Estimating average treatment effects from observational data using teffects

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Outline

1 What do we want to estimate?

2 Estimators: Overview



A question

- Will a mother hurt her child by smoking while she is pregnant?
 - Too vague
- Will a mother reduce the birthweight of her child by smoking while she is pregnant?
 - Less interesting, but more specific
 - There might even be data to help us answer this question
 - The data will be observational, not experimental

Potential outcomes

- Potential outcomes are the data that we wish we had to estimate causal treatment effects
- Suppose that we could see
 - the birthweight of a child born to each mother when she smoked while pregnant, and
 - 2 the birthweight of a child born to each mother when she did not smoke while pregnant

For example, we wish we had data like

. list mother_id bw_smoke bw_nosmoke in 1/5, abbreviate(10)

| | mother_id | bw_smoke | bw_nosmoke |
|----|-----------|----------|------------|
| 1. | 1 | 3183 | 3509 |
| 2. | 2 | 3060 | 3316 |
| 3. | 3 | 3165 | 3474 |
| 4. | 4 | 3176 | 3495 |
| 5. | 5 | 3241 | 3413 |

- There are two treatment levels, the mother smokes and the mother does not smoke
 - For each treatment level, there is an outcome (a baby's birthweight) that would be observed if the mother got that treatment level

Average treatment effect

 If we had data on each potential outcome, the sample-average treatment effect would be the sample average of bw_smoke minus bw_nosmoke

| . mean bw_smoke bw_nosmoke Mean estimation | | Number of obs | | = | 4642 | | | |
|---|------------------------------|----------------------|--------------|-------|--------------|------|-------|-----------|
| | Mean | Std. Err. | [95% | Conf. | Inter | val] | | |
| bw_smoke bw_nosmoke | 3171.72 3402.599 | .9088219 1.529189 | 3169 3399 | | 3173 3405 | | | |
| . lincom _b[bt (1) bw_smol | w_smoke]b ke - bw_nosmok | | •] | | | | | |
| Mean | Coef. | Std. Err. | t | P> t | | [95% | Conf. | Interval] |
| (1) | -230.8791 | 1.222589 | -188.84 | 0.000 |) | -233 | . 276 | -228.4823 |

• In population terms, the average treatment effect is

$$ATE = \mathbf{E}[bw_{smoke} - bw_{nosmoke}] = \mathbf{E}[bw_{smoke}] - \mathbf{E}[bw_{nosmoke}]$$



Missing data

- The "fundamental problem of causal inference" (Holland (1986)) is that we only observe one of the potential outcomes
 - The other potential outcome is missing
 - 1 We only see bw_{smoke} for mothers who smoked
 - 2 We only see $bw_{nosmoke}$ for mothers who did not smoked
- We can use the tricks of missing-data analyis to estimate treatment effects
- For more about potential outcomes Rubin (1974), Holland (1986), Heckman (1997), Imbens (2004), (Cameron and Trivedi, 2005, chapter 2.7), Imbens and Wooldridge (2009), and (Wooldridge, 2010, chapter 21)

Random-assignment case

- Many questions require using observational data, because experimental data would be unethical
 - We could not ask a random selection of mothers to smoke while pregnant
- The random-assignment methods used with experimental data are useful, because observational-data methods build on them
- When the treatment is randomly assigned, the potential outcomes are independent of the treatment
- If smoking were randomly assigned to mothers, the missing potential outcome would be missing completely at random
 - The average birthweight of babies born to mothers who smoked would be a good estimator for mean of the smoking potential outcome of all mothers in the population
 - The average birthweight of babies born to mothers who did not smoke would be a good estimator for mean of the not-smoking potential outcome of all mothers in the population
 - 3 The difference in the two averages computed from ARRA RELATIONS OF THE STATE OF T

