Usefulness and estimation of proportionality constraints

The propcnsreg package

Maarten L. Buis

Department of Social Research Methodology
Vrije Universiteit Amsterdam
http://home.fsw.vu.nl/m.buis/
Outline

usefulness
  proportionality constraint
  a latent variable
  scale for a categorical variable

estimation
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proportionality constraint
a latent variable
scale for a categorical variable

estimation
Hypothesis: Effect of father’s and mother’s socioeconomic status on child’s education can change over cohorts,
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\[ ed = \beta_0 + \beta_1 coh + (1 + \lambda_1 coh)(\gamma_1 pasei + \gamma_2 masei) \]
example

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Effect of father’s and mother’s socioeconomic status on child’s education can change over cohorts, but the relative contribution of the father and the mother remains constant.

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empirical example

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- Variable *byr*: cohort centered in 1940 and measuring time in decades, ranges between 1929 and 1979.
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- Variable *degree*: educational attainment in pseudo years
- Variable *byr*: cohort centered in 1940 and measuring time in decades, ranges between 1929 and 1979.
- Variables *pasei* and *masei*: Father’s and mother’s occupational status, ranges between 0 and 1.
Usefulness and estimation of proportionality constraints

```
. propcnsreg degree byr, lambda(byr) constrained(masei pasei) lcons
Constraint: [lambda]_cons = 1

+--------------------------------------------------+
| degree | Coef.   | Std. Err. | z       | P>|z|  | [95% Conf. Interval] |
|--------|---------|-----------|---------|------|---------------------|
| unconstrained | | | | | |
| byr    | .0392033 | .1418648 | 0.28    | 0.782 | -.2388465   .3172531 |
| _cons  | 10.2406  | .2762536  | 37.07   | 0.000 | 9.699157     10.78205 |
| constrained | | | | | |
| masei  | 3.363018 | .3688164  | 9.12    | 0.000 | 2.640152     4.085885 |
| pasei  | 3.948723 | .3972388  | 9.94    | 0.000 | 3.170149     4.727296 |
| lambda | | | | | |
| byr    | -.0323712 | .037854   | -0.86   | 0.392 | -.1065637   .0418212 |
| _cons  | 1        |          |         |      | .         .        |
| ln_sigma | | | | | |
| _cons  | .837853  | .014199  | 59.01   | 0.000 | .8100234    .8656826 |
+--------------------------------------------------+
LR test vs. unconstrained model: chi2(1) = 0.04 Prob > chi2 = 0.849
```
alternative way of looking

\[ ed = \beta_0 + \beta_1 coh + (\lambda_0 + \lambda_1 coh) \left( \gamma_1 pasei + \gamma_2 masei \right) \]

latent family sei

Need to identify the latent variable by fixing the origin and the scale.

If the minimum value of \( pasei \) and \( masei \) is 0 then the origin is fixed to when both variables are minimum.
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\[ ed = \beta_0 + \beta_1 coh + (\lambda_0 + \lambda_1 coh) (\gamma_1 pasei + \gamma_2 masei) \]

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\[ \text{latent family sei} = \gamma_1 \text{pasei} + \gamma_2 \text{masei} \]
alternative way of looking

$$ed = \beta_0 + \beta_1 coh + (\lambda_0 + \lambda_1 coh) (\gamma_1 pasei + \gamma_2 masei)$$

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latent family sei $= \gamma_1 0 + \gamma_2 0 = 0$
alternative way of looking

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- If the minimum value of \( \text{pasei} \) and \( \text{masei} \) is 0 then the origin is fixed to when both variables are minimum.
- If the maximum value of \( \text{pasei} \) and \( \text{masei} \) is 1, and
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\[ ed = \beta_0 + \beta_1 \text{coh} + (\lambda_0 + \lambda_1 \text{coh}) (\gamma_1 \text{pasei} + \gamma_2 \text{masei}) \]

