# ${\color{black} 20} \hspace{0.1in} {\rm Estimation} \hspace{0.1in} {\rm and} \hspace{0.1in} {\rm postestimation} \hspace{0.1in} {\rm commands} \\$

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# 20.1 All estimation commands work the same way

All Stata commands that fit statistical models—commands such as regress, logit, sureg, and so on—work the same way. Most single-equation estimation commands have the syntax

command varlist [if] [in] [weight] [, options]

and most multiple-equation estimation commands have the syntax

command (varlist) (varlist) ... (varlist) [if] [in] [weight] [, options]

Adopt a loose definition of single and multiple equation in interpreting this. For instance, heckman is a two-equation system, mathematically speaking, yet we categorize it, syntactically, with single-equation commands because most researchers think of it as a linear regression with an adjustment for the censoring. The important thing is that most estimation commands have one or the other of these two syntaxes.

In single-equation commands, the first variable in the *varlist* is the dependent variable, and the remaining variables are the independent variables, with some exceptions. For instance, mixed allows special variable prefixes to identify random factors.

Prefix commands may be specified in front of an estimation command to modify what it does. The syntax is

prefix: command ...

where the prefix commands are

Prefix command	Description	Manual entry
by	repeat command on subsets of data	[D] <b>by</b>
statsby	collect results across subsets of data	[D] statsby
rolling	time-series rolling estimation	[TS] rolling
*svy	estimation for complex survey data	[SVY] svy
*mi estimate	multiply imputed data and multiple imputation	[MI] mi estimate
*nestreg	nested model statistics	[R] nestreg
*stepwise	stepwise estimation	[R] stepwise
*xi	interaction expansion	[R] <b>xi</b>
*fp	fractional polynomials	[R] <b>fp</b>
*mfp	multiple fractional polynomials	[R] <b>mfp</b>

\*Available for some but not all estimation commands

Two other prefix commands—bootstrap and jackknife—also work with estimation commands see [R] **bootstrap** and [R] **jackknife**—but usually it is easier to specify the estimation-command option vce(bootstrap) or vce(jackknife).

Also, all estimation commands—whether single or multiple equation—share the following features:

- 1. You can use the standard features of Stata's syntax—if *exp* and in *range*—to specify the estimation subsample; you do not have to make a special dataset.
- 2. You can retype the estimation command without arguments to redisplay the most recent estimation results. For instance, after fitting a model with regress, you can see the estimates again by typing regress by itself. You do not have to do this immediately—any number of commands can occur between the estimation and the replaying, and, in fact, you can even replay the last estimates after the data have changed or you have dropped the data altogether. Stata never forgets (unless you type discard; see [P] discard).

- 3. You can specify the level() option at the time of estimation, or when you redisplay results if that makes sense, to specify the width of the confidence intervals for the coefficients. The default is level(95), meaning 95% confidence intervals. You can reset the default with set level; see [R] level.
- 4. You can use the postestimation command margins to display model results in terms of marginal effects (dy/dx) or even df(y)/dx, which can be displayed as either derivatives or elasticities; see [R] margins.
- 5. You can use the postestimation command margins to obtain tables of estimated marginal means, adjusted predictions, and predictive margins; see [U] 20.16 Obtaining conditional and average marginal effects and [R] margins.
- 6. You can use the postestimation command pwcompare to obtain pairwise comparisons across levels of factor variables. You can compare estimated cell means, marginal means, intercepts, marginal intercepts, slopes, or marginal slopes—collectively called margins. See [U] 20.17 Obtaining pairwise comparisons, [R] margins, and [R] margins, pwcompare.
- 7. You can use the postestimation command contrast to obtain contrasts, which is to say, to compare levels of factor variables and their interactions. This command can also produce ANOVA-style tests of main effects, interactions effects, simple effects, and nested effects; and it can be used after most estimation commands. See [U] 20.18 Obtaining contrasts, tests of interactions, and main effects, [R] contrast, and [R] margins, contrast.
- 8. You can use the postestimation command marginsplot to graph any of the results produced by margins. And because margins can replicate any result produced by pwcompare and contrast, you can graph any result produced by them, too. See [R] marginsplot.
- 9. You can use the postestimation command estat to obtain common statistics associated with the model. The available statistics are documented in the postestimation section following the documentation of the estimation command, for instance, in [R] regress postestimation following [R] regress.

