友万科技

Conditional Average Treatment-Effects Estimation using Stata

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ATE versus CATE

- ATE is a popular way to measure the treatment effects.

$$ATE = \mathbf{E}[y(1) - y(0)] \tag{1}$$

When each individual or group has different (heterogeneous) treatment effects, ATE may oversimplify the treatment effects.

- Conditional Average Treatment Effects (CATE) measure the treatment effects conditional on a set of variables.

$$CATE = \mathbf{E}[y(1) - y(0)|\mathbf{x}] \tag{2}$$

CATE measures the treatment effects as a function of $\mathbf{x}$.

Advantages of studying CATE

1. It improves the understanding of the treatment-effect heterogeneity.
   - Are the treatment effects heterogeneous?
   - How do the treatment effects vary with some variables?
   - Do the treatment effects vary between prespecified groups?
   - Do the data discover groups where treatment effects are different?
2. It helps to evaluate the treatment-assignment policy.
   - If we implement a treatment-assignment policy, how would the average outcome in the population change?
   - Which policy is better among a candidate set of policies?

Different versions of CATE

$$CATE = \mathbf{E}[y(1) - y(0)|\mathbf{x}]$$

Depending on the definition of $\mathbf{x}$, CATE helps us to understand the heterogeneous treatment effects at different levels.

- **IATE**: Individualized average treatment effects when $\mathbf{x}$ is individual characteristics (finest level of treatment effects).
- **GATE**: Group average treatment effects when $\mathbf{x}$ is a group (prespecified group analysis).
- **GATES**: Sorted group average treatment effects when $\mathbf{x}$ ranks IATEs (data-driven group hypothesis testing).

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The `cate` suite (I)

- **Estimation**:
  - `cate po` estimates IATE function (partialling-out model)
  - `cate aipw` estimates IATE function (AIPW model)
  - `cate ..., group(`*varname*`)` estimates GATE
  - `cate ..., group(`*#*`)` estimates GATES
- **Prediciton**: `predict` observational level IATEs, its standard error and CI
- **Visualization**
  - `categraph histogram`: histogram of predictions of IATEs
  - `categraph gateplot`: plot of GATE or GATES
  - `categraph iateplot`: plot of the IATE function

The `cate` suite (II)

- **Inference**:
  - `estat heterogeneity`: Heterogeneous treatment-effects test
  - `estat gatetest`: GATE or GATES heterogeneity test
  - `estat classification`: Classification analysis of the data-driven groups
  - `estat ate`: ATE for a subpopulation
  - `estat projection`: IATE function linear approximation
  - `estat series`: IATE function series approximation
  - `estat policyeval`: Treatment-assignment policy evaluation

Methodological building blocks

- Generalized random forest: estimates the IATE function $\tau(\mathbf{x}) = \mathbf{E}[y(1) - y(0)|\mathbf{x}]$ (Athey et al., 2019)
- Debiased/double machine learning: partialling-out and AIPW estimators + cross-fitting (Athey et al. 2019, Semenova and Chernozhukov 2021, Nie and Wager 2021, Kennedy 2020, and Knaus 2022)

Benefits of modern methods:

1. Flexible IATE estimation without assuming parametric assumptions
2. High-dimensional controls in both the outcome and the treatment models
3. Guard against machine learning bias

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Partial linear outcome model

- We want to estimate the effect of 401(k) eligibility on net financial assets.
$$\mathbf{E}[\text{asset}(1) - \text{asset}(0)|\mathbf{x}]$$
where $\mathbf{x}$ are individual characteristics such as income, age, education, pension, marital status, etc.
- The outcome model is
$$\text{asset} = \text{e401k} * \tau(\mathbf{x}) + g(\mathbf{x}, \mathbf{w}) + \epsilon$$
where $\mathbf{w}$ is high-dimensional controls.
- So the potential outcomes are
$$\text{asset}(1) = \tau(\mathbf{x}) + g(\mathbf{x}, \mathbf{w}) + \epsilon$$
$$\text{asset}(2) = g(\mathbf{x}, \mathbf{w}) + \epsilon$$
- Thus,
$$\mathbf{E}[\text{asset}(1) - \text{asset}(0)|\mathbf{x}] = \tau(\mathbf{x})$$

