`simarwilson`:

DEA based Two-Step Efficiency Analysis

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Efficiency Measurement

- Efficiency measurement industry in empirical research
  - ✓ Thousands of applications
- Two major methodological approaches
  1. Parametric approaches
     - Most important: **stochastic frontier** (SF; Aigner et al., 1977) → `frontier`, `xtfrontier` (real Stata); `sfcross` and `sfpanel` (user written programs implementing additional model variants; Belotti et al., 2013)
  2. Non-parametric approaches
     - Most important: **DEA** (Data Envelopment Analysis; Charnes et al., 1978) → `dea` (user written Stata command implementing most common DEA models; Ji and Lee, 2010)
     - Less often applied: FDH (Free Disposal Hull; Deprins et al., 1984), partial frontier (Cazals et al., 2002; Aragon et al., 2005) → `orderm`, `orderalpha` (user written Stata commands implementing FDH and partial frontier models; Tauchmann, 2012)

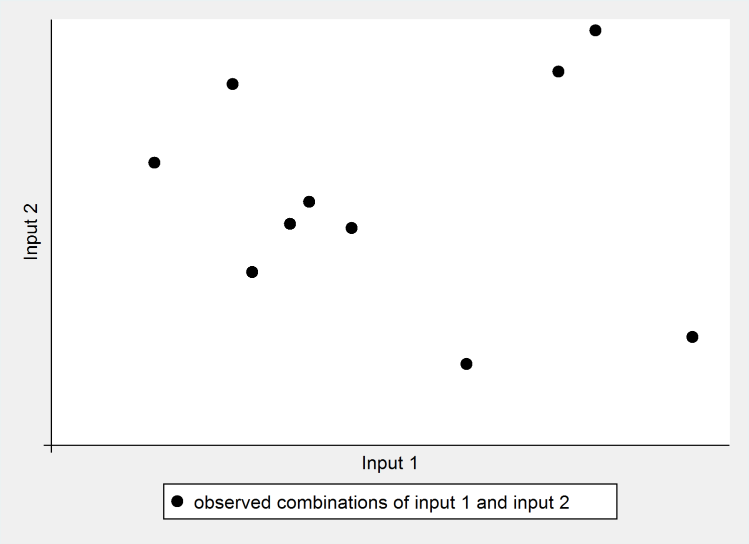
Stochastic Frontier Models

- SF embedded in familiar regression framework
  $y_i = x_i'\beta + \varepsilon_i - \nu_i$ with i indexing DMUs (decision making unit)
- y_i: log-output from production
- x_i: log-inputs to production
- ε_i: conventional normal error
  - ✓ Unexplained heterogeneity in production possibility frontier
- ν_i support on the $[0, \infty)$ interval (exponential, half-normal, truncated normal)
  - ✓ Deviation from production possibility frontier ($\rightarrow$ inefficiency)
- Efficiency measured as $\text{E}(\exp(-\nu_i)|\varepsilon_i - \nu_i)$
- $\text{E}(\nu_i)$ or $\text{Var}(\nu_i)$ can be specified as a function of DMU specific characteristics z_i
- Stochastic Frontier model allows for both
  1. Estimating individual efficiency
  2. Identifying effects DMU characteristics exert on (in)efficiency