You can always use the postestimation command estat vce to obtain the variance-covariance matrix of the estimators (VCE), presented as either a correlation matrix or a covariance matrix. (You can also obtain the estimated coefficients and covariance matrix as vectors and matrices and manipulate them with Stata's matrix capabilities; see [U] 14.5 Accessing matrices created by Stata commands.)

- 10. You can use the postestimation command predict to obtain predictions, residuals, influence statistics, and the like, either for the data on which you just estimated or for some other data. You can use postestimation command predictnl to obtain point estimates, standard errors, etc., for customized predictions. See [R] predict and [R] predictnl.
- 11. You can use the postestimation command forecast to perform dynamic and static forecasts, with optional forecast confidence intervals. This includes the ability to produce forecasts from multiple estimation commands, even when estimates imply simultaneous systems. An example of a simultaneous system is when y2 predicts y1 in estimation 1 and y1 predicts y2 in estimation 2. forecast provides many facilities for creating comparative forecast scenarios. See [TS] forecast.
- 12. You can refer to the values of coefficients and standard errors in expressions (such as with generate) by using standard notation; see [U] 13.5 Accessing coefficients and standard errors. You can refer in expressions to the values of other estimation-related statistics by using e(*resultname*). For instance, all commands define e(N) recording the number of observations in the estimation subsample. After estimation, type ereturn list to see a list of all that is

available. See the *Stored results* section in the estimation command's documentation for their definitions.

An especially useful e() result is e(sample): it returns 1 if an observation was used in the estimation and 0 otherwise, so you can add if e(sample) to the end of other commands to restrict them to the estimation subsample. You could type, for instance, summarize if e(sample).

- 13. You can use the postestimation command test to perform tests on the estimated parameters (Wald tests of linear hypotheses), testnl to perform Wald tests of nonlinear hypotheses, and lrtest to perform likelihood-ratio tests. You can use the postestimation command lincom to obtain point estimates and confidence intervals for linear combinations of the estimated parameters and the postestimation command nlcom to obtain nonlinear combinations.
- 14. You can specify the coeflegend option at the time of estimation or when you redisplay results to see how to type your coefficients in postestimation commands, such as test and lincom (see [R] test and [R] lincom), and in expressions.
- 15. You can use the statsby prefix command (see [D] statsby) to fit models over each category in a categorical variable and collect the results in a Stata dataset.
- 16. You can use the postestimation command estimates to store estimation results by name for later retrieval or for displaying/comparing multiple models by using estimates, or to save estimation results in a file; see [R] estimates.
- 17. You can use the postestimation command \_estimates to hold estimates, perform other estimation commands, and then restore the prior estimates. This is of particular interest to programmers. See [P] \_estimates.
- 18. You can use the postestimation command suest to obtain the joint parameter vector and variance-covariance matrix for coefficients from two different models by using seemingly unrelated estimation. This is especially useful for testing the equality, say, of coefficients across models. See [R] suest.
- 19. You can use the postestimation command hausman to perform Hausman model-specification tests by using hausman; see [R] hausman.
- 20. With some exceptions, you can specify the vce(robust) option at the time of estimation to obtain the Huber/White/robust alternate estimate of variance, or you can specify the vce(cluster *clustvar*) option to relax the assumption of independence of the observations; see [R] vce\_option.

Most estimation commands also allow a vce(*vcetype*) option to specify other alternative variance estimators—the allowed alternative variance estimators are documented with the estimator—and usually vce(opg), vce(bootstrap), and vce(jackknife) are available.

As a rule, the points discussed briefly above and in more detail later in this entry do not apply to the Bayesian analysis commands. For more information about Bayesian analysis commands, see the *Stata Bayesian Analysis Reference Manual*.

# 20.2 Standard syntax

You can combine Stata's if *exp* and in *range* with any estimation command. Estimation commands also allow by *varlist*:, where it would be sensible.

