Tools to Analyze Interest Rates and Value Bonds

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DISCOVER

Executive summary

- Bond markets contain a wealth of information about investor expectations
  - Observed market rates tell us returns bond investors require today to invest for various periods
  - We want to know what these rates will be in the future, but we can't directly observe them
    - *Forward rates* — market participants' expectations of future interest rates
    - E.g., yield on a 6-month Treasury bill six months from now
- Extracting such information from market interest rates is computationally burdensome
- Three new Stata commands to analyze term structure of interest rates and value bonds
  - **`genspot`** – Generates a spot rate curve from a few market rates
  - **`genfwd`** – Generates a forward rate curve from a spot rate curve
  - **`pricebond`** – Values a bond using forward (or spot) rates
  - …and one bonus command (**`splinert`**) that generates a cubic spline from a few "knots"

What is a bond?

- Bond: financial contract to borrow money for a specified period of time

- Bond: a borrower’s promise to return an investor’s money in the future with interest
    - *Principal (P)* – Investor’s loan to borrower; returned when bond matures
    - *Coupon (C)* – Periodic payments from borrower to investor over the life of the bond
        - Compensation for the investor’s risk (e.g., credit risk, interest rate risk, etc.)

Illustrative bond cash flows

2-year tenor; 3% annual coupon rate, paid semi-annually; $1,000 face value

Investor's cash flows:						**Sum**
Face value	($1,000)				$1,000	**$0**
Coupon		$15	$15	$15	$15	**$60**
Sum	($1,000)	$15	$15	$15	$1,015	
Date	Jan-19	Jul-19	Jan-20	Jul-20	Jan-21	
Time	0	1	2	3	4	

How does one value a bond?

- Finance is all about valuing future cash flows
  - Money has time value (a dollar today is worth more than a dollar tomorrow)
  - Value (price) of any financial instrument is the present value (PV) of its future cash flows (FV) at discount rate *r*

$$PV = FV \frac{1}{(1+r)^n}$$

- Bonds can be thought of as a series of zero-coupon (single payment) cash flows

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$P_t = C \frac{1}{(1+r)^t}$		$\frac{\$15}{(1+r_1)^1}$	$\frac{\$15}{(1+r_2)^2}$	$\frac{\$15}{(1+r_3)^3}$	$\frac{\$1{,}015}{(1+r_4)^4}$	

How does one value a bond?

- Proper discount rate for each cash flow is the Spot Rate (S_t) – yield on a zero-coupon bond maturing at time *t*

- Bond price is the sum of the discounted future cash flows:

$$p_T = C \sum_{t=1}^{T-1} \frac{1}{(1+S_t)^t} + \frac{(C+P)}{(1+S_T)^T}$$

- To price a bond, one needs the spot rate corresponding to each future cash flow

- *Problem*: One can only directly observe a few spot rates
  - Rates on T-bills (Treasury securities maturing in one year or less) are spot rates, for example

- *Solution*: Use observable spot rates to construct (“bootstrap”) other spot rates

New command to generate spot rate curve: **genspot**

- **genspot** – Generates a spot rate curve from a yield curve of market rates
  - *Syntax*: `genspot` ***`newvar`***`,` `principal(`***`real`***`)` `tenor(`***`tenorvar`***`)` `coupon(`***`couponvar`***`)` `ytm(`***`ytmvar`***`)` `price(`***`pricevar`***`)` `freq(`***`integer`***`)` `fn(`***`filename`***`)`
    - ***`newvar`*** — new variable to store calculated spot rates
    - All "options" are required:
      - `principal(`***`real`***`)` — principal amount of one "bond" (usually 100.0)
      - `tenor(`***`tenorvar`***`)` — variable name of bond tenor (in years)
      - `coupon(`***`couponvar`***`)` — variable name of bond coupon (in percent)
      - `ytm(`***`ytmvar`***`)` — variable name of bond yield to maturity (in percent)
      - `price(`***`pricevar`***`)` — variable name of bond price
      - `freq(`***`integer`***`)` — number of coupon payments per year
      - `fn(`***`filename`***`)` — name of file to store spot rate curve

  - Run on a dataset of bonds with tenor, coupon, yield to maturity and price variables; *requires at least two spot rates (bonds with zero coupons) in the shortest tenors*
  - Utilizes a "bootstrap" method under a no-arbitrage assumption to construct theoretical spot rate curve (a.k.a., term structure of interest rates)

$$P_T = C\sum_{t=1}^{T-1}\frac{1}{(1+S_t)^t} + \frac{(C+P)}{(1+S_T)^T}$$

$$S_T = \left[\frac{C+P}{P_T - C\sum_{t=1}^{T-1}\frac{1}{(1+S_t)^t}}\right]^{1/T} - 1$$

New command to generate forward rate curve: **genfwd**

- *Forward rate* — market participants' expectation of future interest rates
  - One can derive forward rates from spot rates
  - E.g., let ${}_4F_1$ be the 6-month forward rate (one 6-month period) two years (four 6-month periods) from now; in general:

$$ {}_nF_f = \left[\frac{(1+S_{n+f})^{n+f}}{(1+S_n)^n}\right]^{1/f} - 1 $$

- **genfwd** – Generates a forward rate curve from a yield curve of spot rates
  - *Syntax*: `genfwd` ***`newvar`***`,` `spotrate(`***`spotvar`***`)` `tenor(`***`tenorvar`***`)` `nperiods(`***`int`***`)`
    - ***`newvar`*** new variable to store forward rate curve
    - All "options" are required:
      - `spotrate(`***`spotvar`***`)` variable name of spot rate (in percent)
      - `tenor(`***`tenorvar`***`)` variable name of tenor (in years)
      - `nperiods(`***`integer`***`)` forward term: number of (6-month) periods from now