Difference in means

```
. regress bweight ibn.mbsmoke, noconstant
     Source
                     SS
                              df
                                       MS
                                                        Number of obs =
                                                                           4642
                                                        F( 2, 4640) =81131.59
      Model
                5.2512e+10
                                  2.6256e+10
                                                        Prob > F
                                                                      = 0.0000
   Residual
                1.5016e+09 4640
                                  323622.478
                                                        R-squared
                                                                      = 0.9722
                                                        Adj R-squared =
                                                                         0.9722
                                                        Root MSE
      Total
                5.4014e+10 4642 11635851.6
                                                                         568.88
    bweight
                    Coef.
                            Std. Err.
                                                P>|t|
                                                           [95% Conf. Interval]
                                            t
    mbsmoke
 nonsmoker
                 3412.912
                            9.255254
                                       368.75
                                                0.000
                                                           3394.767
                                                                       3431.056
    smoker
                  3137.66
                            19.35363
                                       162.12
                                                0.000
                                                           3099.717
                                                                       3175.602
```

. contrast r.mbsmoke, nowald Contrasts of marginal linear predictions Margins : asbalanced

| | Contrast | Std. Err. | [95% Conf. | Interval] |
|----------------------------------|-----------|-----------|------------|-----------|
| mbsmoke (smoker vs nonsmoker) | -275.2519 | 21.4528 | -317.3096 | -233.1942 |

As good as random

- Instead of assuming that the treatment is randomly assigned, we will
 now assume that the after conditioning on covariates the treatment is
 as good as randomly assigned
- Formally, this assumption is known as conditional independence
- Even more formally, we only need conditional mean independence which says that after conditioning on covariates, the treatment does not affect the means of the potential outcomes

Assumptions used with observational data

- The assumptions we need vary over estimator and effect parameter, but some version of the following assumptions are required.
 - CMI The conditional mean-independence CMI assumption restricts the dependence between the treatment model and the potential outcomes
- Overlap The overlap assumption ensures that each individual could get any treatment level
 - IID The independent-and-identically-distributed (IID) sampling assumption ensures that the potential outcomes and treatment status of each individual are unrelated to the potential outcomes and treatment statuses of all the other individuals in the population

The overlap assumption

- The overlap assumption requires that each individual has a positive probability of receiving each treatment level.
- Formally, the overlap assumption requires that for each possible \mathbf{x}_i in the population and each treatment level t, $0 < \mathbf{P}(t_i = t | \mathbf{x}) < 1$.

The IID assumption

- We also make the standard assumption that we have an independently and identically distributed (IID) sample from the population
- In potential-outcome models, IID sampling implies that the potential outcomes and treatment status of each individual are unrelated to the potential outcomes and treatment statuses of all the other individuals in the population
 - IID sampling rules out interactions among the individuals
 - For instance, models of vaccinations in epidemiology and spatially-dependent outcomes in economics violate the independence assumption

Some references for assumptions

- Versions of the CMI assumption are also known as unconfoundedness and selection-on-observables in the literature; see Rosenbaum and Rubin (1983), Heckman (1997), Heckman and Navarro-Lozano (2004), (Cameron and Trivedi, 2005, section 25.2.1), (Tsiatis, 2006, section 13.3), (Angrist and Pischke, 2009, chapter 3), Imbens and Wooldridge (2009), and (Wooldridge, 2010, section 21.3)
- Rosenbaum and Rubin (1983) call the combination of conditional independence and overlap assumptions strong ignorability; see also (Abadie and Imbens, 2006, pp 237-238) and Imbens and Wooldridge (2009).
- The IID assumption is a part of what is known as the stable unit treatment value assumption (SUTVA); see (Wooldridge, 2010, p.905) and Imbens and Wooldridge (2009)

Model

Choice of auxiliary model

- Recall that the potential-outcomes framework formulates the estimation of the ATE as a missing-data problem
- We use the parameters of an auxiliary model to solve the missing-data problem

```
Regression adjustment (RA)
                     outcome
                                       Inverse-probability weighted (IPW)
                                 \rightarrow
                    treatment
                                       Augmented IPW (AIPW)
     outcome and treatment
                                 \rightarrow
     outcome and treatment
                                       IPW RA (IPWRA)
                                  \rightarrow
outcome (nonparametrically)
                                       Nearest-neighbor matching (NNMATCH)
                                  \rightarrow
                                       Propensity-score matching (PSMATCH)
                    treatment
                                 \rightarrow
```

