New multivariate time-series estimators in Stata 11

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Outline

1. Stata 11 has new command `sspace` for estimating the parameters of state-space models
2. Stata 11 has new command `dfactor` for estimating the parameters of dynamic-factor models
3. Stata 11 has new command `dvech` for estimating the parameters of diagonal vech multivariate GARCH models

What are state-space models

- Flexible modeling structure that encompasses many linear time-series models
  - VARMA with or without exogenous variables
    - ARMA, ARMAX, VAR, and VARX models
  - Dynamic-factor models
  - Unobserved component (Structural time-series) models
- Models for stationary and non-stationary data
- Hamilton (1994b,a); Brockwell and Davis (1991); Hannan and Deistler (1988) provide good introductions

The state-space modeling process

- Write your model as a state-space model
- Express your state-space space model in sspace syntax
  - sspace will estimate the parameters by maximum likelihood
  - For stationary models, sspace uses the Kalman filter to predict the conditional means and variances for each time period
  - For nonstationary models, sspace uses the De Jong diffuse Kalman filter to predict the conditional means and variances for each time period
  - These predicted conditional means and variances are used to compute the log-likelihood function, which sspace maximizes

Definition of a state-space model

$$
\begin{aligned}
\mathbf{z}_t &= \mathbf{A}\mathbf{z}_{t-1} + \mathbf{B}\mathbf{x}_t + \mathbf{C}\boldsymbol{\epsilon}_t && \text{(State Equations)} \\
\mathbf{y}_t &= \mathbf{D}\mathbf{z}_t + \mathbf{F}\mathbf{w}_t + \mathbf{G}\boldsymbol{\nu}_t && \text{(Observation equations)}
\end{aligned}
$$

$\mathbf{z}_t$ is an $m \times 1$ vector of unobserved state variables;
$\mathbf{x}_t$ is a $k_x \times 1$ vector of exogenous variables;
$\boldsymbol{\epsilon}_t$ is a $q \times 1$ vector of state-error terms, $(q \leq m)$;
$\mathbf{y}_t$ is an $n \times 1$ vector of observed endogenous variables;
$\mathbf{w}_t$ is a $k_w \times 1$ vector of exogenous variables; and
$\boldsymbol{\nu}_t$ is an $r \times 1$ vector of observation-error terms, $(r \leq n)$;
$\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$, $\mathbf{F}$, and $\mathbf{G}$ are parameter matrices.

The error terms are assumed to be zero mean, normally distributed, serially uncorrelated, and uncorrelated with each other

Specify model in covariance or error form

An AR(1) model

- Consider a first-order autoregressive (AR(1)) process

$$y_t - \mu = \alpha(y_{t-1} - \mu) + \epsilon_t$$

- Letting the state be $u_t = y_t - \mu$ allows us to write the AR(1) in state-space form as

$$\begin{aligned} u_t &= \alpha u_{t-1} + \epsilon_t && \text{(state equation)} && (1) \\ y_t &= \mu + u_t && \text{(observation equation)} && (2) \end{aligned}$$

- If you are in doubt, you can obtain the AR(1) model by substituting equation (1) into equation (2) and then plugging $y_{t-1} - \mu$ in for u_{t-1}

Covariance-form syntax for sspace

sspace *state_ceq* [*state_ceq* ... *state_ceq*]
obs_ceq [*obs_ceq* ... *obs_ceq*] [*if*] [*in*] [, *options*]

where each *state_ceq* is of the form

(*statevar* [*lagged_statevars*] [*indepvars*], state [noerror noconstant])

and each *obs_ceq* is of the form

(*depvar* [*statevars*] [*indepvars*], [noerror noconstant])

some of the available options are

covstate(*covform*)	specifies the covariance structure for the errors in the state variables
covobserved(*covform*)	specifies the covariance structure for the errors in the observed dependent variable
constraints(*constraints*)	apply linear constraints
vce(*vcetype*)	*vcetype* may be oim, or robust