- Need to identify the latent variable by fixing the origin and the scale.
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- If the maximum value of \textit{pasei} and \textit{masei} is 1, and their parameters are constrained to sum to 1,
alternative way of looking

\[ ed = \beta_0 + \beta_1 coh + (\lambda_0 + \lambda_1 coh) (\gamma_1 pasei + \gamma_2 masei) \]

-latent family sei

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- If the minimum value of \textit{pasei} and \textit{masei} is 0 then the origin is fixed to when both variables are minimum.
- If the maximum value of \textit{pasei} and \textit{masei} is 1, and their parameters are constrained to sum to 1, then the unit is fixed to the distance between both variables at minimum and both variables at maximum.
alternative way of looking

\[ ed = \beta_0 + \beta_1 coh + (\lambda_0 + \lambda_1 coh) (\gamma_1 pasei + \gamma_2 masei) \]

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latent family sei = \( \gamma_1 pasei + \gamma_2 masei \)
alternative way of looking

$$ed = \beta_0 + \beta_1 coh + (\lambda_0 + \lambda_1 coh) (\gamma_1 pasei + \gamma_2 masei)$$

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- If the minimum value of $pasei$ and $masei$ is 0 then the origin is fixed to when both variables are minimum.
- If the maximum value of $pasei$ and $masei$ is 1, and their parameters are constrained to sum to 1, then the unit is fixed to the distance between both variables at minimum and both variables at maximum.

$$\text{latent family sei} = \gamma_1 1 + \gamma_2 1 = 1$$
Usefulness and estimation of proportionality constraints

A latent variable scale for a categorical variable

### Example Output

```
.propcnsreg degree byr, lambda(byr) constrained(masei pasei) unit(masei pasei)
Constraint: [constrained]masei + [constrained]pasei = 1

| degree | Coef.   | Std. Err. | z     | P>|z|  | [95% Conf. Interval] |
|--------|---------|-----------|-------|------|---------------------|
| unconstrained |       |           |       |      |                     |
| byr    | 0.0392033 | 0.1418647 | 0.28  | 0.782 | [-0.2388464, 0.3172529] |
| _cons  | 10.2406  | 2.762534  | 37.07 | 0.000 | [9.699158, 10.78205]   |
| constrained |     |           |       |      |                     |
| masei  | 0.4599477 | 0.0323745 | 14.21 | 0.000 | [0.3964949, 0.5234005]  |
| pasei  | 0.5400523 | 0.0323745 | 16.68 | 0.000 | [0.4765995, 0.6035051]  |
| lambda |         |           |       |      |                     |
| byr    | -0.2366899 | 0.2935214 | -0.81 | 0.420 | [-0.8119814, 0.3386015]  |
| _cons  | 7.311741   | 6.01956   | 12.15 | 0.000 | [6.131929, 8.491553]    |
| ln_sigma |       |           |       |      |                     |
| _cons  | 0.837853   | 0.014199  | 59.01 | 0.000 | [0.8100234, 0.8656826]   |

LR test vs. unconstrained model: chi2(1) = 0.04  Prob > chi2 = 0.849
```
scale for a categorical variable

Example

- Differences in the effect of education in 5 dummies on occupational status between white and black US men:
  - < highschool (reference)
  - highschool ($hs$)
  - some college ($sc$)
  - college ($c$)
  - graduate ($g$)
scale for a categorical variable

Example

- Differences in the effect of education in 5 dummies on occupational status between white and black US men:
  - < highschool (reference)
  - highschool (hs)
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\[
isei = \beta_0 + (\lambda_0 + \lambda_1 \text{black})(\gamma_1 hs + \gamma_2 sc + \gamma_3 c + \gamma_4 g)
\]
Example

Differences in the effect of education in 5 dummies on occupational status between white and black US men:

- < highschool (reference)
- highschool (hs)
- some college (sc)
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isei = \beta_0 + (\lambda_0 + \lambda_1 black)(\gamma_1 hs + \gamma_2 sc + \gamma_3 c + 1g)
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Example

