Heterogeneous Difference-in-Differences in Stata

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"TWFE" and heterogeneous treatment effects

Setup

- Estimate treatment effects using panel data or repeated cross-section
- Treatments start at different times
- Staggered treatment

Problems with using "TWFE"

$$y_{it} = \theta_t + \eta_i + d_{it}\alpha + v_{it}$$

1. Treatment effects heterogeneity in two dimensions (cohort and time)?
2. Summarize treatment effects with a single number?

We are asking too much from "TWFE"! See Goodman-Bacon (2021) and de Chaisemartin and D'Haultfœuille (2020)

Simulation example

$$ATET(g,t) = \begin{cases} t - g + 1, & \text{if } g <= 3 \\ -(t - g + 1), & \text{if } g > 3 \end{cases}$$

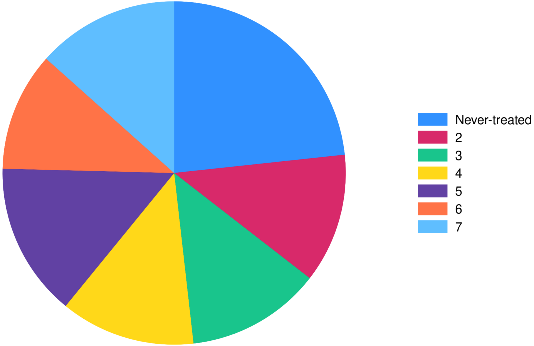
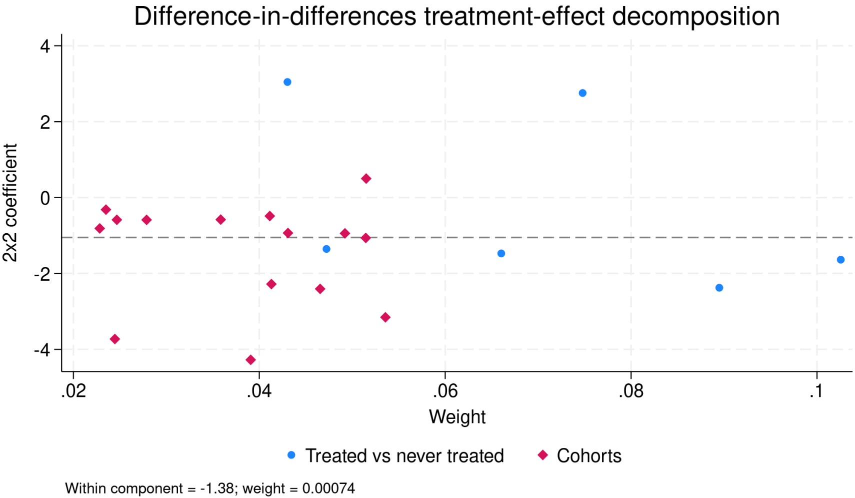


Figure 1: Cohorts distribution

Try "TWFE"

