

Nonlinear dynamic stochastic general equilibrium models

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Motivation

- Models used in macroeconomics for policy analysis
- Dynamic
- Stochastic
- General equilibrium
- Nonlinear

Here's a model

- Households demand output, given inflation and interest rates:

$$1 = \beta E_t \left[\left(\frac{X_{t+1}}{X_t} \right)^{-1} \left(\frac{R_t}{\pi_{t+1} Z_{t+1}} \right) \right]$$

- Firms set prices, given output demand:

$$\phi + (\pi_t - 1) = \frac{1}{\phi} X_t + \beta E_t [\pi_{t+1} - 1]$$

- Central bank sets interest rate, given inflation

$$\beta R_t = \pi_t^{1/\beta} M_t$$

Here's a model

- The model's control variables are determined by equations:

$$1 = \beta E_t \left[\left(\frac{X_{t+1}}{X_t} \right)^{-1} \left(\frac{R_t}{\Pi_{t+1} Z_{t+1}} \right) \right]$$
$$\phi + (\Pi_t - 1) = \frac{1}{\phi} X_t + \beta E_t [\Pi_{t+1} - 1]$$
$$\beta R_t = \Pi_t^{1/\beta} M_t$$

- The model is completed by adding equations for the state variables:

$$\ln(Z_{t+1}) = \rho_z \ln(Z_t) + \xi_{t+1}$$
$$\ln(M_{t+1}) = \rho_m \ln(M_t) + e_{t+1}$$

Here's a model in Stata

```
. dsngen1 (1 = {beta}*(F.x/x)^(-1)*(r/(F.p*F.z)))          ///  
          ({phi}+(p-1) = 1/{phi}*x + {beta}*(F.p-1))        ///  
          ({beta}*r = p^(1/{beta})*m)                       ///  
          (ln(F.m) = {rhom}*ln(m))                          ///  
          (ln(F.z) = {rhoz}*ln(z))                          ///  
          , exostate(z m) observed(p r) unobserved(x)
```

Parameter estimation

```
. dsngen1 (1 = {beta}*(F.x/x)^(-1)*(r/(F.p*F.z)))    ///
>          ({phi}+(p-1) = 1/{phi}*x + {beta}*(F.p-1))  ///
>          ({beta}*r = p^(1/{beta})*m)                ///
>          (ln(F.m) = {rho}m*ln(m))                   ///
>          (ln(F.z) = {rho}z*ln(z))                   ///
>          , exostate(z m) observed(p r) unobserved(x)
```

Solving at initial parameter vector ...

Checking identification ...

First-order DSGE model

Sample: 1955q1 - 2015q4

Number of obs = 244

Log likelihood = -753.57131

	OIM				[95% Conf. Interval]	
	Coef.	Std. Err.	z	P> z		
<hr/>						
/structural						
beta	.5146672	.0783493	6.57	0.000	.3611054	.668229
phi	.1659058	.0474002	3.50	0.000	.0730032	.2588083
rho	.7005483	.0452634	15.48	0.000	.6118335	.789263
rhoz	.9545256	.0186417	51.20	0.000	.9179886	.9910627
<hr/>						
sd(e.z)	.650712	.1123897			.4304321	.8709918
sd(e.m)	2.318204	.3047452			1.720914	2.915493
<hr/>						

Policy questions

What is the effect of an unexpected increase in interest rates?

Estimated DSGE model provides an answer to this question. We can subject the model to a shock, then see how that shock feeds through the rest of the system.