Partialling-out estimator

$$\text{asset} = \text{e401k} * \tau(\mathbf{x}) + g(\mathbf{x}, \mathbf{w}) + \epsilon \tag{3}$$

$$\text{e401k} = f(\mathbf{x}, \mathbf{w}) + u \tag{4}$$

Taking conditional expecation in eq. (3) on both sides

$$\mathbf{E}[\text{asset}|\mathbf{x}, \mathbf{w}] = f(\mathbf{x}, \mathbf{w}) * \tau(\mathbf{x}) + g(\mathbf{x}, \mathbf{w}) \tag{5}$$

Eq. (3) minus (5) partialled-out $g(\mathbf{x}, \mathbf{w})$

$$\overbrace{\text{asset} - \mathbf{E}[\text{asset}|\mathbf{x}, \mathbf{w}]}^{\widetilde{\text{asset}}} = \overbrace{(\text{e401k} - f(\mathbf{x}, \mathbf{w}))}^{\widetilde{\text{e401k}}} * \tau(\mathbf{x}) + \epsilon \tag{6}$$

Estimate $\tau(\mathbf{x})$ by solving a local moment condition via generalized random forest.

$$\mathbf{E}\left[\alpha(\mathbf{x}) * \widetilde{\text{e401k}} * \left(\widetilde{\text{asset}} - \widetilde{\text{e401k}} * \tau(\mathbf{x})\right)\right] = 0$$

Load data

```
. webuse assets3
(Excerpt from Chernozhukov and Hansen (2004))
. global catecovars age educ i.(incomecat pension married twoearn ira ownhome)
.
. global fvars incomecat pension married twoearn ira ownhome
. global controls c.(educ age)#i.($fvars)
```

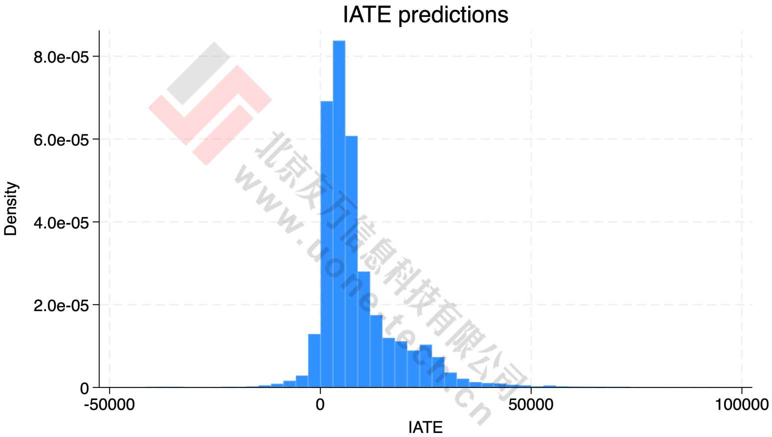
- `catecovars` refers to $\mathbf{x}$
- `controls` refers to $\mathbf{w}$

Using `cate` to estimate IATE

```
. cate po (assets $catecovars) (e401k), rseed(12345671) controls($controls) nol
> og

Conditional average treatment effects     Number of observations         = 9,913
Estimator:        Partialing out          Number of folds in cross-fit =    10
Outcome model:    Linear lasso            Number of outcome controls    =    47
Treatment model:  Logit lasso             Number of treatment controls =    47
CATE model:       Random forest           Number of CATE variables      =    17

------------------------------------------------------------------------------
             |               Robust
      assets | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
ATE          |
       e401k |
   (Eligible |
          vs |
 Not elig..) |   8107.563   1144.817     7.08   0.000     5863.763    10351.36
-------------+----------------------------------------------------------------
POmean       |
       e401k |
 Not eligi.. |   13902.88   838.5924    16.58   0.000     12259.27    15546.49
------------------------------------------------------------------------------
```

The output shows ATE. Under the hood, `cate` also estimates a nonparametric function for IATE via generalized random forest.

`categraph histogram`: IATE predictions histogram



IATE distribution has a fat right tail, so the ATE possibly overestimates the treatment effects.

`estat heterogeneity`: test of treatment-effects heterogeneity

```
. estat heterogeneity

Treatment-effects heterogeneity test
H0: Treatment effects are homogeneous

    chi2(1) =   4.19
Prob > chi2 = 0.0406
```

We reject the null hypothesis of homogenenous treatment effects. In other words, treatment effects are heterogeneous.