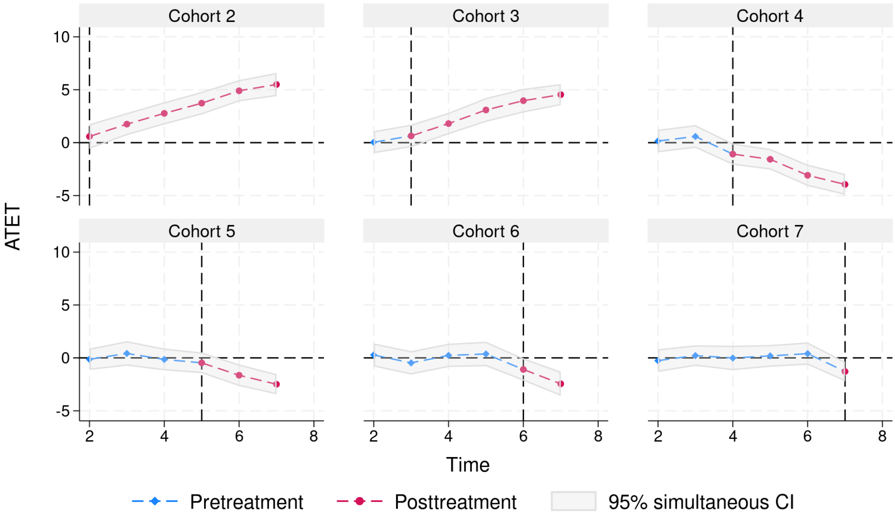

```
. xtdidregress (y x*) (treat), group(id) time(time)
Treatment and time information

Time variable: time
Control:       treat = 0
Treatment:     treat = 1
-----------------------------------
             |   Control  Treatment
-------------+---------------------
Group        |
          id |       233        767
-------------+---------------------
Time         |
     Minimum |         1          2
     Maximum |         1          7
-----------------------------------

Difference-in-differences regression                 Number of obs = 7,000
Data type: Longitudinal

                                (Std. err. adjusted for 1,000 clusters in id)
------------------------------------------------------------------------------
             |               Robust
           y | Coefficient  std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
ATET         |
       treat |
   (1 vs 0)  |  -1.052117   .0747087   -14.08   0.000    -1.198721    -.905513
------------------------------------------------------------------------------
Note: ATET estimate adjusted for covariates, panel effects, and time effects.
Note: Treatment occurs at different times.
```

- `estat bdcomp, graph`

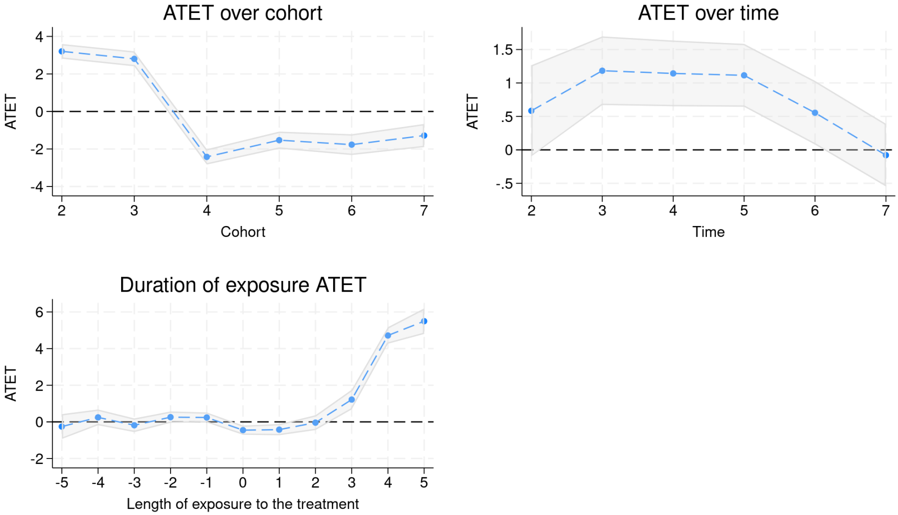


xthdidregress

- `xthdidregress aipw (y x*) (treat), group(id)`
- `estat atetplot, sci`



estat aggregation



Overview of heterogeneous DID in Stata 18

Estimation:

1. `xthdidregress` and `hdidregress` for panel data and repeated cross-section data
2. Four estimators: `ra`, `ipw`, `aipw` in Callaway and Sant'Anna (2021) and `twfe` in Wooldridge (2021)

Post-estimation:

1. `estat atetplot`: visualize ATETs
2. `estat aggregation`: aggregate ATETs along different dimensions
3. `estat ptrends`: pre-treatment parallel trend tests
4. `estat sci`: simultaneous CI for RA, IPW, and AIPW estimators

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Potential outcome framework

Some notations:

- $y_{i,t}(g)$ is unit i's potential outcome at time t if it starts treatment at time g
- $y_{i,t}(\infty)$ is unit i's potential outcome at time t if it is never treated
- G_i indicates unit i's cohort (when the treatment starts), and it is one element in $\mathbf{G} = \{2, \ldots, G, \infty\}$
- Thus, there are G potential outcomes for each unit i at time t
- $y_{i,t}$ is unit i's observed outcome at time t

$$y_{i,t} = \underbrace{\mathbb{1}(t < G_i)y_{i,t}(\infty)}_{\text{before treatment}} + \underbrace{\mathbb{1}(t \geq G_i)\sum_{g \in \mathbf{G}} \mathbb{1}(G_i = g)y_{i,t}(g)}_{\text{after treatment}}$$

Heterogeneous ATETs

$$ATET(g,t) = \mathbf{E}[y_{i,t}(g) - y_{i,t}(\infty)|G_i = g]$$

Remarks

- $ATET(g,t)$ is a function of two arguments: cohort g and time t
- *ATETs* can be heterogeneous over cohorts, across time, across both time and cohorts
- Objective: consistently estimate ATETs and summarize them

Key assumptions

- Observe I.I.D samples of $\{y_{i,t}, \mathbf{x}_{i,t}, \mathbf{z}_{i,t}, d_{i,t}\}_{i=1,t=1}^{i=N,t=T}$, where $\mathbf{x}_{i,t}$ and $\mathbf{z}_{i,t}$ are covariates, and $d_{i,t}$ is observational level treatment indicator
- No one is treated in the first period
- No anticipation in pre-treatment periods $t < g$

$$\mathbf{E}[y_{i,t}(g)|\mathbf{x}, G_i = g] = \mathbf{E}[y_{i,t}(\infty)|\mathbf{x}, G_i = g]$$

- Conditional parallel trend

$$\mathbf{E}[y_{i,t}(\infty) - y_{i,t-1}(\infty)|\mathbf{x}, G_i = g] = \mathbf{E}[y_{i,t}(\infty) - y_{i,t-1}(\infty)|\mathbf{x}, G_i = \infty]$$

- Overlap assumption for propensity scores

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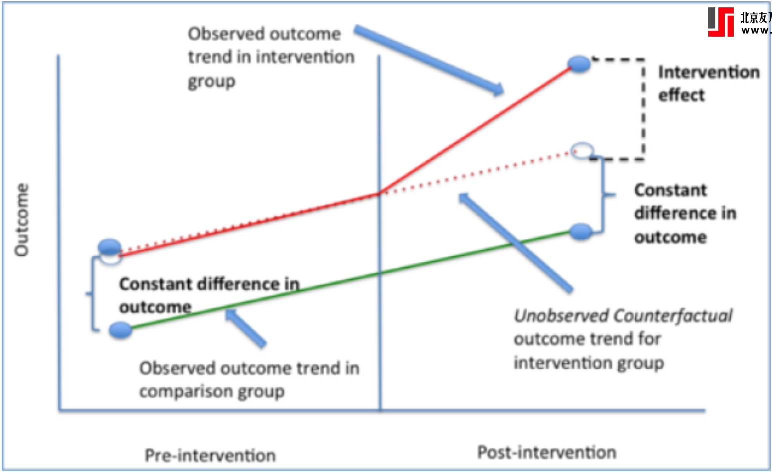


$$ATET = E[\underbrace{Y_t(treat=1)}_{Y_{t,treat}} - \underbrace{Y_t(treat=0)}_{Y_{t,control}+(Y_{t-1,treat}-Y_{t-1,control})} \,|treat=1]$$

$$= E[\left(Y_{t,treat} - Y_{t-1,treat}\right) - \left(Y_{t,control} - Y_{t-1,control}\right) |treat=1]$$