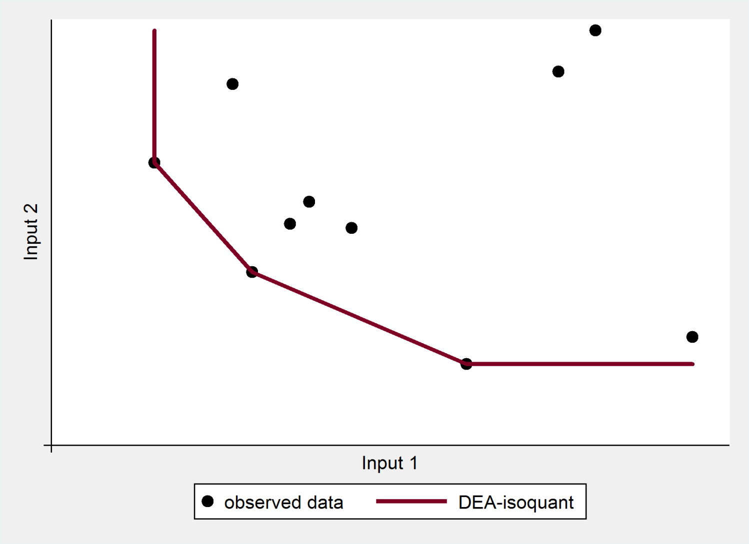
Data Envelopment Analysis

- DEA not a regression model
- Estimation of production possibility frontier by non-parametrically enveloping a given sample of data
- Major advantages as compared to SF-Models
  - ✓ No distributional assumptions required
  - ✓ Straight forward modeling of multi-output processes ($\rightarrow$ no cost-efficiency approach required)
  - ✓ Not a causal model ($\rightarrow$ endogeneity of inputs no issue)
- Various different DEA variants available
  - ✓ Assumptions about frontier ($\rightarrow$ return to scale)
  - ✓ Efficient counterpart of observed DMU at frontier ($\rightarrow$ orientation, treatment of slacks)
- Solving linear program yields eff. score θ_i for each DMU i
  1. $\theta_i^{in} \in (0, 1]$: possible prop. input reduction (input orient.)
  2. $\theta_i^{out} \in [0, \infty)$: possible prop. output increase (output orient.)

