Title

> **xtprobit —** Random-effects and population-averaged probit models

Description

xtprobit fits random-effects and population-averaged probit models for a binary dependent variable. The probability of a positive outcome is assumed to be determined by the standard normal cumulative distribution function.

Quick start

Random-effects probit model of y as a function of x1, x2, and indicators for levels of categorical variable a using xtset data

    xtprobit y x1 x2 i.a

Population-averaged model with robust standard errors

    xtprobit y x1 x2 i.a, pa vce(robust)

As above, but specify an autoregressive correlation structure of order 1

    xtprobit y x1 x2 i.a, pa vce(robust) corr(ar 1)

Random-effects model with cluster–robust standard errors for panels nested within cvar

    xtprobit y x1 x2 i.a, vce(cluster cvar)

Menu

Statistics > Longitudinal/panel data > Binary outcomes > Probit regression (RE, PA)

Syntax

Random-effects (RE) model

> xtprobit *depvar* [*indepvars*] [*if*] [*in*] [*weight*] [, re *RE_options*]

Population-averaged (PA) model

> xtprobit *depvar* [*indepvars*] [*if*] [*in*] [*weight*], pa [*PA_options*]

RE_options	Description
Model	
noconstant	suppress constant term
re	use random-effects estimator; the default
offset(*varname*)	include *varname* in model with coefficient constrained to 1
constraints(*constraints*)	apply specified linear constraints
collinear	keep collinear variables
asis	retain perfect predictor variables
SE/Robust	
vce(*vcetype*)	*vcetype* may be oim, robust, cluster *clustvar*, bootstrap, or jackknife
Reporting	
level(*#*)	set confidence level; default is level(95)
lrmodel	perform the likelihood-ratio model test instead of the default Wald test
nocnsreport	do not display constraints
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Integration	
intmethod(*intmethod*)	integration method; *intmethod* may be mvaghermite (the default) or ghermite
intpoints(*#*)	use *#* quadrature points; default is intpoints(12)
Maximization	
maximize_options	control the maximization process; seldom used
coeflegend	display legend instead of statistics

PA_options	Description
[Model]	
<u>nocons</u>tant	suppress constant term
pa	use population-averaged estimator
<u>off</u>set(*varname*)	include *varname* in model with coefficient constrained to 1
asis	retain perfect predictor variables
[Correlation]	
<u>corr</u>(*correlation*)	within-panel correlation structure
force	estimate even if observations unequally spaced in time
[SE/Robust]	
vce(*vcetype*)	*vcetype* may be conventional, robust, <u>boot</u>strap, or <u>jack</u>knife
nmp	use divisor $N - P$ instead of the default N
scale(*parm*)	overrides the default scale parameter; *parm* may be x2, dev, phi, or #
[Reporting]	
<u>l</u>evel(*#*)	set confidence level; default is level(95)
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
[Optimization]	
optimize_options	control the optimization process; seldom used
coeflegend	display legend instead of statistics

correlation	Description
<u>exch</u>angeable	exchangeable
<u>ind</u>ependent	independent
<u>uns</u>tructured	unstructured
<u>fix</u>ed *matname*	user-specified
ar *#*	autoregressive of order *#*
<u>sta</u>tionary *#*	stationary of order *#*
<u>nonsta</u>tionary *#*	nonstationary of order *#*

A panel variable must be specified. For xtprobit, pa, correlation structures other than exchangeable and independent require that a time variable also be specified. Use xtset; see [XT] **xtset**.

indepvars may contain factor variables; see [U] **11.4.3 Factor variables**.

depvar and *indepvars* may contain time-series operators; see [U] **11.4.4 Time-series varlists**.

by, mi estimate, and statsby are allowed; see [U] **11.1.10 Prefix commands**. fp is allowed for the random-effects model.

vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] **mi estimate**.

iweights, fweights, and pweights are allowed for the population-averaged model, and iweights are allowed for the random-effects model; see [U] **11.1.6 weight**. Weights must be constant within panel.

coeflegend does not appear in the dialog box.

See [U] **20 Estimation and postestimation commands** for more capabilities of estimation commands.

Options for RE model

    │ Model │

noconstant; see [R] **estimation options**.

re requests the random-effects estimator. re is the default if neither re nor pa is specified.

offset(*varname*), constraints(*constraints*), collinear; see [R] **estimation options**.

asis forces retention of perfect predictor variables and their associated, perfectly predicted observations and may produce instabilities in maximization; see [R] **probit**.

    │ SE/Robust │

vce(*vcetype*) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster *clustvar*), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] *vce_options*.

Specifying vce(robust) is equivalent to specifying vce(cluster *panelvar*); see *xtprobit, re and the robust VCE estimator* in *Methods and formulas*.

    │ Reporting │

level(*#*), lrmodel, nocnsreport; see [R] **estimation options**.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(*#*), fvwrapon(*style*), cformat(*%fmt*), pformat(*%fmt*), sformat(*%fmt*), and nolstretch; see [R] **estimation options**.

    │ Integration │

intmethod(*intmethod*), intpoints(*#*); see [R] **estimation options**.

