xtcluster: To pool or not to pool? A partially heterogeneous framework for short panel data models

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1. Estimation problem

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2. `xtcluster` estimation command

Outline

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- Sarafidis and Weber (2015, *Oxford Bulletin of Econ and Stat*) propose a middle ground, that of imposing partially heterogeneous restrictions with respect to the individuals, N. That is, individuals may behave in clusters with homogeneous slope parameters and the intra-cluster heterogeneity is attributed to unobserved fixed effects.
- The method is useful for exploring data in the absence of knowledge about parameter structures. It is also useful for examining the validity of *a priori* imposed structures, such as industry or risk classification, or some other economically-driven structure.

Estimation setting

- For a given linear short panel data model with exogenous regressors, the estimation problem is concerned with discovering potentially heterogeneous clusters of individuals, $i_\omega = 1, 2, \ldots, N_\omega$, each repeatedly observed over a fixed time period, $t = 1, 2, \ldots, T$.

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- Note that it is the individual that belongs to a cluster $\omega = 1, 2, \ldots, \Omega$ and not a single observation, hence no individual can be classified in more than one cluster.
- The focus is on the analysis of 'short panels' where $N >> T$, with $N \rightarrow \infty$ and T fixed. In practical applications, T can be unbalanced hence with individual-specific T_i in which case $\overline{T}$ is fixed.

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- The method has been originally developed for linear static panel data analysis with no endogenous regressors (i.e. can be applied with `xtreg`).

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- $\boldsymbol{\beta}_\omega$ is a $M \times 1$ vector of fixed parameters homogeneous to each cluster and heterogeneous across clusters.
- All remaining intra-cluster parameter heterogeneity is attributed to individual-specific and time-specific fixed effects, as part of the composite error term, $\epsilon_{i_\omega t} = u_{i_\omega} + t_s + v_{i_\omega t}$, where $v_{i_\omega t} \sim N\left(0, \sigma^2\right)$.

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6. Repeat steps 1 to 5 for different Ω sizes. The optimal Ω is the one which optimises the MIC.

Model Information Criterion (MIC)

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- where $\bar{T} = \frac{1}{N}\sum_{i=1}^{N} T_i$ is the average time series length for panels with unequal time series. For panels with equal time-series it holds that $\bar{T} = T$ (i.e. strongly or weakly balanced panels).

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- $f(\Omega)\, \theta_N$ is a penalty function for overfitting Ω. $f(\Omega)$ is a strictly increasing function, and the defaults are set to $f(\Omega) = \Omega$ and $\theta_N = 13 \log N + 23N$.
- Note that $1 \geq \Omega \geq \xi$ where $\xi << N$. Again, we do not know Ω but for $N \to \infty$ and T fixed, the MIC will point to the true Ω.

Outline

Stata estimation command

`xtcluster` requires the initialisation of cluster partition and then iterates the reclassification of individuals to all clusters up to the convergence of the RSS. The `eclass` command has the following syntax:

`xtcluster` *depvar indepvars* [if] [in] , [*options*]

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`xtcluster` *depvar indepvars* `[if] [in] , [`*options*`]`

where *options*, with their default values, include:

- `name`(om)
- `iterate`(100)
- `theta`(default)
- `initpart`(*suboptions*)

```
random seed(123)
omega(3)
preclass(var)
ktype(kmeans)
prevars(varlist)
```

Stata estimation command

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Alternatively, the user may specify a predetermined initial partition and the size of Ω on the basis of some theoretical classification:

- `initpart(preclass(`*indicator_var*`))`

Finally, the initial partition may be obtained on the basis of pre-specified variables, using the Calinski-Harabasz clustering criterion:

- `initpart(prevars(X) omega(3))`
- `initpart(prevars(`*varlist*`) omega(3) ktype(kmedians))`
- `initpart(prevars(b) omega(3))`

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- `ereturn list` returns two scalars: the model information criterion as `e(mic)` and the specified size of Ω as `e(omega)`. It also returns a matrix: the *RSS* at every iteration in vector form as `e(rss)`.

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- `ereturn list` returns two scalars: the model information criterion as `e(mic)` and the specified size of Ω as `e(omega)`. It also returns a matrix: the *RSS* at every iteration in vector form as `e(rss)`.
- `xtcluster` generates an indicator variable as specified in option `name()` that takes the values $\omega = 1, 2, \ldots, \Omega$ (the default name is `om`). This can then be used for subsequent analysis, e.g.:

```
.  forvalues i = 1/`=e(omega)' {
2.  xtreg y x1 x2 if om==`i', fe vce(robust)
3.  }
```

Outline

Application 1: supporting evidence for homogeneity assumption

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- `xtcluster` can be used to examine for the underlying hypothesis in `xtreg` that the slopes are homogeneous across all panels.
- Munnell (1990) and Baltagi, Song and Jung (2001) apply a Cobb-Douglas production function for modelling the productivity of public capital at the state level, as a function of private capital stock, highway component, water component, building, and unemployment rate. The `productivity.dta` dataset contains observations on 48 U.S. states (panels) over 1970-1986. and the xtcluster command suggests that the slopes are homogeneous.
- The `productivity.dta` dataset is available from the StataPress website and is discussed in the manual entry of the `mixed` command.