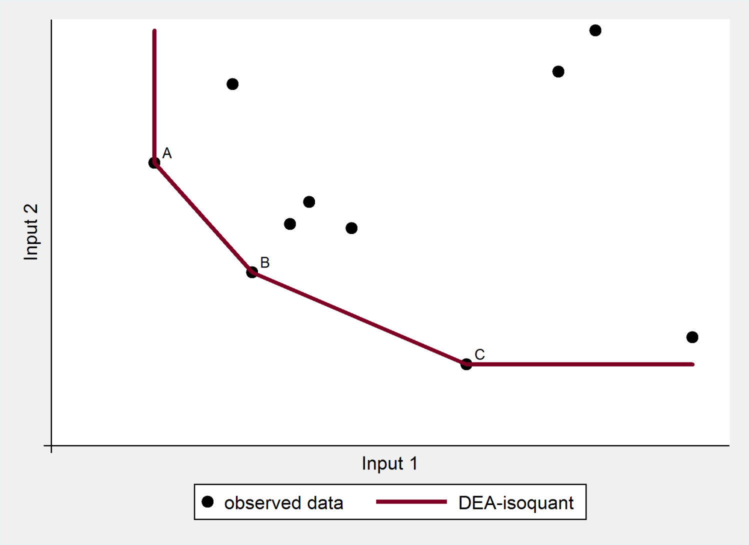
Graphical Illustration of DEA



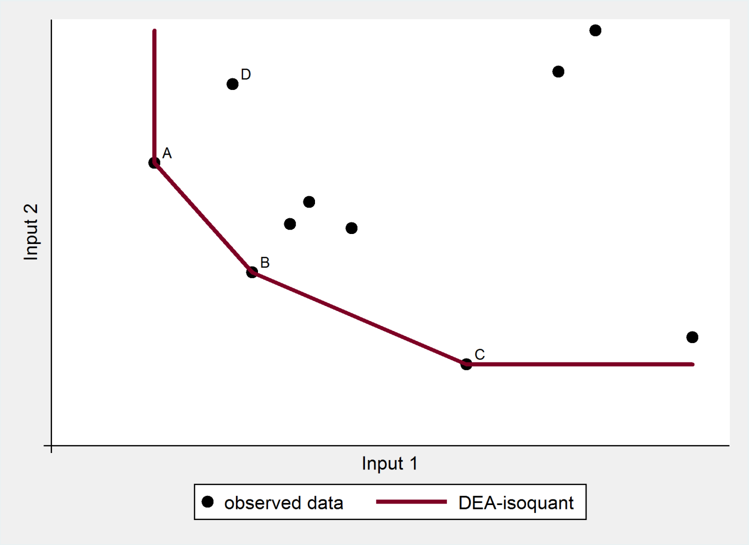
Graphical Illustration of DEA



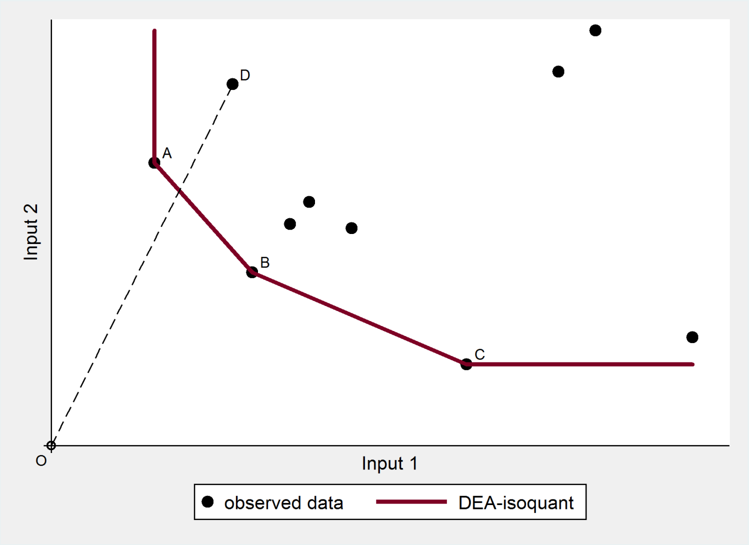
Graphical Illustration of DEA



Graphical Illustration of DEA



Graphical Illustration of DEA



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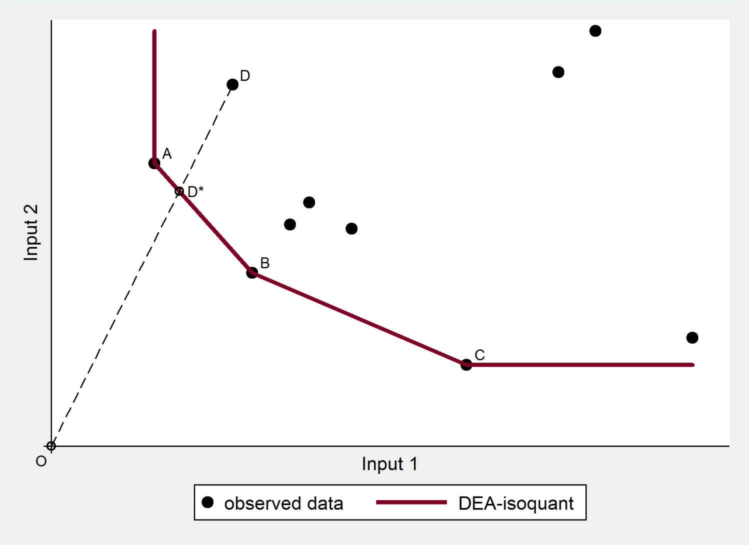
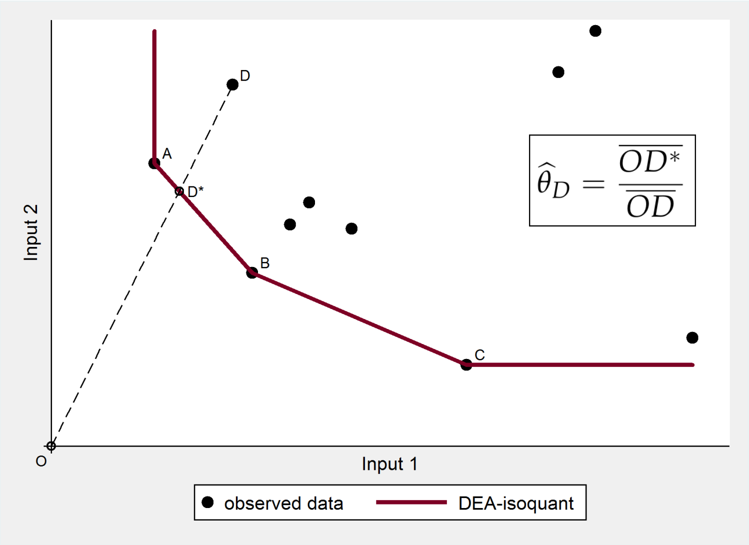


Graphical Illustration of DEA



DEA & Explaining Efficiency Differentials

- DEA focussed on **measuring** efficiency
  - ✓ Distance to estimated frontier
  - ✓ Benchmarking major field of applications
- DEA **does not explain** efficiency differentials
- Two-step approach intuitive
  1. Estimating θ_i using DEA ($\rightarrow$ yields certain share M/N of DMUs for which $\widehat{\theta}_i = 1$ holds)
  2. Regressing $\widehat{\theta}_i$ (or transformation of $\widehat{\theta}_i$) on DMU characteristics z_i (OLS, censored regression, ...)
- Numerous applications of such two-step approaches

