

Difference-in-Differences in Stata 17

StataCorp LLC

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Difference-in-differences (DID)

- One of the most popular causal effects estimators (1855)
- Understand the effect of a treatment on an outcome for the treated group
 - Subsidy on productivity
 - A drug on cholesterol levels
 - An after-school program on GPA

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 - Identification hinges on control for group and time unobservable characteristics

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 - Identification does not depend on controlling for covariates
 - Identification hinges on control for group and time unobservable characteristics
- Estimate of causal effect of a treatment controlling for unobservables

Stata implementation

- Two-way fixed effects also known as generalized DID (default)
- Allows 2x2 design
- Provides a wide range of standard errors
- Provides diagnostics and tests
- Binary or continuous treatment
- Difference-in-difference-in-differences (DDD) with group and time interactions

Stata implementation

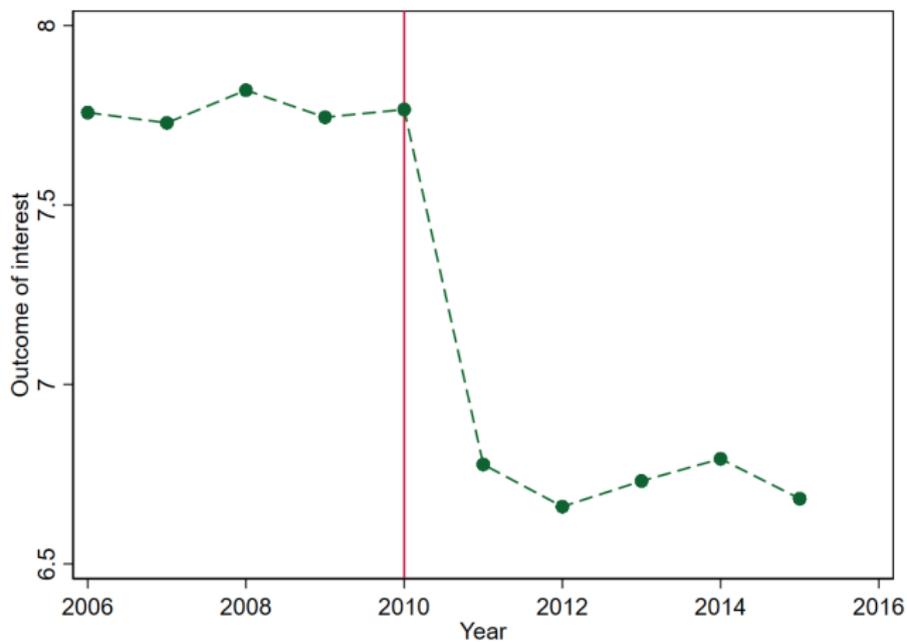
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- Allows 2x2 design
- Provides a wide range of standard errors
- Provides diagnostics and tests
- Binary or continuous treatment
- Difference-in-difference-in-differences (DDD) with group and time interactions
- Caveats
 - Treatment effects are homogeneous
 - Standard error literature is large and growing

