Bayesian Dynamic Stochastic General Equilibrium models in Stata 17

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Stata

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Bayesian econometrics in Stata

- Stata 17 introduces Bayesian estimation of a variety of time-series and panel-data econometric models
  - `bayes: dsge`
  - `bayes: dsgenl`
  - `bayes: var`
  - `bayesirf`
  - `bayesfcast`
  - `bayes: xt`
Outline

- DSGE models, dsge, and bayes: dsge
- AR(1) model
- Linear New Keynesian model
- Nonlinear stochastic growth model
A DSGE model is a system of equations that describes an economy.

A model consists of three kinds of variables:
- Control variables, whose values are determined by the system of equations each period
- State variables, which are fixed at the beginning of any given period but whose laws of motion are part of the system of equations
- Stochastic shocks, which drive the system

DSGE models come from economic theory. Theories are forward-looking, so equations are forward-looking.

Models are used for policy analysis: explore different policy alternatives or how different parameter values affect model outcomes.
An example model

- Suppose we wished to model the effect of monetary policy on macroeconomic variables
- We model relationships among output, inflation, and the interest rate
- These variables are linked to state variables representing monetary shocks, and perhaps other factors
- State variables, in turn, are driven by shocks
An example model

- The following system of equations is a DSGE model:

\[
x_t = E_t x_{t+1} - (r_t - E_t \pi_{t+1} - z_t)
\]

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa x_t
\]

\[
r_t = \frac{1}{\beta} \pi_t + w_t
\]

\[
w_{t+1} = \rho_w w_t + e_{t+1}
\]

\[
z_{t+1} = \rho_z z_t + \varepsilon_{t+1}
\]

- Control variables: \((x_t, \pi_t, r_t)\)
- State variables: \((w_t, z_t)\)
- Stochastic shock: \((e_t, \varepsilon_t)\)
- Parameters: \((\kappa, \beta, \rho_w, \rho_z, \sigma_e^2, \sigma_\varepsilon^2)'\)
DSGE models in Stata

Model:

\[ x_t = E_t x_{t+1} - \left( r_t - E_t \pi_{t+1} - z_t \right) \]  
\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \]  
\[ r_t = \frac{1}{\beta} \pi_t + w_t \]

\[ w_{t+1} = \rho_w w_t + e_{t+1} \]
\[ z_{t+1} = \rho_z z_t + \varepsilon_{t+1} \]

In Stata: dsge (model equations)

```
. dsge (x = F.x - (r - F.p - z) , unobserved) ///
   (p = {beta}*F.p + {kappa}*x) ///
   (r = (1/{beta})*p + w) ///
   (F.z = {rhoz}*z , state) ///
   (F.w = {rhow}*w , state)
```
Two views on the impact of monetary shocks

. quietly webuse usmacro2
. matrix param1 = (0.1, 0.5, 0.9, 0.7)
. matrix colnames param1 = kappa beta rhoz rhow
. matrix param2 = (1, 0.5, 0.9, 0.7)
. matrix colnames param2 = kappa beta rhoz rhow

. quietly irf set dsge_irf.irf, replace

. quietly dsge (x = F.x - (r - F.p - z) , unobserved ) ///
> (p = {beta}*F.p + {kappa}*x ) ///
> (r = (1/{beta})*p + w ) ///
> (F.z = {rhoz}*z , state ) ///
> (F.w = {rhow}*w , state ) , ///
> from(param1) solve

. quietly irf create keynes

. quietly dsge (x = F.x - (r - F.p - z) , unobserved ) ///
> (p = {beta}*F.p + {kappa}*x ) ///
> (r = (1/{beta})*p + w ) ///
> (F.z = {rhoz}*z , state ) ///
> (F.w = {rhow}*w , state ) , ///
> from(param2) solve

. quietly irf create classical
Two views on the impact of monetary shocks

Impulse responses

- Graphs showing impulse responses for:
  - \( w \rightarrow x \)
  - \( w \rightarrow p \)
  - \( w \rightarrow r \)
  - \( w \rightarrow w \)