Regression adjustment (RA)

$$
\begin{aligned}
ATET(g,t) &= \mathbf{E}\left[\frac{K_g}{\mathbf{E}(K_g)}\left(y_t - y_{g-1} - m_{g,t}\right)\right] \\
&= \underbrace{\mathbf{E}\left[y_t - y_{g-1} | K_g = 1\right]}_{\text{treated differences}} - \underbrace{\mathbf{E}\left[m_{g,t}(\mathbf{x}) | K_g = 1\right]}_{\text{untreated differences}}
\end{aligned}
$$

where

- $K_g = \mathbb{1}(G_i = g)$ and $m_{g,t}(\mathbf{x}) = \mathbf{E}(y_t - y_{g-1} | \mathbf{x}, G_i = \infty)$
- It is 2×2 difference-in-differences (two groups $\times$ two periods)
- Benchmark time: one period before treatment $(g - 1)$
- Benchmark group: never-treated group $(G_i = \infty)$

In Stata, we type

- `xthdidregress ra` $\underbrace{(y \quad \mathbf{x})}_{m_{g,t}(\mathbf{x})}$ `(d), group(id)`

Inverse probability weighting (IPW)

$$ATET(g,t) = \mathbf{E}\left[\left(\frac{K_g}{\mathbf{E}(K_g)} - \frac{\frac{p_g(\mathbf{z})K_\infty}{1-p_g(\mathbf{z})}}{\mathbf{E}\left[\frac{p_g(\mathbf{z})K_\infty}{1-p_g(\mathbf{z})}\right]}\right)(Y_t - Y_{g-1})\right]$$

where

- $p_g(\mathbf{z}) = \Pr(K_g = 1|\mathbf{z}, K_g + K_\infty = 1) = \frac{\Pr(K_g=1|\mathbf{z})}{\Pr(K_g+K_\infty=1|\mathbf{z})}$
- $\frac{p_g(\mathbf{z})}{1-p_g(\mathbf{z})} = \frac{\Pr(K_g=1|\mathbf{z})}{\Pr(K_\infty=1|\mathbf{z})}$. Thus, in the benchmark group (never treated), attach more weights to observations that are more probably observed in the cohort g
- We estimate $p_g(\mathbf{z})$ by a logit regression

In Stata, we type

- `xthdidregress ipw (y)` $\underbrace{(d\ \ \mathbf{z})}_{p_g(\mathbf{z})}$`, group(id)`

Augmented inverse probability weighting (AIPW)

$$ATET(g,t) = \mathbf{E}\left[\underbrace{\left(\frac{K_g}{\mathbf{E}(K_g)} - \frac{\frac{p_g(\mathbf{z})K_\infty}{1-p_g(\mathbf{z})}}{\mathbf{E}\left[\frac{p_g(\mathbf{z})K_\infty}{1-p_g(\mathbf{z})}\right]} \right) \left(Y_t - Y_{g-1} - \right.}_{\text{IPW}} \left. \overbrace{m_{g,t}(\mathbf{x})}^{\text{augmented term}} \right) \right]$$

- AIPW is doubly robust: only one of the outcome model or the treatment model needs to be correctly specified

In Stata, we type

- `xthdidregress aipw` $\underbrace{(y\ \ \mathbf{x})}_{m_{g,t}(\mathbf{x})}$ $\underbrace{(d\ \ \mathbf{z})}_{p_g(\mathbf{z})}$, `group(id)`

TWFE

Traditional TWFE:

$$y_{it} = \theta_t + \eta_i + d_{it}\alpha + v_{it}$$

TWFE in Wooldridge (2021):

$$y_{it} = \theta_t + \eta_i + \sum_{g \in \mathbf{G}} \sum_{s=g}^{T} \alpha_{g,t} \mathbb{1}(G_i = g, t = s) + v_{it}$$

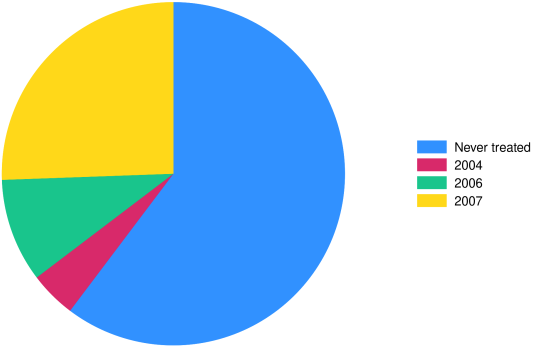
With covariates $\mathbf{x}$, add full interactions with θ_t, η_i, and $\mathbb{1}(G_i = g, t = s)$.