- Differences in the effect of education in 5 dummies on occupational status between white and black US men:
  - < highschool (reference)
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  - college (c)
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\[
isei = \beta_0 + (\lambda_0 + \lambda_1 \text{black})(\gamma_1 hs + \gamma_2 sc + \gamma_3 c + 1 g)
\]

\(\gamma_1, \gamma_2, \text{ and } \gamma_3\) now measure the position of highschool, some college, and college education, relative to less than highschool (0) and graduate (1).
```stata
.example output

. propcnsreg sei black, lambda(black) constrained(hs sc c g) unit(g)
Constraint: [constrained]g = 1

|       | Coef.  | Std. Err. | z     | P>|z|  | [95% Conf. Interval] |
|-------|--------|-----------|-------|------|---------------------|
| sei   |        |           |       |      |                     |
| unconstrained |        |           |       |      |                     |
| black | -.042371 | .009563   | -4.43 | 0.000 | -.0611141 -.0236279 |
| _cons | .3638307 | .0076114  | 47.80 | 0.000 | .3489126 .3787488  |
| constrained |        |           |       |      |                     |
| hs    | .2226429 | .016662   | 13.36 | 0.000 | .1899861 .2552997  |
| sc    | .4411229 | .0206904  | 21.32 | 0.000 | .4005705 .4816753  |
| c     | .7185653 | .01676    | 42.87 | 0.000 | .6857163 .7514144  |
| g     | 1       |           |       |      |                     |
| lambda |        |           |       |      |                     |
| black | .0458751 | .0227816  | 2.01  | 0.044 | .0012239 .0905263  |
| _cons | .38541  | .0099432  | 38.76 | 0.000 | .3659217 .4048983  |
| ln_sigma |        |           |       |      |                     |
| _cons | -1.859163 | .0090043  | -206.48 | 0.000 | -1.876811 -1.841515 |

LR test vs. unconstrained model: chi2(3) = 5.42  Prob > chi2 = 0.144
```

Usefulness and estimation of proportionality constraints
Scaling of education

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Usefulness and estimation of proportionality constraints
Scaling of education

Scaling of education in pseudo years

- Graduate: ◆
- Bachelor: ◊
- Junior college: +
- Highschool: ●
- < Highschool: ○

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Usefulness and estimation of proportionality constraints
Outline

usefulness
- proportionality constraint
- a latent variable
- scale for a categorical variable

estimation
EM-algorithm for starting values

\[ y = \beta_0 + \beta_1 x_1 + (\lambda_0 + \lambda_1 x_1)(\gamma_1 z_1 + \gamma_2 z_2) \]
EM-algorithm for starting values

\[ y = \beta_0 + \beta_1 x_1 + (\lambda_0 + \lambda_1 x_1)(\gamma_1 z_1 + \gamma_2 z_2) \]

1. Given current estimates/start value for \( \gamma \), create a new variable containing the latent variable.
EM-algorithm for starting values

\[ y = \beta_0 + \beta_1 x_1 + (\lambda_0 + \lambda_1 x_1)(\gamma_1 z_1 + \gamma_2 z_2) \]

1. Given current estimates/starting value for \( \gamma \), create a new variable containing the latent variable. This simplifies the problem to: \( y = \beta_0 + \beta_1 x_1 + \lambda_0 latent + \lambda_1 x_1 latent \)
EM-algorithm for starting values

\[ y = \beta_0 + \beta_1 x_1 + (\lambda_0 + \lambda_1 x_1)(\gamma_1 z_1 + \gamma_2 z_2) \]

1. Given current estimates/starting value for \( \gamma \), create a new variable containing the latent variable. This simplifies the problem to: \( y = \beta_0 + \beta_1 x_1 + \lambda_0 \text{latent} + \lambda_1 x_1 \text{latent} \)