Legend:
- Red line: Keynesian impulse response
- Blue line: Classical impulse response
Two views on the impact of monetary shocks

Maximum likelihood estimation

\[
\begin{align*}
&. \text{dsge} (x = F.x - (r - F.p - z), \text{unobserved}) \\
&> (p = \{beta\} \cdot F.p + \{kappa\} \cdot x) \\
&> (r = (1/{beta}) \cdot p + w) \\
&> (F.z = \{rhoz\} \cdot z, \text{state}) \\
&> (F.w = \{rhow\} \cdot w, \text{state}), \text{nolog}
\end{align*}
\]

DSGE model

Sample: 1955q1 thru 2015q4  
Number of obs = 244

Log likelihood = -753.57131

|                | Coefficient | Std. err. | z    | P>|z| | [95% conf. interval] |
|----------------|-------------|-----------|------|-----|----------------------|
| /structural    |             |           |      |     |                      |
| beta           | .5146664    | .0783491  | 6.57 | 0.000 | .3611051 .6682278    |
| kappa          | .1659059    | .0474074  | 3.50 | 0.000 | .0729891 .2588226    |
| rhoz           | .9545256    | .0186424  | 51.20| 0.000 | .9179872 .991064     |
| rhow           | .7005484    | .0452604  | 15.48| 0.000 | .6118396 .7892572    |
| sd(e.z)        | .6211222    | .1015082  |      |     | .4221699 .8200745    |
| sd(e.w)        | 2.318207    | .3047457  |      |     | 1.720916 2.915497    |
Bayesian DSGE models

- Estimation of DSGE models has shifted towards the Bayesian approach in the past 15 years
- Substantive:
  - Priors reflect genuine prior beliefs about the distribution of parameter values
  - Priors allow the incorporation of other evidence that is hard to incorporate into the likelihood, e.g. micro-evidence on price adjustment
- Technical:
  - Mapping from structural parameters to solution matrices is highly nonlinear, and identification issues are pervasive
  - Priors can aid in isolating identification problems
  - Priors useful for imposing bounds on parameters
Bayesian analysis and the `bayes` prefix

- The `bayes:` prefix allows for Bayesian estimation of likelihood-based models
- Just attach `bayes:` to the existing Stata command
  - `bayes: regress y x`
  - default priors provided (can be overwritten)
- For DSGE models:
  - `bayes, prior(prior_spec): dsge (model_equations)`
  - some priors are required
DSGE models, dsge, and bayes: dsge
AR(1) model
Linear New Keynesian model
Nonlinear stochastic growth model
US macro data

. webuse usmacro2
(Federal Reserve Economic Data - St. Louis Fed, 2017-01-15)
. describe
Contains data from https://www.stata-press.com/data/r17/usmacro2.dta
Observations: 244
Variables: 11
1 May 2020 17:52
(_dta has notes)

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Storage type</th>
<th>Display format</th>
<th>Value label</th>
</tr>
</thead>
<tbody>
<tr>
<td>daten</td>
<td>int</td>
<td>%td</td>
<td>Numeric (daily) date</td>
</tr>
<tr>
<td>year</td>
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<td>%9.0g</td>
<td>Year</td>
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<td>quarter</td>
<td>byte</td>
<td>%9.0g</td>
<td>Quarter</td>
</tr>
<tr>
<td>dateq</td>
<td>int</td>
<td>%tq</td>
<td>Date (quarters)</td>
</tr>
<tr>
<td>y</td>
<td>double</td>
<td>%10.0g</td>
<td>Growth rate of real GDP (GDPC96)</td>
</tr>
<tr>
<td>p</td>
<td>double</td>
<td>%10.0g</td>
<td>Growth rate of prices (GDPDEF)</td>
</tr>
<tr>
<td>r</td>
<td>double</td>
<td>%10.0g</td>
<td>Federal funds rate (FEDFUNDS)</td>
</tr>
<tr>
<td>c</td>
<td>double</td>
<td>%10.0g</td>
<td>Growth rate of consumption (PCECC96)</td>
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<tr>
<td>n</td>
<td>double</td>
<td>%10.0g</td>
<td>Growth rate of hours worked (HOANBS)</td>
</tr>
<tr>
<td>i</td>
<td>double</td>
<td>%10.0g</td>
<td>Corporate bond interest rate (AAA)</td>
</tr>
<tr>
<td>e</td>
<td>double</td>
<td>%10.0g</td>
<td>Percentage change in US exchange rate (TWEXBMTH)</td>
</tr>
</tbody>
</table>