`estat projection`: linear projection of IATE

```
. estat projection
Treatment-effects linear projection                 Number of obs =      9,913
                                                    F(11, 9901)   =       5.12
                                                    Prob > F      =     0.0000
                                                    R-squared     =     0.0047
                                                    Adj R-squared =     0.0036
                                                    Root MSE      = 1.138e+05
```

	Coefficient	Robust std. err.	t	P>|t|	[95% conf.	interval]
age	164.3654	116.2698	1.41	0.157	-63.54715	392.2779
educ	-440.1495	472.5372	-0.93	0.352	-1366.419	486.1197
incomecat						
1	-3093.247	1981.377	-1.56	0.119	-6977.15	790.6558
2	2216.006	2195.87	1.01	0.313	-2088.346	6520.359
3	6116.068	3244.506	1.89	0.059	-243.8253	12475.96
4	18355.28	5321.146	3.45	0.001	7924.749	28785.81
pension						
Receives ..	4320.983	2439.087	1.77	0.076	-460.1247	9102.09
married						
Married	-2103.475	3370.329	-0.62	0.533	-8710.007	4503.056
twoearn						
Yes	-1957.787	4326.422	-0.45	0.651	-10438.45	6522.88
ira						
Yes	-1284.949	3578.426	-0.36	0.720	-8299.392	5729.495
ownhome						
Yes	2963.537	1630.756	1.82	0.069	-233.0765	6160.15
_cons	1728.235	7880.15	0.22	0.826	-13718.46	17174.93

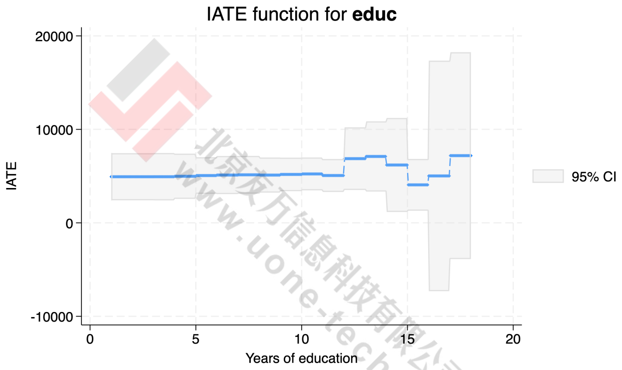
`categraph iateplot` (I): IATE function plot

```
. categraph iateplot educ
Note: IATE estimated at fixed values of covariates other than educ.
```

Variable	Statistic	Value	Type
age	mean	41.05891	continuous
incomecat	base	0	factor
ira	base	0	factor
married	base	0	factor
ownhome	base	0	factor
pension	base	0	factor
twoearn	base	0	factor

Notice that $\tau(\mathbf{x})$ is a function of several parameters when $dim(\mathbf{x}) > 1$. To plot a multiple dimension function, we fix all the variables to specific values except `educ`.

`categraph iateplot` (II)



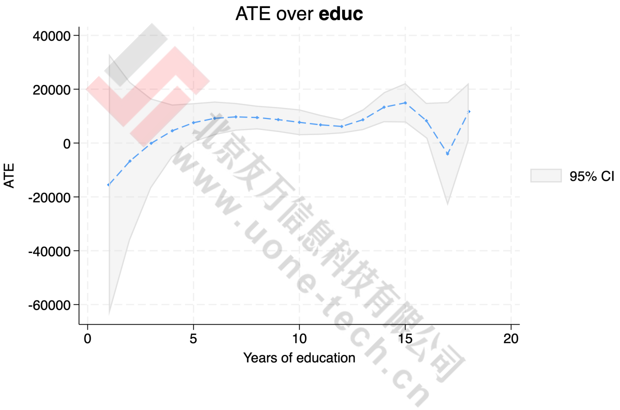
Think about this graph as a slice in a bread in a specific direction. Each point is $\tau(\mathbf{x})$ when $\mathbf{x}$ takes a specific value. For example, $\mathbf{E}[y(1) - y(0)|educ = 10, others = fixed]$.

