)` specifies the weight matrix for the spatial-autocorrelated error term.
- `error(#)` defines the structure of the model. # is equal to 1 when $\lambda \neq \phi \neq 0$, # is equal to 2 when $\lambda = 0$, # is equal to 3 when $\phi = 0$, # is equal to 4 when $\lambda = \phi$.

Postestimation command allows to post-estimate spatial fixed or random effects. The methods implemented in this command are the panel data extensions of those available in Drukker, Prucha, and Raciborski (2011)

`predict` [*type*] *newvar* [*if*] [*in*] [, `statistic`]

where `statistic` includes:

- `rform` the default, calculates predicted values from the reduced-form equation: $y_{it} = (I_n - \rho W)^{-1}(x_{it}\beta + \alpha_i)$
- `limited` predicted values based on the limited information set. This option is available only when `model(sac)`.
- `naive` predicted values based on the observed values of $y_{it} = \rho W y_{it} + x_{it}\beta + \alpha_i$
- `xb` calculates the linear prediction including the fixed or random effect $x_{it}\beta + \alpha_i$.
- `a` estimates α_i, the fixed or random-effect. In the case of fixed-effects models, this statistic is allowed only when `type(`*ind*`)`

DGP - Fixed effects SDM

$$
\begin{aligned}
y_{it} &= 0.3\sum_{j=1}^{n} w_{ij} y_{jt} + 0.5x_{1it} - 0.3x_{2it} - 0.2x_{3it} + 0.3\sum_{j=1}^{n} w_{ij} x_{1it} + \\
+ &\quad 0.6\sum_{j=1}^{n} w_{ij} x_{2it} + 0.9\sum_{j=1}^{n} w_{ij} x_{3it} + \mu_i + \gamma_t + \epsilon_{it} \quad n = 1, ..., 188 \;\; t = 1, ..., 5
\end{aligned}
$$

where the nuisance parameters μ_i $(i = 1, \ldots, n)$ are drawn from an iid standard Gaussian random variable. To allow for dependence between the unit-specific effects and the regressors, we generate the latter as follows

$$
x_{kit} = 0.4\mu_i + (1 - 0.4^2)^{1/2} z_{kit},
$$

where $k = 1, 2, 3$, z_{1it} is standard Gaussian, z_{2it} is $N(0, 1.5^2)$ and z_{3it} is $N(0, 2^2)$.

```
.. *** load a dta dataset containing the spatial contiguity matrix
. use ASL_contiguity_mat_ns.dta, clear

. *** get an spmat objects from dta
. spmat dta W W*, replace

. *** Summarize the spmat obj
. spmat summarize W, links

Summary of spatial-weighting object W
--------------------------------------
        Matrix |          Description
---------------+----------------------
    Dimensions |            188 x 188
     Stored as |            188 x 188
 Links         |
         total |                  906
           min |                    1
          mean |             4.819149
           max |                   13
--------------------------------------
```

```
. ** Fixed-effects Durbin model (correctly specified, row normalized W)
. xsmle y x1 x2 x3, wmat(W) model(sdm) fe type(ind) nsim(500) nolog
Warning: All regressors will be spatially lagged


SDM with spatial fixed-effects                    Number of obs =        940

Group variable: id                              Number of groups =        188
Time variable: t                                    Panel length =          5

R-sq:    within  = 0.5727
         between = 0.3663
         overall = 0.4554

Mean of fixed-effects = -0.0137

Log-likelihood = -1230.7734
------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Main         |
          x1 |   .5186041   .0364303    14.24   0.000     .4472019    .5900062
          x2 |  -.2946314   .0236541   -12.46   0.000    -.3409925   -.2482702
          x3 |  -.1923373   .0192912    -9.97   0.000    -.2301474   -.1545272
-------------+----------------------------------------------------------------
Wx           |
          x1 |   .3772047    .075502     5.00   0.000     .2292235    .5251859
          x2 |   .5765484   .0449332    12.83   0.000     .4884809    .6646159
          x3 |   .8692021   .0372769    23.32   0.000     .7961408    .9422634
-------------+----------------------------------------------------------------
Spatial      |
         rho |   .2519025   .0374278     6.73   0.000     .1785454    .3252596
-------------+----------------------------------------------------------------
Variance     |
    sigma2_e |   .7915998   .0366863    21.58   0.000     .7196959    .8635037
-------------+----------------------------------------------------------------
[CONTINUES]
```