Estimator

Regression adjustment estimators

- Regression adjustment (RA) estimators:
 - RA estimators run separate regressions for each treatment level, then
 - means of predicted outcomes using all the data and the estimated coefficients for treatment level i all the data estimate POM_i
 - use differences of POMs, or conditional on the treated POMs, to estimate ATEs or ATETs
 - Formally, the CMI assumption implies that we can we can estimate $\mathbf{E}(y_t|\mathbf{x}_i)$ directly from the observations for which person i gets treatment t
 - \bullet y_t is the potential outcome for treatment level t
 - Averages of predicted $\mathbf{E}(y_t|\mathbf{x}_i)$ yield estimates of the POM $\mathbf{E}[y_t]$
- See (Cameron and Trivedi, 2005, chapter 25), (Wooldridge, 2010, chapter 21), and (Vittinghoff et al., 2012, chapter 9)

RA example I

```
. use cattaneo2
(Excerpt from Cattaneo (2010) Journal of Econometrics 155: 138-154)
. teffects ra (bweight mmarried prenatal1 fbaby medu) (mbsmoke)
              EE criterion = 2.336e-23
Iteration 0:
              EE criterion = 5.702e-26
Iteration 1:
Treatment-effects estimation
                                                Number of obs
                                                                           4642
Estimator
               : regression adjustment
Outcome model : linear
Treatment model: none
                             Robust
                                                           [95% Conf. Interval]
     bweight
                    Coef
                            Std. Err.
                                                P>|z|
ATE
     mbsmoke
    (smoker
         WS.
 nonsmoker)
                -230 9541
                            24 34012
                                        -9 49
                                                 0.000
                                                          -278 6599
                                                                      -183.2484
POmean
     mbsmoke
  nonsmoker
                 3402.548
                          9.546721
                                       356.41
                                                0.000
                                                           3383.836
                                                                       3421.259
```

RA with linear regression to model outcome



RA example II

```
. teffects ra (bweight mmarried prenatal1 fbaby medu, poisson) (mbsmoke)
Iteration 0:
               EE criterion = 3.926e-17
               EE criterion = 1.612e-23
Iteration 1:
Treatment-effects estimation
                                                Number of obs
                                                                           4642
Estimator
               : regression adjustment
Outcome model : Poisson
Treatment model: none
                             Robust
     bweight
                    Coef.
                            Std. Err.
                                                P>|z|
                                                           [95% Conf. Interval]
ATF.
     mbsmoke
    (smoker
         vs
                -230.7723
                                                          -278.6213
 nonsmoker)
                            24.41324
                                        -9.45
                                                0.000
                                                                      -182.9232
POmean
     mbsmoke
 nonsmoker
                 3402.497
                            9.547989
                                       356.36
                                                0.000
                                                           3383.783
                                                                       3421.211
```

• RA with exponential conditional mean to model outcome



RA other models

 teffects ra can also model the outcome using probit, logit, or heteroskedastic probit

Inverse-probability-weighted estimators

- Inverse-probability-weighted (IPW) estimators:
 - IPW estimators weight observations on the outcome variable by the inverse of the probability that it is observed to account for the missingness process
 - Observations that are not likely to contain missing data get a weight close to one; observations that are likely to contain missing data get a weight larger than one, potentially much larger
 - IPW estimators model the probability of treatment without any assumptions about the functional form for the outcome model
 - In contrast, RA estimators model the outcome without any assumptions about the functional form for the probability of treatment model
- See Horvitz and Thompson (1952) Robins and Rotnitzky (1995),
 Robins et al. (1994), Robins et al. (1995), Imbens (2000), Wooldridge (2002), Hirano et al. (2003), (Tsiatis, 2006, chapter 6), Wooldridge (2007) and (Wooldridge, 2010, chapters 19 and 21)

IPW example I

```
. teffects ipw (bweight ) (mbsmoke mmarried prenatal1 fbaby medu)
Tteration 0: EE criterion = 1.701e-23
              EE criterion = 6.339e-27
Iteration 1:
Treatment-effects estimation
                                                Number of obs
                                                                          4642
Estimator
               : inverse-probability weights
Outcome model : weighted mean
Treatment model: logit
                             Robust
    bweight
                   Coef.
                            Std. Err.
                                           z
                                                P>|z|
                                                          [95% Conf. Interval]
ATF.
    mbsmoke
    (smoker
         vs
 nonsmoker)
                -231.1516
                            24.03183
                                       -9.62
                                                0.000
                                                         -278.2531
                                                                     -184.0501
POmean
    mbsmoke
 nonsmoker
                 3402.219
                          9.589812
                                      354.77
                                                0.000
                                                          3383.423
                                                                      3421.015
```

