Classification using stochastic ensembles

Linden McBride and Austin Nichols

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Topics

- Discriminant analysis and classification
- Classification and Regression Trees
- Stochastic ensemble methods
- Our application: USAID Poverty Assessment Tools
- Other applications
Discriminant analysis and classification

Classification, or predictive discriminant analysis, involves the assignment of observations to classes.

Predictions are based on a model trained in a dataset in which class membership is known (Huberty 1994, Rencher 2002, Hastie et al. 2009).

- Prediction of qualitative response
- With class $>2$, linear regression methods generally not appropriate
- Methods available in statistics, machine learning, predictive analytics
Our classification problem: identifying poor from nonpoor

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This is a stylized example of the problem faced by USAID, the World Bank, and other institutions attempting to target the poor in developing countries where income status is difficult to assess.
Discriminant methods available in Stata

Many discrimination methods are available, including linear, quadratic, logistic, and nonparametric methods:

**MV** `discrim lda` and [MV] `candisc`.

**MV** `discrim qda` provides quadratic discriminant analysis.

**MV** `discrim logistic` provides logistic discriminant analysis.

**MV** `discrim knn` provides kth-nearest-neighbor discrimination.
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Methods not available in Stata include SVM, CART (limited), various ensemble methods.
Classification and regression trees recursively partition a feature space to meet some criteria (entropy reduction, minimized prediction error, etc).

Predictions for a given set of features are made based on the relative proportion of classes found in a terminal node (classification) or on the mean response for the data in that partition (regression).
Classification using stochastic ensembles

CART

hh head age >=32

roof = straw

hh head age <32

roof = tin

hh size < 5

hh size >= 5

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Stochastic ensemble methods

Ensemble methods construct many models on subsets of data (e.g. via resampling with replacement); they then average across these models (or allow them to vote) to obtain a less noisy prediction.

One version of this is known as **bootstrap aggregation**, or bagging (Breiman 1996a).

A stochastic ensemble method adds randomness to the construction of the models. This has the advantage of “de-correlating” models across subsets, which can reduce total variance (Breiman 2001).

Out-of-sample error is estimated by training the model in each randomly selected subset, and using the balance of the data to test the model. This estimated out-of-sample error is an unbiased estimator of the true out-of-sample prediction error (Breiman 1996b).
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This combination of bagging and decorrelating ensembles of trees produces classification and regression forests.

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Algorithms in R and Stata

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Discriminant analysis
CART
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Algorithms in R and Stata

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  - Regression Forest analog of quantile regression
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- Ensemble of Perfect Random Trees
  - Grows trees randomly
  - Averages over the most influential voters
Poverty Assessment Tools

The Poverty Assessment Tools were developed by the University of Maryland IRIS Center for USAID.

The IRIS tool is typically developed via quantile regression in a randomly selected subset of the data. Accuracy (out of sample prediction error) is assessed on the data not used for model development.
Our methods

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We randomly divide the data into training and testing sets, estimate the model in the training data and then assess accuracy in the testing data. We iterate this process 1000 times. We report the means and the $2.5^{th}$ and $97.5^{th}$ percentile confidence intervals.
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We use the 2005 Bolivia Household Survey, the 2004/5 Malawi Integrated Household Survey, and the 2001 East Timor Living Standards Survey.
## Classification error

<table>
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For our application, we’re interested in five accuracy measures:

- **Total Accuracy (TA)**:
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The next five slides present the comparative out-of-sample accuracy of discriminant analysis and stochastic ensemble methods in these datasets.
Total Accuracy

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Classification using stochastic ensembles
Poverty Accuracy

- Bolivia
  - KNN-5
  - LDA
  - QDA
  - LD
  - CF-R
  - QRF (40)-R
  - IRIS Q(42)

- Malawi
  - KNN-3
  - LDA
  - QDA
  - LD
  - CF-R
  - RF-R
  - IRIS Q(57)

- East Timor
  - KNN-3
  - LDA
  - QDA
  - LD
  - CF-R
  - QRF (50)-R
  - IRIS Probit

Poverty Accuracy Scale: 0 to 100
Leakage

![Leakage Diagram]

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*Classification using stochastic ensembles*
Undercoverage

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Balanced Poverty Accuracy
Stochastic ensemble methods in general, and random forests in particular, have become essential tools in a variety of applications.

**Bioinformatics:** In comparison with DL, KNN, and SVM, Diaz-Uriarte and Alvarez de Andres (2006) conclude, "because of its performance and features, random forest and gene selection using random forest should probably become part of the 'standard tool-box' of methods for class prediction and gene selection with microarray data."

**Kaggle and predictive analytics:** the following kaggle competitions were won using random forests

- Semi-supervised feature learning (computer science)
- Air quality prediction (environmental science)
- RTA freeway travel time prediction (urban development/economics)

**Other applications:** remote sensing, diagnostics, spam filters
Stochastic ensemble methods have broad applicability to classification and prediction problems; we find their use promising in poverty assessment tool development.

Such methods would be additional assets in the Stata classification tool kit.


Classification error

For a two-group classification problem, when misclassification costs are equal,

\[ MSE = \frac{1}{N} \sum_{i=0}^{n} (\hat{P}_i - P_i)^2 = \frac{1}{N} (FN + FP) \]

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