    │ Maximization │

maximize_options: difficult, technique(*algorithm_spec*), iterate(*#*), [no]log, trace, gradient, showstep, hessian, showtolerance, tolerance(*#*), ltolerance(*#*), nrtolerance(*#*), nonrtolerance, and from(*init_specs*); see [R] **maximize**. These options are seldom used.

The following option is available with xtprobit but is not shown in the dialog box:

coeflegend; see [R] **estimation options**.

Options for PA model

    │ Model │

noconstant; see [R] **estimation options**.

pa requests the population-averaged estimator.

offset(*varname*); see [R] **estimation options**.

asis forces retention of perfect predictor variables and their associated, perfectly predicted observations and may produce instabilities in maximization; see [R] **probit**.

      Correlation

corr(*correlation*) specifies the within-panel correlation structure; the default corresponds to the equal-correlation model, corr(exchangeable).

When you specify a correlation structure that requires a lag, you indicate the lag after the structure's name with or without a blank; for example, corr(ar 1) or corr(ar1).

If you specify the fixed correlation structure, you specify the name of the matrix containing the assumed correlations following the word fixed, for example, corr(fixed myr).

force specifies that estimation be forced even though the time variable is not equally spaced. This is relevant only for correlation structures that require knowledge of the time variable. These correlation structures require that observations be equally spaced so that calculations based on lags correspond to a constant time change. If you specify a time variable indicating that observations are not equally spaced, the (time dependent) model will not be fit. If you also specify force, the model will be fit, and it will be assumed that the lags based on the data ordered by the time variable are appropriate.

      SE/Robust

vce(*vcetype*) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional), that are robust to some kinds of misspecification (robust), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] *vce_options*.

vce(conventional), the default, uses the conventionally derived variance estimator for generalized least-squares regression.

nmp, scale(x2 | dev | phi | #); see [XT] *vce_options*.

      Reporting

level(#); see [R] **estimation options**.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(*style*), cformat(%*fmt*), pformat(%*fmt*), sformat(%*fmt*), and nolstretch; see [R] **estimation options**.

      Optimization

optimize_options control the iterative optimization process. These options are seldom used.

iterate(#) specifies the maximum number of iterations. When the number of iterations equals #, the optimization stops and presents the current results, even if convergence has not been reached. The default is iterate(100).

tolerance(#) specifies the tolerance for the coefficient vector. When the relative change in the coefficient vector from one iteration to the next is less than or equal to #, the optimization process is stopped. tolerance(1e-6) is the default.

nolog suppresses display of the iteration log.

trace specifies that the current estimates be printed at each iteration.

The following option is available with xtprobit but is not shown in the dialog box:

coeflegend; see [R] **estimation options**.

Remarks and examples

xtprobit may be used to fit a population-averaged model or a random-effects probit model. There is no command for a conditional fixed-effects model, as there does not exist a sufficient statistic allowing the fixed effects to be conditioned out of the likelihood. Unconditional fixed-effects probit models may be fit with the probit command with indicator variables for the panels. However, unconditional fixed-effects estimates are biased. We do not discuss fixed-effects further in this entry.

By default, the population-averaged model is an equal-correlation model; that is, xtprobit, pa assumes corr(exchangeable). Thus, xtprobit is a convenience command for obtaining the population-averaged model using xtgee; see [XT] **xtgee**. Typing

    . xtprobit ..., pa ...

is equivalent to typing

    . xtgee ..., ... family(binomial) link(probit) corr(exchangeable)

See also [XT] **xtgee** for information about xtprobit.

By default or when re is specified, xtprobit fits via maximum likelihood the random-effects model

$$\Pr(y_{it} \neq 0 | \mathbf{x}_{it}) = \Phi(\mathbf{x}_{it}\boldsymbol{\beta} + \nu_i)$$

for $i = 1, \dots, n$ panels, where $t = 1, \dots, n_i$, ν_i are i.i.d., $N(0, \sigma_\nu^2)$, and Φ is the standard normal cumulative distribution function.

Underlying this model is the variance components model

$$y_{it} \neq 0 \iff \mathbf{x}_{it}\boldsymbol{\beta} + \nu_i + \epsilon_{it} > 0$$

where ϵ_{it} are i.i.d. Gaussian distributed with mean zero and variance $\sigma_\epsilon^2 = 1$, independently of ν_i.

▷ Example 1: Random-effects model

We are studying unionization of women in the United States and are using the union dataset; see [XT] **xt**. We wish to fit a random-effects model of union membership:

```
. use http://www.stata-press.com/data/r15/union
(NLS Women 14-24 in 1968)
. xtprobit union age grade i.not_smsa south##c.year
Fitting comparison model:
Iteration 0:   log likelihood =  -13864.23
Iteration 1:   log likelihood = -13545.541
Iteration 2:   log likelihood = -13544.385
Iteration 3:   log likelihood = -13544.385
Fitting full model:
rho =  0.0     log likelihood = -13544.385
rho =  0.1     log likelihood = -12237.655
rho =  0.2     log likelihood = -11590.282
rho =  0.3     log likelihood = -11211.185
rho =  0.4     log likelihood = -10981.319
rho =  0.5     log likelihood = -10852.793
rho =  0.6     log likelihood = -10808.759
rho =  0.7     log likelihood =  -10865.57
Iteration 0:   log likelihood = -10807.712
Iteration 1:   log likelihood = -10599.332
Iteration 2:   log likelihood = -10552.287
Iteration 3:   log likelihood = -10552.225
Iteration 4:   log likelihood = -10552.225
```

