

# Multinomial logit models for longitudinal data

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## The multinomial logit model

- Primary use case: unordered categorical outcome variable.
- Panel data model in utility-maximization form:  $U_{ijt} = x_{it}\beta_j + u_{ij} + \epsilon_{ijt}$
- $u_{ij}$  is a panel-level error term, also referred to as panel-level unobservables, that captures (time-invariant) unobserved heterogeneity.
- Assuming a type-I extreme value distribution for  $\epsilon_{ijt}$  and normalizing with respect to a base category gives rise to the logit model:

$$\Pr(y_{it} = m | x_{it}, \beta_j, u_{ij}) = F(y_{it} = m, x_{it}\beta_j + u_{ij}) =$$
$$\frac{1}{1 + \sum_{j=2}^J \exp(x_{it}\beta_j + u_{ij})} \text{ if } m = 1$$
$$\frac{\exp(x_{it}\beta_m + u_{im})}{1 + \sum_{j=2}^J \exp(x_{it}\beta_j + u_{ij})} \text{ if } m > 1$$

## The random-effects estimator

- `xtmlogit` implements a random-effects and conditional fixed-effects estimator.
- The random-effects estimator assumes that the panel-level unobservables are uncorrelated with the covariates.
- The fixed-effects estimator relaxes this assumption and the unobservables can be arbitrarily correlated with the covariates.
- Random-effects estimator:

$$l_i = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \left\{ \prod_{t=1}^{T_i} F(y_{it} = m, \mathbf{x}_{it}\beta_j + u_{ij}) \right\} \phi(u_i, \Sigma_u) du_i$$

- This integral has no closed form and we approximate it using quadrature.

# The fixed-effects estimator

- Fixed-effects estimator:

$$\log I_i = \sum_{t=1}^{T_i} \sum_{j=2}^J Y_{ijt} \mathbf{x}_{it} \beta_j - \log \sum_{\tilde{Y}_{ijt} \in \Psi(c_i)} \exp \left( \sum_{t=1}^{T_i} \sum_{j=2}^J \tilde{Y}_{ijt} \mathbf{x}_{it} \beta_j \right)$$

- $\Psi(c_i)$  is a set of all permutations of individual i's observed sequence of outcomes that satisfies the condition  $\sum_{t=1}^{T_i} \tilde{Y}_{it} = c_i$ . That is,

$$\Psi(c_i) = \left\{ \tilde{Y}_i = (\tilde{Y}_{i1}, \dots, \tilde{Y}_{iT_i}) \mid \sum_{t=1}^{T_i} \tilde{Y}_{it} = c_i \right\}$$

- where  $\tilde{Y}_{it} = (\tilde{Y}_{i1t}, \dots, \tilde{Y}_{iJt})$  is a vector of indicators with respect to the permutations of the observed outcome sequence  $Y_i$ .
- For example, the sequence  $Y_i = (3, 2, 3)$  has a total of three permutations (including the original sequence):  $(2, 3, 3)$ ,  $(3, 2, 3)$ , and  $(3, 3, 2)$

## Curse of dimensionality: random-effects estimator

- For the random-effects estimator, the curse is rooted in  $J$ , the number of outcomes, because we have a  $J - 1$  dimensional integral.
- Computation time can be high for more than just three or four outcomes.
- For example, with six outcomes, we have to approximate a five-dimensional integral.
- Using seven quadrature integration points, we would end up with  $7^5 = 16,807$  integration points.
- If computation time becomes infeasible one might consider using a single, shared variance component, if appropriate (option covariance (shared)).

## Curse of dimensionality: fixed-effects estimator

- The curse of dimensionality in case of the fixed-effects estimator is rooted mainly in  $T_i$ , the number of repeated observations, and potentially in  $J$ .
- The number of permutations in  $\Psi(c_i)$  grows exponentially with  $T_i$  and can become infeasibly large.
- The number of permutations of panel  $i$ 's observed vector of outcomes is:

$$K_i = \frac{T_i!}{c_{i1}! \cdots c_{ij}! \cdots c_{iJ}!}$$

- For example, for an individual with 15 repeated observations in a dataset with 6 outcomes,  $j = 1, 2, \dots, 6$ , with the sequence of outcomes  $Y_i = (3, 3, 3, 2, 4, 1, 1, 5, 4, 6, 6, 1, 1, 2, 4)$  the size of the set of permutations of this outcome vector is

$$K_i = \frac{15!}{4! 2! 3! 3! 1! 2!} = 378,378,000$$

- Realistically,  $T_i$  should not be larger than around 9 or 10.

