Analyzing Proportions

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- A proportion is bounded between 0 and 1, this means that:
  - the effect of explanatory variables tends to be non-linear, and
  - the variance tends to decrease when the mean gets closer to one of the boundaries.
- This makes linear regression unattractive.

Solutions

- model the distribution of the dependent variable(s) with either
  - a beta distribution, `betafit`
  - a zero/one inflated beta distribution, `zoib`
  - a Dirichlet distribution, `dirifit`

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- model the distribution of the dependent variable(s) with either
  - a beta distribution, `betafit`
  - a zero/one inflated beta distribution, `zoib`
  - a Dirichlet distribution, `dirifit`
- model how the mean proportion relates to explanatory variables using
  - a fractional logit, `glm`
  - a fractional multinomial logit, `fmlogit`

Outline

A single proportion

Multiple proportions

the beta distribution

- A flexible distribution bounded between 0 and 1 (excluding 0 and 1)

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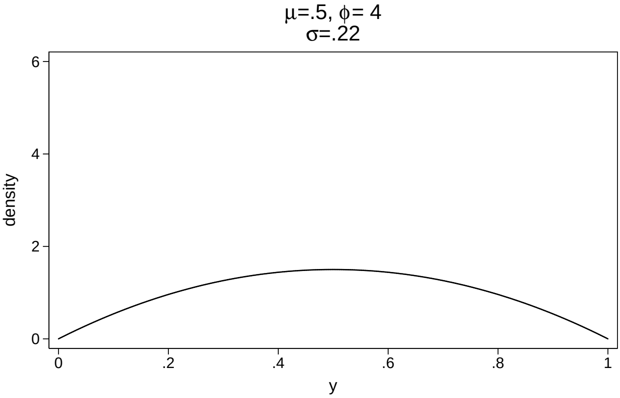
- Two parameters: the mean and a scale parameter.
- The variance is a function of the mean and the scale parameter: the variance is largest when the mean is 0.5.

Some pictures



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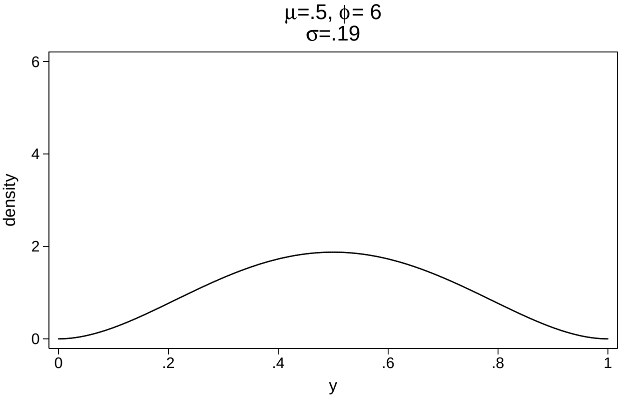

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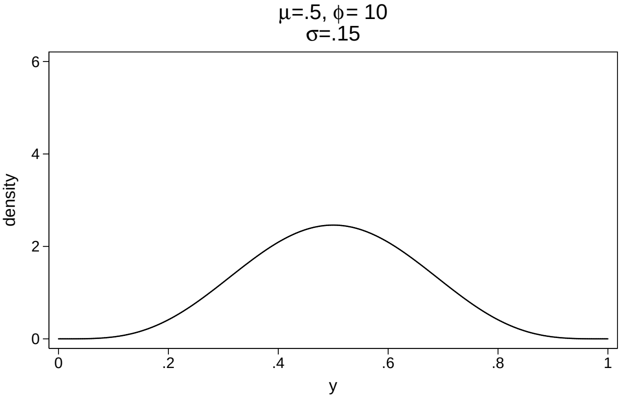

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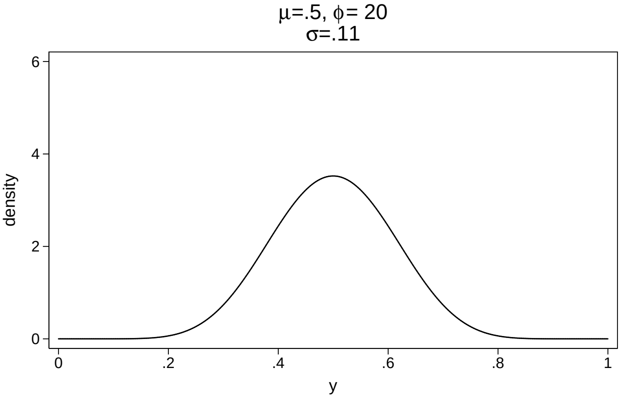