Random-effects probit regression				Number of obs	=	26,200
Group variable: idcode				Number of groups	=	4,434
Random effects u_i ~ Gaussian				Obs per group:		
				min =		1
				avg =		5.9
				max =		12
Integration method: mvaghermite				Integration pts.	=	12
				Wald chi2(6)	=	220.91
Log likelihood = -10552.225				Prob > chi2	=	0.0000

union	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	
age	.0082967	.0084599	0.98	0.327	-.0082843	.0248778
grade	.0482731	.0099469	4.85	0.000	.0287776	.0677686
1.not_smsa	-.139657	.0460548	-3.03	0.002	-.2299227	-.0493913
1.south	-1.584394	.358473	-4.42	0.000	-2.286989	-.8818002
year	-.0039854	.0088399	-0.45	0.652	-.0213113	.0133406
south#c.year						
1	.0134017	.0044622	3.00	0.003	.0046559	.0221475
_cons	-1.668202	.4751819	-3.51	0.000	-2.599542	-.7368628
/lnsig2u	.6103616	.0458783			.5204418	.7002814
sigma_u	1.35687	.0311255			1.297217	1.419267
rho	.6480233	.0104643			.6272511	.6682502

```
LR test of rho=0: chibar2(01) = 5984.32                Prob >= chibar2 = 0.000
```

The output includes the additional panel-level variance component, which is parameterized as the log of the variance $\ln(\sigma_\nu^2)$ (labeled lnsig2u in the output). The standard deviation σ_ν is also included in the output (labeled sigma_u) together with ρ (labeled rho), where

$$\rho = \frac{\sigma_\nu^2}{\sigma_\nu^2 + 1}$$

which is the proportion of the total variance contributed by the panel-level variance component.

When rho is zero, the panel-level variance component is unimportant, and the panel estimator is not different from the pooled estimator. A likelihood-ratio test of this is included at the bottom of the output. This test formally compares the pooled estimator (probit) with the panel estimator.

◁

❑ Technical note

The random-effects model is calculated using quadrature, which is an approximation whose accuracy depends partially on the number of integration points used. We can use the quadchk command to see if changing the number of integration points affects the results. If the results change, the quadrature approximation is not accurate given the number of integration points. Try increasing the number of integration points using the intpoints() option and run quadchk again. Do not attempt to interpret the results of estimates when the coefficients reported by quadchk differ substantially.

```
. quadchk, nooutput
Refitting model intpoints() =  8
Refitting model intpoints() = 16
```

Quadrature check

	Fitted quadrature 12 points	Comparison quadrature 8 points	Comparison quadrature 16 points	
Log likelihood	-10552.225	-10554.496 -2.2712569 .00021524	-10552.399 -.17396615 .00001649	 Difference Relative difference
union: age	.00829671	.00828745 -9.265e-06 -.0011167	.00831488 .00001817 .00218987	 Difference Relative difference
union: grade	.0482731	.04860277 .00032967 .00682917	.04826287 -.00001023 -.00021188	 Difference Relative difference
union: 1.not_smsa	-.13965702	-.14057441 -.00091739 .00656891	-.13953521 .00012181 -.00087218	 Difference Relative difference
union: 1.south	-1.5843944	-1.5909857 -.00659135 .00416017	-1.5843375 .00005689 -.00003591	 Difference Relative difference
union: year	-.00398535	-.00397811 7.237e-06 -.00181578	-.00400181 -.00001646 .00412982	 Difference Relative difference
union: 1.south#c.~r	.01340169	.01344457 .00004288 .00319946	.01340388 2.193e-06 .0001636	 Difference Relative difference
union: _cons	-1.6682022	-1.6757524 -.00755024 .00452597	-1.6665327 .00166948 -.00100077	 Difference Relative difference
/: lnsig2u	.61036163	.61780789 .00744626 .01219976	.60974814 -.00061349 -.00100513	 Difference Relative difference

The results obtained for 12 quadrature points were closer to the results for 16 points than to the results for eight points. Although the relative and absolute differences are a bit larger than we would like, they are not large. We can increase the number of quadrature points with the intpoints() option; if we choose intpoints(20) and do another quadchk we will get acceptable results, with relative differences around 0.01%.

This is not the case if we use nonadaptive quadrature. Then the results we obtain are

```
. xtprobit union age grade i.not_smsa south##c.year, intmethod(ghermite)

Fitting comparison model:

Iteration 0:   log likelihood =  -13864.23
Iteration 1:   log likelihood = -13545.541
Iteration 2:   log likelihood = -13544.385
Iteration 3:   log likelihood = -13544.385

Fitting full model:

rho =  0.0    log likelihood = -13544.385
rho =  0.1    log likelihood = -12237.655
rho =  0.2    log likelihood = -11590.282
rho =  0.3    log likelihood = -11211.185
rho =  0.4    log likelihood = -10981.319
rho =  0.5    log likelihood = -10852.793
rho =  0.6    log likelihood = -10808.759
rho =  0.7    log likelihood =  -10865.57

Iteration 0:   log likelihood = -10808.759
Iteration 1:   log likelihood = -10594.349
Iteration 2:   log likelihood = -10560.913
Iteration 3:   log likelihood = -10560.876
Iteration 4:   log likelihood = -10560.876
```