#### Example 1

We have data on 74 automobiles that record the mileage rating (mpg), weight (weight), and whether the car is domestic or foreign produced (foreign). We can fit a linear regression model of mpg on weight and the square of weight, using just the foreign-made automobiles, by typing

. use http://www.stata-press.com/data/r14/auto2 (1978 Automobile Data)

. regress mpg	weight c.weig	ht#c.weigh	t if foreig	'n			
Source	SS	df	MS	Numl	per of obs	s =	22
				• F(2	, 19)	=	8.31
Model	428.256889	2	214.128444	Prol	b > F	=	0.0026
Residual	489.606747	19	25.7687762	R-so	quared	=	0.4666
				• Adj	R-squared	l =	0.4104
Total	917.863636	21	43.7077922	Root	t MSE	=	5.0763
	r						
mpg	Coef.	Std. Err.	t	P> t	[95% C	Conf.	Interval]
weight	0132182	.0275711	-0.48	0.637	07092	252	.0444888
c.weight# c.weight	5.50e-07	5.41e-06	0.10	0.920	00001	.08	.0000119
_cons	52.33775	34.1539	1.53	0.142	-19.147	19	123.8227

. regress mpg weight c.weight#c.weight if foreign

We use the factor-variable notation c.weight#c.weight to add the square of weight to our regression; see [U] 11.4.3 Factor variables.

We can run separate regressions for the domestic and foreign-produced automobiles with the by *varlist*: prefix:

-> foreign = I	Domestic					
Source	SS	df	MS		OI OI ODD	= 52
Model	905.395466	2	452.697733	· F(2,		= 91.64 = 0.0000
Residual	242.046842	49	4.93973146		-	= 0.7891
					n byuurou	= 0.7804
Total	1147.44231	51	22.4988688	Root	MSE	= 2.2226
	Coef.	Std. Err.	t	P> t	[95% Conf	. Interval]
mpg						. incervarj
weight	0131718	.0032307	-4.08	0.000	0196642	0066794
c.weight#						
c.weight	1.11e-06	4.95e-07	2.25	0.029	1.19e-07	2.11e-06
0						
_cons	50.74551	5.162014	9.83	0.000	40.37205	61.11896
-> foreign = H	Foreign					
Source	SS	df	MS	Numb	er of obs	= 22
				· F(2,		= 8.31
Model	428.256889	2	214.128444		· · ·	= 0.0026
Residual	489.606747	19	25.7687762		uurou	= 0.4666 = 0.4104
Total	917.863636	21	43.7077922		n bquur ou	= 0.4104 = 5.0763
mpg	Coef.	Std. Err.	t	P> t	[95% Conf	. Interval]
weight	0132182	.0275711	-0.48	0.637	0709252	.0444888
-						
c.weight#						
c.weight	5.50e-07	5.41e-06	0.10	0.920	0000108	.0000119
_cons	52.33775	34.1539	1.53	0.142	-19.14719	123.8227

. by foreign: regress mpg weight c.weight#c.weight

Although all estimation commands allow if *exp* and in *range*, only some allow the by *varlist*: prefix. For by(), the duration of Stata's memory is limited: it remembers the last set of estimates only. This means that, if we were to use any of the other features described below, they would use the last regression estimated, which right now is mpg on weight and square of weight for the Foreign subsample.

We can instead collect the statistics from each of the by-groups by using the statsby prefix; see [D] statsby.

statsby runs the regression first on domestic cars and then on foreign cars, and it saves the coefficients by overwriting our dataset. Do not worry; if the dataset has not been previously saved, statsby will refuse to run unless we also specify the clear option.

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Here is what we now have in memory.

. list

	foreign	_b_weight	_stat_2	_b_cons
1.	Domestic	0131718	1.11e-06	50.74551
2.	Foreign	0132182	5.50e-07	52.33775

These are the coefficients from the two regressions above. statsby does not know how to name the coefficient for c.weight#c.weight, so it labels the coefficient with the generic name \_stat\_2. We can also save the standard errors and other statistics from the regressions; see [D] statsby.

# 20.3 Replaying prior results

When you type an estimation command without arguments, it redisplays prior results.

### Example 2

. regress

To perform a regression of mpg on the variables weight and displacement, we could type

```
. use http://www.stata-press.com/data/r14/auto2, clear (1978 Automobile Data)
```

```
. regress mpg weight displacement
```

. 1001000 mPO	"oreno aropra	o om on o					
Source	SS	df	MS	Numbe	er of obs	=	74
				- F(2,	71)	=	66.79
Model	1595.40969	2	797.704846	6 Prob	> F	=	0.0000
Residual	848.049768	71	11.9443629	) R-squ	lared	=	0.6529
				- Adj H	R-squared	=	0.6432
Total	2443.45946	73	33.4720474		-	=	3.4561
mpg	Coef.	Std. Err.	t	P> t	[95% Co	nf	Interval]
шРб	0001.	Dtu: HII.		12101	[30% 00	·	Incervarj
weight	0065671	.0011662	-5.63	0.000	008892	5	0042417
displacement	.0052808	.0098696	0.54	0.594	014398	6	.0249602
_cons	40.08452	2.02011	19.84	0.000	36.0565	4	44.11251

We now go on to do other things—summarizing data, listing observations, performing hypothesis tests, or anything else. If we decide that we want to see the last set of estimates again, we type the estimation command without arguments.