In a spatial setting, the effect of an explanatory variable change in a particular unit affects not only that unit but also its neighbors (LeSage and Pace, 2009).

$$\left[\frac{\partial Y}{\partial x_{nk}}\right] = (I-\rho W)^{-1}\begin{bmatrix} \beta_k & w_{12}\theta_k & \cdot & w_{1n}\theta_k \\ w_{21}\theta_k & \beta_k & \cdot & w_{2n}\theta_k \\ \cdot & \cdot & \cdot & \cdot \\ w_{n1}\theta_k & w_{n2}\theta_k & \cdot & \beta_k \end{bmatrix}$$

If we have only 2 units and 1 regressor:

- SAR and SAC $\rightarrow (I-\rho W)^{-1}\begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_1 \end{bmatrix}$
- SEM $\rightarrow \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_1 \end{bmatrix}$
- SDM $\rightarrow (I-\rho W)^{-1}\begin{bmatrix} \beta_1 & w_{12}\theta_1 \\ w_{21}\theta_1 & \beta_1 \end{bmatrix}$

```
[CONTINUES]
-------------+----------------------------------------------------------------
Direct       |
          x1 |   .5481382   .0362326    15.13   0.000     .4771237    .6191527
          x2 |  -.2642811   .0231199   -11.43   0.000    -.3095953   -.2189669
          x3 |  -.1422518   .0176968    -8.04   0.000    -.1769369   -.1075668
-------------+----------------------------------------------------------------
Indirect     |
          x1 |   .6480929    .090572     7.16   0.000      .470575    .8256108
          x2 |   .6450951   .0599307    10.76   0.000     .5276331    .7625571
          x3 |   1.050599    .058257    18.03   0.000     .9364176    1.164781
-------------+----------------------------------------------------------------
Total        |
          x1 |   1.196231   .1038425    11.52   0.000     .9927034    1.399759
          x2 |    .380814   .0677252     5.62   0.000     .2480751     .513553
          x3 |   .9083474   .0660288    13.76   0.000     .7789334    1.037761
------------------------------------------------------------------------------
. estimates store sdm_fe
```

```
 ** Fixed-effects Durbin model (correctly specified, row normalized W)
 . xsmle y x1 x2 x3, wmat(W) model(sdm) re type(ind) nsim(500) nolog noeff
Warning: Option type(ind) will be ignored
Warning: All regressors will be spatially lagged


SDM with random-effects                               Number of obs =       940

Group variable: id                                 Number of groups =       188
Time variable: t                                       Panel length =         5

R-sq:    within  = 0.5666
         between = 0.4543
         overall = 0.4936

Log-likelihood = -1513.7006
------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Main         |
          x1 |   .6230976   .0408605    15.25   0.000     .5430126    .7031826
          x2 |  -.2439834   .0264129    -9.24   0.000    -.2957518    -.192215
          x3 |  -.1688081   .0211584    -7.98   0.000    -.2102778   -.1273385
       _cons |  -.0169191   .0811545    -0.21   0.835    -.1759791    .1421409
-------------+----------------------------------------------------------------
Wx           |
          x1 |   .3706183   .0824133     4.50   0.000     .2090911    .5321454
          x2 |    .557779   .0493092    11.31   0.000     .4611347    .6544234
          x3 |   .8845199   .0411496    21.50   0.000     .8038681    .9651717
-------------+----------------------------------------------------------------
Spatial      |
         rho |   .2472432   .0376366     6.57   0.000     .1734769    .3210096
-------------+----------------------------------------------------------------
Variance     |
   lgt_theta |  -.3920581   .1040247    -3.77   0.000    -.5959428   -.1881735
     sigma_e |   1.005536   .0528831    19.01   0.000     .9018867    1.109185
------------------------------------------------------------------------------

. estimates store sdm_re
```

```
hausman sdm_fe sdm_re, eq(1:1 2:2 3:3)

                  ---- Coefficients ----
             |      (b)          (B)            (b-B)     sqrt(diag(V_b-V_B))
             |     sdm_fe       sdm_re        Difference          S.E.
-------------+----------------------------------------------------------------
comp1        |
          x1 |    .5186041     .6230976       -.1044935              .
          x2 |   -.2946314    -.2439834        -.050648              .
          x3 |   -.1923373    -.1688081       -.0235292              .
-------------+----------------------------------------------------------------
comp2        |
          x1 |    .3772047     .3706183        .0065864              .
          x2 |    .5765484      .557779        .0187694              .
          x3 |    .8692021     .8845199       -.0153178              .
-------------+----------------------------------------------------------------
comp3        |
         rho |    .2519025     .2472432        .0046593              .
------------------------------------------------------------------------------
                           b = consistent under Ho and Ha; obtained from xsmle
            B = inconsistent under Ha, efficient under Ho; obtained from xsmle

    Test:  Ho: difference in coefficients not systematic
                 chi2(7) = (b-B)'[(V_b-V_B)^(-1)](b-B)
                         =    -47.08    chi2<0 ==> model fitted on these
                                        data fails to meet the asymptotic
                                        assumptions of the Hausman test;
                                        see suest for a generalized test
```

```
. ** Fixed-effects Durbin model (correctly specified, row normalized W)
. xsmle y x1 x2 x3, wmat(W) model(sdm) fe type(ind) hausman noeff nolog
Warning: All regressors will be spatially lagged
... estimating random-effects model to perform Hausman test
SDM with spatial fixed-effects                       Number of obs =       940

Group variable: id                                Number of groups =       188
Time variable: t                                      Panel length =         5

R-sq:    within  = 0.5727
         between = 0.3663
         overall = 0.4554

Mean of fixed-effects = -0.0137

Log-likelihood = -1230.7734
------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Main         |
          x1 |   .5186041   .0364303    14.24   0.000     .4472019    .5900062
          x2 |  -.2946314   .0236541   -12.46   0.000    -.3409925   -.2482702
          x3 |  -.1923373   .0192912    -9.97   0.000    -.2301474   -.1545272
-------------+----------------------------------------------------------------
Wx           |
          x1 |   .3772047    .075502     5.00   0.000     .2292235    .5251859
          x2 |   .5765484   .0449332    12.83   0.000     .4884809    .6646159
          x3 |   .8692021   .0372769    23.32   0.000     .7961408    .9422634
-------------+----------------------------------------------------------------
Spatial      |
         rho |   .2519025   .0374278     6.73   0.000     .1785454    .3252596
-------------+----------------------------------------------------------------
Variance     |
     sigma2_e |   .7915998   .0366863    21.58   0.000     .7196959    .8635037
------------------------------------------------------------------------------
Ho: difference in coeffs not systematic chi2(7) = 89.58    Prob>=chi2 = 0.0000
------------------------------------------------------------------------------
```