Sorted by: dateq
GDP growth rate

```
.tsline y
```
An AR(1) model

Model:

\[ y_t = \rho y_{t-1} + u_t \]

State-space formulation:

\[ y_t = z_t \quad \text{(Observation equation)} \]
\[ z_t = \rho z_{t-1} + u_t \quad \text{(State transition equation)} \]

Stata specification:

```
. dsge (y=z) (f.z = \{rho\}*z, state)
```

With the `bayes` prefix:

```
. bayes, prior({rho}, uniform(-1,1)): dsge (y=z) (f.z = \{rho\}*z, state)
```
An AR(1) model

Output

```
. bayes, prior({rho}, uniform(-1,1)) rseed(20) : ///
>       dsge (y = z) (f.z = {rho}*z, state)

```

note: initial parameter vector set to means of priors.

Burn-in ...
Simulation ...
Model summary

Likelihood:
   y ~ dsgell({rho},{sd(e.z)})

Priors:
   {rho} ~ uniform(-1,1)
   {sd(e.z)} ~ igamma(.01,.01)

(Output continues on next slide)
An AR(1) model
Output II

Bayesian linear DSGE model
Random-walk Metropolis-Hastings sampling
Sample: 1955q1 thru 2015q4

MCMC iterations = 12,500
Burn-in = 2,500
MCMC sample size = 10,000
Number of obs = 244
Acceptance rate = .266
Efficiency: min = .1109
avg = .1181
max = .1253

Log marginal-likelihood = -648.62049

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. dev.</th>
<th>MCSE</th>
<th>Median</th>
<th>Equal-tailed [95% cred. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>rho</td>
<td>.3361959</td>
<td>.0592612</td>
<td>.001674</td>
<td>.3351618</td>
<td>.2176139 to .4571248</td>
</tr>
<tr>
<td>sd(e.z)</td>
<td>3.340431</td>
<td>.150282</td>
<td>.004513</td>
<td>3.336891</td>
<td>3.060339 to 3.649831</td>
</tr>
</tbody>
</table>
An AR(1) model

Diagnostics

```
.bayesgraph diagnostics {rho}
```

![Trace and Histogram of rho](image)

**Trace**

- **Iteration number**: 0, 2000, 4000, 6000, 8000, 10000
- **Lag**: 0, 2, 4, 6, 8
- **Density**: rho

![Autocorrelation and Density](image)

**Autocorrelation**

- **Lag**: 0, 1, 2, 3, 4, 5, 6

**Density**

- **Legend**: all, 1-half, 2-half
An AR(1) model
More diagnostics

```
.bayesstats ess

Efficiency summaries      MCMC sample size = 10,000
Efficiency:              min = 0.1109
                    avg = 0.1181
                    max = 0.1253

<table>
<thead>
<tr>
<th></th>
<th>ESS</th>
<th>Corr. time</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>rho</td>
<td>1252.97</td>
<td>7.98</td>
<td>0.1253</td>
</tr>
<tr>
<td>sd(e.z)</td>
<td>1108.99</td>
<td>9.02</td>
<td>0.1109</td>
</tr>
</tbody>
</table>
```
Summary

- Basic syntax of DSGE models:
  . `dsge (model_equations)`

- Basic syntax of Bayesian DSGE models:
  . `bayes, prior(prior_spec) : dsge (model_equations)`

- Parameter estimation
- Postestimation diagnostics
Outline

- DSGE models, dsge, and bayes: dsge
- AR(1) model
- Linear New Keynesian model
- Nonlinear stochastic growth model
Linear DSGE models