## When to use which estimator

- If the assumptions of the random-effects estimator hold, both the random-effects and fixed-effects estimators are consistent, but the random-effects estimator is more efficient. If the assumptions do not hold, the random-effects estimator becomes inconsistent.
- If all we care about is estimating  $\beta_j$ , we should use the fixed-effects estimator because we have to make fewer assumptions.
- Practical consideration: beyond the gain in efficiency, if we want to perform marginal predictions or include time-constant covariates, the random-effects estimator is preferable.
- With the fixed-effects estimator, we cannot make predictions that account for the panel-level unobservables because these are not parameterized.
- A potential way to decide between the two is to perform a Hausman test.

## Some notable features

- In the context of the **random-effects** estimator, we can decide what restrictions to place on the variance-covariance matrix of the random effects:
  - ▶ covariance (independent); distinct variances, the default
  - ▶ covariance (unstructured); no restrictions
  - ▶ covariance (exchangeable); equal variances and covariances
  - ▶ covariance (identity); equal variances, covariance 0
  - ▶ covariance (shared); one common random effect
- With the **fixed-effects** estimator, the curse of dimensionality can be alleviated to some degree by taking random samples from the permutation sets using option `rsample()`.
- `xtmlogit` can be used with `svy`
- `bayes: xtmlogit (random-effects estimator)`

# Example data (1)

```
. webuse estatus  
(Fictional employment status data)
```

```
. describe
```

Contains data from <https://www.stata-press.com/data/r17/estatus.dta>  
Observations: 4,761 Fictional employment status data  
Variables: 8 26 Feb 2021 15:17

Variable name	Storage type	Display format	Value label	Variable label
id	int	%9.0g		Respondent ID
year	int	%9.0g		Year of survey
estatus	byte	%18.0g	alt	Employment status
hhchild	byte	%9.0g	noyes	Children <18 years old in household
hhincome	int	%10.0g		Annual household income (in \$1,000s)
hhsigno	byte	%10.0g	noyes	Significant other living in household
bwinner	byte	%10.0g	noyes	Primary/sole breadwinner in household
age	byte	%10.0g		Age (in years)

Sorted by: id year

```
. tab estatus
```

Employment status	Freq.	Percent	Cum.
Out of labor force	1,682	35.33	35.33
Unemployed	703	14.77	50.09
Employed	2,376	49.91	100.00
Total	4,761	100.00	

## Example data (2)

```
. list in 1/10, compress noobs
```

			id	year	estatus	hhc_d	hhi_e	hhs_o	bwi_r	age
1	1	2002	Out of labor force		Yes	56	Yes	Yes	34	
	1	2004	Out of labor force		Yes	62	No	Yes	36	
	1	2006	Employed		Yes	68	Yes	No	38	
	1	2008	Employed		No	73	No	Yes	40	
	1	2010	Out of labor force		No	85	Yes	No	42	
2	2	2002	Employed		No	28	Yes	No	19	
	2	2004	Out of labor force		Yes	26	Yes	Yes	21	
	2	2006	Employed		Yes	31	No	No	23	
	2	2008	Employed		Yes	36	Yes	Yes	25	
	2	2010	Unemployed		Yes	42	Yes	No	27	

# Using mlogit

```
. mlogit estatus i.hhchild age hhincome i.hhsigno i.bwinner
Iteration 0: log likelihood = -4746.2288
Iteration 1: log likelihood = -4586.1414
Iteration 2: log likelihood = -4581.5114
Iteration 3: log likelihood = -4581.496
Iteration 4: log likelihood = -4581.496
Multinomial logistic regression                                         Number of obs = 4,761
Log likelihood = -4581.496                                         LR chi2(10) = 329.47
                                                               Prob > chi2 = 0.0000
                                                               Pseudo R2 = 0.0347
```

