The Assessment of Fit in the Class of Logistic Regression Models: A Pathway out of the Jungle of Pseudo-$R^2$'s Using Stata

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“Models are to be used, but not to be believed.”

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1. What is the problem?

Current situation in applied research:

- An increasing number of people uses logistic models for qualitative dependent variables
- But users often complain about the bad fit of logistic models especially for the multinomial ones
- There is no general agreement on how to assess their fit corresponding to practical significance
- Let me show you the pathway out of the jungle of the pseudo-coefficients of determination
Which solutions does Stata provide?

- Indeed, for binary, ordinal and multinomial logit model Stata calculates only the McFadden Pseudo-$R^2$

- but J. Scott Long & Jeremy Freese have published their fitstat.ado in 2000. It calculates a set of Pseudo-$R^2$s for binary, ordinal, multinomial logit or limited dependent variable models discussed by Long in 1997
2. Summary of the econometric Monte-Carlo studies for testing Pseudo-$R^2$s

- Econometricians made a lot of Monte-Carlo studies in the early 90s:
  - Hagle & Mitchell 1992
  - Windmeijer 1995
  - DeMaris 2002
- They tested systematically the most common Pseudo-$R^2$s for binary and ordinal probit / logit models
Which Pseudo-$R^2$s were tested in these studies?

- Likelihood-based measures:

- Log-Likelihood-based measures:
  - McFadden Pseudo-$R^2$ (1974)
  - Aldrich & Nelson Pseudo-$R^2$ with the Veall & Zimmermann correction (1992)

- Basing on the estimated probabilities:

- Basing on the variance decomposition of the estimated Probits / Logits:
  - McKelvey & Zavoina Pseudo-$R^2$ (1975)
Results of the Monte-Carlo-Studies for binary and ordinal logits or probits

- The McKelvey & Zavoina Pseudo-$R^2$ is the best estimator for the “true $R^2$” of the OLS regression.
- The Aldrich & Nelson Pseudo-$R^2$ with the Veall & Zimmermann correction is the best approximation of the McKelvey & Zavoina Pseudo-$R^2$.
- Lave / Efron, Aldrich & Nelson, McFadden and Cragg & Uhler Pseudo-$R^2$ severely underestimate the “true $R^2$” of the OLS regression.

My personal advice:
- Use the McKelvey & Zavoina Pseudo-$R^2$ to assess the fit of binary and ordinal logit models.
Let’s have a detailed look at the winner

- McKelvey & Zavoina Pseudo-$R^2$ (M&Z Pseudo-$R^2$)

\[
M \& Z \, Pseudo - R^2 = \frac{\text{Var}(\hat{y}^*)}{\text{Var}(\hat{y}^*) + \text{Var}(\varepsilon)} = \frac{\sum_{i=1}^{n} (\hat{y}_{i}^{*} - \overline{\hat{y}}^{*})^2}{\sum_{i=1}^{n} (\hat{y}_{i}^{*} - \overline{\hat{y}}^{*})^2 + \frac{\pi^2}{3}}
\]

Range: \(0 \leq M\&Z\text{-Pseudo-}R^2 \leq 1\)

Legend:
- \(\text{Var}(\hat{y}^*)\): Variance of the estimated logits (latent variable \(Y^*\))
- \(\hat{y}_{i}^{*}\): Estimated logit of case \(i\)
- \(\overline{\hat{y}}^{*}\): Mean of the estimated logits
- \(\frac{\pi^2}{3}\): Variance of logistic density function
3. Generalization of McKelvey & Zavoina Pseudo-$R^2$ to multinomial logit model

- Equations of a multinomial logit model (MNL) for a dependent variable $Y$ with 3 categories
  - Simultaneous estimation of the parameters of two logit equations instead of 2 separate binary logit models

\[
\begin{align*}
\text{(1) } \log \left( \frac{P_{3i}}{P_{1i}} \right) &= \sum_{k=0}^{K} \beta_{31k} X_{ki} \left\{ +\varepsilon_{31i} \right\} \\
\text{(2) } \log \left( \frac{P_{2i}}{P_{1i}} \right) &= \sum_{k=0}^{K} \beta_{21k} X_{ki} \left\{ +\varepsilon_{21i} \right\}
\end{align*}
\]
Conditions of getting unbiased estimates