In Stata, we type

- `xthdidregress twfe` $\underbrace{(y \quad \mathbf{x})}_{\text{TWFE outcome}}$ `(d), group(id)`

Example: minimum wage and young employment

- **Outcome**: county-level employment for young workers
- **Treatment**: minimum wage restrictions introduced by State government; see Callaway and Sant'Anna (2021)
- **Multiple periods**: 2002 - 2007 (6 years)
- **Multiple treatment timings**: 2004, 2006, 2007



xthdidregress aipw

Define covariates

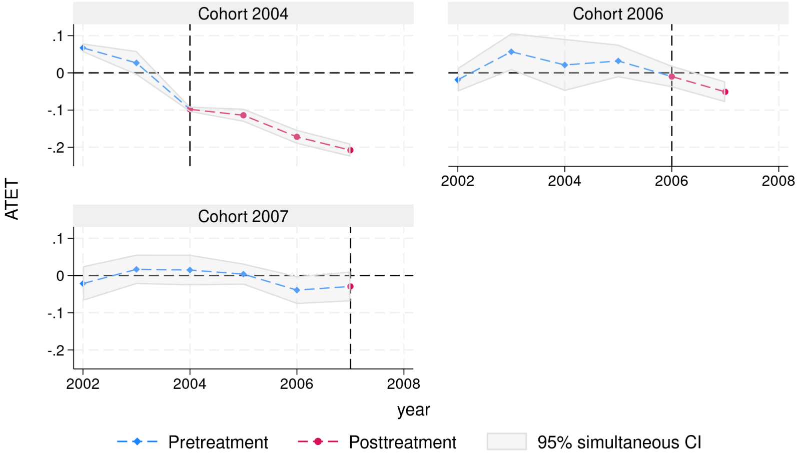
```
global covars i.region pop medinc white hs pov ///
       c.pop#c.pop c.medinc#c.medinc
```

Use AIPW estimator

```
xthdidregress aipw (lemp $covars) (treat $covars), group(state)
```

- Adding covariates for conditional parallel trend
- There are 18 ATET(g,t)'s (6 years $\times$ 3 cohorts)
- Standard errors are adjusted by clusters of state

estat atetplot, sci



- Specify option `sci` for simultaneous confidence intervals
- For cohorts 2004 and 2006, minimum wage restriction decreases the employment rate for young workers

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Summarize ATET(g,t)'s

- How do the ATETs vary with the length of exposure to the treatment? (event study)
- How do the ATETs vary with cohorts? (does start treatment earlier matter?)
- How do the ATETs vary with time? (Good year vs. lousy year)
- Overall ATETs across time and cohorts

We can express the aggregations as a weighted mean of all ATETs

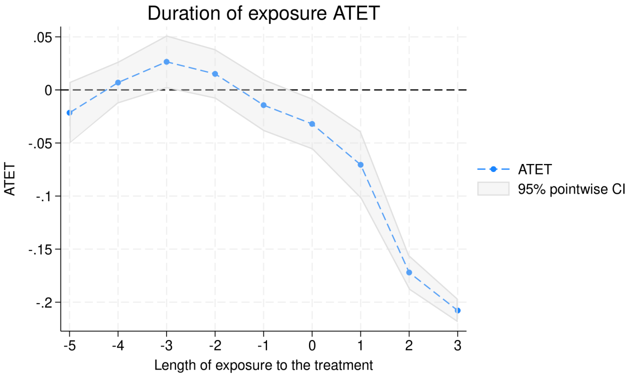
$$\theta = \sum_{g \in \mathbf{G}} \sum_{t=2}^{T} \underbrace{w(g,t)}_{\text{weight}} ATET(g,t)$$