Application 1: randomized initial partition

```
. use http://www.stata-press.com/data/r14/productivity.dta
. xtcluster gsp private emp hwy water other unemp,
> init(random omega(2) seed(123)) name(om2)

Initial partition through randomized classification

Partitional clustering iterations:

Iteration 1:   Total RSS = .7157044
Iteration 2:   Total RSS = .6881156
Iteration 3:   Total RSS = .6862053
Iteration 4:   Total RSS = .6744427
Iteration 5:   Total RSS = .6588415
Iteration 6:   Total RSS = .6588415
```

Specified Number of clusters	Total Residual Sum of Squares	Model Information Criterion
2	0.659	1738.298

Application 1: initial partition via prespecified variation

```
. xtcluster gsp private emp hwy water other unemp,
> init(prevars(X) omega(4))

Initial partition given prespecified variables:

Calinski & Harabasz pseudo-F for 1 groups:   .
Calinski & Harabasz pseudo-F for 2 groups:   1653.544
Calinski & Harabasz pseudo-F for 3 groups:   3419.6376
Calinski & Harabasz pseudo-F for 4 groups:   6893.3135

Partitional clustering iterations:

Iteration 1:   Total RSS = .4971343
Iteration 2:   Total RSS = .4665229
Iteration 3:   Total RSS = .4665229
```

Specified Number of clusters	Total Residual Sum of Squares	Model Information Criterion
4	0.467	4030.381

Application 1: initial partition via prespecified classes

```
. xtcluster gsp private emp hwy water other unemp,
> init(preclass(region)) name(om_region)

Initial partition given prespecified classification: region

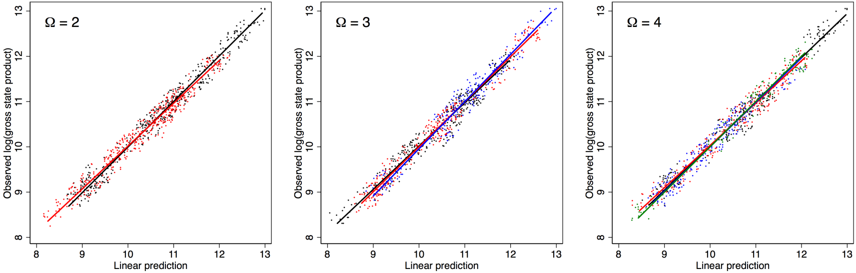
Partitional clustering iterations:

Iteration 1:   Total RSS = .3527132
Iteration 2:   Total RSS = .3457506
Iteration 3:   Total RSS = .3428682
Iteration 4:   Total RSS = .3428682
```

Specified Number of clusters	Total Residual Sum of Squares	Model Information Criterion
9	0.343	9787.227

Application 1: different sizes for Ω

```
. forvalues i = 3/8 {
2. xtcluster gsp private emp hwy water other unemp,
>  init(random omega(`i') seed(123)) name(om`i')
3. }
```



- xtcluster pretty much confirms slope homogeneity across states.