Outline

- Basic concepts
- Stata examples

Basic Concepts

Treated group



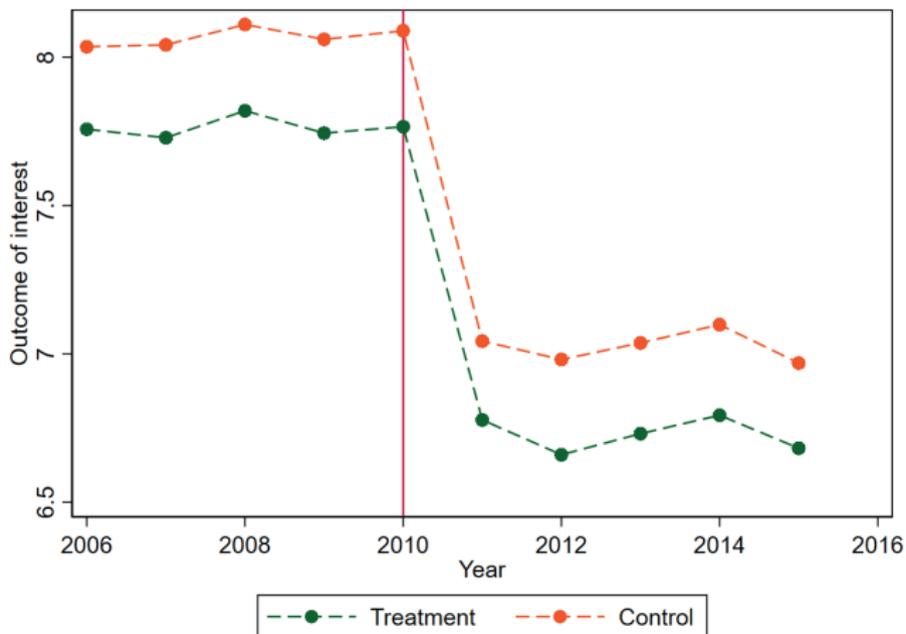
What have we learned

- Clearly there is a change in the outcome after treatment for the treated
- Is it causal?
 - Time specific effects. Another policy. Covid-19.
 - Group unobservable characteristics correlated to covariates. Jargon.

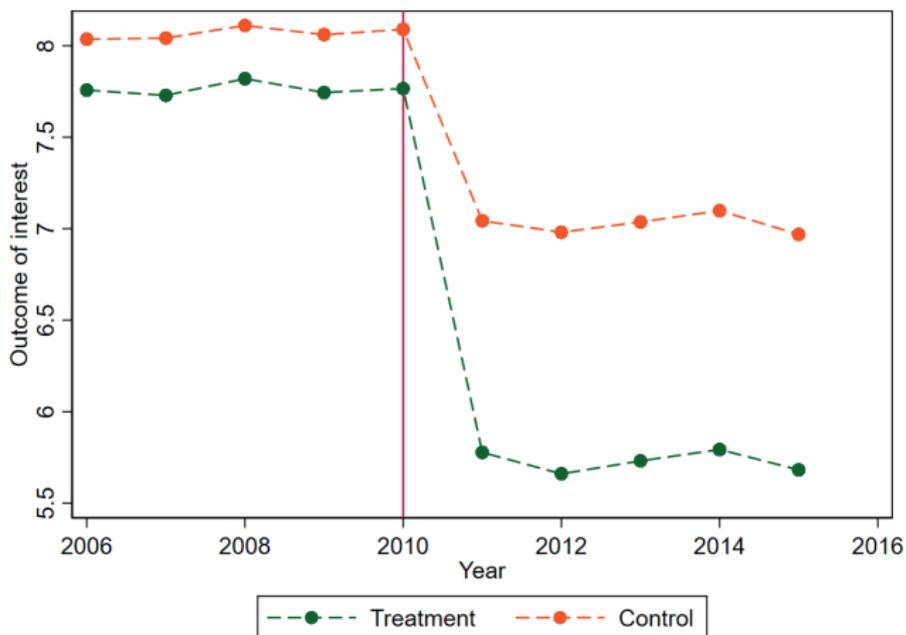
What have we learned

- Clearly there is a change in the outcome after treatment for the treated
- Is it causal?
 - Time specific effects. Another policy. Covid-19.
 - Group unobservable characteristics correlated to covariates. Jargon.
- What can we do?
 - Control for time-specific effects
 - Control for group-specific unobservables (fixed-effects)
 - Use a causal-inference framework

Graphical representation I



Graphical representation II



Card and Krueger (1994)

- Intervention: Increase in the minimum wage
- Group: New Jersey and Pennsylvania
- Outcome: Employment

Linear Framework: Card and Krueger (1994)

- Individuals (i) in a state (s) at two time period $t \in \{0, 1\}$
- Potential outcomes (for now no covariates):

$$E(y_{i0}|s, t) = \lambda_t + \gamma_s$$

$$E(y_{i1}|s, t) = \lambda_t + \gamma_s + \beta$$

- λ_t is a time effect
- γ_s is a state effect

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- y_{i1} is only observed if state s at time t receives the treatment, an increase in minimum wage, $D_{st} = 1$

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- λ_t is a time effect
- γ_s is a state effect
- y_{i1} is only observed if state s at time t receives the treatment, an increase in minimum wage, $D_{st} = 1$
- y_{i0} is only observed if state s at time t does not receive the treatment, $D_{st} = 0$