Event study

- Let $e = t - g$ be the length of exposure to the treatment.

$$\theta(\mathbf{e}) = \sum_{g \in \mathbf{G}} \underbrace{\mathbb{1}(g + e \leq T) \Pr(G = g | g + e \leq T)}_{\text{propotions used to estimate ATET(g, g+e)}} ATET(g, g + e)$$

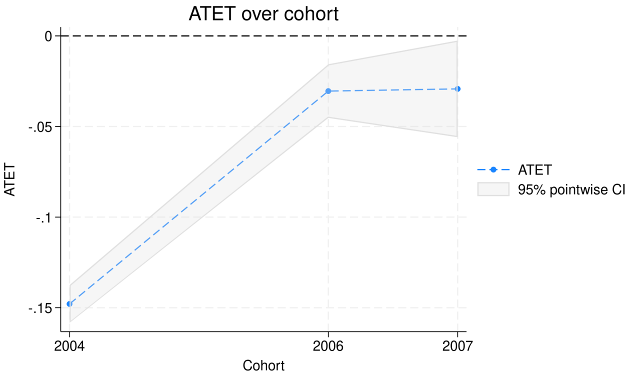
- `estat aggregation,` **dynamic** `graph`



ATETs over cohort

$$\theta(\mathbf{g}) = \sum_{t=g}^{T} \underbrace{\Pr(G = g | G = g, t \geq g)}_{\text{proportions used to estimate post-treatment ATET(g,t)}} \; ATET(g,t)$$

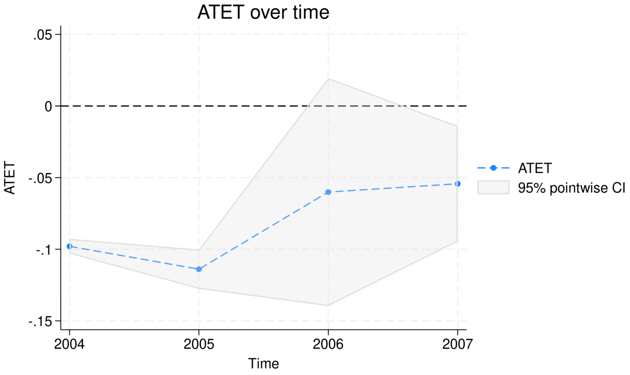
- `estat aggregation,` **cohort** `graph`



ATETs across time

$$\theta(\mathbf{t}) = \sum_{g \in \mathbf{G}} \mathbb{1}(t \geq g)\,\Pr(G = g | G \leq t) ATET(g, t)$$

- `estat aggregation,` **time** `graph`



Overall aggregations

- A single number to summarize ATET's

$$\theta = \frac{1}{\kappa} \sum_{g \in \mathbf{G}} \sum_{t=2}^{T} \mathbb{1}(t \geq g) \Pr(G = g | G \leq T) ATET(g, t)$$

- `estat aggregation,` **overall**

```
. estat aggreg, overall
Overall ATET                                        Number of obs = 15,988
                                  (Std. err. adjusted for 29 clusters in state)
```

lemp	ATET	Robust std. err.	z	P>|z|	[95% conf.	interval]
treat (1 vs 0)	-.062811	.0256879	-2.45	0.014	-.1131582	-.0124637