estatus	Coefficient	Std. err.	z	P> z	[95% conf. interval]
Out_of_lab_e hhchild					
Yes	.3603056	.0767921	4.69	0.000	.2097958 .5108155
age	-.0051251	.0050107	-1.02	0.306	-.0149459 .0046958
hhincome	-.0041118	.0012464	-3.30	0.001	-.0065546 -.001669
hhsigno					
Yes	.4251824	.085001	5.00	0.000	.2585835 .5917814
bwinner					
Yes	-.4341223	.0646731	-6.71	0.000	-.5608793 -.3073654
_cons	-.3054547	.2219434	-1.38	0.169	-.7404558 .1295464
Unemployed					
hhchild					
Yes	-.0421006	.1026229	-0.41	0.682	-.2432378 .1590366
age	.0027158	.0066839	0.41	0.685	-.0103845 .015816
hhincome	-.0278535	.0020728	-13.44	0.000	-.0319161 -.0237909
hhsigno					
Yes	.1070661	.1100785	0.97	0.331	-.1086839 .3228161
bwinner					
Yes	-.2185759	.0879215	-2.49	0.013	-.3908989 -.046253
_cons	.0142404	.293933	0.05	0.961	-.5618577 .5903385
Employed	(base outcome)				

# Panel data model with random effects (1)

```
. xtset id
Panel variable: id (unbalanced)
. xtmlogit estatus i.hhchild age hhincome i.hhsigno i.bwinner
Fitting comparison model ...
Refining starting values:
Grid node 0:    log likelihood = -4483.1721
Grid node 1:    log likelihood = -4516.6753
Fitting full model:
Iteration 0:    log likelihood = -4483.1721
Iteration 1:    log likelihood = -4474.3849
Iteration 2:    log likelihood = -4468.9353
Iteration 3:    log likelihood = -4468.8415
Iteration 4:    log likelihood = -4468.8413
Random-effects multinomial logistic regression          Number of obs     = 4,761
Group variable: id                                     Number of groups  = 800
Random effects u_i ~ Gaussian                         Obs per group:
                                                       min =      5
                                                       avg =     6.0
                                                       max =      7
Integration method: mvaghermite                      Integration pts. =    7
Log likelihood = -4468.8413                           Wald chi2(10)    = 239.26
                                                       Prob > chi2     = 0.0000

```

estatus	Coefficient	Std. err.	z	P> z	[95% conf. interval]

# Panel data model with random effects (2)

estatus	Coefficient	Std. err.	z	P> z	[95% conf. interval]
Out_of_lab_e					
hhchild					
Yes	.4628125	.0962758	4.81	0.000	.2741154 .6515096
age	-.004825	.0066428	-0.73	0.468	-.0178446 .0081946
hhincome	-.0046922	.001839	-2.55	0.011	-.0082965 -.0010879
hhsigno					
Yes	.4967056	.0946442	5.25	0.000	.3112063 .6822049
bwinner					
Yes	-.4740919	.0727992	-6.51	0.000	-.6167756 -.3314082
_cons	-.4787579	.2845139	-1.68	0.092	-1.036395 .0788792
Unemployed					
hhchild					
Yes	-.0401989	.119596	-0.34	0.737	-.2746027 .1942049
age	.0042644	.0081818	0.52	0.602	-.0117716 .0203004
hhincome	-.0308468	.0026529	-11.63	0.000	-.0360463 -.0256473
hhsigno					
Yes	.0968	.1192659	0.81	0.417	-.1369568 .3305568
bwinner					
Yes	-.2252587	.0951984	-2.37	0.018	-.4118441 -.0386733
_cons	-.0953821	.3508736	-0.27	0.786	-.7830817 .5923175
Employed	(base outcome)				
var(u1)	.8587807	.1090216		.6696113	1.101392
var(u2)	.7370366	.1388917		.5094287	1.066338

LR test vs. multinomial logit: chi2(2) = 225.31 Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.