`estat series`: ATE over a continuous variable

```
. estat series educ, graph
warning: you have entered variable educ as continuous but it only has 18
        distinct values.  The estimates may differ substantially if you
        inadvertently include a discrete variable as continuous

Computing approximating function

Minimizing cross-validation criterion

Iteration 0:  Cross-validation criterion =  1.30e+10
Iteration 1:  Cross-validation criterion =  1.30e+10

Computing average derivatives

Nonparametric series regression for IATE
Cubic B-spline estimation                  Number of obs      =          9,913
Criterion: cross-validation                Number of knots    =              3
------------------------------------------------------------------------------
             |               Robust
             |     Effect   std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
        educ |   2532.489   1377.915     1.84   0.066    -168.1735    5233.152
------------------------------------------------------------------------------
Note: Effect estimates are averages of derivatives.
```

The output shows the marginal effects of education on the treatment effects.

`estat series`: ATE over a continuous variable



Notice that each point shows the ATE if the education is fixed at a specific value. For example, $E[y(1) - y(0)|educ = 10]$.

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Using `cate ..., group(`*varname*`)` for GATE

We want to know the effects of `e401k` on `asset` for each income category.

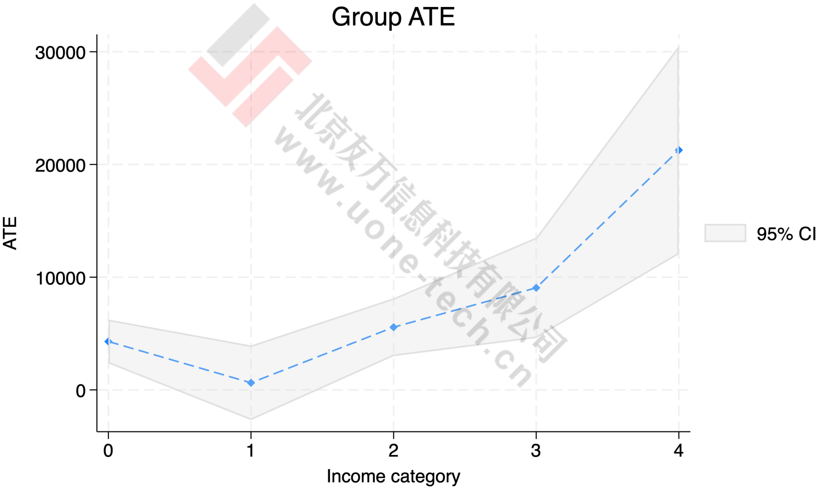
```
. cate aipw (assets $catecovars) (e401k), rseed(12345671) ///
>         controls($controls) group(incomecat) nolog

Conditional average treatment effects     Number of observations        = 9,913
Estimator:          Augmented IPW          Number of folds in cross-fit =    10
Outcome model:      Linear lasso           Number of outcome controls   =    47
Treatment model:    Logit lasso            Number of treatment controls =    47
CATE model:         Random forest          Number of CATE variables     =    17

------------------------------------------------------------------------------
             |               Robust
      assets | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
GATE         |
   incomecat |
          0  |   4295.829   992.7063     4.33   0.000      2350.16    6241.497
          1  |   628.2236   1690.636     0.37   0.710    -2685.362    3941.809
          2  |    5562.85   1310.006     4.25   0.000     2995.284    8130.415
          3  |   9058.087   2276.042     3.98   0.000     4597.125    13519.05
          4  |   21275.42   4716.757     4.51   0.000     12030.74    30520.09
-------------+----------------------------------------------------------------
ATE          |
       e401k |
   (Eligible |
          vs |
Not elig..)  |   8164.364   1151.125     7.09   0.000       5908.2    10420.53
-------------+----------------------------------------------------------------
POmean       |
       e401k |
Not eligi..  |   13910.87   842.0945    16.52   0.000     12260.39    15561.34
------------------------------------------------------------------------------
```

`categraph gateplot`: Visualize GATE

```
. categraph gateplot
```



`estat gatetest`: Test GATE homogeneity

```
. estat gatetest

Group treatment-effects heterogeneity test
H0: Group average treatment effects are homogeneous

 ( 1)  [GATE]0bn.incomecat - [GATE]1.incomecat = 0
 ( 2)  [GATE]0bn.incomecat - [GATE]2.incomecat = 0
 ( 3)  [GATE]0bn.incomecat - [GATE]3.incomecat = 0
 ( 4)  [GATE]0bn.incomecat - [GATE]4.incomecat = 0

    chi2(4) =   22.39
Prob > chi2 = 0.0002
```