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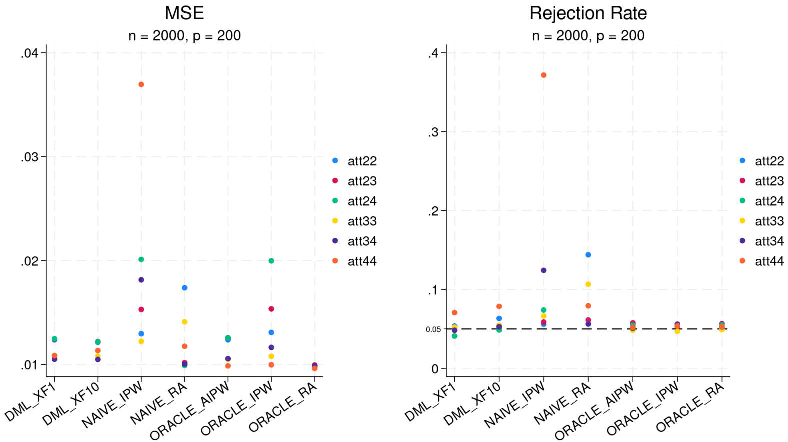
Double machine learning AIPW estimator

- Recall the AIPW estimator

$$ATET(g,t) = \mathbf{E}\left[\left(\frac{K_g}{\mathbf{E}(K_g)} - \frac{\frac{p_g(\mathbf{z})K_\infty}{1-p_g(\mathbf{z})}}{\mathbf{E}\left[\frac{p_g(\mathbf{z})K_\infty}{1-p_g(\mathbf{z})}\right]}\right)(Y_t - Y_{g-1} - m_{g,t}(\mathbf{x}))\right]$$

- AIPW is not only doubly robust but also Neyman orthognal
- Allowing high-dimensional covariates in $p_g(\mathbf{z})$ and $m_{g,t}(\mathbf{x})$
- Combining the cross-fitting with the AIPW scores
- For details, see the working paper Callaway, Drukker, Liu, and Sant'Anna (2023)

Simulations with high-dimensional covariates



- Double machine learning AIPW (DML_XF1, DML_XF10)
- Naive estimators for IPW and RA
- Oracle estimators for RA, IPW, and AIPW

References

Callaway, B., D. Drukker, D. Liu, and P. Sant'Anna. 2023. Double/Debiased Machine-learning estimator for Difference-in-Difference with Multiple Periods. URL `https://www.doi.org/10.13140/RG.2.2.33815.65447`.

Callaway, B., and P. H. Sant'Anna. 2021. Difference-in-Differences with multiple time periods. *Journal of Econometrics* 225: 200–230.

de Chaisemartin, C., and X. D'Haultfœuille. 2020. Two-Way Fixed Effects Estimators with Heterogeneous Treatment Effects. *American Economic Review* 110: 2964–2996.

Goodman-Bacon, A. 2021. Difference-in-differences with variation in treatment timing. *Journal of Econometrics* 225: 254–277.

Wooldridge, J. M. 2021. Two-Way Fixed Effects, the Two-Way Mundlak Regression, and Difference-in-Differences Estimators. URL `https://ssrn.com/abstract=3931952`.

Appendix

Contents

1 xthdidregress

```
. global covars i.region pop medinc white hs pov c.pop#c.pop c.medinc#c.medinc

. xthdidregress aipw (lemp $covars) (treat $covars), group(state)
note: variable _did_cohort, containing cohort indicators formed by treatment
      variable treat and group variable state, was added to the dataset.

Computing ATET for each cohort and time:
Cohort 2004 (6): ...... done
Cohort 2006 (6): ...... done
Cohort 2007 (6): ...... done

Treatment and time information

Time variable: year
Time interval: 2001 to 2007
Control:       _did_cohort = 0
Treatment:     _did_cohort > 0
```

	_did_cohort
Number of cohorts	4
Number of obs	
Never treated	9639
2004	700
2006	1561
2007	4088

```
Heterogeneous-treatment-effects regression          Number of obs    = 15,988
                                                    Number of panels =     29
Estimator:        Augmented IPW
Panel variable:   countyreal
Treatment level:  state
Control group:    Never treated

                                (Std. err. adjusted for 29 clusters in state)
```