# Displaying relative-risk ratios

```
. xtmglogit, rrr  
Random-effects multinomial logistic regression  
Group variable: id  
Random effects u_i ~ Gaussian  
Number of obs = 4,761  
Number of groups = 800  
Obs per group:  
    min = 5  
    avg = 6.0  
    max = 7  
Integration method: mvaghermite  
Integration pts. = 7  
Wald chi2(10) = 239.26  
Prob > chi2 = 0.0000  
Log likelihood = -4468.8413
```

estatus	RRR	Std. err.	z	P> z	[95% conf. interval]
Out_of_lab_e					
hhchild					
Yes	1.588535	.1529375	4.81	0.000	1.315367 1.918435
age	.9951866	.0066108	-0.73	0.468	.9823137 1.008228
hhincome	.9953188	.0018303	-2.55	0.011	.9917379 .9989127
hhsigno					
Yes	1.643299	.1555288	5.25	0.000	1.365071 1.978235
bwinner					
Yes	.6224501	.0453138	-6.51	0.000	.5396818 .7179121
_cons	.6195525	.1762713	-1.68	0.092	.3547312 1.082074
Unemployed					
hhchild					
Yes	.9605983	.1148837	-0.34	0.737	.7598739 1.214345
age	1.004274	.0082168	0.52	0.602	.9882974 1.020508
hhincome	.9696241	.0025723	-11.63	0.000	.9645956 .9746788
hhsigno					
Yes	1.10164	.1313881	0.81	0.417	.8720079 1.391743
bwinner					
Yes	.7983097	.0759978	-2.37	0.018	.6624275 .9620649
_cons	.9090255	.3189531	-0.27	0.786	.4569955 1.808174
Employed	(base outcome)				
var(u1)	.8587807	.1090216		.6696113	1.101392
var(u2)	.7370366	.1388917		.5094287	1.066338

Note: Estimates are transformed only in the first 3 equations to relative-risk ratios.

Note: `_cons` estimates baseline relative risk (conditional on zero random effects).

LR test vs. multinomial logit: chi2(2) = 225.31 Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

# Unstructured covariance estimation (1)

```
. xtmglogit estatus i.hhchild age hhincome i.hhsigno i.bwinner, covariance(unstr  
> uctured)  
Fitting comparison model ...  
Refining starting values:  
Grid node 0: log likelihood = -4483.1721  
Grid node 1: log likelihood = -4516.6753  
Fitting full model:  
Iteration 0: log likelihood = -4483.1721  
Iteration 1: log likelihood = -4454.5313  
Iteration 2: log likelihood = -4438.9076  
Iteration 3: log likelihood = -4438.2905  
Iteration 4: log likelihood = -4438.2887  
Iteration 5: log likelihood = -4438.2887  
Random-effects multinomial logistic regression  
Number of obs = 4,761  
Group variable: id  
Number of groups = 800  
Random effects u_i ~ Gaussian  
Obs per group:  
min = 5  
avg = 6.0  
max = 7  
Integration method: mvaghermite  
Integration pts. = 7  
Wald chi2(10) = 242.93  
Prob > chi2 = 0.0000  
Log likelihood = -4438.2887
```

---

estatus	Coefficient	Std. err.	z	P> z	[95% conf. interval]
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# Unstructured covariance estimation (2)

estatus	Coefficient	Std. err.	z	P> z	[95% conf. interval]
<hr/>					
Out_of_lab_e					
hhchild					
Yes	.4924799	.1002988	4.91	0.000	.295898 .6890619
age	-.004219	.0070064	-0.60	0.547	-.0179513 .0095133
hhincome	-.006046	.001992	-3.04	0.002	-.0099503 -.0021417
hhsigno					
Yes	.5036976	.0966982	5.21	0.000	.3141726 .6932225
bwinner					
Yes	-.489057	.0745454	-6.56	0.000	-.6351632 -.3429507
_cons	-.3930378	.298386	-1.32	0.188	-.9778636 .191788
<hr/>					
Unemployed					
hhchild					
Yes	.0399687	.1238417	0.32	0.747	-.2027565 .2826939
age	.0045538	.0085081	0.54	0.592	-.0121219 .0212294
hhincome	-.0315377	.0027426	-11.50	0.000	-.0369131 -.0261624
hhsigno					
Yes	.1495817	.1214242	1.23	0.218	-.0884053 .3875687
bwinner					
Yes	-.2552257	.0968165	-2.64	0.008	-.4449826 -.0654689
_cons	-.0417024	.3633406	-0.11	0.909	-.7538368 .670432
Employed	(base outcome)				
var(u1)	1.132081	.1331468		.899012	1.425572
var(u2)	1.102612	.1698422		.8152803	1.49121
cov(u1,u2)	.7871916	.1222148	6.44	0.000	.547655 1.026728

LR test vs. multinomial logit: chi2(3) = 286.41

Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.