$$u_t = \alpha u_{t-1} + \epsilon_t \quad \text{(state equation)}$$
$$y_t = \mu + u_t \quad \text{(observation equation)}$$

```
. webuse manufac
(St. Louis Fed (FRED) manufacturing data)
. constraint define 1 [D.lncaputil]u = 1
. sspace (u L.u, state noconstant) (D.lncaputil u , noerror ), constraints(1)
searching for initial values ..........
(setting technique to bhhh)
Iteration 0:   log likelihood =  1483.3603
  (output omitted)
Refining estimates:
Iteration 0:   log likelihood =    1516.44
Iteration 1:   log likelihood =    1516.44
State-space model
Sample: 1972m2 - 2008m12                          Number of obs   =        443
                                                  Wald chi2(1)    =      61.73
Log likelihood =    1516.44                       Prob > chi2     =     0.0000
 ( 1)  [D.lncaputil]u = 1
```

lncaputil	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
u						
u L1.	.3523983	.0448539	7.86	0.000	.2644862	.4403104
D.lncaputil						
u	1	.	.	.	.	.
_cons	-.0003558	.0005781	-0.62	0.538	-.001489	.0007773
var(u)	.0000622	4.18e-06	14.88	0.000	.000054	.0000704

Note: Tests of variances against zero are conservative and are provided only for reference.

Estimation by arima

```
. arima D.lncaputil, ar(1) technique(nr) nolog
ARIMA regression
Sample:   1972m2 - 2008m12                      Number of obs     =        443
                                                Wald chi2(1)      =      61.73
Log likelihood =   1516.44                      Prob > chi2       =     0.0000
```

D.lncaputil	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
lncaputil						
_cons	-.0003558	.0005781	-0.62	0.538	-.001489	.0007773
ARMA						
ar						
L1.	.3523983	.0448539	7.86	0.000	.2644862	.4403104
/sigma	.0078897	.0002651	29.77	0.000	.0073701	.0084092

An ARMA(1,1) model

Harvey (1993, 95–96) wrote a zero-mean, first-order, autoregressive moving-average (ARMA(1,1)) model

$$y_t = \alpha y_{t-1} + \theta \epsilon_{t-1} + \epsilon_t$$

as a state-space model with state equations

$$\begin{pmatrix} y_t \\ \theta \epsilon_t \end{pmatrix} = \begin{pmatrix} \alpha & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} y_{t-1} \\ \theta \epsilon_{t-1} \end{pmatrix} + \begin{pmatrix} 1 \\ \theta \end{pmatrix} \epsilon_t$$

and observation equation

$$y_t = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} y_t \\ \theta \epsilon_t \end{pmatrix}$$

This state-space model is in error form

An ARMA(1,1) model (continued)

Letting $u_{1t} = y_t$ and $u_{2t} = \theta\epsilon_t$ allows use to write the ARMA(1,1) model

$$y_t = \alpha y_{t-1} + \theta\epsilon_{t-1} + \epsilon_t$$

as a state-space model with state equations

$$\begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} = \begin{pmatrix} \alpha & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_{1(t-1)} \\ u_{2(t-1)} \end{pmatrix} + \begin{pmatrix} 1 \\ \theta \end{pmatrix} \epsilon_t$$

and observation equation

$$y_t = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}$$

Error-form syntax for sspace

sspace *state_efeq* [*state_efeq* ... *state_efeq*]
obs_efeq [*obs_efeq* ... *obs_efeq*] [*if*] [*in*] [, *options*]

where each *state_efeq* is of the form

(*statevar* [*lagged_statevars*] [*indepvars*] [*state_errors*], state [noconstant])

and each *obs_ceq* is of the form

(*depvar* [*statevars*] [*indepvars*] [*obs_errors*], [noconstant])

state_errors is a list of state-equation errors that enter a state equation. Each state error has the form e.*statevar*, where *statevar* is the name of a state in the model.