- Independence of Irrelevant Alternatives (IIA)-Axiom:
  - Comparison of two alternatives is independent of the existence of a third one
  - By using the MNL as a nonlinear probability model the IIA-assumption is fulfilled by the discrete and disjunctive categories of the dependent variable Y

- IID-Axiom formulated by Hensher, Rose & Greene (2005: 77):
  - The error terms $\varepsilon$ are independently and identically distributed
    - Stochastic independence of $\varepsilon_{21}$ and $\varepsilon_{31}$
    - Identical density function of $\varepsilon_{21}$ and $\varepsilon_{31}$
Reasons to apply M&Z-Pseudo-$R^2$ to MNL

- The multinomial logit model (MNL) is...
  - A multi-equation model
  - It has independent error terms $\varepsilon_{21}$ and $\varepsilon_{31}$
  - $\varepsilon_{21}$ and $\varepsilon_{31}$ follow the logistic density function
- Therefore we can calculate the McKelvey & Zavoina Pseudo-$R^2$ separately for each comparison of categories
  - Simultaneous estimation by the multinomial logit model
  - Estimation by $k-1$ separate binary logit models (Begg & Gray 1984)
- Therefore I use the binary McKelvey-Zavoina-Pseudo-$R^2$'s to validate the ones of the MNL
4. Application of the generalized M&Z Pseudo-\(R^2\) in an election study

The Student Election Survey 1998 in Sachsen-Anhalt

- Population
  - 31,000 Students in 150 schools
  - All 5th thru 12th classes in all educational tracks
  - Age 10 thru 18 years

- Sample
  - Representative probability sample of 3,500 students in 22 schools
  - Survey date: 4 days after the general federal election (October 1st, 1998)
Independent variables

- C_AGE in years (centered)
- GENDER: boys vs. girls
- SCHOOL TYPE: GRAMMAR school, VOCATIONAL school vs. secondary school,
- Internal and external political C_EFFICACY (centered)
- Perceived influence of the peers on the vote (PEERS)
- Perceived influence of the parents (PARENTS)
- Perceived influence of the media (MEDIA)
- Perceived influence of the teachers (TEACHERS)
- Countryside vs. city (LOCATION)
Dependent variable

- Voting for party
  - Social Democratic Party (SPD) [0]
  - Christian Democratic Union (CDU) [1]
  - Party of Democratic Socialism / Ex-SED communist party (PDS) [2]
  - Free Demokratic Party / Liberals (FDP) [3]
  - Alliance 90 / the Green (B90) [4]
  - Right-wing extremist parties (DVU, REP, NPD) [5]
Students’ party votes in LSA 1998

- SPD: 46.88%
- CDU: 19.54%
- PDS: 12.57%
- FDP: 6.864%
- B90: 11.09%
- DVU, REP, NDP: 3.062%

Sample size = 1894
### Estimated multinomial logit model for voting

- **Choice of the base outcome category**
  - The comparison of right wing extremist vs. established parties marks the main political conflict line in East-Germany