Card and Krueger (1994) continued

- New Jersey increased minimum wage in April (treatment)
- Neighboring Pennsylvania did not (control)
- Before wage change in February:

$$E(y_{i0}|PA, Feb) = \lambda_{Feb} + \gamma_{PA}$$

$$E(y_{i0}|NJ, Feb) = \lambda_{Feb} + \gamma_{NJ}$$

$$E(y_{i0}|NJ, Feb) - E(y_{i0}|PA, Feb) = \gamma_{NJ} - \gamma_{PA}$$

Card and Krueger (1994) continued

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- The model assumes a common time trend and differing state effects
- Differencing eliminates unobserved time effects

Card and Krueger (1994) continued

- After the minimum wage change, in November:

$$E(y_{i1}|NJ, Nov) - E(y_{i1}|PA, Nov) = \gamma_{NJ} - \gamma_{PA} + \beta$$

- Difference-in-differences looks at differences before and after the policy:

$$[E(y_{i1}|., Nov) - E(y_{i1}|., Nov)] - [E(y_{i0}|., Feb) - E(y_{i0}|., Feb)]$$

Card and Krueger (1994) continued

- After the minimum wage change, in November:

$$E(y_{i1}|NJ, Nov) - E(y_{i1}|PA, Nov) = \gamma_{NJ} - \gamma_{PA} + \beta$$

- Difference-in-differences looks at differences before and after the policy:

$$[E(y_{i1}|., Nov) - E(y_{i1}|., Nov)] - [E(y_{i0}|., Feb) - E(y_{i0}|., Feb)]$$

- The difference in these two differences is β
- It is also the average treatment effect on the treated (ATT)

Parallel trends

- y_{i0} potential outcome of not being treated
- $D_{st} \equiv D$ if group s was treated at time t , $D \in \{0, 1\}$
- s and t are $\in \{0, 1\}$
- At $t = 0$ no one is treated
- Parallel trends:

$$\underbrace{E(y_{i0} | s = 1, D = 1, t = 1)}$$

potential outcome of treated in group $s = 1$ had they remained untreated at $t = 1$

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$$\begin{aligned} E(y_{i0} | s = 1, D = 1, t = 1) - E(y_{i0} | s = 1, D = 1, t = 0) = \\ E(y_{i0} | s = 0, D = 1, t = 1) - E(y_{i0} | s = 0, D = 1, t = 0) \end{aligned}$$

- Could be conditional on covariates

Observed Outcome and Estimating equation

$$E(y_i|s, t) = D_{st}E(y_{i1}|s, t) + (1 - D_{st})E(y_{i0}|s, t)$$

$$E(y_i|s, t) = D_{st}(\lambda_t + \gamma_s + \beta) + (1 - D_{st})(\lambda_t + \gamma_s)$$

$$E(y_i|s, t) = \lambda_t + \gamma_s + D_{st}\beta$$

- This suggests fitting a regression model with a dummy variable D_{st}
- The specification could have regressors

Generalized DID or two-way fixed effects

$$y_{its} = \gamma_s + \gamma_t + D_{st}\beta + \varepsilon_{its}$$

- D_{st} is an observation level indicator of treatment $D_{st} \in \{0, 1\}$
- In panel data if individuals are nested in s individual effect absorb state effects
- You may include covariates in the specification above

2 x 2 specification DID

$$y_{its} = \gamma \mathbb{1}_{treated} + \gamma \mathbb{1}_{post} + \mathbb{1}_{treated} \times \mathbb{1}_{post} \beta + \varepsilon_{its}$$

- Works when all units are treated at the same time (balanced)
- This model is nested in the generalized DID
 - $\mathbb{1}_{treated}$ is a linear combination of the group dummies
 - $\mathbb{1}_{post}$ is a linear combination of the time dummies
- This model assumes all post periods and all treatment groups are equivalent.

