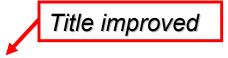
Rescaling results of nonlinear probability models to compare regression coefficients or variance components across hierarchically nested models

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Rescaling of fixed and random effects in hierarchically nested multilevel models

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Outline

- The problem: Increase of fixed or random effects in nonlinear probability models
- A solution: Rescaling of fixed and random effects
- Example of implementation in Stata

 Adding a random intercept or variables with fixed effects to a logistic or probit model may increase effects of earlier included variables.

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Logistic regression models for taking a science subject (Snijders & Bosker, 1999, p. 266 f.)						266 f.)		
	Single Le	vel	ML Mode	el 1	ML Mode	el 2	ML Mode	13
Fixed Effect	Coeff.	SE	Coeff.	SE	Coeff.	SE	Coeff.	SE
γ ₀ Intercept	2.246	0.090	2.440	0.109	1.448	0.058	2.487	0.110
γ_1 Gender	-1.397	0.102	-1.507	0.102			-1.515	0.102
γ_2 Minority Status					-0.644	0.174	-0.727	0.195
Random Effect	VComp	SE	VComp	SE	VComp	SE	VComp	SE
$\tau_0{}^2 = var(u_{0j})$			0.514	0.084	0.293	0.043	0.481	0.082
Deviance	3345.2		3251.9		3476.1		3238.3	

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	VComp 3345.2	SE	1					
$\tau_0{}^2 = var(u_{0j})$		SE	0.514		0.293		0.481	

- The ultimate reason for this phenomenon is the fact that in nonlinear probability models the variance of the residual variance (on the individual level) is fixed at a constant (see Long & Freese, 2006):
 - $\pi^2/3 = 3.29$ (logistic regression models)
 - 1.0 (probit regression models)
- Therefore, the residual variance cannot decrease when adding fixed effects of other variables to the model. Instead, the estimates of other regression coefficients (and random effects) will become larger in absolute value.
- As a consequence, changes (or a lack of change) of fixed effects of earlier included variables may not be interpreted as in OLS or multilevel linear regression models: Decreases due to a correlation of the independent variables are obscured by increases due to this phenomenon.

A solution

- Hox (2010) (based on Fielding, 2004) suggests to rescale the fixed and random effects so that real changes in parameter values can be assessed.
- By using the extent of real changes in the level 1 variance when moving from one model to the next, a scaling factor is computed which effectively holds the implicit scaling of the response constant to that of a base model.
- The procedure includes to
 - calculate the total variance of the null model σ_0^2
 - calculate the total variance of model *m* including the first level predictor variables σ_m^2
 - rescale the fixed effects and random effects by using the scale correction factor $\int \sigma_0^2$

$$\sqrt{rac{\sigma_0^-}{\sigma_m^2}}$$

A solution

 For a multilevel logistic regression model with a random intercept, the scale correction factor is calculated by

$$\sigma_0^2 = \sigma_{u0}^2 + \sigma_R^2 = \sigma_{u0}^2 + 3.29$$

with σ_{u0}^{2} = second level intercept variance and σ_{R}^{2} = lowest level residual variance

-
$$\sigma_m^2 = \sigma_F^2 + \sigma_{u0}^2 + \sigma_R^2 = \sigma_F^2 + \sigma_{u0}^2 + 3.29$$

with σ_{F}^{2} = variance of the linear predictor of model *m*, using the coefficients of the predictors of the fixed part of the equation

$$-SCF = \sqrt{\frac{\sigma_0^2}{\sigma_m^2}}$$

with SCF = scale correction factor

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Implementation in Stata

 The rescaling procedure is implemented in Stata by –meresc– (Enzmann & Kohler, 2012), available on SSC:

<u>Title</u>

meresc Rescaled results for nonlinear mixed models

<u>Syntax</u>

meresc [, <u>v</u>erbose]

Description

meresc rescales the results of mixed nonlinear probability models such as <u>xtmelogit</u>, <u>xtlogit</u>, or <u>xtprobit</u> to the same scale as the intercept-only model. The technique applied is described in chapter 6.5 of Hox (2010: 133--139).

The technique rescales all random and fixed effects of a multilevel model. The variance scale correction factor for random effect parameters is the total variance of the intercept only model devided by the total variance of the model with lowest level variables only. The fixed effects are rescaled using the square root of the variance scale correction factor (i.e. using the scale correction factor).