2. Estimate \( \beta \) and \( \lambda \) using `regress`

3. Given current estimates of \( \lambda \), create a new variable containing the effect of the latent variable. This simplifies the problem to: \( y = \beta_0 + \beta_1 x_1 + \gamma_1 \text{effect} z_1 + \gamma_2 \text{effect} z_2 \)

4. Estimate \( \beta \) and \( \gamma \) using `cnsreg` imposing the constraint specified in the `unit` option.

5. Repeat steps 1-4 till convergence.
EM-algorithm for starting values

\[ y = \beta_0 + \beta_1 x_1 + (\lambda_0 + \lambda_1 x_1)(\gamma_1 z_1 + \gamma_2 z_2) \]

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EM-algorithm for starting values

\[ y = \beta_0 + \beta_1 x_1 + (\lambda_0 + \lambda_1 x_1)(\gamma_1 z_1 + \gamma_2 z_2) \]

1. Given current estimates/starting value for \( \gamma \), create a new variable containing the latent variable. This simplifies the problem to: \( y = \beta_0 + \beta_1 x_1 + \lambda_0 latent + \lambda_1 x_1 latent \)

2. Estimate \( \beta \) and \( \lambda \) using `regress`

3. Given current estimates of \( \lambda \), create a new variable containing the effect of the latent variable. This simplifies the problem to: \( y = \beta_0 + \beta_1 x_1 + \gamma_1 effectz_1 + \gamma_2 effectz_2 \)
EM-algorithm for starting values

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EM-algorithm for starting values

\[ y = \beta_0 + \beta_1 x_1 + (\lambda_0 + \lambda_1 x_1)(\gamma_1 z_1 + \gamma_2 z_2) \]

1. Given current estimates/starting value for \( \gamma \), create a new variable containing the latent variable. This simplifies the problem to: \[ y = \beta_0 + \beta_1 x_1 + \lambda_0 \text{latent} + \lambda_1 x_1 \text{latent} \]

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4. Estimate \( \beta \) and \( \gamma \) using \texttt{cnsreg} imposing the constraint specified in the \texttt{unit} option.

5. Repeat steps 1-4 till convergence.
speed and standard errors
speed and standard errors

- To speed up convergence every 5th iteration will consist of two \texttt{ml} iterations for the complete model.
speed and standard errors

- To speed up convergence every 5\textsuperscript{th} iteration will consist of two \texttt{ml} iterations for the complete model.
- Once the EM has converged, these estimates are fed into \texttt{ml} for the complete model to get the variance covariance matrix.
example iteration log

<table>
<thead>
<tr>
<th>iteration</th>
<th>unconstrained part only</th>
<th>constrained part only</th>
<th>full model part only</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2712.7047</td>
<td>2716.1367</td>
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<tr>
<td>2</td>
<td>2716.4376</td>
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<tr>
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<td>4</td>
<td>2716.674</td>
<td>2716.6825</td>
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</tr>
<tr>
<td>5</td>
<td>two iterations from full model</td>
<td></td>
<td>2716.6914</td>
</tr>
<tr>
<td>6</td>
<td>2716.6914</td>
<td>2716.6914</td>
<td></td>
</tr>
</tbody>
</table>

estimating full model

Iteration 0: log likelihood = 2716.6899
Iteration 1: log likelihood = 2716.6914
Iteration 2: log likelihood = 2716.6914
Conclusion

▶ A proportionality constraint means that the effects of a group of variables changes, but that the relative differences in the sizes of the effect remain constant.
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- This can be of interest in its own right, e.g. the effect on child’s education of father’s and mother’s status change, but the relative contribution of each parent can remain constant.
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- It can also be interpreted in terms of a latent variable, e.g. father’s and mother’s status both measure family status.
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▶ This can be of interest in its own right, e.g. the effect on child’s education of father’s and mother’s status change, but the relative contribution of each parent can remain constant.

▶ It can also be interpreted in terms of a latent variable, e.g. father’s and mother’s status both measure family status.

▶ Standard ml can have a hard time converging, so starting values are created using a EM algorithm.