# Standard deviation and correlation

```
. estat sd
```

estatus	Coefficient	Std. err.	z	P> z	[95% conf. interval]
sd(u1)	1.063993	.0625694		.9481624	1.193973
sd(u2)	1.050053	.0808731		.9029287	1.221151
corr(u1,u2)	.7045801	.0632646	11.14	0.000	.5581225 .8084624

# Panel data model with conditional fixed effects (1)

```
. xtmglogit estatus i.hhchild age hhincome i.hhsigno i.bwinner, fe  
note: 80 groups (451 obs) omitted because of no variation in the outcome  
variable over time.  
Computing initial values ...  
Setting up 26,168 permutations:  
....10%....20%....30%....40%....50%....60%....70%....80%....90%....100%  
Fitting full model:  
Iteration 0: log likelihood = -2136.5919  
Iteration 1: log likelihood = -2136.2728  
Iteration 2: log likelihood = -2136.2728  
Fixed-effects multinomial logistic regression  
Number of obs = 4,310  
Group variable: id Number of groups = 720  
Obs per group:  
min = 5  
avg = 6.0  
max = 7  
LR chi2(10) = 103.29  
Prob > chi2 = 0.0000  
Log likelihood = -2136.2728
```

estatus	Coefficient	Std. err.	z	P> z	[95% conf. interval]

## Panel data model with conditional fixed effects (2)

estatus	Coefficient	Std. err.	z	P> z	[95% conf. interval]
Out_of_lab_e					
hhchild					
Yes	.5881852	.1258696	4.67	0.000	.3414854 .834885
age	-.0003842	.0147741	-0.03	0.979	-.0293409 .0285725
hhincome	-.0122043	.0088464	-1.38	0.168	-.029543 .0051344
hhsigno					
Yes	.5090034	.100111	5.08	0.000	.3127893 .7052174
bwinner					
Yes	-.4655745	.0782841	-5.95	0.000	-.6190085 -.3121406
Unemployed					
hhchild					
Yes	.163612	.1638934	1.00	0.318	-.1576132 .4848372
age	.0063355	.019404	0.33	0.744	-.0316957 .0443667
hhincome	-.029742	.0120031	-2.48	0.013	-.0532676 -.0062164
hhsigno					
Yes	.1173192	.1301364	0.90	0.367	-.1377435 .3723819
bwinner					
Yes	-.2489958	.1030027	-2.42	0.016	-.4508773 -.0471142
Employed	(base outcome)				

# Fixed-effects model with permutation sampling (1)

```
. xtset id year
Panel variable: id (unbalanced)
Time variable: year, 2002 to 2014, but with gaps
    Delta: 1 unit
. xtmlogit estatus i.hhchild age hhincome i.hhsigno i.bwinner, fe rsample(10, r
> seed(123))
note: option vce() set to vce(robust) because of permutation sampling.
note: 80 groups (451 obs) omitted because of no variation in the outcome
      variable over time.
Computing initial values ...
Setting up 3,495 permutations:
....10%....20%....30%....40%....50%....60%....70%....80%....90%....100%
Fitting full model:
Iteration 0:  log pseudolikelihood = -908.26163
Iteration 1:  log pseudolikelihood = -906.4585
Iteration 2:  log pseudolikelihood = -906.45801
Iteration 3:  log pseudolikelihood = -906.45801
Fixed-effects multinomial logistic regression          Number of obs     =  4,310
Group variable: id                                  Number of groups  =    720
                                                       Obs per group:
                                                       min =           5
                                                       avg =           6.0
                                                       max =           7
                                                       Wald chi2(10)    =   72.91
Log pseudolikelihood = -906.45801                  Prob > chi2     = 0.0000
                                                       (Std. err. adjusted for 720 clusters in id)
```

estatus	Robust				
	Coefficient	std. err.	z	P> z	[95% conf. interval]