- **Stata mlogit output formatted with Ben Jann esttab.ado**

<table>
<thead>
<tr>
<th></th>
<th>voting spd</th>
<th>voting cdu</th>
<th>voting pds</th>
<th>voting fdp</th>
<th>voting b90</th>
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<tbody>
<tr>
<td><code>c_age</code></td>
<td>-0.206***</td>
<td>-0.248***</td>
<td>-0.0872</td>
<td>-0.0271</td>
<td>-0.258***</td>
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<td>(-4.34)</td>
<td>(-4.74)</td>
<td>(-1.54)</td>
<td>(-0.31)</td>
<td>(-3.85)</td>
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<td><code>gender</code></td>
<td>-1.275***</td>
<td>-0.765***</td>
<td>-0.893***</td>
<td>-0.756*</td>
<td>-1.275***</td>
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<td></td>
<td>(-6.77)</td>
<td>(-3.68)</td>
<td>(-4.02)</td>
<td>(-2.32)</td>
<td>(-4.94)</td>
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<td><code>grammar</code></td>
<td>0.628</td>
<td>1.498***</td>
<td>1.559***</td>
<td>1.526**</td>
<td>1.710***</td>
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<tr>
<td></td>
<td>(1.82)</td>
<td>(4.02)</td>
<td>(3.92)</td>
<td>(2.75)</td>
<td>(4.02)</td>
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<td><code>vocational</code></td>
<td>0.327</td>
<td>1.083**</td>
<td>0.493</td>
<td>0.0864</td>
<td>-0.0607</td>
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<td></td>
<td>(0.88)</td>
<td>(2.61)</td>
<td>(1.08)</td>
<td>(0.12)</td>
<td>(-0.10)</td>
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<tr>
<td><code>c_efficacy</code></td>
<td>-0.109***</td>
<td>-0.120***</td>
<td>-0.0595</td>
<td>-0.0213</td>
<td>-0.192***</td>
</tr>
<tr>
<td></td>
<td>(-3.69)</td>
<td>(-3.72)</td>
<td>(-1.70)</td>
<td>(-0.40)</td>
<td>(-4.74)</td>
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<tr>
<td><code>peers</code></td>
<td>-0.838***</td>
<td>-0.869***</td>
<td>-0.814***</td>
<td>-0.778***</td>
<td>-0.776***</td>
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<tr>
<td></td>
<td>(-8.68)</td>
<td>(-7.86)</td>
<td>(-6.67)</td>
<td>(-3.99)</td>
<td>(-5.16)</td>
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<tr>
<td><code>parents</code></td>
<td>0.488***</td>
<td>0.514***</td>
<td>0.550***</td>
<td>0.454**</td>
<td>0.324*</td>
</tr>
<tr>
<td></td>
<td>(4.80)</td>
<td>(4.63)</td>
<td>(4.62)</td>
<td>(2.58)</td>
<td>(2.28)</td>
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<tr>
<td><code>media</code></td>
<td>0.219*</td>
<td>0.0731</td>
<td>0.102</td>
<td>-0.0279</td>
<td>-0.0803</td>
</tr>
<tr>
<td></td>
<td>(2.55)</td>
<td>(0.77)</td>
<td>(0.98)</td>
<td>(-0.16)</td>
<td>(-0.65)</td>
</tr>
<tr>
<td><code>teachers</code></td>
<td>0.0324</td>
<td>-0.0397</td>
<td>-0.269</td>
<td>-0.193</td>
<td>-0.0303</td>
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<tr>
<td></td>
<td>(0.30)</td>
<td>(-0.33)</td>
<td>(-1.94)</td>
<td>(-0.88)</td>
<td>(-0.18)</td>
</tr>
<tr>
<td><code>location</code></td>
<td>-0.699**</td>
<td>-0.403</td>
<td>-0.340</td>
<td>-0.468</td>
<td>-1.315***</td>
</tr>
<tr>
<td></td>
<td>(-2.84)</td>
<td>(-1.43)</td>
<td>(-1.08)</td>
<td>(-0.95)</td>
<td>(-3.55)</td>
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<tr>
<td><code>_cons</code></td>
<td>2.450***</td>
<td>1.151**</td>
<td>0.740</td>
<td>-0.448</td>
<td>1.015*</td>
</tr>
<tr>
<td></td>
<td>(7.70)</td>
<td>(3.24)</td>
<td>(1.91)</td>
<td>(-0.78)</td>
<td>(2.37)</td>
</tr>
</tbody>
</table>

- **N**: 1894
- **LR-chi2(50)**: 452.2916
- **Prob**: 0.0000
- **McFadden R2**: 0.0813