Alternative specifications

- D_{St} is not binary but continuous (intensity of treatment)
- Differences occur between two groups (differencing two group unobservables)
- DDD or triple differences. It incorporates unobservables from two control groups.
 - Number of parameters is large
 - Identification is more challenging

Standard error computation

Treatment occurs at the group level, state, county, country, etc. and time

- Cluster at the group level Bertrand, Dufflo, Mullainathan (2004)
- Few number of elements in the group:
 - Donald and Lang (2007) aggregation and other aggregation methods
 - Wild-cluster bootstrap
 - Bias-corrected standard errors with Bell and McCaffrey (2002) degrees of freedom adjustment
 - Other suggestions

Stata Examples

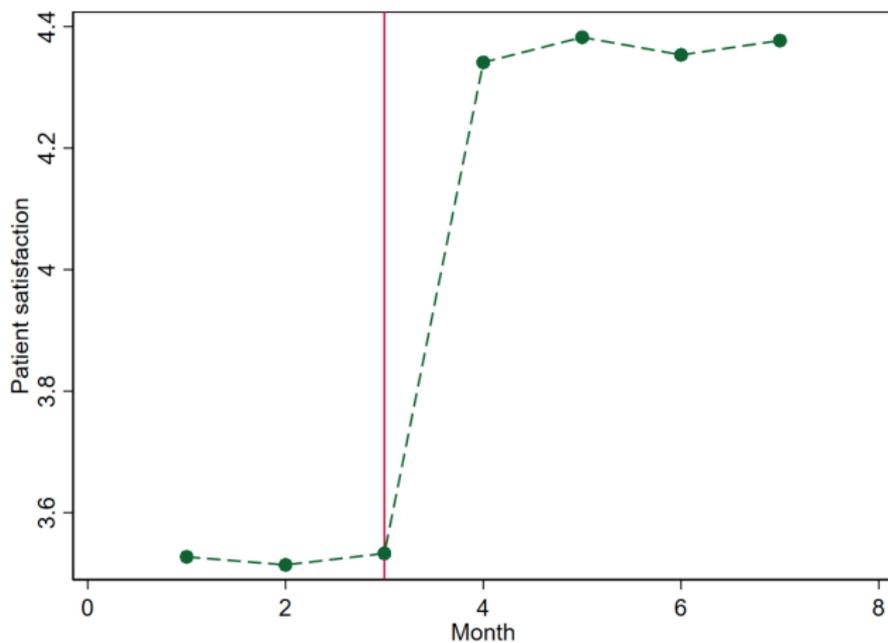
Artificial data

```
. webuse hospdd, clear
(Artificial hospital admission procedure data)
. describe
Contains data from https://www.stata-press.com/data/r17/hospdd.dta
Observations:           7,368           Artificial hospital admission
                               procedure data
Variables:                5           7 Mar 2021 19:52
```

Variable name	Storage type	Display format	Value label	Variable label
hospital	byte	%9.0g		Hospital ID
frequency	byte	%9.0g	size	Hospital visit frequency
month	byte	%8.0g	mnth	Month
procedure	byte	%9.0g	pol	Admission procedure
satis	float	%9.0g		Patient satisfaction score

Sorted by: hospital

Graphical representation III



Estimation

```
. didregress (satis) (procedure), group(hospital) time(month)
Number of groups and treatment time
Time variable: month
Control:      procedure = 0
Treatment:    procedure = 1
```

	Control	Treatment
Group		
hospital	28	18
Time		
Minimum	1	4
Maximum	1	4

```
Difference-in-differences regression          Number of obs = 7,368
Data type: Repeated cross-sectional
              (Std. err. adjusted for 46 clusters in hospital)
```

satis	Coefficient	Robust std. err.	t	P> t	[95% conf. interval]	
ATEE procedure (New vs Old)	.8479879	.0321121	26.41	0.000	.7833108	.912665

Note: ATEE estimate adjusted for group effects and time effects.

