Estimation of Average Causal Effects of Dichotomous Treatments by Propensity Score Weighting and Regression Adjustment

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Estimation of causal effects

- The estimation of average causal effects of treatments is a central goal of many disciplines

Estimation of causal effects

- Let:
  - T denote the dichotomous treatment of interest (1=treated , 0=control)
  - Y_{1i} denote the value taken on by outcome variable Y when subject i is treated
  - Y_{0i} denote the value taken on by outcome variable Y when subject i is not treated

Estimation of causal effects

- Researchers are often interested in estimating the average causal effect of treatment *T* in the whole population, defined as:

$$\tau = E(Y_1 - Y_0)$$

Estimation of causal effects

- This definition of τ makes it difficult to estimate, since for each subject *i* we only observe either Y_1 or Y_0, but never both
- To tackle this estimation problem we must be ready to make some assumptions, the most important of which is the unconfoundedness assumption

Estimation of causal effects

- The unconfoundedness assumption asserts that the distribution of outcomes Y_1 and Y_0 is independent of treatment T conditional on the values taken on by a set of pre-treatment (control) variables X. Formally:

$$(Y_1, Y_0) \perp T \mid X$$

Estimation of causal effects

- If the unconfoundedness assumption holds, then the average causal effect of treatment *T* in the whole population can be defined as:

$$\tau = E_X\left(E(Y_1 \mid T = 1, X = x) - E(Y_0 \mid T = 0, X = x)\right)$$

- Contrary to the previous one, this definition of τ has the merit of being based on fully observable quantities

Estimation of causal effects

- In Stata, average causal effects of treatments can be estimated by means of several user-written commands that implement some kind of matching:
  - **`nnmatch`** (Abadie, Leber Herr, Imbens, Drukker)
  - **`pscore`** (Becker, Ichino)
  - **`psmatch2`** (Leuven, Sianesi)

Estimation of causal effects

- Here I propose a new Stata command, called **`treateff`**, designed to estimate average causal effects of dichotomous treatments by means of a combination of regression adjustment and weighting based on the propensity score
- **`treateff`** is a close implementation of the estimators described in Hirano and Imbens (2001)

Estimation of causal effects

- Following Robins and Rotnitzky (1995), Hirano and Imbens (2001) have proposed a class of estimators of τ based on weighted estimation of the regression function

$$\eta_i = \beta_0 + \tau \cdot T_i + \beta_1 Z_i + \beta_2 (Z_i - \bar{Z}) T_i$$

where $\eta_i = \bullet\left(E(Y_i)\right)$ and Z_i denotes a subset of the control variables X_i with sample mean $\bar{Z}$

Estimation of causal effects

- The weights are defined as follows:

$$\hat{\omega}_i = \frac{T_i}{\hat{e}(V_i)} + \frac{1 - T_i}{1 - \hat{e}(V_i)}$$

where $\hat{e}(V_i)$ denotes the estimated propensity score and V_i denotes a subset of the control variables X_i

Estimation of causal effects

- Researchers are also often interested in estimating the average causal effect of treatment *T* in the treated subpopulation, defined as:

$$\tau_1 = E(Y_1 - Y_0 \mid T = 1)$$

Estimation of causal effects

- In this case, the regression function to be estimated is defined as:

$$\eta_i = \beta_0 + \tau_1 \cdot T_i + \beta_1 Z_i + \beta_2 (Z_i - \bar{Z}_1) T_i$$

and weights are defined as:

$$\hat{\omega}_i = T_i + \frac{(1 - T_i) \cdot \hat{e}(V_i)}{1 - \hat{e}(V_i)}$$

Estimation of causal effects

- Finally, one may be interested in estimating the average causal effect of treatment *T* in the control subpopulation, defined as:

$$\tau_0 = E(Y_1 - Y_0 \mid T = 0)$$

Estimation of causal effects

- In this case, the regression function to be estimated is defined as:

$$\eta_i = \beta_0 + \tau_0 \cdot T_i + \beta_1 Z_i + \beta_2 (Z_i - \bar{Z}_0) T_i$$

and weights are defined as:

$$\hat{\omega}_i = \frac{T_i \cdot \left(1 - \hat{e}(V_i)\right)}{\hat{e}(V_i)} + (1 - T_i)$$

Example

- Data: NSW-PSID1 (Dehejia and Wahba 1999)
- Outcome: re78 (real earnings in 1978)
- Treatment: t (participation in the National Supported Work Program)
- *V* variables: age, age^2, educ, $educ^2$, nodegree, black, hisp, married, re74, $re74^2$, re75, $re75^2$
- *Z* variables: same as *V*
- Goal: estimation of τ_1

Example

```
treateff re78 t, pscore(age-re75_2)
```

Example

Estimates of effects

Number of treated obs. = 185
Number of control obs. = 2490

Effect	Mean difference	Robust Std.Err.	Robust 95% confidence interval	
raw	-1.5e+04	655.914	-1.6e+04	-1.4e+04
adj	1417.821	790.748	-132.017	2967.659
ate	-1.3e+04	2653.015	-1.8e+04	-7.5e+03
att	1214.372	1069.836	-882.469	3311.213
atc	-1.4e+04	2773.994	-1.9e+04	-8.4e+03

Example

```
treateff re78 t,                        ///
          pscore(age-re75_2)    ///
          format(%6.0f)
```

Example

```
Estimates of effects                              Number of treated obs. =    185
                                                  Number of control obs. =   2490

-------------------------------------------------------------------------------
 Effect              Mean              Robust                Robust
                  difference          Std.Err.       95% confidence interval
-------------------------------------------------------------------------------
  raw              -15205               656             -16490       -13919
  adj                1418               791               -132         2968
  ate              -12706              2653             -17906        -7507
  att                1214              1070               -882         3311
  atc              -13833              2774             -19270        -8396
-------------------------------------------------------------------------------
```

Example

```
treateff re78 t,                      ///
         pscore(age-re75_2)           ///
         adj(_PS_)                    ///
         format(%6.0f)
```

Example

```
Estimates of effects                             Number of treated obs. =    185
                                                 Number of control obs. =   2490

-------------------------------------------------------------------------------
 Effect           Mean           Robust                    Robust
               difference        Std.Err.          95% confidence interval
-------------------------------------------------------------------------------
  raw           -15205             656            -16490       -13919
  adj             1418             791              -132         2968
  ate           -10197            2863            -15809        -4586
  att             1818             771               306         3329
  atc           -12002            3748            -19348        -4656
-------------------------------------------------------------------------------
```

Example

- `treateff` is planned to offer the user several options, e.g., estimation of standard errors by bootstrap:

```
treateff re78 t, pscore(age-re75_2) ///
         adj(_PS_) format(%6.0f)    ///
         bootstrap breps(1000)      ///
         bstrata(t)
```

Example

Estimates of effects

Number of treated obs. = 185
Number of control obs. = 2490
Bootstrap replications = 1000

Effect	Mean difference	Bootstrap Std.Err.	Normal 95% confidence interval	
raw	-15205	664	-16508	-13902
adj	1418	803	-157	2993
ate	-10197	21918	-53208	32814
att	1818	898	55	3580
atc	-12002	26001	-63025	39022

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References

- Hirano, K. and Imbens, G. (2001) "Estimation of Causal Effects using Propensity Score Weighting: An Application to Data on Right Heart Catheterization", *Health Services & Outcomes Research Methodology*, 2, 259-278.
- Robins, J. and Rotnitzky, A. (1995) “Semiparametric Efficiency in Multivariate Regression Models with Missing Data”, *Journal of the American Statistical Association*, 90, pp. 122-129.