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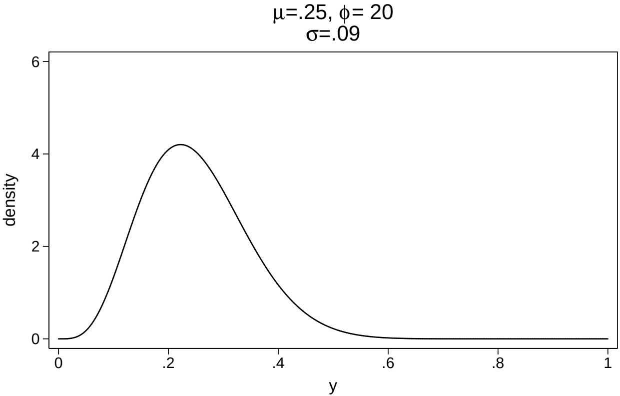

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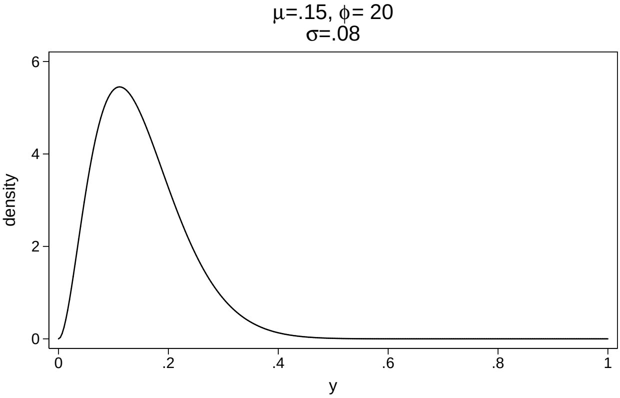

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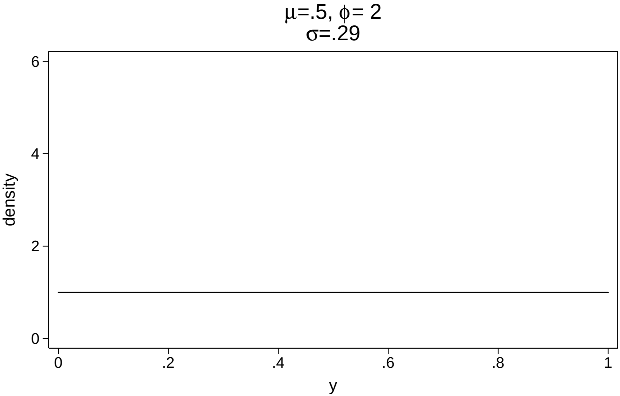

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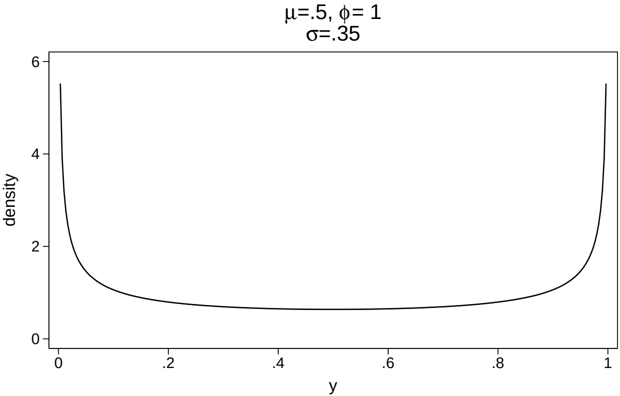