- We now return to a small, fully-featured DSGE model
- Equations:

\[ x_t = E_t x_{t+1} - (r_t - E_t \pi_{t+1} - z_t) \]  \hspace{1cm} \text{(IS)}

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \]  \hspace{1cm} \text{(Phillips Curve)}

\[ r_t = \frac{1}{\beta} \pi_t + w_t \]  \hspace{1cm} \text{(Taylor Rule)}

\[ w_{t+1} = \rho_w w_t + e_{t+1} \]

\[ z_{t+1} = \rho_z z_t + \varepsilon_{t+1} \]

- Control equations for output gap, inflation, interest rate
- State equations for monetary and IS disturbances (AR(1))
- Equations are linear in variables, nonlinear in parameters
- Forward-looking elements in the control equations
- Shocks flow into state variables, then into control variables
Linearized New Keynesian model

Data

```
. webuse usmacro2
(Federal Reserve Economic Data - St. Louis Fed, 2017-01-15)
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Contains data from https://www.stata-press.com/data/r17/usmacro2.dta
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<td>Percentage change in US exchange rate (TWEXBMTH)</td>
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</tbody>
</table>
```

Schenck (Stata)  Bayesian DSGE  June 7, 2021  24 / 55
Linearized New Keynesian model

Data

```
.tsline p r, legend(rows(2))
```

![Graph showing growth rate of prices (GDPDEF) and Federal funds rate (FEDFUNDS) over time.](image)

- **Growth rate of prices (GDPDEF)**
- **Federal funds rate (FEDFUNDS)**
Linear New Keynesian model

Model specification

.bayes, prior({beta}, beta(10, 10)) prior({kappa}, beta(30, 70)) ///
> prior({rhow}, beta(10, 10)) prior({rhoz}, beta(35, 15)) ///
> rseed(17) dots:
> dsge (x = F.x - (r - F.p - z) , unobserved ) ///
> (p = {beta}*F.p + {kappa}*x ) ///
> (r = (1/{beta})*p + w ) ///
> (F.z = {rhoz}*z , state ) ///
> (F.w = {rhow}*w , state )

note: initial parameter vector set to means of priors.

Burn-in 2500 aaaaaaaaa1000........2000..... done
Simulation 10000 ........1000........2000........3000........4000........5
> 000........6000........7000........8000........9000........10000 done
**Model summary**

<table>
<thead>
<tr>
<th>Likelihood:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p(r) \sim \text{dsgell}\left({\beta}, {\kappa}, {\rho z}, {\rho w}, {\text{sd}(e.z)}, {\text{sd}(e.w)}\right))</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Priors:</th>
</tr>
</thead>
<tbody>
<tr>
<td>${\beta}\sim \text{beta}(10,10)$</td>
</tr>
<tr>
<td>${\kappa}\sim \text{beta}(30,70)$</td>
</tr>
<tr>
<td>${\rho z}\sim \text{beta}(35,15)$</td>
</tr>
<tr>
<td>${\rho w}\sim \text{beta}(10,10)$</td>
</tr>
<tr>
<td>${\text{sd}(e.z) \text{ sd}(e.w)}\sim \text{igamma}(.01,.01)$</td>
</tr>
</tbody>
</table>
Bayesian linear DSGE model
Random-walk Metropolis-Hastings sampling
Sample: 1955q1 thru 2015q4

MCMC iterations = 12,500
Burn-in = 2,500
MCMC sample size = 10,000
Number of obs = 244
Acceptance rate = .1523
Efficiency: min = .01375
avg = .01964
max = .02488