• IPW with logit to model treatment



IPW example II

nonsmoker

```
. teffects ipw (bweight) (mbsmoke mmarried prenatal1 fbaby medu, hetprobit(medu
> ))
               EE criterion = 7.158e-16
Iteration 0:
Iteration 1:
             EE criterion = 1.681e-26
Treatment-effects estimation
                                                 Number of obs
                                                                           4642
Estimator
               : inverse-probability weights
Outcome model : weighted mean
Treatment model: heteroskedastic probit
                             Robust
                            Std. Err.
     bweight
                    Coef.
                                                P>|z|
                                                           [95% Conf. Interval]
ATF.
     mbsmoke
    (smoker
         vs
                -217.7521
                             28.5796
 nonsmoker)
                                        -7.62
                                                0.000
                                                          -273.7671
                                                                      -161.7371
POmean
     mbsmoke
```

355.44

0.000

3383.03

3420.546

- IPW with heteroskedastic probit to model treatment
- Could have used probit to model the treatment

9.570692

3401.788



Augmented IPW estimators

- Augmented IPW (AIPW) estimators
 - Augmented-inverse-probability-weighted (AIPW) estimators model both the outcome and the treatment probability
 - The estimating equation that combines both models is essentially an IPW estimating equation with an augmentation term
 - AIPW estimator have the double-robust property
 - only one of the two models must be correctly specified to consistently estimate the treatment effects
 - AIPW estimators can be more efficient than IPW or RA estimators
- See Robins and Rotnitzky (1995), Robins et al. (1995), Lunceford and Davidian (2004), Bang and Robins (2005), (Tsiatis, 2006, chapter 13), Cattaneo (2010), Cattaneo, Drukker, and Holland (2013)

AIPW example I

```
. teffects aipw (bweight mmarried prenatal1 fbaby medu)
          (mbsmoke mmarried prenatal1 fbaby medu)
              EE criterion = 4.031e-23
Iteration 0:
              EE criterion = 2.179e-26
Iteration 1:
Treatment-effects estimation
                                               Number of obs
                                                                         4642
Estimator
             : augmented IPW
Outcome model : linear by ML
Treatment model: logit
                            Robust
    bweight
                   Coef.
                           Std. Err.
                                               P>|z|
                                                         [95% Conf. Interval]
ATF.
    mbsmoke
    (smoker
 nonsmoker)
               -229.7809
                           24.96839
                                       -9.20
                                               0.000
                                                         -278.718
                                                                    -180.8437
POmean
    mbsmoke
  nonsmoker
                 3403.122 9.564165
                                      355.82
                                               0.000
                                                         3384.376
                                                                     3421.867
```

AIPW with linear model for outcome and logit for treatment



AIPW example II

```
. teffects aipw (bweight mmarried prenatal1 fbaby medu, poisson) ///
          (mbsmoke mmarried prenatal1 fbaby medu, hetprobit(medu))
               EE criterion = 7.551e-16
Iteration 0:
Iteration 1:
               EE criterion = 8.662e-24
Treatment-effects estimation
                                                 Number of obs
                                                                            4642
Estimator
               : augmented IPW
Outcome model : Poisson by ML
Treatment model: heteroskedastic probit
                             Robust
                            Std. Err.
     bweight
                    Coef.
                                                 P>|z|
                                                           [95% Conf. Interval]
ATF.
     mbsmoke
    (smoker
 nonsmoker)
                 -220.496
                            28 30292
                                         -7.79
                                                 0.000
                                                          -275 9687
                                                                       -165.0233
POmean
     mbsmoke
                                                 0.000
  nonsmoker
                 3402.429
                            9 557345
                                        356.00
                                                           3383.697
                                                                        3421.161
```