Cohort	ATET	Robust std. err.	z	P>|z|	[95% conf.	interval]
2004						
year						
2002	.0672458	.0061125	11.00	0.000	.0552655	.079226
2003	.0266718	.0122508	2.18	0.029	.0026608	.0506829
2004	-.0979371	.002649	-36.97	0.000	-.103129	-.0927451
2005	-.1139248	.0070092	-16.25	0.000	-.1276627	-.1001869
2006	-.1719979	.0082852	-20.76	0.000	-.1882366	-.1557592
2007	-.2078132	.0056814	-36.58	0.000	-.2189485	-.196678
2006						
year						
2002	-.0186685	.0105915	-1.76	0.078	-.0394274	.0020904
2003	.056737	.0181748	3.12	0.002	.0211151	.0923589
2004	.0212315	.0363779	0.58	0.559	-.0500679	.092531
2005	.0319911	.0158191	2.02	0.043	.0009863	.0629959
2006	-.009851	.0117487	-0.84	0.402	-.0328781	.013176
2007	-.0510452	.0092241	-5.53	0.000	-.069124	-.0329664

2007						
year						
2002	-.0215125	.014779	-1.46	0.145	-.0504788	.0074538
2003	.0167167	.0132905	1.26	0.208	-.0093322	.0427655
2004	.0149363	.0133763	1.12	0.264	-.0112809	.0411534
2005	.0038453	.0092391	0.42	0.677	-.014263	.0219537
2006	-.0390546	.0114977	-3.40	0.001	-.0615896	-.0165196
2007	-.0292338	.0136042	-2.15	0.032	-.0558976	-.00257

Note: ATET computed using covariates.

2 estat aggregation, dynamic

```
. estat aggreg, dynamic graph(name(d1))
```

Duration of exposure ATET   Number of obs = 15,988

(Std. err. adjusted for 29 clusters in state)

Exposure	ATET	Robust std. err.	z	P>|z|	[95% conf.	interval]
-5	-.0215125	.014779	-1.46	0.145	-.0504788	.0074538
-4	.0069386	.0100519	0.69	0.490	-.0127627	.0266399
-3	.0264872	.0126915	2.09	0.037	.0016122	.0513621
-2	.0151101	.0118987	1.27	0.204	-.0082109	.0384311
-1	-.0143403	.0124878	-1.15	0.251	-.0388159	.0101353
0	-.032043	.0122219	-2.62	0.009	-.0559975	-.0080885
1	-.0705126	.0161956	-4.35	0.000	-.1022553	-.0387699
2	-.1719979	.0082852	-20.76	0.000	-.1882366	-.1557592
3	-.2078132	.0056814	-36.58	0.000	-.2189485	-.196678

Note: Exposure is the number of periods since the first treatment time.

3 estat aggregation, cohort

```
. estat aggreg, cohort graph(name(c1))
```

ATET over cohort   Number of obs = 15,988

(Std. err. adjusted for 29 clusters in state)

Cohort	ATET	Robust std. err.	z	P>|z|	[95% conf.	interval]
2004	-.1479183	.0053113	-27.85	0.000	-.1583283	-.1375082
2006	-.0304481	.0075561	-4.03	0.000	-.0452578	-.0156384
2007	-.0292338	.0136042	-2.15	0.032	-.0558976	-.00257

4 estat aggregation, time

```
. estat aggreg, time graph(name(t1))

ATET over time                                    Number of obs = 15,988

                               (Std. err. adjusted for 29 clusters in state)
```

Time	ATET	Robust std. err.	z	P>|z|	[95% conf.	interval]
2004	-.0979371	.002649	-36.97	0.000	-.103129	-.0927451
2005	-.1139248	.0070092	-16.25	0.000	-.1276627	-.1001869
2006	-.0600513	.0406199	-1.48	0.139	-.1396648	.0195622
2007	-.0542855	.0206191	-2.63	0.008	-.0946981	-.0138728