Log marginal-likelihood = -784.87902

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. dev.</th>
<th>MCSE</th>
<th>Median</th>
<th>Equal-tailed [95% cred. interval]</th>
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<tr>
<td>beta</td>
<td>.4731495</td>
<td>.0506063</td>
<td>.003631</td>
<td>.4739432</td>
<td>.3692667 to .5735173</td>
</tr>
<tr>
<td>kappa</td>
<td>.261336</td>
<td>.0374749</td>
<td>.002812</td>
<td>.2595999</td>
<td>.1971056 to .341189</td>
</tr>
<tr>
<td>rhoz</td>
<td>.9084647</td>
<td>.0151727</td>
<td>.000962</td>
<td>.9083686</td>
<td>.8783911 to .9359358</td>
</tr>
<tr>
<td>rhow</td>
<td>.6309735</td>
<td>.033607</td>
<td>.002866</td>
<td>.6322336</td>
<td>.563363 to .6972567</td>
</tr>
<tr>
<td>sd(e.z)</td>
<td>.7389675</td>
<td>.0738833</td>
<td>.005184</td>
<td>.7357219</td>
<td>.60302 to .8981278</td>
</tr>
<tr>
<td>sd(e.w)</td>
<td>2.53286</td>
<td>.2537029</td>
<td>.017219</td>
<td>2.495344</td>
<td>2.128193 to 3.108239</td>
</tr>
</tbody>
</table>
Linear New Keynesian model

Diagnostics

.bayesgraph diagnostics kappa

Trace

Histogram

Autocorrelation

Density

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Linear New Keynesian model

Prior-posterior plot

```
.bayesgraph kdensity {kappa}, lcolor(red) ///
>    addplot(function Prior = betaden(30,70, x), ///
>    legend(on label(1 "Posterior")) lcolor(blue))
```

Density of \textit{kappa}
### Linear New Keynesian model

#### Diagnostics

```
.bayesstats ess
Efficiency summaries

<table>
<thead>
<tr>
<th></th>
<th>ESS</th>
<th>Corr. time</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
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<tr>
<td>kappa</td>
<td>177.56</td>
<td>56.32</td>
<td>0.0178</td>
</tr>
<tr>
<td>rhoz</td>
<td>248.80</td>
<td>40.19</td>
<td>0.0249</td>
</tr>
<tr>
<td>rhow</td>
<td>137.55</td>
<td>72.70</td>
<td>0.0138</td>
</tr>
<tr>
<td>sd(e.z)</td>
<td>203.12</td>
<td>49.23</td>
<td>0.0203</td>
</tr>
<tr>
<td>sd(e.w)</td>
<td>217.08</td>
<td>46.07</td>
<td>0.0217</td>
</tr>
</tbody>
</table>

MCMC sample size = 10,000
Efficiency: min = 0.01375
avg = 0.01964
max = 0.02488
```
One key object of interest in DSGE analysis is the impulse response function (IRF).

IRFs plot the response of model variables to unexpected shocks to a state variable.

A type of counterfactual analysis.

Answers to policy questions: what is the effect of a change in monetary policy (or fiscal policy, oil prices, etc) on variables of interest?
Stata has the `irf` suite of commands to create, plot, and make tables of IRFs.

Available after `dsge`, `var`, and some other time-series commands.

Now, this suite is extended to work after Bayesian estimation with `bayesirf`:

- `bayesirf set` to set a results file
- `bayesirf create` to compute IRFs
- `bayesirf graph` for plots
- `bayesirf table` for tables
- and other utility commands
Linear New Keynesian model

Impulse responses

. bayesirf set nkirf.irf, replace
(file nkirf.irf created)
(file nkirf.irf now active)

. bayesirf create model1
(file nkirf.irf updated)

. bayesirf graph irf, impulse(w) response(x p r w) byopts(yrescale)
Linear New Keynesian model

Impulse responses

Graphs by irfname, impulse variable, and response variable

95% equal-tailed CrI

Posterior mean of IRF
Linear New Keynesian model

Underidentified model

- Bayesian estimation can be performed even when the model parameters are not classically identified
- Identification then comes off the prior
- Consider the model:

\[
\begin{align*}
    x_t &= E_t x_{t+1} - \sigma (r_t - E_t \pi_{t+1} - z_t) \\
    \pi_t &= \beta E_t \pi_{t+1} + \kappa x_t \\
    r_t &= \frac{1}{\psi} \pi_t + w_t \\
    w_{t+1} &= \rho_w w_t + e_{t+1} \\
    z_{t+1} &= \rho_z z_t + \varepsilon_{t+1}
\end{align*}
\]