- AIPW with exponential conditional mean model for outcome and heteroskedastic probit for treatment
- Could have used linear, poisson, logit, probit, or heteroskedastic probit to model the outcome and probit, logit, or heteroskedastic logit to model the treatment



- IPWRA estimators combine models for the outcome and the treatment
- IPWRA estimators are double-robust
- IPWRA use the inverse of the estimated treatment-probability weights to estimate missing-data-corrected regression coefficients that are subsequently used to compute the POMs
 - The ATE is estimated by a difference in the estimated POMs
- See Wooldridge (2007) and (Wooldridge, 2010, section 21.3.4)

IPWRA example I

```
. teffects ipwra (bweight mmarried prenatal1 fbaby medu) ///
          (mbsmoke mmarried prenatal1 fbaby medu)
              EE criterion = 3.016e-22
Iteration 0:
Iteration 1: EE criterion = 1.361e-25
Treatment-effects estimation
                                                Number of obs
                                                                         4642
Estimator
              : IPW regression adjustment
Outcome model : linear
Treatment model: logit
                            Robust
    bweight
                   Coef.
                           Std. Err.
                                               P>|z|
                                                         [95% Conf. Interval]
ATF.
    mbsmoke
    (smoker
 nonsmoker)
                -227.4408
                           25.62591
                                       -8.88
                                               0.000
                                                        -277.6667
                                                                     -177.215
POmean
    mbsmoke
  nonsmoker
                3403.027
                            9.56025
                                      355.96
                                               0.000
                                                         3384.289
                                                                     3421.765
```

• IPWRA with linear model for outcome and logit for treatment



IPWRA example II

```
. teffects ipwra (bweight mmarried prenatal1 fbaby medu, poisson) ///
          (mbsmoke mmarried prenatal1 fbaby medu, hetprobit(medu))
               EE criterion = 7.496e-16
Iteration 0:
Iteration 1:
               EE criterion = 6.131e-24
Treatment-effects estimation
                                                 Number of obs
                                                                            4642
Estimator
               : IPW regression adjustment
Outcome model : Poisson
Treatment model: heteroskedastic probit
                             Robust
                            Std. Err.
     bweight
                    Coef.
                                                 P>|z|
                                                            [95% Conf. Interval]
ATF.
     mbsmoke
    (smoker
 nonsmoker)
                -221 2331
                            27 66194
                                        -8.00
                                                 0.000
                                                          -275 4495
                                                                      -167.0166
POmean
     mbsmoke
                                                 0.000
  nonsmoker
                 3402.416
                            9 558767
                                        355.95
                                                           3383.682
                                                                        3421.151
```

- IPWRA with exponential conditional mean model for outcome and heteroskedastic probit for treatment
- Could have used linear, poisson, logit, probit, or heteroskedastic probit to model the outcome and probit, logit, or heteroskedastic logit to model the treatment

Matching estimators

- Matching estimators use an average of the outcomes of the nearest individuals to impute the missing potential outcome for each sampled individual
- The difference between the observed outcome and the imputed potential outcome is essentially an estimate of the expected individual-level treatment effect conditional on the covariates
- These estimated expected individual-level treatment effects are averaged to estimate the ATE

Nearest-neighbor matching

- Nearest-neighbor matching (NNM) determines "nearest" using a weighted function of the covariates for each observation
- NNM is nonparametric
 - No explicit functional form for either the outcome model or the treatment model is specified
 - The estimator needs more data to get to the true value than an estimator that imposes a functional form
 - The NNM estimator converges to the true value at a rate slower than the parametric rate, when matching on more than one continuous covariate
 - teffects nnmatch uses bias-correction to fix this problem

Nearest-neighbor matching II

- See Abadie and Imbens (2006) and Abadie and Imbens (2011) for formal results, rates of convergence, and the details of the bias-correction methods
- Rubin (1973), Rubin (1977), Quade (1982) did early work on matching estimators with formal results in Abadie and Imbens (2006) and Abadie and Imbens (2011)
- tefffect nnmatch is based on the results in Abadie and Imbens (2006) and Abadie and Imbens (2011) and a previous implementation in Abadie et al. (2004)