The argument of Simar and Wilson (2007)

Conventional two-step approaches inappropriate

1. Two-step approaches lack a well defined data generating mechanism
   - ✓ Censored regression model not appropriate
   - ✓ Probability mass at $\theta = 1$ artifact of efficiency measurement by DEA (finite sample problem)
   - ✓ No strictly positive probability for DMU being located on true production possibility frontier ($\neq$ estimated DEA frontier)
2. DEA generates complex (unknown) pattern of correlation between the estimated efficiency scores
   - ✓ $\widehat{\theta}_i$ with $i = 1, \ldots, N$ by construction not independent
   - ✓ Misleading inference based on two-step approaches
   - ✓ Naive bootstrap no solution because of boundary estimation nature of DEA

The Simar and Wilson (2007) Approach

1. Constructing and simulating a ‘sensible’ data generating process
2. Generating artificial iid bootstrap samples from artificial data generating process
3. Construction standard errors and confidence through bootstrapping/simulation

The Simar and Wilson (2007) Procedure

1. Estimate θ_i with $i = 1, \ldots, N$ using **DEA**
2. Fitting $\widehat{\theta}_i = \beta' z_i + \epsilon_i$ using **truncated regression** (ML) ($\rightarrow$ obtain estimates $\widehat{\beta}$ and $\widehat{\sigma}_\varepsilon$)
   - ✓ **Efficient DMUs** j ($\widehat{\theta}_j = 1, j = 1, \ldots, M$) **excluded**
   - ✓ $\epsilon_i \equiv \varepsilon_i + \zeta_i$ with $\zeta_i \equiv \widehat{\theta}_i - \theta_i$
   - ✓ $\widehat{\theta}_i^{in} \in (0, 1]$ (input orient.): right-truncation at 1
   - ✓ $\widehat{\theta}_i^{out} \in [0, \infty)$ (output orient.): left-truncation at 1

The Simar and Wilson (2007) Procedure II

3. **Loop** over the next three steps B times ($b = 1, \ldots, B$)
   3.1 **Draw** ε_i^b from $N(0, \widehat{\sigma}_\varepsilon)$ with **left-truncation** (output orient.) or **right-truncation** (input orient.) at $(1 - \widehat{\beta}' z_i)$ for $i = M + 1, \ldots, N$
   3.2 **Compute** $\theta_i^b = \widehat{\beta}' z_i + \varepsilon_i^b$ for $i = M + 1, \ldots, N$
   3.3 Estimate $\widehat{\beta}^b$ and $\widehat{\sigma}_\varepsilon^b$ by **truncated regression** using the artificial efficiency scores θ_i^b as *lhs*-variable
4. Construct **standard errors** for $\widehat{\beta}$ and $\widehat{\sigma}_\varepsilon$ (conf. interv. for β and σ_ε) from **simulated distribution** of $\widehat{\beta}^b$ and $\widehat{\sigma}_\varepsilon^b$

The `simarwilson` command

- `simarwilson` implements above procedure in Stata
  - ✓ Except for step 1
  - ✓ Efficiency scores have to be obtained prior to running `simarwilson` ($\rightarrow$ e.g. using `dea`)
    - Implemented procedure is ‘algorithm #1’ (Simar and Wilson, 2007)
    - Alternative (more involved) ‘algorithm #2’ requires looping over DEA
- `simarwilson` requires user written mata modul `RTNORM` Belotti and Ilardi (2010) to drawn from the truncated normal distribution

Syntax of `simarwilson`

`simarwilson` *depvar indepvars* [if] [in], [nounit reps(#) dots level(#)]

- **depvar** is assumed to be an efficiency score estimated in a preceding step. *depvar* needs to be a numeric nonnegative variable.
- **nounit** indicates that *depvar* > 1 holds for inefficient dmus, **unit** indicates that for indicates that *depvar* < 1 holds for inefficient dmus. If *depvar* is is well coded, `simarwilson` recognizes if efficiency scores originate form an input or an output-oriented DEA. Specifying **nounit** is required for poorly coded data or if the data contain superefficient dmus
- With **dots** specified one dot character is displayed for each bootstrap replication
- **reps(#)** specifies the number of bootstrap replications to be performed. The default is 50. For simulating meaningful confidence intervals a much larger number of replications is required
- **level(#)** set confidence level; default is `level(95)`