(IS) (Phillips Curve) (Taylor Rule)

- New parameters \((\sigma, \psi)\) not identified just from data on inflation and interest rate
- We can bring in prior information to aid estimation
Linear New Keynesian model
Underidentified model: setup

.bayes, prior({beta}, beta(95, 5)) prior({kappa}, beta(30,70)) ///
> prior({sigma}, beta(10,90)) prior({psi}, beta(67,33)) ///
> prior({rhow}, beta(10, 10)) prior({rhoz}, beta(35,15)) ///
> rseed(17) dots burnin(5000) mcmcsize(30000):
> dsge (x = F.x - {sigma}*(r - F.p - z) , unobserved ) ///
> (p = {beta}*F.p + {kappa}*x ) ///
> (r = (1/{psi})*p + w ) ///
> (F.z = {rhoz}*z , state ) ///
> (F.w = {rhow}*w , state )

note: initial parameter vector set to means of priors.

Burn-in 5000 aaaaaaaaa1000aaaaaaaaaa2000aaaa.....3000.........4000........5000
> done
Simulation 30000 ........1000........2000........3000.........4000........5
> 000........6000 ........7000........8000 ..........9000 ..........10000......
> .11000.........12000........13000.........14000.........15000.........16000.
> ........17000.........18000.........19000.........20000.........21000......
> .22000.........23000.........24000.........25000.........26000.........27000.
> ........28000.........29000.........30000 done
Linear New Keynesian model

Underidentified model: header output

Model summary

Likelihood:
\[ p \ r \ r \ \text{dsge}\{\text{sigma}, \text{beta}, \text{kappa}, \text{psi}, \text{rhoz}, \text{rhow}, \text{sd(e.z)}, \text{sd(e.w)}\} \]

Priors:
- \{sigma\} \sim \text{beta}(10, 90)
- \{beta\} \sim \text{beta}(95, 5)
- \{kappa\} \sim \text{beta}(30, 70)
- \{psi\} \sim \text{beta}(67, 33)
- \{rhoz\} \sim \text{beta}(35, 15)
- \{rhow\} \sim \text{beta}(10, 10)
- \{sd(e.z) \ sd(e.w)\} \sim \text{igamma}(0.01, 0.01)
Linear New Keynesian model
Underidentified model: estimation output

Bayesian linear DSGE model
Random-walk Metropolis-Hastings sampling
Sample: 1955q1 thru 2015q4

Log marginal-likelihood = -787.73905

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>MCSE</th>
<th>Median</th>
<th>Equal-tailed [95% cred. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>sigma</td>
<td>.1443227</td>
<td>.0292876</td>
<td>.001318</td>
<td>.1432416</td>
<td>.088498 to .2043906</td>
</tr>
<tr>
<td>beta</td>
<td>.9547238</td>
<td>.0203592</td>
<td>.001276</td>
<td>.9576523</td>
<td>.9077723 to .9848212</td>
</tr>
<tr>
<td>kappa</td>
<td>.3419745</td>
<td>.0457376</td>
<td>.003318</td>
<td>.3415295</td>
<td>.2517864 to .4357346</td>
</tr>
<tr>
<td>psi</td>
<td>.6527897</td>
<td>.043041</td>
<td>.003403</td>
<td>.6529768</td>
<td>.5686343 to .7351054</td>
</tr>
<tr>
<td>rhoz</td>
<td>.9078086</td>
<td>.0157278</td>
<td>.00091</td>
<td>.9080455</td>
<td>.8749843 to .9369648</td>
</tr>
<tr>
<td>rhow</td>
<td>.7546737</td>
<td>.0269813</td>
<td>.001109</td>
<td>.7541327</td>
<td>.7017534 to .8085894</td>
</tr>
<tr>
<td>sd(e.z)</td>
<td>.6048148</td>
<td>.0950875</td>
<td>.005623</td>
<td>.5951787</td>
<td>.4475909 to .8237128</td>
</tr>
<tr>
<td>sd(e.w)</td>
<td>1.955303</td>
<td>.1265823</td>
<td>.008904</td>
<td>1.948905</td>
<td>1.734296 to 2.230285</td>
</tr>
</tbody>
</table>
Linear New Keynesian model