obs_errors is a list of observation-equation errors that enter an equation for an observed variable. Each error has the form e.*depvar*, where *depvar* is an observed dependent variable in the model.

```
. constraint 3 [u1]e.u1       = 1
. constraint 4 [D.lncaputil]u1 = 1
. sspace (u1 L.u1 L.u2 e.u1,      state noconstant)        ///
>        (u2 e.u1,                state noconstant)        ///
>        (D.lncaputil  u1,        noconstant ),            ///
>        constraints(2/4)  covstate(diagonal)  nolog
State-space model
Sample: 1972m2 - 2008m12                       Number of obs    =       443
                                               Wald chi2(2)     =    333.84
Log likelihood =   1531.255                    Prob > chi2      =    0.0000
 ( 1)  [u1]L.u2 = 1
 ( 2)  [u1]e.u1 = 1
 ( 3)  [D.lncaputil]u1 = 1

------------------------------------------------------------------------------
             |                 OIM
    lncaputil |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
u1           |
          u1 |
         L1. |   .8056815   .0522661    15.41   0.000     .7032418    .9081212
             |
          u2 |
         L1. |          1          .        .       .            .           .
        e.u1 |          1          .        .       .            .           .
-------------+----------------------------------------------------------------
u2           |
        e.u1 |  -.5188453   .0701985    -7.39   0.000    -.6564317   -.3812588
-------------+----------------------------------------------------------------
D.lncaputil  |
          u1 |          1          .        .       .            .           .
-------------+----------------------------------------------------------------
     var(u1) |   .0000582   3.91e-06    14.88   0.000     .0000505    .0000659
------------------------------------------------------------------------------
te: Tests of variances against zero are conservative and are provided only
```

Estimation by arima

```
. arima D.lncaputil, ar(1) ma(1) tech(nr) noconstant nolog nrtolerance(1e-9)
ARIMA regression
Sample:   1972m2 - 2008m12                        Number of obs      =       443
                                                  Wald chi2(2)       =    333.84
Log likelihood =  1531.255                        Prob > chi2        =    0.0000
```

D.lncaputil	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
ARMA						
ar						
L1.	.8056814	.0522662	15.41	0.000	.7032415	.9081213
ma						
L1.	-.5188451	.0701986	-7.39	0.000	-.6564318	-.3812584
/sigma	.0076289	.0002563	29.77	0.000	.0071266	.0081312

A VARMA(1,1) model

We are going to model the changes in the natural log of capacity utilization and the changes in the log of hours as a first-order vector autoregressive moving-average (VARMA(1,1)) model

$$\begin{pmatrix} \Delta\texttt{lncaputil}_t \\ \Delta\texttt{lnhours}_t \end{pmatrix} = \begin{pmatrix} \alpha_1 & 0 \\ \alpha_2 & \alpha_3 \end{pmatrix} \begin{pmatrix} \Delta\texttt{lncaputil}_{t-1} \\ \Delta\texttt{lnhours}_{t-1} \end{pmatrix} + \begin{pmatrix} \theta_1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \epsilon_{1(t-1)} \\ \epsilon_{2(t-1)} \end{pmatrix} + \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix}$$

We simplify the problem by assuming that

$$\text{Var}\begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix} = \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix}$$

State-space form of a VARMA(1,1) model

Letting $s_{1t} = \Delta \texttt{lncaputil}_t$, $s_{2t} = \theta_1 \epsilon_{1t}$, and $s_{3t} = \Delta \texttt{lnhours}_t$ implies that the state equations are

$$\begin{pmatrix} s_{1t} \\ s_{2t} \\ s_{3t} \end{pmatrix} = \begin{pmatrix} \alpha_1 & 1 & 0 \\ 0 & 0 & 0 \\ \alpha_2 & 0 & \alpha_3 \end{pmatrix} \begin{pmatrix} s_{1(t-1)} \\ s_{2(t-1)} \\ s_{3(t-1)} \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ \theta_1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix}$$

with observation equations

$$\begin{pmatrix} \Delta \texttt{lncaputil} \\ \Delta \texttt{lnhours} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s_{1t} \\ s_{2t} \\ s_{3t} \end{pmatrix}$$

```
. constraint 5  [u1]L.u2        = 1
. constraint 6  [u1]e.u1        = 1
. constraint 7  [u3]e.u3        = 1
. constraint 8 [D.lncaputil]u1  = 1
. constraint 9 [D.lnhours]u3    = 1
```

```
>        (u2 e.u1,           state noconstant)      ///
>        (u3 L.u1 L.u3 e.u3,  state noconstant)      ///
>        (D.lncaputil u1,     noconstant)            ///
>        (D.lnhours u3,       noconstant),           ///
>        constraints(5/9) covstate(diagonal) nolog vsquish nocnsreport
State-space model
Sample: 1972m2 - 2008m12                          Number of obs   =        443
                                                  Wald chi2(4)    =     427.55
Log likelihood =  3156.0564                       Prob > chi2     =     0.0000
```