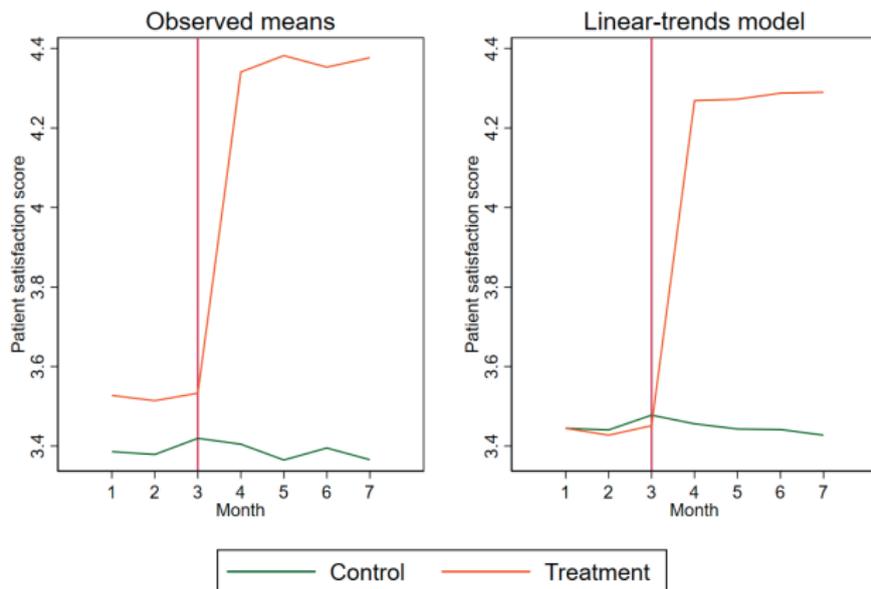
Diagnostic plots

```
estat trendplot
```

- First plot: Mean of the outcome for treated and untreated units
- Second plot: Trend of treated and control groups (group interacted with time)

Diagnostic plots

Graphical diagnostics for parallel trends



Tests: estat ptrends

```
. estat ptrends
Parallel-trends test (pretreatment time period)
H0: Linear trends are parallel
F(1, 45) = 0.55
Prob > F = 0.4615
```

- Augmented model with trends for treated vs. control group before and after treatment. Test if the pretreatment trends are parallel.

Tests: estat granger

```
. estat granger
Granger causality test
H0: No effect in anticipation of treatment
F(2, 45) = 0.33
Prob > F = 0.7239
```

- Augment the model to include dummies as if treatment had occurred in the past. Test coefficients jointly.

A 2×2 specification

- Create dummy variables for treated group and post time period
- Tell `didregress` not to include group and time effects
- Add dummies to the outcome equation

A 2×2 specification

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- Tell `didregress` not to include group and time effects
- Add dummies to the outcome equation

```
. bysort hospital: egen treated = mean(procedure)
. replace treated = 1 if treated>0
(3,064 real changes made)
. generate post = 0
. replace post = 1 if month>3
(3,684 real changes made)
```

A 2×2 specification

```
. didregress (satis treated post) (procedure), ///  
>          group(hospital) time(month) nogteffects
```

Number of groups and treatment time

Time variable: month

Control: procedure = 0

Treatment: procedure = 1

	Control	Treatment
Group		
hospital	28	18
Time		
Minimum	1	4
Maximum	1	4

Difference-in-differences regression

Number of obs = 7,368

Data type: Repeated cross-sectional

(Std. err. adjusted for 46 clusters in hospital)

	Coefficient	Robust std. err.	t	P> t	[95% conf. interval]	
satis						
ATET						
procedure						
(New						
vs						
Old)	.8479879	.0320051	26.50	0.000	.7835263	.9124494

Note: ATET estimate adjusted for covariates.

Difference-in-difference-in-differences DDD

- Augmented DID

Difference-in-difference-in-differences DDD

- Augmented DID
- Selection on unobservables provides identification
- What if there are unobservables that vary at the group and time level
- Find a new group not exposed to treatment but exposed to the problematic time-varying confounder
- Subtract the effect of that group from the original DID

Difference-in-difference-in-differences DDD

- Augmented DID
- Selection on unobservables provides identification
- What if there are unobservables that vary at the group and time level
- Find a new group not exposed to treatment but exposed to the problematic time-varying confounder
- Subtract the effect of that group from the original DID
- In our example think about individuals frequency of visit affecting satisfaction

DDD preparing my data

```
. generate hightrt = procedure==1 & (frequency==3 | frequency==4)
. label define trt 0 "Untreated" 1 "Treated"
. label values hightrt trt
```

DDD estimation

```
. didregress (satis) (hightrt), group(hospital frequency) time(month)  
  (output omitted)
```

Number of groups and treatment time

Time variable: month

Control: hightrt = 0

Treatment: hightrt = 1

	Control	Treatment
Group		
hospital	28	18
frequency	2	2
Time		
Minimum	1	4
Maximum	1	4

Triple-differences regression

Number of obs = 7,368

Data type: Repeated cross-sectional

(Std. err. adjusted for 46 clusters in hospital)

satis	Coefficient	Robust std. err.	t	P> t	[95% conf. interval]	
ATET						
hightrt (Treated vs Untreated)	.764154	.0402603	18.98	0.000	.6830655	.8452425

Note: ATET estimate adjusted for group effects, time effects, and group- and time-effects interactions.