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- Various types of partial and marginal effects: `dbetafit`
- Can be installed by typing in Stata `ssc install betafit`

example

```
. use http://fmwww.bc.edu/repec/bocode/c/citybudget.dta, clear
(Spending on different categories by Dutch cities in 2005)

. betafit governing , mu(minorityleft noleft houseval popdens) nolog

ML fit of beta (mu, phi)                       Number of obs   =        394
                                               Wald chi2(4)    =     109.99
Log likelihood =  768.06704                    Prob > chi2     =     0.0000

------------------------------------------------------------------------------
   governing |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
minorityleft |   -.102143    .059603    -1.71   0.087    -.2189627    .0146768
      noleft |   .1047123   .0611709     1.71   0.087    -.0151804    .2246049
    houseval |   .2970051   .0483488     6.14   0.000     .2022432     .391767
     popdens |  -.1247097   .0262695    -4.75   0.000     -.176197   -.0732223
       _cons |  -2.607601   .0856001   -30.46   0.000    -2.775374   -2.439828
-------------+----------------------------------------------------------------
     /ln_phi |   4.168403   .0715323    58.27   0.000     4.028202    4.308604
-------------+----------------------------------------------------------------
         phi |   64.61219   4.621857                      56.15986    74.33662
------------------------------------------------------------------------------
```

example

```
. dbetafit, at(minorityleft 0 noleft 0)
```

discrete change	Min --> Max coef.	se	+-SD/2 coef.	se	+-1/2 coef.	se
minorityleft	-.0084	.0047				
noleft	.0094	.0057				
houseval	.0879	.0217	.0101	.0022	.0255	.0056
popdens	-.0494	.0075	-.0099	.0019	-.0107	.0021

Marginal Effects	MFX at x coef.	se	Max MFX coef.	se
houseval	.0254	.0056	.0743	.0121
popdens	-.0107	.0021	-.0312	.0066

E(governing|x) = .0945

	x	mean	sd	min	max
minorityleft	0	.434	.4963	0	1
noleft	0	.3858	.4874	0	1
houseval	1.492	1.492	.3971	.72	3.63
popdens	.7629	.7629	.9303	.025	5.711

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  - 0s and 1s represent distinct processes
  - Implies a zero-one inflated beta, which in Stata can be estimated using `zoib`
- Alternatively, you can transform your dependent variable to "push" your 0s and 1s a tiny bit inwards
- Smithson and Verkuilen (2006) propose
  y' = (y*(N - 1) + .5)/N

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    - less sensitive to errors in other parts of the model, e.g. the variance, but
    - not suitable when interest is in other quantities than the mean, e.g. the variance
- Can be estimated with `glm` in combination with the `link(logit) family(binomial) robust` options.

example

```
. use "http://fmwww.bc.edu/repec/bocode/k/k401.dta", clear
(source: Papke and Wooldridge 1996)

. replace totemp = totemp/10000
(4734 real changes made)

. glm prate mrate totemp age sole, ///
>     family(binomial) link(logit) vce(robust) nolog
note: prate has noninteger values

Generalized linear models                          No. of obs      =      4734
Optimization     : ML                              Residual df     =      4729
                                                   Scale parameter =         1
Deviance         =  1023.737134                    (1/df) Deviance =  .2164807
Pearson          =  1377.971352                    (1/df) Pearson  =  .2913875

Variance function: V(u) = u*(1-u/1)                [Binomial]
Link function    : g(u) = ln(u/(1-u))              [Logit]

                                                   AIC             =  .5794217
Log pseudolikelihood = -1366.491144                BIC             = -38995.55

------------------------------------------------------------------------------
             |               Robust
       prate |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       mrate |   .5734427   .0799253     7.17   0.000      .416792    .7300934
      totemp |  -.0577987    .011467    -5.04   0.000    -.0802736   -.0353238
         age |   .0308946   .0027881    11.08   0.000     .0254301    .0363591
        sole |   .3635964   .0476003     7.64   0.000     .2703017    .4568912
       _cons |   1.074062   .0489076    21.96   0.000     .9782051    1.169919
------------------------------------------------------------------------------
```

example

```
. mfx, at(mean sole=0)
Marginal effects after glm
      y  = Predicted mean prate (predict)
         =  .86775841
```

variable	dy/dx	Std. Err.	z	P>|z|	[95%	C.I.]	X
mrate	.0658047	.00803	8.19	0.000	.050058	.081551	.746335
totemp	-.0066326	.00132	-5.02	0.000	-.009224	-.004041	.462107
age	.0035453	.00033	10.69	0.000	.002895	.004195	13.1398
sole*	.0364495	.00471	7.73	0.000	.027209	.04569	0

(*) dy/dx is for discrete change of dummy variable from 0 to 1

zoib: zero one inflated beta

- A zero/one inflated beta model consists of three parts:
  - a logistic regression model for whether or not the proportion equals 0,
  - a logistic regression model for whether or not the proportion equals 1,
  - a beta model for the proportions between 0 and 1.