NNM example

```
. teffects nnmatch (bweight mmarried prenatal1 fbaby medu) (mbsmoke)
Treatment-effects estimation
                                                Number of obs
                                                                          4642
              : nearest-neighbor matching
                                                Matches: requested =
Estimator
Outcome model : matching
                                                               min =
Distance metric: Mahalanobis
                                                               max =
                                                                           645
                           AT Robust
                           Std. Err.
                                               P>|z|
                                                          [95% Conf. Interval]
    bweight
                   Coef.
                                          z
ATE
    mbsmoke
    (smoker
         VS
                -220.5255
                            28.0835
                                       -7.85
                                               0.000
                                                         -275.5681
                                                                     -165.4828
nonsmoker)
```

Propensity-score matching

- Propensity-score matching (PSM) determines "nearest" using the estimated treatment probabilities, which are known as the propensity scores
 - PSM is implemented in teffects psmatch
- PSM provides an alternative to bias-correction because it matches on a single continuous covariate, the estimated treatment probabilities
- Abadie and Imbens (2012) derived the standard errors that account for the error in estimating the propensity scores

PSM example I

```
. teffects psmatch (bweight) (mbsmoke mmarried prenatal1 fbaby medu)
Treatment-effects estimation
                                                 Number of obs
                                                                            4642
               : propensity-score matching
                                                 Matches: requested =
Outcome model : matching
Treatment model: logit
                                                                 may =
                                                                             645
                            AI Robust
     bweight
                    Coef
                            Std. Err.
                                                 P>|z|
                                                            [95% Conf. Interval]
ATE
     mbsmoke
    (smoker
 nonsmoker)
                -217.3852
                                         -7.50
                            28.98542
                                                 0.000
                                                          -274.1956
                                                                       -160.5748
```

- Used logit for propensity score
- Other choices were probit or heteroskedastic probit

PSMATCH example I

```
. teffects psmatch (bweight) (mbsmoke mmarried prenatal1 fbaby medu)
Treatment-effects estimation
                                                 Number of obs
                                                                            4642
               : propensity-score matching
                                                 Matches: requested =
Outcome model : matching
Treatment model: logit
                                                                 may =
                                                                             645
                             AI Robust
     bweight
                    Coef
                            Std. Err.
                                                 P>|z|
                                                            [95% Conf. Interval]
ATE
     mbsmoke
    (smoker
 nonsmoker)
                -217.3852
                            28.98542
                                         -7.50
                                                 0.000
                                                           -274.1956
                                                                       -160.5748
```

- Used heteroskedastic probit for propensity score
- Other choices were logit or probit

Now what?

 Go to http://www.stata.com/manuals13/te.pdf entry teffects intro advanced for more information and lots of links to literature and examples

What are QTE

- Quantile treatment effects (QTE) are differences in the quantiles of the marginal potential outcome distributions
 - $q_1(\tau) = F_{y_1}^{-1}(\tau)$ is the τ (th) quantile of the distribution of the treated potential outcome y_1
 - $q_0(\tau) = F_{y_0}^{-1}(\tau)$ is the $\tau(\text{th})$ quantile of the distribution of the control potential outcome y_0
 - $q_1(\tau)$ and $q_0(\tau)$ are quantiles of the marginal distributions of the potential outcomes
 - $QTE = q_1(\tau) q_0(\tau)$, the QTE is the difference in the marginal quantiles
 - The distributions are marginalized over the distributions of the covariates
 - $\bullet \ F_{y_j}(y) = \mathbf{E}_{\mathbf{x}}[F_{y_j|\mathbf{x}}(y|\mathbf{x})]$
 - Keep in mind that $q_j(\tau) = F_{y_j}^{-1}(\tau) \neq \mathbf{E}[q_j(\tau|\mathbf{x})]$, where $q_j(\tau|\mathbf{x})$ is condition-on- \mathbf{x} quantile of the potential-outcome distribution

QTEs for survival data

- Imagine a study that followed middle-aged men for two years after suffering a heart attack
 - Does exercise affect the time to a second heart attack?
 - Some observations on the time to second heart attack are censored
 - Observational data implies that treatment allocation depends on covariates
 - We use a model for the outcome to adjust for this dependence