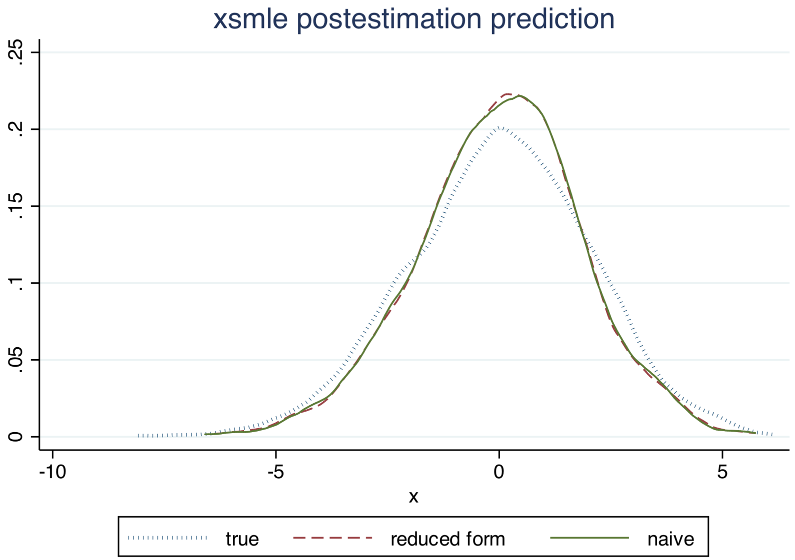
```
. ********* Postestimation
. predict yhat, rform

. predict yhat1, naive

. predict alphahat, a

. sum alpha alphahat

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
       alpha |       940     .037577    1.058726  -2.261747   3.343453
    alphahat |       940    -.013692    1.182919  -2.688471   4.028156
```


xsmle postestimation prediction
.25
.2
.15
.1
.05
0
-10
-5
0
5
x
true
reduced form
naive