	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
u1						
u1 L1.	.8058031	.0522493	15.42	0.000	.7033964	.9082098
u2 L1.	1	.	.	.	.	.
e.u1	1	.	.	.	.	.
u2						
e.u1	-.518907	.0701848	-7.39	0.000	-.6564667	-.3813474
u3						
u1 L1.	.1734868	.0405156	4.28	0.000	.0940776	.252896
u3 L1.	-.4809376	.0498574	-9.65	0.000	-.5786563	-.3832188
e.u3	1	.	.	.	.	.
D.lncaputil						
u1	1	.	.	.	.	.
D.lnhours						
u3	1	.	.	.	.	.
var(u1)	.0000582	3.91e-06	14.88	0.000	.0000505	.0000659
var(u3)	.0000382	2.56e-06	14.88	0.000	.0000331	.0000432

```
Note: Tests of variances against zero are conservative and are provided only
      for reference.
```

A local linear-trend model

- The local linear-trend model is a standard unobserved component (UC) model
- Harvey (1989) popularized UC models under the name structural time-series models
- The local-level model

$$
\begin{aligned}
y_t &= \mu_t + \epsilon_t \\
\mu_t &= \mu_{t-1} + \nu_t
\end{aligned}
$$

  models the dependent variable as a random walk plus an idiosyncratic noise term
- The local-level model is already in state-space form

A local-level model for the S&P 500

```
. webuse sp500w, clear
. constraint 10 [z]L.z    = 1
. constraint 11 [close]z = 1
. sspace (z L.z,      state noconstant)           ///
>        (close z,    noconstant),                ///
>        constraints(10 11) nolog
State-space model
Sample: 1 - 3093                               Number of obs    =       3093
Log likelihood =  -12576.99
 ( 1)   [z]L.z = 1
 ( 2)   [close]z = 1
```

close	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
z						
z L1.	1	.	.	.	.	.
close						
z	1	.	.	.	.	.
var(z)	170.3456	7.584909	22.46	0.000	155.4794	185.2117
var(close)	15.24858	3.392457	4.49	0.000	8.599486	21.89767

```
Note: Model is not stationary.
Note: Tests of variances against zero are conservative and are provided only
    for reference.
```

Dynamic-factor models

- Dynamic-factor models model multivariate time series as linear functions of
  - unobserved factors,
  - their own lags,
  - exogenous variables, and
  - disturbances, which may be autoregressive
- The unobserved factors may follow a vector autoregressive structure
- These models are used in forecasting and in estimating the unobserved factors
  - Economic indicators
  - Index estimation
  - Stock and Watson (1989) and Stock and Watson (1991) discuss macroeconomic applications

A dynamic-factor model has the form

$$\mathbf{y}_t = \mathbf{P}\mathbf{f}_t + \mathbf{Q}\mathbf{x}_t + \mathbf{u}_t$$
$$\mathbf{f}_t = \mathbf{R}\mathbf{w}_t + \mathbf{A}_1\mathbf{f}_{t-1} + \mathbf{A}_2\mathbf{f}_{t-2} + \cdots + \mathbf{A}_{t-p}\mathbf{f}_{t-p} + \boldsymbol{\nu}_t$$
$$\mathbf{u}_t = \mathbf{C}_1\mathbf{u}_{t-1} + \mathbf{C}_2\mathbf{u}_{t-2} + \cdots + \mathbf{C}_{t-q}\mathbf{u}_{t-q} + \boldsymbol{\epsilon}_t$$