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Using `cate ..., group(#)` for GATES

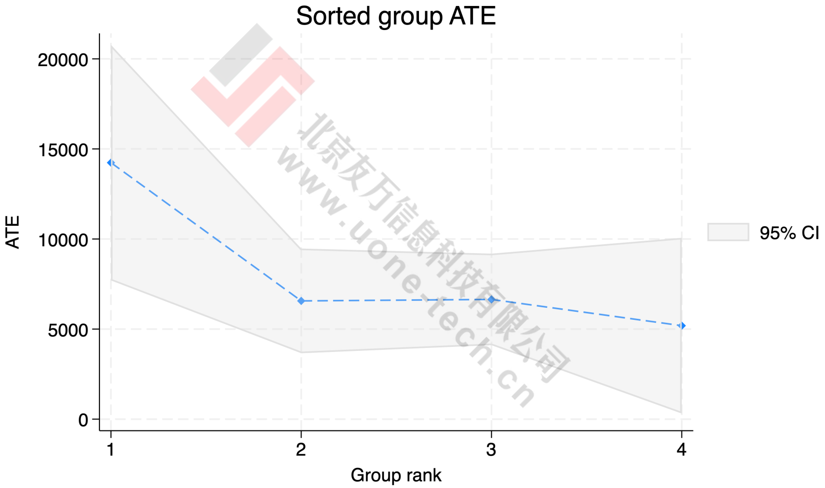
```
. cate aipw (assets $catecovars) (e401k), rseed(12345671) ///
>         controls($controls) group(4) nolog

Conditional average treatment effects     Number of observations       = 9,913
Estimator:        Augmented IPW           Number of folds in cross-fit =    10
Outcome model:    Linear lasso            Number of outcome controls   =    47
Treatment model: Logit lasso              Number of treatment controls =    47
CATE model:       Random forest           Number of CATE variables     =    17
```

assets	Coefficient	Robust std. err.	z	P>|z|	[95% conf.	interval]
GATES						
rank						
1	14238.01	3335.108	4.27	0.000	7701.317	20774.7
2	6565.533	1482.069	4.43	0.000	3660.732	9470.334
3	6646.957	1294.802	5.13	0.000	4109.191	9184.723
4	5190.023	2487.992	2.09	0.037	313.6494	10066.4
ATE						
e401k (Eligible vs Not elig..)	8164.364	1151.125	7.09	0.000	5908.2	10420.53
POmean						
e401k Not eligi..	13910.87	842.0945	16.52	0.000	12260.39	15561.34

The group is defined by the IATE quantiles in a cross-fitting manner.
So higher rank does not necessarily imply higher ATE.

`categraph gateplot`: Visualize GATES

```
. categraph gateplot
```



In this example, group 1 has higher ATE than group 4. We can test it!

`estat gatetest`: Test GATES homogeneity

```
. estat gatetest 1 4

Sorted group treatment-effects heterogeneity test
H0: Sorted group average treatment effects are homogeneous

 ( 1)  [GATES]1bn.rank - [GATES]4.rank = 0

    chi2(1) =   4.73
Prob > chi2 = 0.0297
```

- The test rejects the null hypothesis of homogeneous GATE between groups 1 and 4. So people in group 1 have higer ATE than those in group 4.
- Question: Do the people in groups 1 and 4 have different characteristics?