Random-effects probit regression				Number of obs	=	26,200
Group variable: idcode				Number of groups	=	4,434
Random effects u_i ~ Gaussian				Obs per group:		
				min =		1
				avg =		5.9
				max =		12
Integration method: ghermite				Integration pts.	=	12
				Wald chi2(6)	=	218.99
Log likelihood = -10560.876				Prob > chi2	=	0.0000

union	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	
age	.0093488	.0083385	1.12	0.262	-.0069945	.025692
grade	.0488014	.0101168	4.82	0.000	.0289728	.06863
1.not_smsa	-.1364862	.0462831	-2.95	0.003	-.2271995	-.045773
1.south	-1.592711	.3576715	-4.45	0.000	-2.293734	-.8916877
year	-.0053723	.0087219	-0.62	0.538	-.0224668	.0117223
south#c.year						
1	.0136764	.0044532	3.07	0.002	.0049482	.0224046
_cons	-1.575539	.4639881	-3.40	0.001	-2.484939	-.6661388
/lnsig2u	.5615976	.0432021			.476923	.6462722
sigma_u	1.324187	.0286038			1.269295	1.381453
rho	.6368221	.0099918			.617021	.6561699

LR test of rho=0: chibar2(01) = 5967.02   Prob >= chibar2 = 0.000

We now check the stability of the quadrature technique for this nonadaptive quadrature model. We expect it to be less stable.

```
. quadchk, nooutput
Refitting model intpoints() =  8
Refitting model intpoints() = 16
                          Quadrature check
```

	Fitted quadrature 12 points	Comparison quadrature 8 points	Comparison quadrature 16 points	
Log likelihood	-10560.876	-10574.239	-10555.792	
		-13.362535	5.0839579	Difference
		.00126529	-.0004814	Relative difference
union: age	.00934876	.01264615	.00731888	
		.0032974	-.00202987	Difference
		.35270966	-.21712744	Relative difference
union: grade	.04880139	.05710089	.04432417	
		.00829951	-.00447722	Difference
		.17006703	-.09174372	Relative difference
union: 1.not_smsa	-.13648624	-.13327724	-.14094541	
		.003209	-.00445917	Difference
		-.0235115	.03267123	Relative difference
union: 1.south	-1.592711	-1.5275627	-1.6059143	
		.06514823	-.01320331	Difference
		-.04090399	.00828983	Relative difference
union: year	-.00537226	-.00867673	-.00307042	
		-.00330447	.00230184	Difference
		.61509968	-.4284678	Relative difference
union: 1.south#c.~r	.01367641	.01278071	.01369009	
		-.0008957	.00001368	Difference
		-.06549266	.00100054	Relative difference
union: _cons	-1.5755388	-1.4888646	-1.6505526	
		.08667418	-.0750138	Difference
		-.0550124	.04761152	Relative difference
/: lnsig2u	.56159763	.49290978	.58068904	
		-.06868786	.0190914	Difference
		-.12230795	.03399481	Relative difference

Once again, the results obtained for 12 quadrature points were closer to the results for 16 points than to the results for eight points. However, here the convergence point seems to be sensitive to the number of quadrature points, so we should not trust these results. We should increase the number of quadrature points with the intpoints() option and then use quadchk again. We should not use the results of a random-effects specification when there is evidence that the numeric technique for calculating the model is not stable (as shown by quadchk).

Generally, the relative differences in the coefficients should not change by more than 1% if the quadrature technique is stable. See [XT] **quadchk** for details. Increasing the number of quadrature points can often improve the stability, and for models with high rho we may need many. We can also switch between adaptive and nonadaptive quadrature. As a rule, adaptive quadrature, which is the default integration method, is much more flexible and robust.

Because the xtprobit, re likelihood function is calculated by Gauss–Hermite quadrature, on large problems the computations can be slow. Computation time is roughly proportional to the number of points used for the quadrature.

❑

▷ Example 2: Equal-correlation model

As an alternative to the random-effects specification, we can fit an equal-correlation probit model:

```
. xtprobit union age grade i.not_smsa south##c.year, pa

Iteration 1: tolerance = .12544249
Iteration 2: tolerance = .0034686
Iteration 3: tolerance = .00017448
Iteration 4: tolerance = 8.382e-06
Iteration 5: tolerance = 3.997e-07
```

GEE population-averaged model		Number of obs	=	26,200
Group variable:	idcode	Number of groups	=	4,434
Link:	probit	Obs per group:		
Family:	binomial	min =		1
Correlation:	exchangeable	avg =		5.9
		max =		12
		Wald chi2(6)	=	242.57
Scale parameter:	1	Prob > chi2	=	0.0000

union	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	
age	.0089699	.0053208	1.69	0.092	-.0014586	.0193985
grade	.0333174	.0062352	5.34	0.000	.0210966	.0455382
1.not_smsa	-.0715717	.027543	-2.60	0.009	-.1255551	-.0175884
1.south	-1.017368	.207931	-4.89	0.000	-1.424905	-.6098308
year	-.0062708	.0055314	-1.13	0.257	-.0171122	.0045706
south#c.year						
1	.0086294	.00258	3.34	0.001	.0035727	.013686
_cons	-.8670997	.294771	-2.94	0.003	-1.44484	-.2893592