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  - a logistic regression model for whether or not the proportion equals 0,
  - a logistic regression model for whether or not the proportion equals 1,
  - a beta model for the proportions between 0 and 1.
- This model is for situations where you believe that the decisions for proportions of 0 and/or 1 are governed by a different process as the other proportions.

example

```
. zoib prate mrate totemp age sole, ///
>       oneinflate( mrate totemp age sole) robust nolog

ML fit of oib                                   Number of obs   =       4734
                                                Wald chi2(4)    =     136.47
Log pseudolikelihood = -1293.6594               Prob > chi2     =     0.0000
```

prate	Coef.	Robust Std. Err.	z	P>|z|	[95% Conf.	Interval]
proportion						
mrate	.1524644	.0466905	3.27	0.001	.0609527	.243976
totemp	-.0265332	.0092522	-2.87	0.004	-.0446673	-.0083992
age	.0216248	.0020206	10.70	0.000	.0176645	.0255852
sole	.0604715	.0376378	1.61	0.108	-.0132972	.1342402
_cons	.8738362	.0354738	24.63	0.000	.8043088	.9433636
oneinflate						
mrate	.7935556	.0653962	12.13	0.000	.6653814	.9217297
totemp	-.1416409	.0354509	-4.00	0.000	-.2111235	-.0721584
age	.020835	.003494	5.96	0.000	.0139869	.0276832
sole	.9044132	.0654829	13.81	0.000	.7760692	1.032757
_cons	-1.472011	.0702084	-20.97	0.000	-1.609617	-1.334405
ln_phi						
_cons	1.77591	.0358677	49.51	0.000	1.705611	1.84621

example

```
. mfx, predict(pr) at(mean sole = 0)

Marginal effects after zoib
      y  = Proportion (predict, pr)
         =  .85369833
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
   mrate |   .0566366      .00679    8.34   0.000   .043326  .069947   .746335
 totemp  |  -.0100315      .00208   -4.83   0.000  -.014104 -.005959   .462107
     age |   .0034952      .00031   11.37   0.000   .002893  .004098   13.1398
   sole* |    .053115       .0047   11.31   0.000   .043908  .062322         0
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1
```

example

```
. mfx, predict(pr1) at(mean sole = 0)
Marginal effects after zoib
      y  = probability of having value 1 (predict, pr1)
         =  .33817515
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
   mrate |   .1776078      .01555   11.42   0.000   .147125  .208091   .746335
  totemp |  -.031701       .00796   -3.98   0.000  -.047304 -.016098   .462107
     age |   .0046631      .00078    6.01   0.000   .003143  .006184   13.1398
   sole* |   .2198069      .01548   14.20   0.000   .189476  .250138         0
------------------------------------------------------------------------------
(*) dy/dx is for discrete change of dummy variable from 0 to 1
.
. mfx, predict(prcond) at(mean sole = 0)
Marginal effects after zoib
      y  = proportion conditional on not having value 0 or 1 (predict, prcond)
         =    .778942
------------------------------------------------------------------------------
variable |      dy/dx    Std. Err.     z    P>|z|  [    95% C.I.   ]      X
---------+--------------------------------------------------------------------
   mrate |   .0262531      .00781    3.36   0.001   .010948  .041558   .746335
  totemp |  -.0045688       .0016   -2.86   0.004  -.007698 -.001439   .462107
     age |   .0037236      .00035   10.57   0.000   .003033  .004414   13.1398
   sole* |   .0102369      .00632    1.62   0.105  -.002149  .022623         0
------------------------------------------------------------------------------
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```

Comparing models

	beta	beta with transformed y	flogit	zoib
mrate	0.027	0.033	0.066	0.057
	(3.30)	(17.20)	(8.19)	(8.34)
totemp	-0.005	-0.006	-0.007	-0.010
	(-2.86)	(-5.23)	(-5.02)	(-4.83)
age	0.004	0.001	0.004	0.003
	(10.54)	(8.38)	(10.69)	(11.37)
sole (d)	0.011	0.038	0.036	0.053
	(1.62)	(13.74)	(7.73)	(11.31)
N	2711	4734	4734	4734

Marginal effects; z statistics in parentheses

(d) for discrete change of dummy variable from 0 to 1

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Multiple proportions

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  - Have not been implemented in Stata.
- How are the proportions related to explanatory variables?
  - Two options:
    - `dirifit`: Fits a Dirichlet distribution, which is an extension of the beta distribution to multiple proportions.
    - `fmlogit`: Fits a fractional multinomial logit, which is an extension of the fractional logit to multiple proportions.
  - Both assume that all correlation between proportions is due to the 'automatic correlation'

example