## Fixed-effects model with permutation sampling (2)

estatus	Robust					
	Coefficient	std. err.	z	P> z	[95% conf. interval]	
Out_of_lab_e						
hhchild	.5827048	.1487376	3.92	0.000	.2911845	.8742251
Yes	.0055092	.0168589	-0.33	0.744	-.0385521	.0275337
age	-.0142493	.0100457	-1.42	0.156	-.0339386	.0054399
hhincome						
hhsigno						
Yes	.4441511	.1213382	3.66	0.000	.2063325	.6819696
bwinner						
Yes	-.4613157	.0978061	-4.72	0.000	-.6530122	-.2696193
Unemployed						
hhchild	.1714144	.1831194	0.94	0.349	-.187493	.5303217
Yes	-.0046656	.0217006	-0.21	0.830	-.0471979	.0378668
age	-.0344681	.0131707	-2.62	0.009	-.0602821	-.0086541
hhincome						
hhsigno						
Yes	-.0760533	.1396542	-0.54	0.586	-.3497705	.197664
bwinner						
Yes	-.2889772	.1178606	-2.45	0.014	-.5199797	-.0579747
Employed	(base outcome)					

# Performing a Hausman test

```
. qui xtmlogit estatus i.hhchild age hhincome i.hhsigno i.bwinner, fe  
. estimates store FE  
. qui xtmlogit estatus i.hhchild age hhincome i.hhsigno i.bwinner  
. estimates store RE  
. hausman FE RE, alleqs
```

	Coefficients			
	(b) FE	(B) RE	(b-B) Difference	sqrt(diag(V_b-V_B)) Std. err.
Out_of_lab_e				
1.hhchild	.5881852	.4628125	.1253727	.0810809
age	-.0003842	-.004825	.0044408	.0131965
hhincome	-.0122043	-.0046922	-.0075122	.0086532
1.hhsigno	.5090034	.4967056	.0122977	.0326296
1.bwinner	-.4655745	-.4740919	.0085173	.0287868
Unemployed				
1.hhchild	.163612	-.0401989	.203811	.1120618
age	.0063355	.0042644	.0020711	.0175947
hhincome	-.029742	-.0308468	.0011048	.0117062
1.hhsigno	.1173192	.0968	.0205192	.0520686
1.bwinner	-.2489958	-.2252587	-.0237371	.0393297

b = Consistent under H0 and Ha; obtained from **xtmlogit**.

B = Inconsistent under Ha, efficient under H0; obtained from **xtmlogit**.

Test of H0: Difference in coefficients not systematic

```
chi2(10) = (b-B)'[(V_b-V_B)^(-1)](b-B)  
= 8.05  
Prob > chi2 = 0.6238
```

# Using margins after xtmlogit, re

```
. qui xtmlogit estatus i.hhchild age hhincome i.hhsigno i.bwinner  
. margins i.hhchild  
Predictive margins                                         Number of obs = 4,761  
Model VCE: OIM  
1._predict: Pr(estatus==Out_of_labor_force), predict(pr outcome(1))  
2._predict: Pr(estatus==Unemployed), predict(pr outcome(2))  
3._predict: Pr(estatus==Employed), predict(pr outcome(3))
```

	Delta-method					
	Margin	std. err.	z	P> z	[95% conf. interval]	
_predict# hhchild						
1#No	.3021986	.0131047	23.06	0.000	.2765138	.3278834
1#Yes	.3912783	.0119865	32.64	0.000	.3677852	.4147714
2#No	.1630791	.0101239	16.11	0.000	.1432367	.1829216
2#Yes	.139782	.0079417	17.60	0.000	.1242167	.1553474
3#No	.5347223	.0136504	39.17	0.000	.507968	.5614766
3#Yes	.4689397	.0116018	40.42	0.000	.4462006	.4916787

- Note: using margins after xtmlogit, fe is not advisable.

## predict

- `xtmlogit, re`
  - ▶ `pr` marginal probability of the specified outcome; the default
  - ▶ `pcr` conditional probability of the specified outcome
  - ▶ `pu0` probability of the specified outcome, assuming zero random effects
  - ▶ `xb` linear prediction for the specified outcome, including random effects
  - ▶ `xb0` linear prediction for the specified outcome, assuming zero random effects
- `xtmlogit, fe`
  - ▶ `pu0` probability of the specified outcome, assuming zero fixed effects; the default
  - ▶ `xb` linear prediction for the specified outcome, assuming zero random effects
- Option `outcome()` to specify a particular outcome.
- Or use `stub*` to get predictions for all outcomes.

# Thank You!