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▷ Example 3: Population-averaged model

In example 3 of [R] **probit**, we showed the above results and compared them with probit, vce(cluster id). xtprobit with the pa option allows a vce(robust) option, so we can obtain the population-averaged probit estimator with the robust variance calculation by typing

```
. xtprobit union age grade i.not_smsa south##c.year, pa vce(robust) nolog
```

GEE population-averaged model		Number of obs	=	26,200
Group variable:	idcode	Number of groups	=	4,434
Link:	probit	Obs per group:		
Family:	binomial	min =		1
Correlation:	exchangeable	avg =		5.9
		max =		12
		Wald chi2(6)	=	156.33
Scale parameter:	1	Prob > chi2	=	0.0000

(Std. Err. adjusted for clustering on idcode)

union	Coef.	Semirobust Std. Err.	z	P>|z|	[95% Conf. Interval]	
age	.0089699	.0051169	1.75	0.080	-.001059	.0189988
grade	.0333174	.0076425	4.36	0.000	.0183383	.0482965
1.not_smsa	-.0715717	.0348659	-2.05	0.040	-.1399076	-.0032359
1.south	-1.017368	.3026981	-3.36	0.001	-1.610645	-.4240906
year	-.0062708	.0055745	-1.12	0.261	-.0171965	.0046549
south#c.year						
1	.0086294	.0037866	2.28	0.023	.0012078	.0160509
_cons	-.8670997	.3243959	-2.67	0.008	-1.502904	-.2312955

These standard errors are similar to those shown for probit, vce(cluster id) in [R] **probit**.

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▷ Example 4: Random-effects model with stable quadrature

In a previous example, we showed how quadchk indicated that the quadrature technique was numerically unstable. Here we present an example in which the quadrature is stable.

In this example, we have (synthetic) data on whether workers complain to managers at fast-food restaurants. The covariates are age (in years of the worker), grade (years of schooling completed by the worker), south (equal to 1 if the restaurant is located in the South), tenure (the number of years spent on the job by the worker), gender (of the worker), race (of the worker), income (in thousands of dollars by the restaurant), genderm (gender of the manager), burger (equal to 1 if the restaurant specializes in hamburgers), and chicken (equal to 1 if the restaurant specializes in chicken). The model is given by

```
. use http://www.stata-press.com/data/r15/chicken
. xtprobit complain age grade south tenure gender race income genderm burger
> chicken, nolog
```

| Random-effects probit regression | | | | Number of obs | = | 2,763 |
| Group variable: restaurant | | | | Number of groups | = | 500 |

Random effects u_i ~ Gaussian	Obs per group:		
	min =	3
	avg =	5.5
	max =	8

Integration method: mvaghermite	Integration pts.	=	12
	Wald chi2(10)	=	126.59
Log likelihood = −1318.2088	Prob > chi2	=	0.0000

complain	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	
age	−.0430409	.0130211	−3.31	0.001	−.0685617	−.01752
grade	.0330934	.0264572	1.25	0.211	−.0187618	.0849486
south	.1012	.0707196	1.43	0.152	−.037408	.2398079
tenure	−.0440079	.0987099	−0.45	0.656	−.2374758	.14946
gender	.3318499	.0601382	5.52	0.000	.2139812	.4497185
race	.3417901	.0382251	8.94	0.000	.2668703	.4167098
income	−.0022702	.0008885	−2.56	0.011	−.0040117	−.0005288
genderm	.0524577	.0706585	0.74	0.458	−.0860305	.1909459
burger	.0448931	.0956151	0.47	0.639	−.1425091	.2322953
chicken	.1904714	.0953067	2.00	0.046	.0036737	.3772691
_cons	−.2145311	.6240549	−0.34	0.731	−1.437656	1.008594
/lnsig2u	−1.704494	.2502057			−2.194888	−1.214099
sigma_u	.4264557	.0533508			.333723	.5449563
rho	.1538793	.0325769			.1002105	.2289765