Other estimation alternatives

- `didregress (y x1 ... xk) (c, continuous), ...`
- `didregress (y ...) (d...), group(g1 g2)`
- `xtdidregress (y x1 ... xk) (d), group(groupvar) time(timevar)`

Standard error considerations

- Default standard errors are cluster robust standard errors at the group level BDM (2004)
- `didregress` is equivalent to `areg` considers group fixed effects as regressors in the degrees of freedom adjustment
- `xtdidregress` is equivalent to `xtreg` does not consider group fixed effects as regressors
- When the number of elements per groups (states, counties, countries) is small cluster robust standard errors do not work well. Alternatives are:
 - Wild cluster bootstrap
 - Bias corrected standard errors
 - Aggregation methods

Wild-cluster bootstrap

- Covariates remain the same across iteration
- We impose the null hypothesis of $ATE_T = 0$
- What changes is the weights given to residuals at each iteration
 $\tilde{y} = X\tilde{\beta} + \tilde{\varepsilon}$, $\tilde{\beta}$, and $\tilde{\varepsilon} = \hat{\varepsilon} * w$
- No standard errors are computed (rely on normal approximation)
- P-values and confidence intervals are computed
- Algorithm computes p-values and then solves a bisection-algorithm to get CI
- Problem to find CI upper bound and CI lower bound are two separate optimization problems

Error weights

Error weight	Formula
rademacher	-1 with pr 0.5 and 1 with pr 0.5
mammen	$1 - \phi$ with pr $\phi/\sqrt{5}$, ϕ otherwise, $\phi = (1 + \sqrt{5})/2$
webb	$-\sqrt{3/2}, -\sqrt{2/2}, -\sqrt{1/2}, \sqrt{1/2}, \sqrt{2/2}, \sqrt{3/2}$ pr 1/6
normal	standard normal
gamma	shape parameter 4 scale parameter 1/2

Wildbootstrap I

```
. didregress (satis) (procedure), ///  
>         group(hospital) time(month) wildbootstrap(rseed(111))  
computing 1000 replications  
Finding p-value  
..... 50%  
..... 100%  
Confidence interval lower bound  
.....  
Confidence interval upper bound  
.....  
    (output omitted)
```

Wildbootstrap II

```
. didregress (satis) (procedure), ///  
>   group(hospital) time(month) wildbootstrap(rseed(111))  
   (output omitted)
```

Number of groups and treatment time

Time variable: month

Control: procedure = 0

Treatment: procedure = 1

	Control	Treatment
Group		
hospital	28	18
Time		
Minimum	1	4
Maximum	1	4

DID with wild-cluster bootstrap inference

Number of obs = 7,368

No. of clusters = 46

Replications = 1,000

Data type: Repeated cross-sectional

Error weight: rademacher

satis	Coefficient	t	P> t	[95% conf. interval]	
ATET					
procedure					
(New vs Old)	.8479879	26.41	0.000	.7806237	.9157614

Note: ATET estimate adjusted for group effects and time effects.

Bias-corrected standard errors

- Cluster generalization of HC2 (scale residuals inverse of square of diagonals from projection matrix)
- Bell and McCaffrey (2002) suggest a degrees of freedom adjustment (per parameter)

Bias-corrected SEs

```
. didregress (satis) (procedure), group(hospital) time(month) vce(hc2)
Computing degrees-of-freedom:
procedure .....
Number of groups and treatment time
Time variable: month
Control:      procedure = 0
Treatment:    procedure = 1
```

	Control	Treatment
Group		
hospital	28	18
Time		
Minimum	1	4
Maximum	1	4

Difference-in-differences regression

Number of obs = 7,368

No. of clusters = 46

Data type: Repeated cross-sectional

satis	Robust HC2		t	P> t	[95% conf. interval]	
	Coefficient	std. err.				
ATET						
procedure						
(New						
vs						
Old)	.8479879	.0325552	26.05	0.000	.7819941	.9139816

Note: ATET estimate adjusted for group effects and time effects.