- **t statistics in parentheses**
- **Two-tailed tests**: * p<0.05, ** p<0.01, *** p<0.001
- **Reference category of voting**: right-wing extremist parties (DVU, REP, NPD)
Classical fit indices and Pseudo-R²s

Calculated with Long & Freese’s fitstat.ado

.m fitstat

<table>
<thead>
<tr>
<th></th>
<th>mlogit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-likelihood</td>
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<tr>
<td>Model</td>
<td>-2556.642</td>
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<tr>
<td>Intercept-only</td>
<td>-2782.788</td>
</tr>
<tr>
<td>Chi-square</td>
<td></td>
</tr>
<tr>
<td>Deviance (df=1839)</td>
<td>5113.285</td>
</tr>
<tr>
<td>LR (df=50)</td>
<td>452.292</td>
</tr>
<tr>
<td>p-value</td>
<td>0.000</td>
</tr>
<tr>
<td>R²</td>
<td></td>
</tr>
<tr>
<td>McFadden</td>
<td>0.081</td>
</tr>
<tr>
<td>McFadden (adjusted)</td>
<td>0.062</td>
</tr>
<tr>
<td>Cox-Snell/ML</td>
<td>0.212</td>
</tr>
<tr>
<td>Cragg-Uhler/Nagelkerke</td>
<td>0.224</td>
</tr>
<tr>
<td>Count</td>
<td>0.494</td>
</tr>
<tr>
<td>Count (adjusted)</td>
<td>0.048</td>
</tr>
<tr>
<td>IC</td>
<td></td>
</tr>
<tr>
<td>AIC</td>
<td>5223.285</td>
</tr>
<tr>
<td>AIC divided by N</td>
<td>2.758</td>
</tr>
<tr>
<td>BIC (df=55)</td>
<td>5528.339</td>
</tr>
</tbody>
</table>

McKelvey & Zavoina
Pseudo-R² for each of k-1 comparisons of Y using my mnl_mrz2.ado

.m mnl_mrz2
Separate McKelvey Zavoina pseudo R² for mlogit equations

<table>
<thead>
<tr>
<th>Equation</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>spd</td>
<td>0.3501</td>
</tr>
<tr>
<td>cdu</td>
<td>0.3607</td>
</tr>
<tr>
<td>pds</td>
<td>0.3540</td>
</tr>
<tr>
<td>fdp</td>
<td>0.3322</td>
</tr>
<tr>
<td>b90</td>
<td>0.4978</td>
</tr>
<tr>
<td>dvu,rep,~d</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Indicating quite a good fit for the comparison of each established party with the right-wing extremist ones. Explained variance of the estimated logits lies between 33% and 50%.

This table presents the best fit of all possible base outcome categories of voting!
Are the M&Z Pseudo-$R^2$s nearly equal?

Validation by comparison of the overall fit of the multinominal and binary logit models
Validation by comparison of the global McKelvey & Zavoina Pseudo-$R^2$'s using linear regression.

\[
\text{mnlogit} = 0.0021 + 0.9117 \times \text{bilogit}
\]

$R^2 = 0.9776; \ r_{yx} = +0.9887$
Validation by comparison of the partial McKelvey & Zavoina Pseudo-$R^2$s using linear regression

$\text{mnlogit} = -0.0017 + 0.9535 \times \text{bilogit}$

$R^2 = 0.9536; r_{yx} = +0.9765$
5. Conclusions

- **Known**
  - The Monte-Carlo-simulation studies show that the McKelvey&Zavoina Pseudo-$R^2$ is the best fit measure for binary and ordinal logit models

- **New**
  - Generalization of the M&Z-Pseudo-$R^2$ to the multinomial logit model to identify its differential fit for its $k-1$ binary comparisons
  - Successful validation of these global and partial M&Z-Pseudo-$R^2$s by those of the corresponding binary logit models

- **That’s why**
  - I suggest to use my mnl_mzr2.ado file to assess the differential fit of the multinomial logit model
Closing words

- Thank you for your attention
- Do you have some questions?
Contact:

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- Email: wolfgang.langer@soziologie.uni-halle.de