New command to value a bond using spot or forward rates: **pricebond**

- **pricebond** – Values a bond using forward or spot rates
  - *Syntax*: **`pricebond`** ***`ratevar`***, **`principal(`*`real`*`)`** **`tenor(`*`tenorvar`*`)`** **`coupon(`*`real`*`)`** **`freq(`*`integer`*`)`**
    - ***`ratevar`*** variable name of spot or forward rate curve
    - All "options" are required:
      - **`principal(`*`real`*`)`** principal amount of one "bond" (usually 100.0)
      - **`tenor(`*`tenorvar`*`)`** variable name of bond tenor (in years)
      - **`coupon(`*`real`*`)`** annual coupon rate (in percent)
      - **`freq(`*`integer`*`)`** frequency of coupon payments (number per year)
  - Run on a dataset with tenor and spot or forward rates
  - Returns the bond price in a stored value: **r(price)**

Constructing a cubic spline through yield curve points: **splinert**

- **splinert** – Generates a cubic spline to connect (yield curve) points
  - *Syntax*: `splinert` ***`newvar`***`,` `x(`***`tenorvar`***`)` `y(`***`ytmvar`***`)` `inc(`***`real`***`)` `fn(`***`filename`***`)`
    - ***`newvar`*** — new variable to store cubic spline
    - All "options" are required:
      - `x(`***`tenorvar`***`)` — variable name of bond tenor (in years)
      - `y(`***`ytmvar`***`)` — variable name of bond yield to maturity (in percent)
      - `inc(`***`real`***`)` — increment for tenor (in years; inverse of **`freq`** in **`genspot`**)
      - `fn(`***`filename`***`)` — name of file to store cubic spline

6-mo T-bi (D0.5, Y0.5) ← P1 → 1-yr T-bill (D1, Y1) ← P2 → 2-yr T-note (D2, Y2) ← P3 → 5-yr T-note (D5, Y5) ← P4 → 10-yr T-note (D10, Y10) ← P5 → 30-yr T-bond (D30, Y30)

Polynomials pass through their end points (10 equations):

P1(D0.5) = Y0.5	P1(D1) = Y1
P2(D1) = Y1	P2(D2) = Y2
…	…
P5(D10) = Y10	P5(D30) = Y30

First derivatives match at interior points (4 equations):
P1'(D1) = P2'(D1)
P2'(D2) = P3'(D2)
P3'(D5) = P4'(D5)
P4'(D10) = P5'(D10)

Second derivatives match at interior points (4 equations):
P1''(D1) = P2''(D1)
P2''(D2) = P3''(D2)
P3''(D5) = P4''(D5)
P4''(D10) = P5''(D10)

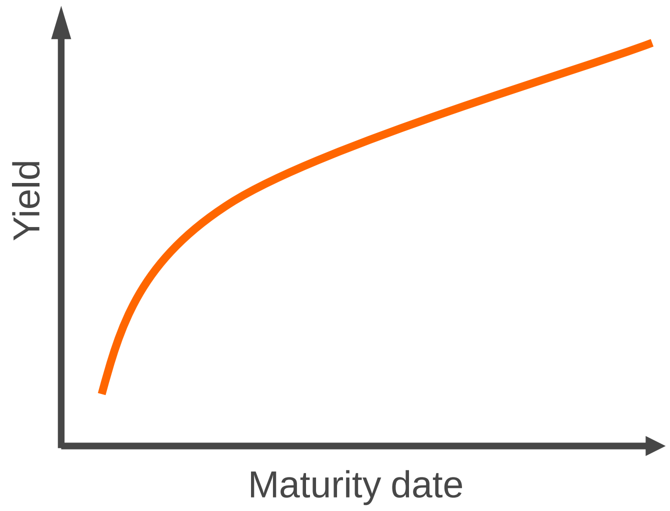
Second derivatives vanish at end points (2 equations):
P1''(D0.5) = 0
P5''(D30) = 0

$$\underset{20 \times 20}{A} \cdot \underset{20 \times 1}{b} = \underset{20 \times 1}{c} \qquad b = A^{-1} \cdot c$$

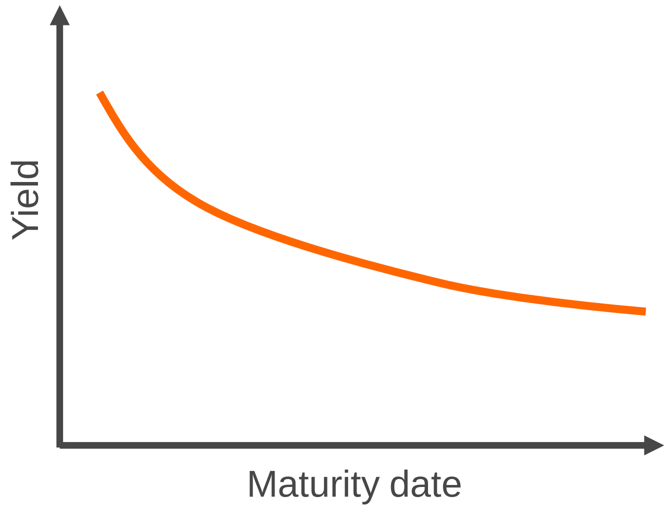
What is a yield curve and what can it tell us about interest rates?

- For bonds of the same credit risk, a *yield curve* plots bond yields against their tenors
  - Financial theory posits that yield curves reflect market participants' expectations of future interest rates

"Normal" (upward-sloping):
Interest rates expected to rise



"Inverted" (downward-sloping):
Interest rates expected to fall



Questions?

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