Using matrix notation the SDM ($\lambda = 0$) may be derived from a SEM model

$$\left\{ \begin{array}{rcl} \mathbf{y} & = & X\boldsymbol{\beta} + \mathbf{u} \\ \mathbf{u} & = & \lambda W\mathbf{u} + \boldsymbol{\epsilon} \end{array} \right.$$

hence

$$\begin{array}{rcl} \mathbf{u}(1-\lambda W) & = & \boldsymbol{\epsilon} \\ \mathbf{y}(1-\lambda W) & = & X\boldsymbol{\beta}(1-\lambda W) + \boldsymbol{\epsilon} \\ \mathbf{y} & = & \lambda W\mathbf{y} + X\boldsymbol{\beta} - \lambda WX\boldsymbol{\beta} + \boldsymbol{\epsilon} \\ \mathbf{y} & = & \lambda W\mathbf{y} + X\boldsymbol{\beta} + \theta WX + \boldsymbol{\epsilon} \end{array}$$

and test the following constraints

1. $\theta = 0$ and $\lambda \neq 0 \Rightarrow$ the model is a SAR
2. $\theta = -\beta\lambda \Rightarrow$ the model is a SEM.

```
 ** Test for SAR
. test [Wx]x1 = [Wx]x2 = [Wx]x3 = 0

 ( 1)  [Wx]x1 - [Wx]x2 = 0
 ( 2)  [Wx]x1 - [Wx]x3 = 0
 ( 3)  [Wx]x1 = 0

           chi2(  3) =  740.80
         Prob > chi2 =    0.0000

.
. ** Test for SEM
. testnl ([Wx]x1 = -[Spatial]rho*[Main]x1) ([Wx]x2 = -[Spatial]rho*[Main]x2) ([
> Wx]x3 = -[Spatial]rho*[Main]x3)

  (1)  [Wx]x1 = -[Spatial]rho*[Main]x1
  (2)  [Wx]x2 = -[Spatial]rho*[Main]x2
  (3)  [Wx]x3 = -[Spatial]rho*[Main]x3

               chi2(3) =      545.31
           Prob > chi2 =        0.0000
```

```
 ** SAC Model
. xsmle y x1 x2 x3, wmat(W) emat(W) model(sac) fe type(ind) noeff nolog

SAC with spatial fixed-effects                      Number of obs =       940

Group variable: id                               Number of groups =       188
Time variable: t                                     Panel length =         5

R-sq:    within  = 0.2652
         between = 0.0011
         overall = 0.0912

Mean of fixed-effects = -0.0117

Log-likelihood = -1386.0860
------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Main         |
          x1 |   .3212791   .0341734     9.40   0.000     .2543005    .3882577
          x2 |  -.3135993   .0232111   -13.51   0.000    -.3590923   -.2681064
          x3 |  -.2997975   .0178884   -16.76   0.000     -.334858   -.2647369
-------------+----------------------------------------------------------------
Spatial      |
         rho |  -.6676721   .0542468   -12.31   0.000    -.7739939   -.5613504
      lambda |   .8426981   .0209346    40.25   0.000      .801667    .8837293
-------------+----------------------------------------------------------------
Variance     |
    sigma2_e |   1.001782   .0440957    22.72   0.000     .9153562    1.088208
------------------------------------------------------------------------------
```

SDM vs SAC Testing

```
. qui  xsmle y x1 x2 x3, wmat(W) model(sdm) fe type(ind)  noeff nologS
. estat ic

-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |    940           .   -1230.773      8     2477.547    2516.314
-----------------------------------------------------------------------------
               Note:  N=Obs used in calculating BIC; see [R] BIC note

. qui xsmle y x1 x2 x3, wmat(W) emat(W) model(sac) fe type(ind) noeff nolog
. estat ic

-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |    940           .   -1386.086      6     2784.172    2813.247
-----------------------------------------------------------------------------
               Note:  N=Obs used in calculating BIC; see [R] BIC note
```

Lee, L. F. and Yu, J. (2010). Estimation of spatial autoregressive panel data models with fixed effects. *Journal of Econometrics*, 154(2):165–185.

LeSage, J. P. and Pace, R. K. (2009). *Introduction to Spatial Econometrics*. Taylor & Francis.

Yu, J., de Jong, R., and Lee, L. F. (2008). Quasi-maximum likelihood estimators for spatial dynamic panel data with fixed effects when both n and t are large. *Journal of Econometrics*, 146:118–134.