Underidentified model: summaries of functions of parameters

```
. bayesstats summary (1/{psi})
Posterior summary statistics
    expr1 : 1/{psi}

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. dev.</th>
<th>MCSE</th>
<th>Median</th>
<th>Equal-tailed [95% cred. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>expr1</td>
<td>1.538681</td>
<td>.1035836</td>
<td>.008064</td>
<td>1.531448</td>
<td>1.360349 - 1.7586</td>
</tr>
</tbody>
</table>
```

MCMC sample size = 30,000
Linear New Keynesian model
Underidentified model: diagnostics of functions of parameters

.bayesgraph diagnostics (1/{psi})
Linear New Keynesian model
Underidentified model: prior/posterior graph setup

.bayesgraph kdensity {kappa}, lcolor(red) ///
>     addplot(function Prior= betaden(30,70, x), ///
>     legend(on label(1 "Posterior")) lcolor(blue)) name(kappa) nodraw

.bayesgraph kdensity {sigma}, lcolor(red) ///
>     addplot(function Prior= betaden(10,90, x), ///
>     legend(on label(1 "Posterior")) lcolor(blue)) name(sigma) nodraw

.bayesgraph kdensity {beta}, lcolor(red) ///
>     addplot(function Prior= betaden(95,5, x), ///
>     legend(on label(1 "Posterior")) lcolor(blue)) name(beta) nodraw

.bayesgraph kdensity {psi}, lcolor(red) ///
>     addplot(function Prior= betaden(67,33, x), ///
>     legend(on label(1 "Posterior")) lcolor(blue)) name(psi) nodraw

.graph combine kappa sigma beta psi
Linear New Keynesian model

Underidentified model: prior/posterior graphs

- Density of $\kappa$
- Density of $\sigma$
- Density of $\beta$
- Density of $\psi$
Outline

- DSGE models, dsge, and bayes: dsge
- AR(1) model
- Linear New Keynesian model
- Nonlinear stochastic growth model
Nonlinear DSGE models

- Linear DSGE models are linear in variables, potentially nonlinear in parameters.
- Nonlinear DSGE models are nonlinear in both variables and parameters.
- `dsgenl` estimates the parameters of nonlinear DSGE models by first-order approximation (linearization).
- `bayes: dsgenl` does the same, with Bayesian methods.
The stochastic growth model

Equations

\[
\frac{1}{c_t} = \beta E_t \left[ \left( \frac{1}{c_{t+1}} \right) (1 + r_{t+1} - \delta) \right] \quad \text{(Consumption)}
\]

\[ r_t = \alpha \frac{y_t}{k_t} \quad \text{(Capital demand)} \]

\[ y_t = z_t k_t^\alpha \quad \text{(Production function)} \]

\[ k_{t+1} = y_t - c_t + (1 - \delta) k_t \quad \text{(Capital accumulation)} \]

\[ \ln z_{t+1} = \rho \ln z_t + e_{t+1} \]
The stochastic growth model

Setup

```stata
.bayes, prior({alpha}, beta(30,70)) prior({beta}, beta(95,5)) ///
> prior({delta}, beta(25,975)) prior({rho}, beta(5, 3)) ///
> rseed(17) burnin(5000) dots : ///
> dsgenl (1/c = {beta}*(1/f.c)*(1+f.r-{delta})) ///
> (r = {alpha}*y/k) ///
> (y = z*k^({alpha})) ///
> (f.k = y - c + (1-{delta})*k) ///
> (ln(f.z) = {rho}*ln(z)) ///
> , exostate(z) endostate(k) observed(y) unobserved(c r)
```

note: initial parameter vector set to means of priors.