Effect on impact

```
. estat policy
```

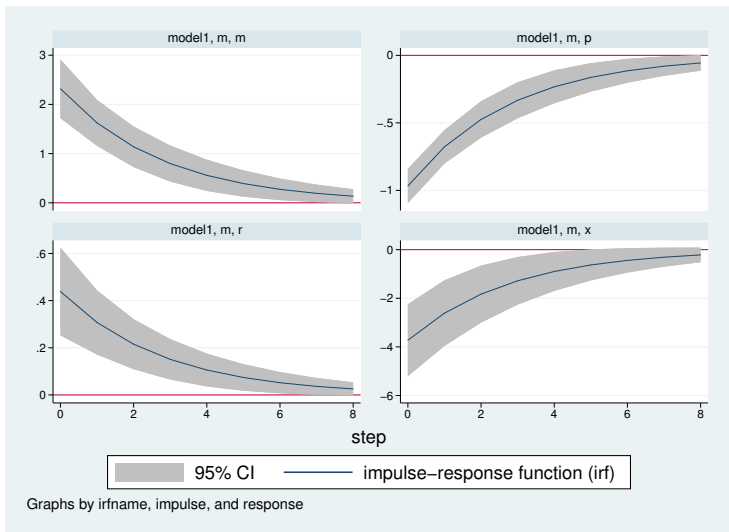
```
Policy matrix
```

		Delta-method			[95% Conf. Interval]		
		Coef.	Std. Err.	z	P> z		
x	z	2.59502	.9077695	2.86	0.004	.8158242	4.374215
	m	-1.608216	.4049684	-3.97	0.000	-2.401939	-.8144921
p	z	.8462697	.2344472	3.61	0.000	.3867617	1.305778
	m	-.4172522	.0393623	-10.60	0.000	-.4944008	-.3401035
r	z	1.644305	.2357604	6.97	0.000	1.182223	2.106387
	m	.1892777	.0591622	3.20	0.001	.0733219	.3052335

Effect over time: impulse response functions

```
. irf set nkirf.irf, replace  
. irf create model1  
. irf graph irf, impulse(m) response(p x r m) byopts(yrescale) yline(0)
```

Impulse responses from the estimated model



Analyzing nonlinear DSGE models

- We can do more than look at impulse responses
- We will switch to a textbook model and explore its features

The stochastic growth model

$$1 = \beta E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-1} (1 + r_{t+1} - \delta) \right]$$

$$y_t = z_t k_t^\alpha$$

$$r_t = \alpha z_t k_t^{\alpha-1}$$

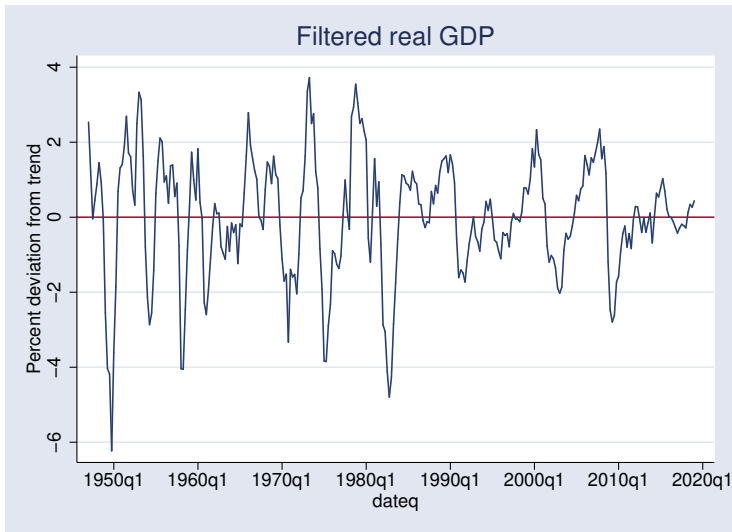
$$k_{t+1} = y_t - c_t + (1 - \delta)k_t$$

$$\ln z_{t+1} = \rho \ln z_t + e_{t+1}$$

The stochastic growth model in Stata

```
. dsngen1 (1={beta}*(c/F.c)*(1+F.r-{delta}))           ///
> (r = {alpha}*y/k)                                     ///
> (y=z*k^{alpha})                                       ///
> (f.k = y - c + (1-{delta})*k)                         ///
> (ln(F.z)={rhoz}*ln(z)),                               ///
> exostate(z) endostate(k) observed(y) unobserved(c r)
```

```
. import fred GDPC1
. generate dateq = qofd(daten)
. tsset dateq, quarterly
. generate lgdp = 100*ln(GDPC1)
. tsfilter hp y = lgdp
```