LR test of rho=0: chibar2(01) = 29.91   Prob >= chibar2 = 0.000

Again we would like to check the stability of the quadrature technique of the model before interpreting the results. Given the estimate of ρ and the small size of the panels (between 3 and 8), we should find that the quadrature technique is numerically stable.

```
. quadchk, nooutput
Refitting model intpoints() =  8
Refitting model intpoints() = 16
```

```
                        Quadrature check

                Fitted        Comparison     Comparison
              quadrature      quadrature     quadrature
              12 points        8 points      16 points
─────────────────────────────────────────────────────────────────────────
Log           -1318.2088      -1318.2088     -1318.2088
likelihood                    -2.002e-06     -1.194e-09   Difference
                               1.519e-09      9.061e-13   Relative difference
─────────────────────────────────────────────────────────────────────────
complain:     -.04304086      -.04304086     -.04304086
     age                      -3.896e-10     -2.625e-12   Difference
                               9.051e-09      6.100e-11   Relative difference
─────────────────────────────────────────────────────────────────────────
complain:      .0330934        .0330934       .0330934
   grade                       2.208e-11      1.867e-12   Difference
                               6.673e-10      5.643e-11   Relative difference
─────────────────────────────────────────────────────────────────────────
complain:      .10119998       .10119999      .10119998
   south                       2.369e-09      3.957e-11   Difference
                               2.341e-08      3.910e-10   Relative difference
─────────────────────────────────────────────────────────────────────────
complain:     -.04400789      -.0440079      -.04400789
  tenure                      -3.362e-09     -2.250e-11   Difference
                               7.640e-08      5.114e-10   Relative difference
─────────────────────────────────────────────────────────────────────────
complain:      .33184986       .33184986      .33184986
  gender                       3.190e-09      2.546e-11   Difference
                               9.612e-09      7.673e-11   Relative difference
─────────────────────────────────────────────────────────────────────────
complain:      .34179006       .34179007      .34179006
    race                       3.801e-09      2.990e-11   Difference
                               1.112e-08      8.749e-11   Relative difference
─────────────────────────────────────────────────────────────────────────
complain:     -.00227021      -.00227021     -.00227021
  income                      -4.468e-11     -9.252e-13   Difference
                               1.968e-08      4.075e-10   Relative difference
─────────────────────────────────────────────────────────────────────────
complain:      .05245769       .05245769      .05245769
 genderm                       1.963e-09      4.481e-11   Difference
                               3.742e-08      8.542e-10   Relative difference
─────────────────────────────────────────────────────────────────────────
complain:      .04489311       .04489311      .04489311
  burger                       4.173e-10      6.628e-12   Difference
                               9.296e-09      1.476e-10   Relative difference
─────────────────────────────────────────────────────────────────────────
complain:      .19047138       .19047139      .19047138
 chicken                       3.096e-09      4.916e-11   Difference
                               1.625e-08      2.581e-10   Relative difference
─────────────────────────────────────────────────────────────────────────
complain:     -.21453112      -.21453111     -.21453112
   _cons                       1.281e-08      2.682e-10   Difference
                              -5.972e-08     -1.250e-09   Relative difference
─────────────────────────────────────────────────────────────────────────
/:            -1.7044935      -1.7044934     -1.7044935
 lnsig2u                       1.255e-07     -4.135e-10   Difference
                              -7.365e-08      2.426e-10   Relative difference
─────────────────────────────────────────────────────────────────────────
```

The relative and absolute differences are all small between the default 12 quadrature points and the result with 16 points. We do not have any coefficients that have a large difference between the default 12 quadrature points and eight quadrature points.

We conclude that the quadrature technique is stable. Because the differences here are so small, we would plan on using and interpreting these results rather than trying to rerun with more quadrature points.

◁

Stored results

xtprobit, re stores the following in e():

Scalars
e(N)	number of observations
e(N_g)	number of groups
e(k)	number of parameters
e(k_aux)	number of auxiliary parameters
e(k_eq)	number of equations in e(b)
e(k_eq_model)	number of equations in overall model test
e(k_dv)	number of dependent variables
e(df_m)	model degrees of freedom
e(ll)	log likelihood
e(ll_0)	log likelihood, constant-only model
e(ll_c)	log likelihood, comparison model
e(chi2)	χ^2
e(chi2_c)	χ^2 for comparison test
e(N_clust)	number of clusters
e(rho)	ρ
e(sigma_u)	panel-level standard deviation
e(n_quad)	number of quadrature points
e(g_min)	smallest group size
e(g_avg)	average group size
e(g_max)	largest group size
e(p)	p-value for model test
e(rank)	rank of e(V)
e(rank0)	rank of e(V) for constant-only model
e(ic)	number of iterations
e(rc)	return code
e(converged)	1 if converged, 0 otherwise

Macros
e(cmd)	xtprobit
e(cmdline)	command as typed
e(depvar)	name of dependent variable
e(ivar)	variable denoting groups
e(model)	re
e(wtype)	weight type
e(wexp)	weight expression
e(title)	title in estimation output
e(clustvar)	name of cluster variable
e(offset)	linear offset variable
e(chi2type)	Wald or LR; type of model χ^2 test
e(chi2_ct)	Wald or LR; type of model χ^2 test corresponding to e(chi2_c)
e(vce)	*vcetype* specified in vce()
e(vcetype)	title used to label Std. Err.
e(intmethod)	integration method
e(distrib)	Gaussian; the distribution of the random effect
e(opt)	type of optimization
e(which)	max or min; whether optimizer is to perform maximization or minimization

e(ml_method)	type of ml method
e(user)	name of likelihood-evaluator program
e(technique)	maximization technique
e(properties)	b V
e(predict)	program used to implement predict
e(marginsdefault)	default predict() specification for margins
e(asbalanced)	factor variables fvset as asbalanced
e(asobserved)	factor variables fvset as asobserved

Matrices

e(b)	coefficient vector
e(Cns)	constraints matrix
e(ilog)	iteration log
e(gradient)	gradient vector
e(V)	variance–covariance matrix of the estimators
e(V_modelbased)	model-based variance

Functions

e(sample)	marks estimation sample

`xtprobit, pa` stores the following in `e()`:

Scalars

e(N)	number of observations
e(N_g)	number of groups
e(df_m)	model degrees of freedom
e(chi2)	χ^2
e(p)	p-value for model test
e(df_pear)	degrees of freedom for Pearson χ^2
e(chi2_dev)	χ^2 test of deviance
e(chi2_dis)	χ^2 test of deviance dispersion
e(deviance)	deviance
e(dispers)	deviance dispersion
e(phi)	scale parameter
e(g_min)	smallest group size
e(g_avg)	average group size
e(g_max)	largest group size
e(rank)	rank of e(V)
e(tol)	target tolerance
e(dif)	achieved tolerance
e(rc)	return code

Macros

e(cmd)	xtgee
e(cmd2)	xtprobit
e(cmdline)	command as typed
e(depvar)	name of dependent variable
e(ivar)	variable denoting groups
e(tvar)	variable denoting time within groups
e(model)	pa
e(family)	binomial
e(link)	probit; link function
e(corr)	correlation structure
e(scale)	x2, dev, phi, or #; scale parameter
e(wtype)	weight type
e(wexp)	weight expression
e(offset)	linear offset variable
e(chi2type)	Wald; type of model χ^2 test
e(vce)	*vcetype* specified in vce()
e(vcetype)	title used to label Std. Err.
e(nmp)	nmp, if specified
e(properties)	b V
e(predict)	program used to implement predict

e(marginsnotok)	predictions disallowed by margins
e(asbalanced)	factor variables fvset as asbalanced
e(asobserved)	factor variables fvset as asobserved

Matrices

e(b)	coefficient vector
e(R)	estimated working correlation matrix
e(V)	variance–covariance matrix of the estimators
e(V_modelbased)	model-based variance

Functions

e(sample)	marks estimation sample

Methods and formulas

xtprobit reports the population-averaged results obtained by using xtgee, family(binomial) link(probit) to obtain estimates.

Assuming a normal distribution, $N(0, \sigma_\nu^2)$, for the random effects ν_i

$$\Pr(y_{i1}, \ldots, y_{in_i} | \mathbf{x}_{i1}, \ldots, \mathbf{x}_{in_i}) = \int_{-\infty}^{\infty} \frac{e^{-\nu_i^2/2\sigma_\nu^2}}{\sqrt{2\pi}\sigma_\nu} \left\{ \prod_{t=1}^{n_i} F(y_{it}, \mathbf{x}_{it}\boldsymbol{\beta} + \nu_i) \right\} d\nu_i$$

where

$$F(y, z) = \begin{cases} \Phi(z) & \text{if } y \neq 0 \\ 1 - \Phi(z) & \text{otherwise} \end{cases}$$

where Φ is the cumulative normal distribution.

The panel-level likelihood l_i is given by

$$l_i = \int_{-\infty}^{\infty} \frac{e^{-\nu_i^2/2\sigma_\nu^2}}{\sqrt{2\pi}\sigma_\nu} \left\{ \prod_{t=1}^{n_i} F(y_{it}, \mathbf{x}_{it}\boldsymbol{\beta} + \nu_i) \right\} d\nu_i$$

$$\equiv \int_{-\infty}^{\infty} g(y_{it}, x_{it}, \nu_i) d\nu_i$$

This integral can be approximated with M-point Gauss–Hermite quadrature

$$\int_{-\infty}^{\infty} e^{-x^2} h(x) dx \approx \sum_{m=1}^{M} w_m^* h(a_m^*)$$

This is equivalent to

$$\int_{-\infty}^{\infty} f(x) dx \approx \sum_{m=1}^{M} w_m^* \exp\left\{ (a_m^*)^2 \right\} f(a_m^*)$$

where the w_m^* denote the quadrature weights and the a_m^* denote the quadrature abscissas. The log likelihood, L, is the sum of the logs of the panel-level likelihoods l_i.

The default approximation of the log likelihood is by adaptive Gauss–Hermite quadrature, which approximates the panel-level likelihood with

$$l_i \approx \sqrt{2}\widehat{\sigma}_i \sum_{m=1}^{M} w_m^* \exp\left\{ (a_m^*)^2 \right\} g(y_{it}, x_{it}, \sqrt{2}\widehat{\sigma}_i a_m^* + \widehat{\mu}_i)$$

where $\widehat{\sigma}_i$ and $\widehat{\mu}_i$ are the adaptive parameters for panel i. Therefore, with the definition of $g(y_{it}, x_{it}, \nu_i)$, the total log likelihood is approximated by

$$L \approx \sum_{i=1}^{n} w_i \log \left[\sqrt{2}\widehat{\sigma}_i \sum_{m=1}^{M} w_m^* \exp\{(a_m^*)^2\} \frac{\exp\{-(\sqrt{2}\widehat{\sigma}_i a_m^* + \widehat{\mu}_i)^2/2\sigma_\nu^2\}}{\sqrt{2\pi}\sigma_\nu} \right.$$

$$\left. \prod_{t=1}^{n_i} F(y_{it}, x_{it}\boldsymbol{\beta} + \sqrt{2}\widehat{\sigma}_i a_m^* + \widehat{\mu}_i) \right]$$

where w_i is the user-specified weight for panel i; if no weights are specified, $w_i = 1$.