Application & Data

- Regional efficiency of health care provision in Bavaria
  - ✓ `simarwilson` originates from project analyzing efficiency of nursing homes
  - ✓ Protected data ($\rightarrow$ not well suited for illustrating the command)
- County level data (N = 96) for year 2006
- Output from health production
  - ✓ Regional survival rate ($\rightarrow$ corrected for demographic composition; normalized to national average)
- Input to health production
  1. General practitioners (per 100 000 inhabitants)
  2. Medical specialists (per 100 000 inhabitants)
  3. Hospital beds (per 10 000 inhabitants)

Descriptives for Input & Outputs

```
.        tabstat survival gps specialists beds, columns(statistics) statistics
> (mean sd median min max) format(%7.0g)
```

variable	mean	sd	p50	min	max
survival	1.0075	.08002	.9978	.84532	1.2205
gps	77.829	11.603	74.392	57.792	109.98
specialists	93.127	62.39	66.709	16.497	245.78
beds	66.402	51.808	49.156	1.7513	227.16

- Variables that enter `dea`
- Substantial heterogeneity across counties

Results from DEA

```
.         foreach direction in i o {
  2.                 quietly: dea gps specialists beds = survival, rts(vrs) ort
> (`direction´)
  3.                 mat deascores = r(dearslt)
  4.                 mat deascores = deascores[1...,"theta"]
  5.                 sort dmu
  6.                 cap drop dea1
  7.                 svmat deascores, names(dea)
  8.                 rename dea1 deascore_`direction´
  9.                 gen efficient_`direction´ = deascore_`direction´ == 1
 10. }
options: RTS(VRS) ORT(IN) STAGE(2)
options: RTS(VRS) ORT(OUT) STAGE(2)

. tabstat deascore_i deascore_o efficient_i efficient_o, columns(statistics) st
> atistics(mean sd median min max) format(%7.0g)
```

variable	mean	sd	p50	min	max
deascore_i	.81203	.12388	.82317	.52548	1
deascore_o	1.1421	.09806	1.1424	1	1.3611
efficient_i	.125	.33245	0	0	1
efficient_o	.125	.33245	0	0	1

Explanatory Variables

- County unemployment rate (*unemployment*)
- Women's share in county population (*female*)
- Indicator for urban county (single town constituting a county, *urban*)
- Share of private hospitals in county hospital beds (*privatehosp*)

```
.        tabstat `reglist´, columns(statistics) statistics(mean sd median min
> max) format(%7.0g)
```

variable	mean	sd	p50	min	max
unemployment	.07069	.02311	.0675	.034	.132
female	.51058	.00881	.50778	.49683	.53575
urban	.26042	.44117	0	0	1
privatehosp	.16448	.29363	0	0	1

(Naive) Censored Regression Analysis

- Estimated input oriented efficiency (*deascore_i*) at lhs

```
.        tobit deascore_i `reglist´, ul(1)

Tobit regression                                  Number of obs   =         96
                                                  LR chi2(4)      =      58.47
                                                  Prob > chi2     =     0.0000
Log likelihood =  62.060332                       Pseudo R2       =    -0.8905

------------------------------------------------------------------------------
  deascore_i |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
unemployment |  -1.498905   .6192545    -2.42   0.017    -2.728798    -.269012
      female |  -4.483155   1.677094    -2.67   0.009    -7.814008   -1.152302
       urban |  -.0657136   .0364946    -1.80   0.075    -.1381952    .0067679
 privatehosp |   .0188993   .0358417     0.53   0.599    -.0522853     .090084
       _cons |   3.226821   .8407472     3.84   0.000     1.557024    4.896618
-------------+----------------------------------------------------------------
      /sigma |   .0976889   .0078113                      .0821749    .1132029
------------------------------------------------------------------------------
  Obs. summary:          0  left-censored observations
                        84     uncensored observations
                        12 right-censored observations at deascore_i>=1
```

Conventional Truncated Regression Analysis

```
.        truncreg deascore_i `reglist´, ul(1)
(note: 12 obs. truncated)

Fitting full model:

Iteration 0:   log likelihood =  102.21542
Iteration 1:   log likelihood =  102.32083
Iteration 2:   log likelihood =  102.32114
Iteration 3:   log likelihood =  102.32114

Truncated regression
Limit:   lower =       -inf                             Number of obs =     84
         upper =          1                             Wald chi2(4)  =  91.26
Log likelihood =  102.32114                             Prob > chi2   = 0.0000
```

deascore_i	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
unemployment	-1.120479	.5156737	-2.17	0.030	-2.13118	-.1097767
female	-5.377884	1.407259	-3.82	0.000	-8.136062	-2.619706
urban	-.0403955	.0289685	-1.39	0.163	-.0971728	.0163818
privatehosp	-.0257407	.0314323	-0.82	0.413	-.0873468	.0358655
_cons	3.634028	.7056814	5.15	0.000	2.250918	5.017138
/sigma	.0753927	.0064815	11.63	0.000	.0626893	.0880962