Item	dimension	definition
$\mathbf{y}_t$	$k \times 1$	vector of dependent variables
$\mathbf{P}$	$k \times n_f$	matrix of parameters
$\mathbf{f}_t$	$n_f \times 1$	vector of unobservable factors
$\mathbf{Q}$	$k \times n_x$	matrix of parameters
$\mathbf{x}_t$	$n_x \times 1$	vector of exogenous variables
$\mathbf{u}_t$	$k \times 1$	vector of disturbances
$\mathbf{R}$	$n_f \times n_w$	matrix of parameters
$\mathbf{w}_t$	$n_w \times 1$	vector of exogenous variables
$\mathbf{A}_i$	$n_f \times n_f$	matrix of autocorrelation parameters for $i \in \{1, 2, \ldots, p\}$
$\boldsymbol{\nu}_t$	$n_f \times 1$	vector of disturbances
$\mathbf{C}_i$	$k \times k$	matrix of autocorrelation parameters for $i \in \{1, 2, \ldots, q\}$
$\boldsymbol{\epsilon}_t$	$k \times 1$	vector of disturbances

Special cases

Dynamic factors with vector autoregressive errors	(DFAR)
Dynamic factors	(DF)
Static factors with vector autoregressive errors	(SFAR)
Static factors	(SF)
Vector autoregressive errors	(VAR)
Seemingly unrelated regression	(SUR)

Syntax for dfactor

`dfactor` *obs_eq* [*fac_eq*] [*if*] [*in*] [, *options*]

obs_eq specifies the equation for the observed dependent variables, and it has the form

(*depvars* = [*exog_d*] [, *sopts*])

fac_eq specifies the equation for the unobserved factors, and it has the form

(*facvars* = [*exog_f*] [, *sopts*])

Among the *sopts* are

`ar(`*numlist*`)`	autoregressive terms
`arstructure(`*arstructure*`)`	structure of autoregressive coefficient matrices
`covstructure(`*covstructure*`)`	covariance structure
`vce(`*vcetype*`)`	*vcetype* may be `oim`, or `robust`

```
. webuse dfex
(St. Louis Fed (FRED) macro data)
. dfactor (D.(ipman income hours unemp) = , noconstant) (f = , ar(1/2)) , nolog
Dynamic-factor model
Sample: 1972m2 - 2008m11                          Number of obs    =       442
                                                  Wald chi2(6)     =    751.95
Log likelihood = -662.09507                       Prob > chi2      =    0.0000
```

	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
f						
f L1.	.2651932	.0568663	4.66	0.000	.1537372	.3766491
L2.	.4820398	.0624635	7.72	0.000	.3596136	.604466
D.ipman						
f	.3502249	.0287389	12.19	0.000	.2938976	.4065522
D.income						
f	.0746338	.0217319	3.43	0.001	.0320401	.1172276
D.hours						
f	.2177469	.0186769	11.66	0.000	.1811407	.254353
D.unemp						
f	-.0676016	.0071022	-9.52	0.000	-.0815217	-.0536816
var(De.ipman)	.1383158	.0167086	8.28	0.000	.1055675	.1710641
var(De.inc~e)	.2773808	.0188302	14.73	0.000	.2404743	.3142873
var(De.hours)	.0911446	.0080847	11.27	0.000	.0752988	.1069903
var(De.unemp)	.0237232	.0017932	13.23	0.000	.0202086	.0272378

```
Note: Tests of variances against zero are conservative and are provided only
      for reference.
```

Multivariate GARCH models

- Multivariate GARCH models allow the conditional covariance matrix of the dependent variables to follow a flexible dynamic structure
- General multivariate GARCH models are under identified
  - There are trade-offs between flexibility and identification
  - Plethora of alternatives
- `dvech` estimates the parameters of diagonal vech GARCH models
  - Each element of the current conditional covariance matrix of the dependent variables depends only on its own past and on past shocks
- Bollerslev, Engle, and Wooldridge (1988); Bollerslev, Engle, and Nelson (1994); Bauwens, Laurent, and Rombouts (2006); Silvennoinen and Teräsvirta (2009) provide good introductions

$$\mathbf{y}_t = \mathbf{C}\mathbf{x}_t + \epsilon_t; \qquad \epsilon_t = \mathbf{H}_t^{1/2}\boldsymbol{\nu}_t$$

$$\mathbf{H}_t = \mathbf{S} + \sum_{i=1}^{p} \mathbf{A}_i \odot \epsilon_{t-i}\epsilon'_{t-i} + \sum_{j=1}^{q} \mathbf{B}_j \odot \mathbf{H}_{t-j}$$