Application 2: discovering potential heterogeneity

- The company audit costs (ln_af) are determined as a exponential function of the size of the audit in terms of company assets (ln_at), as well as the inherent risk of the audit in terms of rate of liquidity (cr) and leverage exposure (d2e).
- The dataset audit.dta holds hand-collected observations on S&P ASX200 company financials over 2000-2007 including audit fees plus other company characteristics. It is an unbalanced panel dataset, and is available from the MEAFA website:

```
. use http://meafa3.econ.usyd.edu.au/dta/audit.dta
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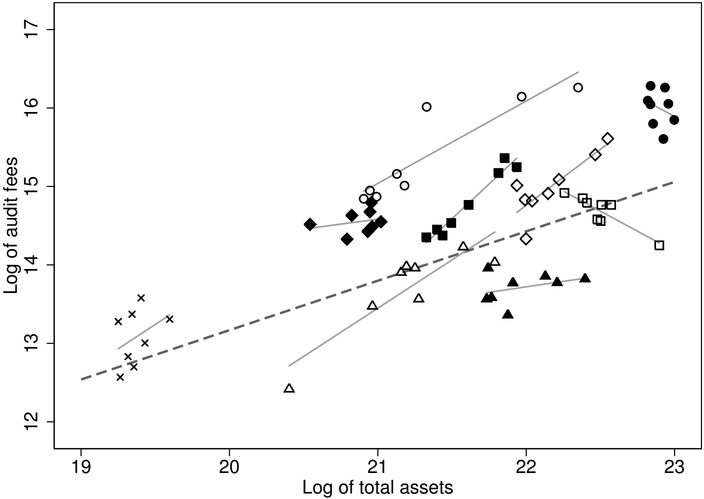
- An initial EDA investigation on parameter structure suggests that `xtcluster` may be well suited for discovering heterogeneity in this data. There is evident heterogeneity in slope coefficients plus remaining heterogeneity in intercepts within clusters of slopes.

Application 2: evidence for potential slope heterogeneity



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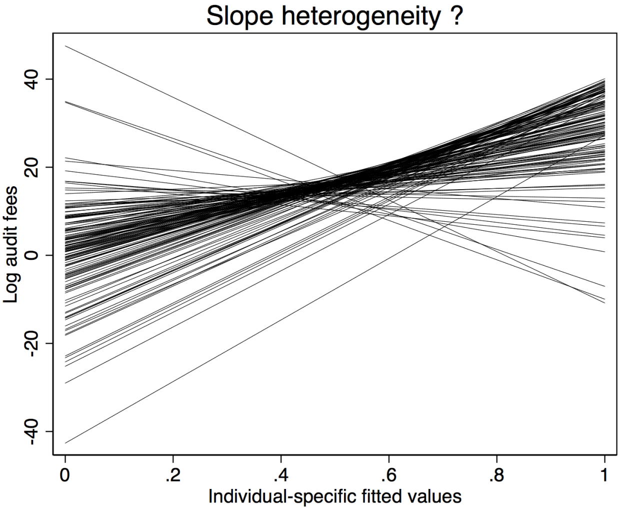

Application 2: examining claimed heterogeneity

- In addition to exploring for potential slope heterogeneity, `xtcluster` can be applied to examine the claims made by the Australian company regulator, ASIC, offering financial reporting cost relief to companies by allowing them to redact disclosure for wholly-owned subsidiary companies. If the relief is effective then companies benefiting should be described by less steep slopes.

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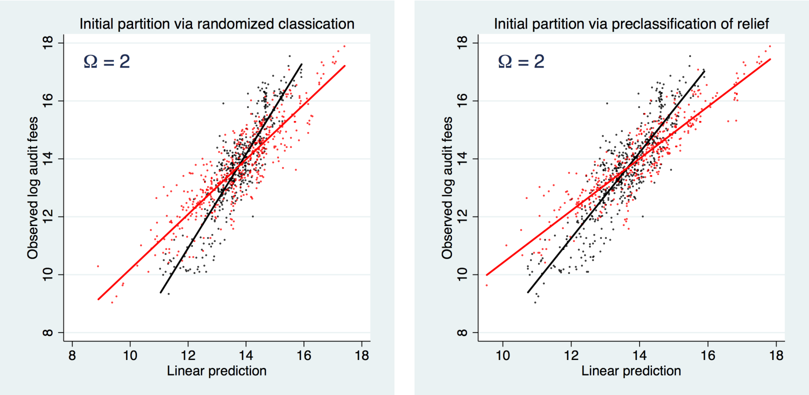
```
. use http://meafa3.econ.usyd.edu.au/dta/audit.dta
. generate ln_af = ln(af)
. generate ln_at = ln(at)
. generate cr = lc/ac
. generate d2e = lt/(at-lt)
. xtcluster ln_af ln_at cr d2e, init(random omega(2)) name(om2)
. xtcluster ln_af ln_at cr d2e, init(preclass(relief))
```

Application 2: heterogeneous slopes

- The MIC suggests optimal partition at $\Omega = 2$.



Application 2: examining the assumption of relief

- Obtaining the initial partition through either randomised classification or preclassification seems to converge to the same final cluster partition. To check if they are the same individuals:

```
. egen tag = tag(id)
. tabulate om2 om_relief if tag
```

om2	om_relief 1	om_relief 2	Total
1	**70**	**7**	**77**
2	**7**	**52**	**59**
Total	**77**	**59**	**136**

Application 2: examining the assumption of relief

- More interestingly, to examine the assumption of `relief`, `tabulate` the optimal partition for $\Omega = 2$ with the binary preclassification of `relief`:

```
. tabulate om_relief relief if tag
. tabulate om2 relief if tag
```

om_relief	relief: Deed	relief: 2	Total
1	45	26	71
2	28	27	55
Total	73	53	126

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- `xtcluster` is computationally intensive so it can be very slow with large data. We are working on increasing computational efficiency.

Concluding remarks

- `xtcluster` can be used to either assess the assumption of slope homogeneity in linear short panel data models, or to discover potential heterogeneous clusters (an exploratory data approach).
- `xtcluster` is computationally intensive so it can be very slow with large data. We are working on increasing computational efficiency.
- We also plan to do the following:
    - Incorporate other methods for obtaining the initial partition, such as on the basis of individual-specific estimated parameters.
    - Extend the application of `xtcluster` and the MIC to other estimators and other objective functions.
    - Allow for more complex factor structures in the error term.

MEAFA forthcoming Stata training

- Google the keyword '**MEAFA**' for forthcoming Stata workshops:

- 28 Sep 2015: **Text Analysis** by Normand Peladeau, Provalis Research.
- 14-16 Dec 2015: **Microeconometrics for Count Data** by Colin Cameron, University of California at Davis.
- 15-19 Feb 2016: **Panel Data Analysis: Linear, Dynamic, Nonlinear, Mixed, Hierarchical** by Vasilis Sarafidis, Monash University.

- Feb 2016: 2 more MEAFA PhD top-up research scholarships.