Degrees of freedom adjustment

```
. mat list r(table)
r(table)[9,9]
      ATET:   Controls:   Controls:   Controls:   Controls:   Controls:
      r1vs0.   1b.         2.         3.         4.         5.
      procedure month      month      month      month      month
      b      .84798786      0      -.00960766      .02196858      -.00328387      -.00940274
      se      .03255515      .      .01836738      .01817606      .02210113      .02325151
      t      26.047731      .      -.52308262      1.2086544      -.14858393      -.40439255
      pvalue  3.558e-25      .      .60348306      .23310851      .88254581      .68783978
      ll      .7819941      .      -.04660147      -.01463989      -.04779783      -.05623368
      ul      .91398163      .      .02738615      .05857705      .04123009      .0374282
      df      36.496106      45      45      45      45      45
      crit   2.0271372      2.0141034      2.0141034      2.0141034      2.0141034      2.0141034
      eform   0      0      0      0      0      0
      Controls:   Controls:   Controls:
      6.         7.
      month      month      _cons
      b      -.00383754      -.01119415      3.444675
      se      .01906173      .0230133      .01140018
      t      -.2013216      -.48642083      302.15965
      pvalue  .84135438      .62902945      4.517e-76
      ll      -.04222984      -.0575453      3.4217139
      ul      .03455476      .03515701      3.4676362
      df      45      45      45
      crit   2.0141034      2.0141034      2.0141034
      eform   0      0      0
```

Aggregation methods

$$y_{its} = \gamma_s + \gamma_t + z_{1ist}\beta_1 + z_{2st}\beta_2 + D_{st}\delta + \varepsilon_{ist}$$

$$y_{ist} = z_{1ist}\beta_2 + C_{st} + \varepsilon_{ist}$$

$$\widehat{C}_{st} = z_{2st}\beta_2 + D_{st}\delta + \nu_{st}$$

- Obtain \widehat{C}_{st}
- Aggregate at the s, t level and regress
 - dlang, constant: regress \widehat{C}_{st} on z_{2st}, D_{st} and time and group fixed effects, degrees of freedom are a function of the level of aggregation st
 - standard: regress \widehat{C}_{st} on z_{2st}, D_{st}
 - dlang, varying: \widehat{C}_{st} is the constant of a regression of each group defined by st , i.e. β_1 is not constant but varying.

aggregate(dlang)

```
. didregress (satis) (procedure), group(hospital) time(month) aggregate(dlang)
Number of groups and treatment time
Time variable: month
Control:      procedure = 0
Treatment:    procedure = 1
```

	Control	Treatment
Group		
hospital	28	18
Time		
Minimum	1	4
Maximum	1	4

```
Difference-in-differences regression      Number of obs = 322
Data type: Repeated cross-sectional
Aggregation: Donald-Lang
```

satis	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
ATET procedure (New vs Old)	.8500467	.0255727	33.24	0.000	.7997311	.9003623

Note: ATET estimate adjusted for group effects and time effects.

aggregate(standard)

```
. didregress `specs', group(hospital) time(month) aggregate(standard) vce(hc2)
Computing degrees-of-freedom:
procedure .....
Number of groups and treatment time
Time variable: month
Control:      procedure = 0
Treatment:    procedure = 1
```

	Control	Treatment
Group		
hospital	28	18
Time		
Minimum	1	4
Maximum	1	4

Difference-in-differences regression

Number of obs = 322

No. of clusters = 46

Data type: Repeated cross-sectional

Aggregation: Standard

	Coefficient	Robust HC2 std. err.	t	P> t	[95% conf. interval]	
ATET						
procedure (New vs Old)	.8500467	.0329513	25.80	0.000	.7832444	.916849

Note: ATET estimate adjusted for group effects and time effects.

Conclusions

- DID and DDD estimation for cross-sectional and panel-data
- Graphical diagnostics and tests to validate identification strategy
- Standard errors for situations with the number of groups is small
- Just a first step from which we will build