The default method of adaptive Gauss–Hermite quadrature is to calculate the posterior mean and variance and use those parameters for $\widehat{\mu}_i$ and $\widehat{\sigma}_i$ by following the method of Naylor and Smith (1982), further discussed in Skrondal and Rabe-Hesketh (2004). We start with $\widehat{\sigma}_{i,0} = 1$ and $\widehat{\mu}_{i,0} = 0$, and the posterior means and variances are updated in the kth iteration. That is, at the kth iteration of the optimization for l_i, we use

$$l_{i,k} \approx \sum_{m=1}^{M} \sqrt{2}\widehat{\sigma}_{i,k-1} w_m^* \exp\{a_m^*)^2\} g(y_{it}, x_{it}, \sqrt{2}\widehat{\sigma}_{i,k-1} a_m^* + \widehat{\mu}_{i,k-1})$$

Letting

$$\tau_{i,m,k-1} = \sqrt{2}\widehat{\sigma}_{i,k-1} a_m^* + \widehat{\mu}_{i,k-1}$$

$$\widehat{\mu}_{i,k} = \sum_{m=1}^{M} (\tau_{i,m,k-1}) \frac{\sqrt{2}\widehat{\sigma}_{i,k-1} w_m^* \exp\{(a_m^*)^2\} g(y_{it}, x_{it}, \tau_{i,m,k-1})}{l_{i,k}}$$

and

$$\widehat{\sigma}_{i,k} = \sum_{m=1}^{M} (\tau_{i,m,k-1})^2 \frac{\sqrt{2}\widehat{\sigma}_{i,k-1} w_m^* \exp\{(a_m^*)^2\} g(y_{it}, x_{it}, \tau_{i,m,k-1})}{l_{i,k}} - (\widehat{\mu}_{i,k})^2$$

and this is repeated until $\widehat{\mu}_{i,k}$ and $\widehat{\sigma}_{i,k}$ have converged for this iteration of the maximization algorithm. This adaptation is applied on every iteration until the log-likelihood change from the preceding iteration is less than a relative difference of 1e–6; after this, the quadrature parameters are fixed.

The log likelihood can also be calculated by nonadaptive Gauss–Hermite quadrature, the `int-method(ghermite)` option, where $\rho = \sigma_\nu^2/(\sigma_\nu^2 + 1)$:

$$L = \sum_{i=1}^{n} w_i \log \left\{ \Pr(y_{i1}, \dots, y_{in_i} | \mathbf{x}_{i1}, \dots, \mathbf{x}_{in_i}) \right\}$$

$$\approx \sum_{i=1}^{n} w_i \log \left[\frac{1}{\sqrt{\pi}} \sum_{m=1}^{M} w_m^* \prod_{t=1}^{n_i} F\left\{ y_{it}, \mathbf{x}_{it}\boldsymbol{\beta} + a_m^* \left(\frac{2\rho}{1-\rho} \right)^{1/2} \right\} \right]$$

Both quadrature formulas require that the integrated function be well approximated by a polynomial of degree equal to the number of quadrature points. The number of periods (panel size) can affect whether

$$\prod_{t=1}^{n_i} F(y_{it}, \mathbf{x}_{it}\boldsymbol{\beta} + \nu_i)$$

is well approximated by a polynomial. As panel size and ρ increase, the quadrature approximation can become less accurate. For large ρ, the random-effects model can also become unidentified. Adaptive quadrature gives better results for correlated data and large panels than nonadaptive quadrature; however, we recommend that you use the quadchk command (see [XT] **quadchk**) to verify the quadrature approximation used in this command, whichever approximation you choose.

xtprobit, re and the robust VCE estimator

Specifying vce(robust) or vce(cluster *clustvar*) causes the Huber/White/sandwich VCE estimator to be calculated for the coefficients estimated in this regression. See [P] **_robust**, particularly *Introduction* and *Methods and formulas*. Wooldridge (2016) and Arellano (2003) discuss this application of the Huber/White/sandwich VCE estimator. As discussed by Wooldridge (2016), Stock and Watson (2008), and Arellano (2003), specifying vce(robust) is equivalent to specifying vce(cluster *panelvar*), where *panelvar* is the variable that identifies the panels.

Clustering on the panel variable produces a consistent VCE estimator when the disturbances are not identically distributed over the panels or there is serial correlation in ϵ_{it}.

The cluster–robust VCE estimator requires that there are many clusters and the disturbances are uncorrelated across the clusters. The panel variable must be nested within the cluster variable because of the within-panel correlation that is generally induced by the random-effects transform when there is heteroskedasticity or within-panel serial correlation in the idiosyncratic errors.

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Also see

[XT] **xtprobit postestimation** — Postestimation tools for xtprobit

[XT] **quadchk** — Check sensitivity of quadrature approximation

[XT] **xtcloglog** — Random-effects and population-averaged cloglog models

[XT] **xtgee** — Fit population-averaged panel-data models by using GEE

[XT] **xtlogit** — Fixed-effects, random-effects, and population-averaged logit models

[XT] **xtset** — Declare data to be panel data

[ME] **meprobit** — Multilevel mixed-effects probit regression

[MI] **estimation** — Estimation commands for use with mi estimate

[R] **probit** — Probit regression

[U] **20 Estimation and postestimation commands**