```
. use http://fmwww.bc.edu/repec/bocode/c/citybudget.dta, clear
(Spending on different categories by Dutch cities in 2005)

. replace social = social + education + recreation
(395 real changes made)

. dirifit governing safety social urbanplanning, ///
>     muvar(minorityleft noleft houseval popdens) nolog

ML fit of Dirichlet (mu, phi)                     Number of obs   =        392
                                                  Wald chi2(12)   =     189.54
Log likelihood =  1725.1477                       Prob > chi2     =     0.0000

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
mu2          |
minorityleft |   .1215461   .0900454     1.35   0.177    -.0549397    .2980319
      noleft |   .0423453   .0925709     0.46   0.647    -.1390903    .2237809
    houseval |   -.104733   .0729776    -1.44   0.151    -.2477665    .0383004
     popdens |   .0030234   .0385816     0.08   0.938    -.0725951    .0786419
       _cons |   .7199195    .129466     5.56   0.000     .4661708    .9736682
-------------+----------------------------------------------------------------
mu3          |
minorityleft |   .0492506    .078676     0.63   0.531    -.1049516    .2034528
      noleft |  -.2055446   .0813187    -2.53   0.011    -.3649264   -.0461629
    houseval |  -.4818642   .0694126    -6.94   0.000    -.6179105   -.3458179
     popdens |   .1282351   .0331747     3.87   0.000      .063214    .1932563
       _cons |   2.237157   .1187476    18.84   0.000     2.004415    2.469898
-------------+----------------------------------------------------------------
mu4          |
minorityleft |   .1530504   .0863215     1.77   0.076    -.0161367    .3222375
      noleft |  -.0205669   .0891623    -0.23   0.818    -.1953218    .1541879
    houseval |  -.1563326    .070531    -2.22   0.027    -.2945709   -.0180943
     popdens |   .1285933   .0354878     3.62   0.000     .0590385    .1981481
       _cons |   .9378096   .1247067     7.52   0.000      .693389     1.18223
-------------+----------------------------------------------------------------
     /ln_phi |    3.60327   .0405736    88.81   0.000     3.523747    3.682792
-------------+----------------------------------------------------------------
         phi |   36.71809   1.489786                      33.91125    39.75726
------------------------------------------------------------------------------
mu2 = safety
mu3 = social
mu4 = urbanplanning

base outcome = governing
```

example

```
. ddirifit, at(minorityleft 0 noleft 0 )
```

discrete change	Min --> Max coef.	Min --> Max se	+-SD/2 coef.	+-SD/2 se	+-1/2 coef.	+-1/2 se
governing						
minorityleft	-.0078	.0066				
noleft	.0099	.0074				
houseval	.0937	.0233	.0115	.0024	.0293	.0062
popdens	-.0461	.0115	-.0087	.0027	-.0093	.0029
safety						
minorityleft	.0072	.0088				
noleft	.0257	.0096				
houseval	.0926	.0254	.013	.003	.0333	.0077
popdens	-.0792	.0152	-.0149	.0035	-.0159	.0037
social						
minorityleft	-.0159	.0114				
noleft	-.0527	.012				
houseval	-.264	.0304	-.0366	.0045	-.0935	.0114
popdens	.0865	.0251	.0164	.0042	.0174	.0045
urbanplann~g						
minorityleft	.0165	.0097				
noleft	.0171	.0102				
houseval	.0777	.0265	.0121	.0033	.0309	.0085
popdens	.0387	.0219	.0073	.0034	.0078	.0036

example

```
Marginal        |     MFX at x
Effects         |  coef.      se
----------------+------------------
governing       |
       houseval |  .0293     .0061
        popdens | -.0093     .0029
----------------+------------------
safety          |
       houseval |  .0334     .0077
        popdens | -.0159     .0037
----------------+------------------
social          |
       houseval | -.0937     .0115
        popdens |  .0174     .0045
----------------+------------------
urbanplann~g    |
       houseval |   .031     .0085
        popdens |  .0078     .0035
-----------------------------------