Saved Results

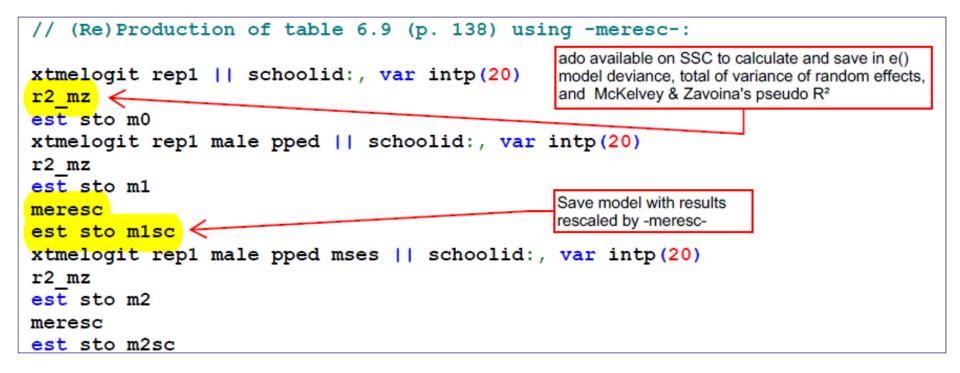
meresc keeps most returned results of the user defined estimation command in memory. However, it stores the rescaled coefficient vector in e(b), and the rescaled variance-covariance matrix in e(V). Moreover it adds the following results to the stored results:

Scalars

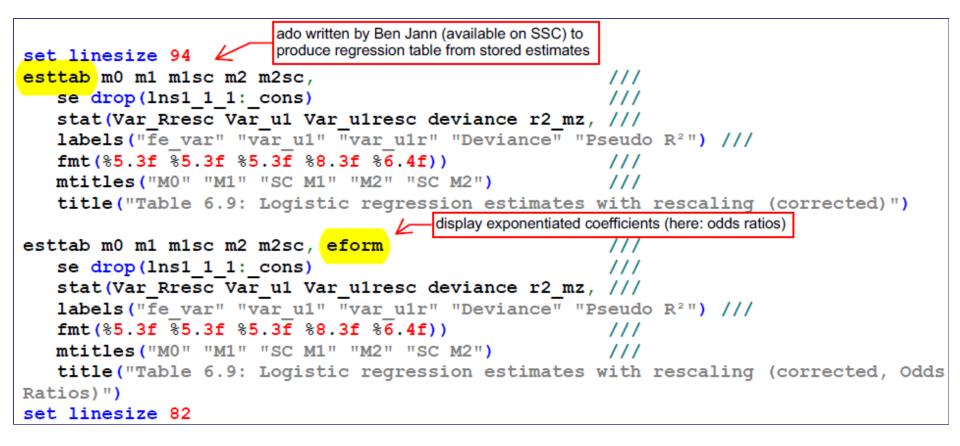
e(SCF)	Scale correction factor
e(VCF)	Variance scale correction factor
e(Var_Flevel1)	Linear Predictor Variance using first level vars only
e(Var_u#)	Variance of Level-# random effect
e(Var_R)	Variance of residuals
e(Var_u0)	Variance of random effects of constant only model
e(Var_u#resc)	Variance of Level-# random effect, rescaled
e(Var_Rresc)	Variance of residuals, rescaled
e(r2_mz)	McKelvy & Zavoina's R2
e(deviance)	Model Deviance
Macros	
e(cmd)	meresc

e(cmd)	meresc				
e(cmdline)	command-line	of	previous	estimation	

• Stata syntax (excerpt):



• Stata syntax (excerpt continued):



• Output produced by –esttab– (1):

	(1)	(2)	(3)	(4)	(5)
	MO	M1	SC M1	M2	SC M2
eq1					
male		0.536***	0.527***	0.535***	0.526***
		(0.0760)	(0.0747)	(0.0760)	(0.0747)
pped		-0.642***	-0.631***	-0.627***	-0.616***
		(0.0996)	(0.0979)	(0.100)	(0.0985)
mses				-0.296	-0.291
				(0.217)	(0.213)
_cons	-2.234***	-2.237***	-2.198***	-2.242***	-2.203***
_	(0.0878)	(0.107)	(0.105)	(0.107)	(0.105)
fe_var			3.178		3.178
var_ul	1.726	1.697	1.697	1.686	1.686
var_ulr			1.640		1.629
Deviance	5537.444	5443.518	5443.518	5441.660	5441.660
Pseudo R²	0.0000	0.0342	0.0342	0.0389	0.0389

• Output produced by –esttab– (2):

	(1) MO	(2) M1	(3) SC M1	(4) M2	(5) SC M2
eql					
male		1.709*** (0.130)		1.708*** (0.130)	1.692*** (0.126)
pped		0.526*** (0.0524)	0.532*** (0.0521)	0.534*** (0.0536)	0.540*** (0.0532)
mses				0.744 (0.161)	0.748 (0.160)
_cons	0.107*** (0.00941)	0.107*** (0.0114)	0.111*** (0.0116)	0.106*** (0.0113)	0.110*** (0.0116)
fe var			3.178		3.178
var_ul var ulr	1.726	1.697	1.697 1.640	1.686	1.686 1.629
_ Deviance	5537.444	5443.518	5443.518	5441.660	5441.660
Pseudo R²	0.0000	0.0342	0.0342	0.0389	0.0389

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GSUG 2012

- Note that the results shown are correct and differ from the results in Hox (2010) because of a mistake in the course of calculations (mixing up a squared and a non-squared scaling factor).
- The moral of the story: Use Stata ados to automate calculations that are error prone.
- Note that although the effect of rescaling is rather small in the example given by Hox, in other instances rescaling of fixed and random effects may change the results quite substantially!

Thanks for your attention!

References

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