$\mathbf{y}_t$ is an $m \times 1$ vector of dependent variables;
$\mathbf{C}$ is an $m \times k$ matrix of parameters;
$\mathbf{x}_t$ is an $k \times 1$ vector of independent variables, which may contain lags of $\mathbf{y}_t$;
$\mathbf{H}_t^{1/2}$ is the Cholesky factor of the time-varying conditional covariance matrix $\mathbf{H}_t$;
$\boldsymbol{\nu}_t$ is an $m \times 1$ vector of normal, independent, and identically distributed (NIID) innovations;
$\mathbf{S}$ is an $m \times m$ symmetric parameter matrix;
each $\mathbf{A}_i$ is an $m \times m$ symmetric parameter matrix;
$\odot$ is the element-wise or Hadamard product;
and each $\mathbf{B}_j$ is an $m \times m$ symmetric parameter matrix

- Bollerslev, Engle, and Wooldridge (1988) proposed a general vech multivariate GARCH model of the form

$$\begin{aligned}
\mathbf{y}_t &= \mathbf{C}\mathbf{x}_t + \boldsymbol{\epsilon}_t \\
\boldsymbol{\epsilon}_t &= \mathbf{H}_t^{1/2}\boldsymbol{\nu}_t \\
\mathbf{h}_t &= \mathrm{vech}(\mathbf{H}_t) = \mathbf{s} + \sum_{i=1}^{p} \mathbf{A}_i \mathrm{vech}(\boldsymbol{\epsilon}_{t-i}\boldsymbol{\epsilon}'_{t-i}) + \sum_{j=1}^{q} \mathbf{B}_j \mathbf{h}_{t-j}
\end{aligned}$$

  - the vech() function stacks the lower diagonal elements of symmetric matrix into a column vector,

$$\mathrm{vech}\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} = (1,\ 2,\ 3)'$$

- Bollerslev, Engle, and Wooldridge (1988) found this form to be under identified and suggested restricting the $\mathbf{A}_i$ and $\mathbf{B}_i$ to be diagonal matrices

Syntax of dvech

`dvech` *eq* [*eq* ⋯ *eq*] [*if*] [*in*] [, *options*]

where each *eq* has the form

(*depvars* = [*indepvars*], [noconstant])

Some of the options are

noconstant	suppress constant term
arch(*numlist*)	ARCH terms
garch(*numlist*)	GARCH terms
constraints(*numlist*)	apply linear constraints
vce(*vcetype*)	*vcetype* may be oim, or robust

- `tbill` is a secondary market rate of a six month U.S. Treasury bill and `bond` is Moody's seasoned AAA corporate bond yield
- Consider a restricted VAR(1) on the first differences with an ARCH(1) term

```
. webuse irates4
(St. Louis Fed (FRED) financial data)
. dvech (D.bond = LD.bond LD.tbill, noconstant)                    ///
>        (D.tbill = LD.tbill, noconstant), arch(1) nolog
Diagonal vech multivariate GARCH model
Sample: 3 - 2456                                    Number of obs   =      2454
                                                    Wald chi2(3)    =   1197.76
Log likelihood =   4221.433                         Prob > chi2     =    0.0000
```

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
D.bond						
bond LD.	.2941649	.0234734	12.53	0.000	.2481579	.3401718
tbill LD.	.0953158	.0098077	9.72	0.000	.076093	.1145386
D.tbill						
tbill LD.	.4385945	.0136672	32.09	0.000	.4118072	.4653817
Sigma0						
1_1	.0048922	.0002005	24.40	0.000	.0044993	.0052851
2_1	.0040949	.0002394	17.10	0.000	.0036256	.0045641
2_2	.0115043	.0005184	22.19	0.000	.0104883	.0125203
L.ARCH						
1_1	.4519233	.045671	9.90	0.000	.3624099	.5414368
2_1	.2515474	.0366701	6.86	0.000	.1796752	.3234195
2_2	.8437212	.0600839	14.04	0.000	.7259589	.9614836

Bibliography

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