Tools to Analyze Interest Rates and Value Bonds

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Executive summary

- Bond markets contain a wealth of information about investor expectations
  - Observed market rates tell us returns bond investors require today to invest for various periods
  - We want to know what these rates will be in the future, but we can’t directly observe them
    - *Forward rates* — market participants’ expectations of future interest rates
    - E.g., yield on a 6-month Treasury bill six months from now

- Extracting such information from market interest rates is computationally burdensome

- Three new Stata commands to analyze term structure of interest rates and value bonds
  - `genspot` — Generates a spot rate curve from a few market rates
  - `genfwd` — Generates a forward rate curve from a spot rate curve
  - `pricebond` — Values a bond using forward (or spot) rates
  - ...and one bonus command (`sphinert`) that generates a cubic spline from a few “knots”
What is a bond?

• Bond: financial contract to borrow money for a specified period of time

• Bond: a borrower’s promise to return an investor’s money in the future with interest
  • Principal (P) – Investor’s loan to borrower; returned when bond matures
  • Coupon (C) – Periodic payments from borrower to investor over the life of the bond
    • Compensation for the investor’s risk (e.g., credit risk, interest rate risk, etc.)

Illustrative bond cash flows
2-year tenor; 3% annual coupon rate, paid semi-annually; $1,000 face value

<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>Face value</th>
<th>Coupon</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan-19</td>
<td>0</td>
<td>($1,000)</td>
<td>$15</td>
<td>$15</td>
</tr>
<tr>
<td>Jul-19</td>
<td>1</td>
<td></td>
<td>$15</td>
<td>$15</td>
</tr>
<tr>
<td>Jan-20</td>
<td>2</td>
<td></td>
<td>$15</td>
<td>$15</td>
</tr>
<tr>
<td>Jul-20</td>
<td>3</td>
<td></td>
<td></td>
<td>$1,015</td>
</tr>
<tr>
<td>Jan-21</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
How does one value a bond?

- Finance is all about valuing future cash flows
  - Money has time value (a dollar today is worth more than a dollar tomorrow)
  - Value (price) of any financial instrument is the present value (PV) of its future cash flows (FV) at discount rate $r$

$$PV = FV \cdot \frac{1}{(1 + r)^n}$$

- Bonds can be thought of as a series of zero-coupon (single payment) cash flows

**Illustrative bond cash flows**
2-year tenor; 3% annual coupon rate, paid semi-annually; $1,000 face value

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<thead>
<tr>
<th>Investor's cash flows:</th>
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$$P_t = C \cdot \frac{1}{(1 + r_t)^t}$$  
$\frac{15}{(1 + r_1)^1}, \frac{15}{(1 + r_2)^2}, \frac{15}{(1 + r_3)^3}, \frac{1,015}{(1 + r_4)^4}$
How does one value a bond?

- Proper discount rate for each cash flow is the Spot Rate \( (S_t) \) – yield on a zero-coupon bond maturing at time \( t \)

- Bond price is the sum of the discounted future cash flows:

\[
p_T = C \sum_{t=1}^{T-1} \frac{1}{(1 + S_t)^t} + \frac{(C + P)}{(1 + S_T)^T}
\]

- To price a bond, one needs the spot rate corresponding to each future cash flow

- **Problem**: One can only directly observe a few spot rates
  - Rates on T-bills (Treasury securities maturing in one year or less) are spot rates, for example

- **Solution**: Use observable spot rates to construct (“bootstrap”) other spot rates
New command to generate spot rate curve: **genspot**

- **genspot** – Generates a spot rate curve from a yield curve of market rates
  - *Syntax:* `genspot newvar, principal(real) tenor(tenorvar) coupon(couponvar) ytm(ytmvar) price(pricevar) freq(integer) fn(filename)`
  - **newvar**
  - All “options” are required:
    - **principal(real)**
    - **tenor(tenorvar)**
    - **coupon(couponvar)**
    - **ytm(ytmvar)**
    - **price(pricevar)**
    - **freq(integer)**
    - **fn(filename)**
  - new variable to store calculated spot rates
  - principal amount of one “bond” (usually 100.0)
  - variable name of bond tenor (in years)
  - variable name of bond coupon (in percent)
  - variable name of bond yield to maturity (in percent)
  - variable name of bond price
  - number of coupon payments per year
  - name of file to store spot rate curve

- Run on a dataset of bonds with tenor, coupon, yield to maturity and price variables; requires at least two spot rates (bonds with zero coupons) in the shortest tenors
- Utilizes a “bootstrap” method under a no-arbitrage assumption to construct theoretical spot rate curve (a.k.a., term structure of interest rates)

\[
P_T = C \sum_{t=1}^{T-1} \frac{1}{(1 + S_t)^t} + \frac{(C + P)}{(1 + S_T)^T}
\]

\[
S_T = \left[ \frac{C + P}{P_T - C \sum_{t=1}^{T-1} \frac{1}{(1 + S_t)^t}} \right]^{1/T} - 1
\]
New command to generate forward rate curve: **genfwd**