`estat classification`: Classification analysis

Question: Is a variable's mean different between groups 1 and 4?

```
. estat classification ownhome
Classification t test with equal variances
------------------------------------------------------------------------------
   Group |     Obs        Mean    Std. err.   Std. dev.   [95% conf. interval]
---------+--------------------------------------------------------------------
       1 |   2,482    .8585818    .0069957    .3485227    .8448638    .8722998
       4 |   2,469    .4641555    .0100387    .4988145    .4444703    .4838407
---------+--------------------------------------------------------------------
Combined |   4,951    .6618865    .0067239    .4731152    .6487047    .6750683
---------+--------------------------------------------------------------------
    diff |            .3944263    .0122248                .3704603    .4183923
------------------------------------------------------------------------------
    diff = mean(1) - mean(4)                                      t =  32.2645
H0: diff = 0                                     Degrees of freedom =     4949

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 1.0000         Pr(|T| > |t|) = 0.0000          Pr(T > t) = 0.0000
.
. estat classification age
Classification t test with equal variances
------------------------------------------------------------------------------
   Group |     Obs        Mean    Std. err.   Std. dev.   [95% conf. interval]
---------+--------------------------------------------------------------------
       1 |   2,482    45.52337    .1785502    8.895315    45.17325    45.87349
       4 |   2,469     37.2017    .2236204    11.11148     36.7632     37.6402
---------+--------------------------------------------------------------------
Combined |   4,951    41.37346    .1547295    10.88729    41.07012     41.6768
---------+--------------------------------------------------------------------
    diff |            8.321667    .2859933                7.760993    8.882341
------------------------------------------------------------------------------
    diff = mean(1) - mean(4)                                      t =  29.0974
H0: diff = 0                                     Degrees of freedom =     4949

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 1.0000         Pr(|T| > |t|) = 0.0000          Pr(T > t) = 0.0000
```

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Treatment-assignment policy

- Policy value:

$$\Pi(\pi) = \mathbf{E}\left[\pi_i y_i(1) + (1-\pi_i) y_i(0)\right] \tag{7}$$

  where $\pi_i \in [0,1]$ is a prespecified treatment-assignment probability for the *i*th observation. π_i is also refered to as the policy.

- Notice that, from IATE estimates, we can already estimate $y(1)$ and $y(0)$. Thus, policy evaluation is closely related to CATE.
- Compare two policies:

$$\Pi\left(\pi_A\right) - \Pi\left(\pi_B\right) \tag{8}$$

ATE is a special case of policy comparison

Let $\pi_A = 1$ and $\pi_B = 0$. Then

$$\begin{aligned} ATE &= \mathbf{E}[y(1)] - \mathbf{E}[y(0)] \\ &= \mathbf{E}[1 * y(1) + 0 * y(0)] - \mathbf{E}[0 * y(1) + 1 * y(0)] \\ &= \Pi(\pi_A) - \Pi(\pi_B) \end{aligned}$$

Thus, ATE is the contrast of the two special policy values. π_A means treat all the units, while π_B means treat none.

Lung transplant treatment-assignment policy evaluation

```
. webuse lung, clear
(Fictional data on lung transplant)

.
. global cvars bmip heightp o2amt lungals centervol walkdist      ///
>         bmid heightd distd lungpo2 hratio ischemict

. global fvars diabetesp karn racep sexp lifesvent assisvent      ///
>         o2rest raced smoked cmv deathcause diabetesd            ///
>         expandd sexd lungalloc genderm racem

.
. global controls c.($cvars)#i.($fvars)

.
. global catecovars c.($cvars) i.($fvars)
```

Treatment: Bilateral lung transplant vs. single lung transplant

Outcome: Forced expiratory volume in one second relative to a healthy person

Using `cate` to estimate IATE

```
. cate aipw (fev1p $catecovars) (transtype), rseed(12345671)      ///
>         controls($controls) nolog

Conditional average treatment effects      Number of observations        = 937
Estimator:        Augmented IPW            Number of folds in cross-fit =  10
Outcome model:    Linear lasso             Number of outcome controls   = 454
Treatment model:  Logit lasso              Number of treatment controls = 454
CATE model:       Random forest            Number of CATE variables     =  46

------------------------------------------------------------------------------
             |               Robust
       fev1p | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
ATE          |
   transtype |
        (BLT |
          vs |
        SLT) |    37.5243   .1646795   227.86   0.000     37.20153    37.84707
-------------+----------------------------------------------------------------
POmean       |
   transtype |
         SLT |   46.49502   .2025403   229.56   0.000     46.09805      46.892
------------------------------------------------------------------------------
```