E(governing|x) =     .0993
E(safety|x) =         .175
E(social|x) =        .5032
E(urbanplann~g|x) =  .2225

                  x     mean      sd     min      max
minorityleft      0    .4337   .4962       0        1
      noleft      0    .3878   .4879       0        1
    houseval  1.483    1.483   .3902     .72     3.63
     popdens  .7839    .7839   .9408    .025    5.711
```

example

```
. fmlogit governing safety social urbanplanning, ///
>     eta(minorityleft noleft houseval popdens) nolog
ML fit of fractional multinomial logit              Number of obs   =        392
                                                    Wald chi2(12)   =     232.02
Log pseudolikelihood = -480.22927                   Prob > chi2     =     0.0000

------------------------------------------------------------------------------
             |               Robust
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
eta_safety   |
minorityleft |   .1893804   .0596091     3.18   0.001     .0725486    .3062121
      noleft |   .0826287   .0617178     1.34   0.181     -.038336    .2035934
    houseval |  -.1389243   .0555301    -2.50   0.012    -.2477614   -.0300872
     popdens |   .0115828    .021217     0.55   0.585    -.0300018    .0531673
       _cons |   .7472656   .0920767     8.12   0.000     .5667985    .9277326
-------------+----------------------------------------------------------------
eta_social   |
minorityleft |   .1274304   .0813587     1.57   0.117    -.0320297    .2868905
      noleft |  -.1631202   .0843848    -1.93   0.053    -.3285114     .002271
    houseval |  -.5152946   .0849965    -6.06   0.000    -.6818845   -.3487046
     popdens |   .1456129   .0257634     5.65   0.000     .0951176    .1961081
       _cons |   2.289081   .1415551    16.17   0.000     2.011638    2.566524
-------------+----------------------------------------------------------------
eta_urbanp~g |
minorityleft |    .234597   .1064367     2.20   0.028     .0259849     .443209
      noleft |   .0302709   .1142185     0.27   0.791    -.1935932     .254135
    houseval |  -.1766753   .0856439    -2.06   0.039    -.3445343   -.0088163
     popdens |   .1601436   .0417171     3.84   0.000     .0783796    .2419076
       _cons |   .9790566   .1526485     6.41   0.000     .6798709    1.278242
------------------------------------------------------------------------------
```

example

```
. dfmlogit, at(minorityleft 0 noleft 0 )
```

discrete change	Min --> Max coef.	se	+-SD/2 coef.	se	+-1/2 coef.	se
governing						
minorityleft	-.0138	.0066				
noleft	.0056	.0072				
houseval	.1036	.0258	.0124	.0027	.0317	.0069
popdens	-.0525	.0085	-.0103	.0021	-.011	.0022
safety						
minorityleft	.0065	.0064				
noleft	.0252	.0069				
houseval	.0847	.0184	.0123	.0024	.0313	.0062
popdens	-.0839	.0105	-.016	.0024	-.017	.0025
social						
minorityleft	-.0122	.0128				
noleft	-.0513	.0129				
houseval	-.2721	.0458	-.0377	.007	-.0963	.0177
popdens	.0792	.0315	.016	.0045	.017	.0047
urbanplann~g						
minorityleft	.0196	.014				
noleft	.0205	.0152				
houseval	.0838	.0422	.0131	.0055	.0333	.0141
popdens	.0572	.038	.0103	.0057	.011	.0061

example

Marginal Effects	MFX at x coef.	se
governing		
houseval	.0317	.0065
popdens	-.011	.0024
safety		
houseval	.0314	.0061
popdens	-.017	.0026
social		
houseval	-.0966	.0178
popdens	.017	.0047
urbanplann~g		
houseval	.0335	.0139
popdens	.011	.006

```
E(governing|x) =      .098
E(safety|x) =        .1699
E(social|x) =        .5046
E(urbanplann~g|x) =  .2276
```

	x	mean	sd	min	max
minorityleft	0	.4337	.4962	0	1
noleft	0	.3878	.4879	0	1
houseval	1.483	1.483	.3902	.72	3.63
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- multiple proportions:
  - relationship between these proportions: no solution in Stata (yet)
  - relationship between mean proportions and explanatory variables: `dirifit` or `fmlogit`

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