QTEs for survival data

- Excercise could help individuals with relatively strong hearts but not help those with weak hearts
- For each treatment level, a strong-heart indidual is in the .75 quantile of the marginal, over the covariates, distribution of time to second heart attack
 - QTE(.75) is difference in .75 marginal quantiles
- Weak-heart individual would be in the .25 quantile of the marginal distribution for each treatment level
 - QTE(.25) is difference in .25 marginal quantiles
- our story indicates that the QTE(.75) should be significantly larger that the QTE(.25)

A regression-adjustment estimator for QTEs

- Estimate the θ_1 parameters of $F(y|\mathbf{x}, t=1, \theta_1)$ the CDF conditional on covariates and conditional on treatment level
 - Conditional independence implies that this conditional on treatment level CDF estimates the CDF of the treated potential outcome
- ullet Similarly, estimate the $oldsymbol{ heta}_0$ parameters of $F(y|\mathbf{x},t=0,oldsymbol{ heta}_1)$
- At the point y,

$$1/N\sum_{i=1}^N F(y|\mathbf{x}_i,\widehat{\boldsymbol{\theta}}_1)$$

estimates the marginal distribution of the treated potential outcome

• The $\widehat{q}_{1..75}$ that solves

$$1/N \sum_{i=1}^{N} F(\widehat{q}_{1,.75} | \mathbf{x}_i, \widehat{\boldsymbol{\theta}}_1) = .75$$

estimates the .75 marginal quantile for the treated potential outcome

A regression-adjustment estimator for QTEs

• The $\hat{q}_{0..75}$ that solves

$$1/N\sum_{i=1}^{N}F(\widehat{q}_{0,.75}|\mathbf{x}_{i},\widehat{\boldsymbol{\theta}}_{0})=.75$$

estimates the .75 marginal quantile for the control potential outcome

- $\widehat{q}_1(.75) \widehat{q}_0(.75)$ consistently estimates QTE(.75)
- See Drukker (2014) for details

mqgamma example

mqgamma is a user-written command documented in Drukker (2014)

```
    use exercise

. mggamma t active, treat(exercise) fail(fail) lns(health) quantile(.25 .75)
Iteration 0:
               EE criterion =
                                 .7032254
Iteration 1:
               EE criterion =
                                .05262105
Iteration 2:
               EE criterion = .00028553
Iteration 3:
               EE criterion =
                                6.892e-07
Iteration 4:
               EE criterion = 4.706e-12
Iteration 5:
               EE criterion = 1.604e-22
Gamma quantile-treatment-effect estimation
                                                  Number of obs
                                                                             2000
                              Robust
                             Std. Err.
           t.
                    Coef.
                                                  P>|z|
                                                             [95% Conf. Interval]
                                             z
q25_0
                                                  0.000
                  .2151604
                             .0159611
                                          13.48
                                                             .1838771
                                                                          .2464436
       _cons
q25_1
                  .2612655
                             .0249856
                                          10.46
                                                  0.000
                                                             .2122946
                                                                          .3102364
       _cons
q75_0
       _cons
                  1.591147
                             .0725607
                                          21.93
                                                  0.000
                                                              1.44893
                                                                         1.733363
q75_1
                 2.510068
                             .1349917
                                          18.59
                                                  0.000
                                                             2.245489
                                                                         2.774647
       _cons
```

mqgamma example

```
. nlcom (_b[q25_1:_cons] - _b[q25_0:_cons]) > (_b[q75_1:_cons] - _b[q75_0:_cons])
                                                          ///
                 _b[q25_1:_cons] - _b[q25_0:_cons]
                 _b[q75_1:_cons] - _b[q75_0:_cons]
                       Coef.
                                Std. Err.
                                                        P>|z|
                                                                    [95% Conf. Interval]
             t
                                                  z
        _nl_1
                    .0461051
                                 .0295846
                                                1.56
                                                        0.119
                                                                   -.0118796
                                                                                  .1040899
                    .9189214
                                 .1529012
                                                6.01
                                                        0.000
        _n1_2
                                                                    .6192405
                                                                                  1.218602
```

poparms estimates QTEs

- poparms is a user-written command documented in Cattaneo, Drukker, and Holland (2013)
- poparms estimates mean and quantiles of the potential-outcome distributions
 - poparms implements an IPW and an AIPW derived in Cattaneo (2010)
 - Cattaneo (2010) and Cattaneo, Drukker, and Holland (2013) call the AIPW estimator an efficient-influence function (EIF) estimator because EIF theory is what produces the augmentation term