Replicate ATE

```
. generate treatall = 1

. generate treatnone = 0

.
. estat policyeval treatall treatnone

Treatment-assignment policy evaluation                    Number of obs = 937

------------------------------------------------------------------------------
             |               Robust
             | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
Value        |
      policy |
    treatall |   84.01932   .3085432   272.31   0.000     83.41459    84.62406
   treatnone |   46.49502   .2025403   229.56   0.000     46.09805      46.892
-------------+----------------------------------------------------------------
Contrast     |
      policy |
   (treatall |
          vs |
  treatnone) |    37.5243   .1646795   227.86   0.000     37.20153    37.84707
------------------------------------------------------------------------------
```

Compare hypothetical policy with the observed policy

Hypothetical policy: Assigns patient to BLT if the patient's walking distance is greater than 500 meters in 6 mins and if the patient does not have diabetes.

```
. generate policy1 = walkdist > 500 & !diabetesp & !missing(walkdist)

.
. estat policyeval policy1 transtype
Treatment-assignment policy evaluation                  Number of obs = 937
```

	Coefficient	Robust std. err.	z	P>|z|	[95% conf.	interval]
Value						
policy						
policy1	72.66426	.714435	101.71	0.000	71.26399	74.06452
transtype	66.53891	.5149955	129.20	0.000	65.52954	67.54828
Contrast						
policy						
(policy1 vs transtype)	6.125348	.9130896	6.71	0.000	4.335725	7.91497

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AIPW scores are useful

- We have PO and AIPW estimators. PO is for the partial linear model, and AIWP is for the fully interactive model. For both models, we can derive the AIPW scores.
- AIPW scores are essential computational elements in the IATE estimator (AIPW estimator), linear projection, series projection, GATE, GATES, and policy evaluation.

We will illustrate the use of AIPW scores using the fully interactive model.

Fully interactive model

$$y(1) = g_1(\mathbf{x}, \mathbf{w}) + \epsilon_1 \tag{9}$$

$$y(0) = g_0(\mathbf{x}, \mathbf{w}) + \epsilon_0 \tag{10}$$

$$d = f(\mathbf{x}, \mathbf{w}_2) + u \tag{11}$$

The AIPW version of the potential outcomes are

$$y(1)_{AIPW} = g_1(\mathbf{x}, \mathbf{w}) + \frac{I(d=1)(y - g_1(\mathbf{x}, \mathbf{w}))}{f(\mathbf{x}, \mathbf{w}_2)} \tag{12}$$

$$y(0)_{AIPW} = g_0(\mathbf{x}, \mathbf{w}) + \frac{I(d=0)(y - g_0(\mathbf{x}, \mathbf{w}))}{1 - f(\mathbf{x}, \mathbf{w}_2)} \tag{13}$$

We can estimate the function $g_1(\mathbf{x}, \mathbf{w})$, $g_0(\mathbf{x}, \mathbf{w})$, and $f(\mathbf{x}, \mathbf{w}_2)$, so we can also estimate $y(1)_{AIPW}$ and $y(0)_{AIPW}$.
Let

$$\widehat{\Gamma} = \widehat{y(1)}_{AIWP} - \widehat{y(0)}_{AIWP} \tag{14}$$

The creative use of AIPW scores

- For IATE, solve $\tau(\mathbf{x})$ in

$$\sum_{i=1}^{N}[\alpha(\mathbf{x}_i)(\widehat{\Gamma}_i - \tau(\mathbf{x}))] = 0$$

  We use $\widehat{\Gamma}$ as the dependent variable in a machine learning prediction problem.

- For GATE or GATES, we

  `regress` $\widehat{\Gamma}$ on `i.groupvar`

  Mean of $\widehat{\Gamma}$ within each group.

- For linear or series projection, we do linear or series projection of $\widehat{\Gamma}$ on the specific variables.

- For policy evaluation, we need to evaluate the weighted mean of the AIPW potential outcomes.

$$\Pi(\pi) = \mathbf{E}[\pi_i y(1)_{AIPW} + (1-\pi_i)y(0)_{AIWP}] \tag{15}$$

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Discussion

What can `cate` do ?

- Study treatment-effects heterogeneity at different levels (IATE, GATE, and GATES) in cross-sectional data
- Policy evaluation
- Nonparametric (GRF), semiparametric (LASSO), or parametric (add linear interaction term) estimation of IATE
- Cross-fitting to guard against machine learning mistakes

The features that I wish to have in the future:

- Clustered data and panel data
- Optimal policy evaluation

Thank you! Your suggestions?

References

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