Burn-in 5000 aaaaaaaaa1000aaaaaaaaaa2000aaaaaaaaaa3000aaaaaaaaa...4000...........5000
> done
Simulation 10000 ...........1000............2000............3000............4000............5
> 000............6000............7000............8000............9000............10000 done
Model summary

Likelihood:
\[ y \sim \text{dsgell}(\{\beta}, \{\delta}, \{\alpha}, \{\rho}, \{\text{sd}(e.z)}) \]

Priors:
\[
\begin{align*}
\{\beta} & \sim \text{beta}(95,5) \\
\{\delta} & \sim \text{beta}(25,975) \\
\{\alpha} & \sim \text{beta}(30,70) \\
\{\rho} & \sim \text{beta}(5,3) \\
\{\text{sd}(e.z)} & \sim \text{igamma}(0.01,0.01)
\end{align*}
\]
The stochastic growth model

Estimation output

Bayesian first-order DSGE model
Random-walk Metropolis-Hastings sampling

MCMC iterations = 15,000
Burn-in = 5,000
MCMC sample size = 10,000

Sample: 1955q1 thru 2015q4
Number of obs = 244
Acceptance rate = .2506
Efficiency: min = .04563
avg = .04999
max = .05552

Log marginal-likelihood = -649.82949

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
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<th>Median</th>
<th>[95% cred. interval]</th>
<th>Equal-tailed</th>
</tr>
</thead>
<tbody>
<tr>
<td>beta</td>
<td>.9554009</td>
<td>.0194803</td>
<td>.00086</td>
<td>.9582636</td>
<td>.9104263</td>
<td>.9847218</td>
</tr>
<tr>
<td>delta</td>
<td>.0250477</td>
<td>.0048846</td>
<td>.00207</td>
<td>.0247132</td>
<td>.0163193</td>
<td>.0357851</td>
</tr>
<tr>
<td>alpha</td>
<td>.2962864</td>
<td>.0442748</td>
<td>.00207</td>
<td>.2958201</td>
<td>.2122214</td>
<td>.3837115</td>
</tr>
<tr>
<td>rho</td>
<td>.3064437</td>
<td>.0584439</td>
<td>.002654</td>
<td>.3047101</td>
<td>.1939408</td>
<td>.422685</td>
</tr>
<tr>
<td>sd(e.z)</td>
<td>3.359195</td>
<td>.152429</td>
<td>.006891</td>
<td>3.360512</td>
<td>3.068458</td>
<td>3.658345</td>
</tr>
</tbody>
</table>
The stochastic growth model

Prior/posterior graphs

```stata
.bayesgraph kdensity {alpha}, lcolor(red) ///
> addplot(function Prior=betaden(30,70,x), ///
> legend(on label(1 "Posterior")) lcolor(blue)) name(alpha) nodraw
.
.bayesgraph kdensity {beta}, lcolor(red) ///
> addplot(function Prior=betaden(95, 5,x), ///
> legend(on label(1 "Posterior")) lcolor(blue)) name(beta) nodraw
.
.bayesgraph kdensity {delta}, lcolor(red) ///
> addplot(function Prior=betaden(25,975,x), ///
> legend(on label(1 "Posterior")) lcolor(blue)) name(delta) nodraw
.
.bayesgraph kdensity {rho}, lcolor(red) ///
> addplot(function Prior=betaden(5,3,x), ///
> legend(on label(1 "Posterior")) lcolor(blue)) name(rho) nodraw
.
.graph combine alpha beta delta rho
```
The stochastic growth model
Prior/posterior graphs

Density of $\alpha$

Density of $\beta$

Density of $\delta$

Density of $\rho$
The stochastic growth model

Impulse responses

. bayesirf set stochmodel, replace
    (file stochmodel.irf created)
    (file stochmodel.irf now active)
. bayesirf create model1
    (file stochmodel.irf updated)
. bayesirf graph irf, impulse(z) byopts(yrescale)
The stochastic growth model

Impulse responses

Graphs by irfname, impulse variable, and response variable

95% equal-tailed CrI  Posterior mean of IRF
Final thoughts

- the bayes: prefix now supports dsge and dsgenl
- Bayesian impulse response functions supported with bayesirf
Thank you!