- *Forward rate* — market participants’ expectation of future interest rates
  - One can derive forward rates from spot rates
  - E.g., let \( F_1 \) be the 6-month forward rate (one 6-month period) two years (four 6-month periods) from now; in general:

  \[
  nF_f = \left[ \frac{(1 + S_{n+f})^{n+f}}{(1 + S_n)^n} \right]^{1/f} - 1
  \]

- **genfwd** — Generates a forward rate curve from a yield curve of spot rates
  - *Syntax:* genfwd newvar, spotrate(spotvar) tenor(tenorvar) nperiods(int)
    - **newvar**
      - new variable to store forward rate curve
    - All “options” are required:
      - **spotrate(spotvar)**
        - variable name of spot rate (in percent)
      - **tenor(tenorvar)**
        - variable name of tenor (in years)
      - **nperiods(integer)**
        - forward term: number of (6-month) periods from now
New command to value a bond using spot or forward rates: **pricebond**

- **pricebond** – Values a bond using forward or spot rates
  
  - **Syntax:** `pricebond ratevar, principal(real) tenor(tenorvar) coupon(real) freq(integer)`
  
  - **ratevar** variable name of spot or forward rate curve
  
  - All “options” are required:
    - **principal(real)** principal amount of one “bond” (usually 100.0)
    - **tenor(tenorvar)** variable name of bond tenor (in years)
    - **coupon(real)** annual coupon rate (in percent)
    - **freq(integer)** frequency of coupon payments (number per year)
  
  - Run on a dataset with tenor and spot or forward rates
  
  - Returns the bond price in a stored value: `r(price)`
Constructing a cubic spline through yield curve points: **splinert**

- **splinert** – Generates a cubic spline to connect (yield curve) points

  - **Syntax:** `splinert newvar, x(tenorvar) y(ytmvar) inc(real) fn(filename)`
    - `newvar` – new variable to store cubic spline
    - All “options” are required:
      - `x(tenorvar)` – variable name of bond tenor (in years)
      - `y(ytmvar)` – variable name of bond yield to maturity (in percent)
      - `inc(real)` – increment for tenor (in years; inverse of `freq` in `genspot`)
      - `fn(filename)` – name of file to store cubic spline

Polynomials pass through their end points (10 equations):

<table>
<thead>
<tr>
<th>6-mo T-bi</th>
<th>1-yr T-bill</th>
<th>2-yr T-note</th>
<th>5-yr T-note</th>
<th>10-yr T-note</th>
<th>30-yr T-bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D0.5, Y0.5)</td>
<td>(D1, Y1)</td>
<td>(D2, Y2)</td>
<td>(D5, Y5)</td>
<td>(D10, Y10)</td>
<td>(D30, Y30)</td>
</tr>
</tbody>
</table>

P1(D0.5) = Y0.5
P2(D1) = Y1
P2(D2) = Y2
...
P5(D10) = Y10
P5(D30) = Y30

First derivatives match at interior points (4 equations):

<table>
<thead>
<tr>
<th>20 x 20</th>
<th>20 x 1</th>
<th>20 x 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1'(D1) = P2'(D1)</td>
<td>P2'(D2) = P3'(D2)</td>
<td>P3'(D5) = P4'(D5)</td>
</tr>
<tr>
<td>P4'(D10) = P5'(D10)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Second derivatives match at interior points (4 equations):

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<thead>
<tr>
<th>20 x 20</th>
<th>20 x 1</th>
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<tbody>
<tr>
<td>P1''(D1) = P2''(D1)</td>
<td>P2''(D2) = P3''(D2)</td>
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</tr>
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<td>P4''(D10) = P5''(D10)</td>
<td></td>
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</table>

Second derivatives vanish at end points (2 equations):

<table>
<thead>
<tr>
<th>20 x 20</th>
<th>20 x 1</th>
<th>20 x 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1''(D0.5) = 0</td>
<td>P5''(D30) = 0</td>
<td></td>
</tr>
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</table>

\[ A \cdot b = c \]

\[ b = A^{-1} \cdot c \]
What is a yield curve and what can it tell us about interest rates?

- For bonds of the same credit risk, a yield curve plots bond yields against their tenors
  - Financial theory posits that yield curves reflect market participants’ expectations of future interest rates

“Normal” (upward-sloping):
Interest rates expected to rise

“Inverted” (downward-sloping):
Interest rates expected to fall
Questions?

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