- Qualitatively similar results as from `tobit`

Simar & Wilson (2007) Procedure

```
.        simarwilson deascore_i `reglist´, reps(500)

Simar & Wilson (2007) truncated regression
DMUs inefficient if deascore_i < unity

Number of obs.               =        96
Number of truncated obs.     =        12
Number of bootstr. reps.     =       500
Wald-test (p-value)          =   5.0e-18
Log-likelihood               =   102.321
```

deascore_i	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
deascore_i						
unemployment	-1.120479	.4943829	-2.27	0.023	-2.089451	-.1515059
female	-5.377884	1.289016	-4.17	0.000	-7.904308	-2.85146
urban	-.0403955	.0284727	-1.42	0.156	-.096201	.0154099
privatehosp	-.0257407	.0275043	-0.94	0.349	-.0796481	.0281667
_cons	3.634028	.6554127	5.54	0.000	2.349443	4.918613
sigma						
_cons	.0753927	.0073111	10.31	0.000	.0610632	.0897223

- Only standard errors differ from `truncreg` (alg. #1)
- (In this application) just small deviation from `truncreg`

simarwilson: output-oriented

```
.         simarwilson deascore_o `reglist´, reps(500)
warning: all efficiency scores deascore_o outside unit-interval, option unit ch
> angened to nounit

Simar & Wilson (2007) truncated regression
DMUs inefficient if deascore_o > unity

Number of obs.                =        96
Number of truncated obs.      =        12
Number of bootstr. reps.      =       500
Wald-test (p-value)           =   1.1e-08
Log-likelihood                =   108.830
```

deascore_o	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
deascore_o						
unemployment	3.315517	.5375734	6.17	0.000	2.261893	4.369142
female	-1.191438	1.384378	-0.86	0.389	-3.90477	1.521893
urban	-.0518028	.0269731	-1.92	0.055	-.1046692	.0010636
privatehosp	-.0301374	.0292012	-1.03	0.302	-.0873707	.0270958
_cons	1.543875	.6942935	2.22	0.026	.1830849	2.904665
sigma						
_cons	.0739952	.0070933	10.43	0.000	.0600927	.0878977

- Results differ from input-oriented analysis
- Estimated effect for *female*, *urban*, and *privatehosp* change direction
- *urban* becomes significant (10% level)

`simarwilson`: output-oriented (inverted score

```
.        gen deascore_oi = 1/deascore_o

.        simarwilson deascore_oi `reglist´, reps(500)

Simar & Wilson (2007) truncated regression
DMUs inefficient if deascore_oi < unity

Number of obs.              =        96
Number of truncated obs.    =        12
Number of bootstr. reps.    =       500
Wald-test (p-value)         =   1.0e-09
Log-likelihood              =   132.557
```

deascore_oi	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
deascore_oi						
unemployment	-2.349899	.3703527	-6.35	0.000	-3.075777	-1.624021
female	.8053843	.9966054	0.81	0.419	-1.147926	2.758695
urban	.0374269	.02103	1.78	0.075	-.0037912	.078645
privatehosp	.0247856	.0212903	1.16	0.244	-.0169425	.0665137
_cons	.6116713	.4991997	1.23	0.220	-.3667421	1.590085
sigma						
_cons	.0530765	.0051011	10.40	0.000	.0430784	.0630745

- Results qualitatively equivalent to using not inverted scores at *lhs*

Conclusions

- Using DEA-scores as *lhs*-variable in regression model questionable
- Simar & Wilson (2007) propose procedure that is not ad hoc but has a basis in statistical theory
  - ✓ Very influential in applied efficiency analysis
- `simarwilson` implements the procedure (alg. #1) in Stata
  - ✓ Also implementing alg. #2 worth considering
  - ✓ Complicated by alg. #2 requiring looping over DEA
- In many application results (inference) do not differ much from simple truncated regression

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