Parameter estimation

```

. constraint 1 _b[beta]=0.96
. constraint 2 _b[alpha]=0.36
. constraint 3 _b[delta]=0.025
. dsge1 (1={beta}*(c/F.c)*(1+F.r-{delta}))          ///
>   (r = {alpha}*y/k)                               ///
>   (y=z*k^{alpha})                                 ///
>   (f.k = y - c + (1-{delta})*k)                   ///
>   (ln(F.z)={rhoz}*ln(z)), constraint(1/3) nocnsreport  ///
>   exostate(z) endostate(k) observed(y) unobserved(c r) nolog
Solving at initial parameter vector ...
Checking identification ...
First-order DSGE model
Sample: 1947q1 - 2019q1          Number of obs   =          289
Log likelihood = -362.93403

```

y	OIM		z	P> z	[95% Conf. Interval]	
	Coef.	Std. Err.				
/structural						
beta	.96	(constrained)				
delta	.025	(constrained)				
alpha	.36	(constrained)				
rhoz	.8391786	.0325307	25.80	0.000	.7754197	.9029375
sd(e.z)	.8470234	.0352336			.7779668	.91608

Steady-state

```
. estat steady
```

```
Location of model steady-state
```

	Delta-method				
	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
k	13.94329
z	1
c	2.233508
r	.0666667
y	2.582091

Note: Standard errors reported as missing for constrained steady-state values.

Policy matrix

```
. estat policy
```

```
Policy matrix
```

		Delta-method		z	P> z	[95% Conf. Interval]	
		Coef.	Std. Err.				
c	k	.6371815
	z	.266745	.0244774	10.90	0.000	.2187701	.3147198
r	k	-.64
	z	1
y	k	.36
	z	1

Note: Standard errors reported as missing for constrained policy matrix values.

State transition matrix

```
. estat transition
```

```
Transition matrix of state variables
```

		Delta-method		z	P> z	[95% Conf. Interval]	
		Coef.	Std. Err.				
F.k	k	.9395996
	z	.1424566	.0039209	36.33	0.000	.1347717	.1501414
F.z	k	0	(omitted)				
	z	.8391786	.0325307	25.80	0.000	.7754197	.9029375

Note: Standard errors reported as missing for constrained transition matrix values.

Model-implied covariances

```
. estat covariance y
```

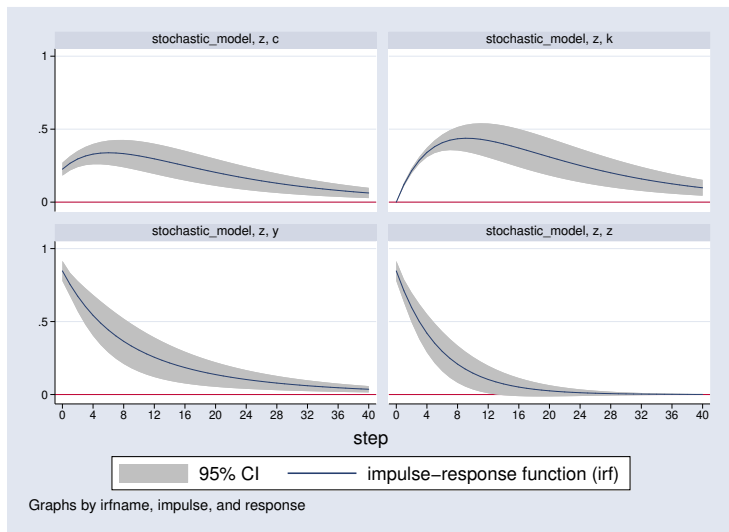
```
Estimated covariances of model variables
```

		Delta-method		z	P> z	[95% Conf. Interval]	
		Coef.	Std. Err.				
y	var(y)	3.872087	.9694708	3.99	0.000	1.971959	5.772215

Impulse responses

```
. irf set stochirf.irf, replace  
. irf create stochastic_model, step(40)  
. irf graph irf, impulse(z) response(y c k z) yline(0) xlabel(0(4)40)
```

Impulse responses



Conclusion

- `dsgen1` estimates the parameters of nonlinear DSGE models
- View steady-state, policy matrix, transition matrix
- View model-implied covariances
- Create and analyze impulse responses

Thank You!