QTE can differ over au

- Suppose that robust babies, those born at the .80 quantile, would not be measurably harmed by the mother smoking a few cigarettes
- Further suppose that at-risk babies, those born at the .20 quantile, could be seriously harmed by the mother smoking a few cigarettes
- ATE and ATET cannot investigate this type of hypothesis
- QTE can investigate this type of hypothesis

poparms

poparms installation

```
. findit poparms
. net install st0303, replace
checking st0303 consistency and verifying not already installed...
all files already exist and are up to date.
. help poparms
```

poparms example

poparms estimates

```
. clear all
. use cattaneo2
(Excerpt from Cattaneo (2010) Journal of Econometrics 155: 138-154)
. poparms (mbsmoke mmarried fbaby medu mage c.medu#c.medu c.mage#c.mage) ///
> (bweight prenatal1 fbaby medu mage), ///
quantiles(.2 .8)
Treatment Mean and Quantiles Estimation Number of obs = 4642
(efficient influence function)
```

| bweight | Coef. | bootstrap Std. Err. | z | P> z | [95% Conf. | Interval] |
|---------------------|---------------------|------------------------|------------------|-------|----------------------|----------------------|
| mean mbsmoke | | | | | | |
| nonsmoker smoker | 3403.35 3183.081 | 9.696517 27.67854 | 350.99 115.00 | 0.000 | 3384.346 3128.832 | 3422.355 3237.33 |
| q20 mbsmoke | | | | | | |
| nonsmoker smoker | 3000 2778 | 13.34484 31.33055 | 224.81 88.67 | 0.000 | 2973.845 2716.593 | 3026.155 2839.407 |
| q80 mbsmoke | | | | | | |
| nonsmoker smoker | 3840 3625 | 9.76136 28.05127 | 393.39 129.23 | 0.000 | 3820.868 3570.021 | 3859.132 3679.979 |

poparms example

poparms estimates

```
. poparms, coeflegend
Treatment Mean and Quantiles Estimation
                                                  Number of obs
                                                                           4642
(efficient influence function)
                    Coef. Legend
     bweight
mean
     mbsmoke
 nonsmoker
                  3403.35 b[mean:Obn.mbsmoke]
                 3183.081 b[mean:1.mbsmoke]
     smoker
a20
     mbsmoke
 nonsmoker
                     3000 _b[q20:0bn.mbsmoke]
                     2778 b[a20:1.mbsmoke]
     smoker
a80
     mbsmoke
 nonsmoker
                          _b[q80:0bn.mbsmoke]
                     3840
                          _b[q80:1.mbsmoke]
     smoker
                     3625
```

poparms example

poparms estimates

. lincom _b[mean:1.mbsmoke] - _b[mean:0.mbsmoke]
(1) - [mean]0bn.mbsmoke + [mean]1.mbsmoke = 0

| (1) [| n, obn.mobmono | · [mounj 11m | 00110110 | | | |
|---------|---------------------------------|--------------|----------|-------|------------|-----------|
| bweight | Coef. | Std. Err. | z | P> z | [95% Conf. | Interval] |
| (1) | -220.2692 | 29.24745 | -7.53 | 0.000 | -277.5931 | -162.9452 |
| | 20:1.mbsmoke]]Obn.mbsmoke + | | | | | |
| bweight | Coef. | Std. Err. | z | P> z | [95% Conf. | Interval] |
| (1) | -222 | 34.18932 | -6.49 | 0.000 | -289.0098 | -154.9902 |
| | 80:1.mbsmoke]]Obn.mbsmoke + | | | | | |
| bweight | Coef. | Std. Err. | z | P> z | [95% Conf. | Interval] |
| (1) | -215 | 29.51215 | -7.29 | 0.000 | -272.8427 | -157.1573 |

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