Source	SS	df	MS	Number of obs	= 74
-				- F(2, 71)	= 66.79
Model	1595.40969	2	797.704846	B Prob > F	= 0.0000
Residual	848.049768	71	11.9443629	R-squared	= 0.6529
				- Adj R-squared	= 0.6432
Total	2443.45946	73	33.4720474	<b>U</b>	= 3.4561
	I				
mpg	Coef.	Std. Err.	t	P> t  [95% C	onf. Interval]
weight	0065671	.0011662	-5.63	0.00000889	250042417
displacement	.0052808	.0098696		0.59401439	
-					
_cons	40.08452	2.02011	19.84	0.000 36.056	54 44.11251

We can also specify options on replay. For example, if we want to see a legend of terms with which to refer to the estimated coefficients in subsequent commands, we can type

. regress, coeflegend (output omitted)

See [U] 20.11 Accessing estimated coefficients for an example using legend terms.

These features work with every estimation command, so we could just as well have used, say, stcox or logit.

# 20.4 Cataloging estimation results

Stata keeps only the results of the most recently fit model in active memory. You can use Stata's estimates command, however, to temporarily store estimation results for displaying, comparing, cross-model testing, etc., during the same session. You can also save estimation results to disk, but that will be the subject of the next section. You may temporarily store up to 300 sets of estimation results.

### Example 3

Continuing with our automobile data, we fit four models and estimates store them. We fit the models quietly to minimize output.

- . quietly regress mpg weight displ
- . estimates store r\_base
- . quietly regress mpg weight displ foreign
- . estimates store r\_alt
- . quietly qreg mpg weight displ
- . estimates store q\_base
- . quietly qreg mpg weight displ foreign
- . estimates store q\_alt

We saved the four models under the names r\_base, r\_alt, q\_base, and q\_alt, but if we forget, we can ask to see a directory of what is stored:

. estimates dir

name	command	depvar	npar	title
r_base	regress	mpg	3	
r_alt	regress	mpg	4	
q_base	qreg	mpg	3	
q_alt	qreg	mpg	4	

We can ask Stata to replay any of the previous models:

```
Model r_base
      Source
                       SS
                                     df
                                               MS
                                                        Number of obs
                                                                         =
                                                                                   74
                                                        F(2, 71)
                                                                         =
                                                                                66.79
       Model
                  1595.40969
                                      2
                                          797.704846
                                                        Prob > F
                                                                         =
                                                                               0.0000
    Residual
                 848.049768
                                          11.9443629
                                                        R-squared
                                                                         =
                                                                               0.6529
                                     71
                                                        Adj R-squared
                                                                         =
                                                                               0.6432
       Total
                 2443.45946
                                     73
                                         33.4720474
                                                        Root MSE
                                                                               3.4561
                                                                         =
                                                    P>|t|
          mpg
                      Coef.
                               Std. Err.
                                               t
                                                                [95% Conf. Interval]
      weight
                  -.0065671
                               .0011662
                                            -5.63
                                                     0.000
                                                               -.0088925
                                                                            -.0042417
displacement
                   .0052808
                               .0098696
                                             0.54
                                                     0.594
                                                              -.0143986
                                                                             .0249602
                   40.08452
                                2.02011
                                            19.84
                                                     0.000
                                                                36.05654
                                                                             44.11251
       _cons
```

Or we can ask to see all the models in a combined table:

. estimates table \_all

. estimates replay r\_base

Variable	r_base	r_alt	q_base	q_alt
weight displacement foreign _cons	00656711 .00528078 40.084522	00677449 .00192865 -1.6006312 41.847949	00581172 .0042841 37.559865	00595056 .00018552 -2.1326005 39.213348

estimates displayed just the coefficients, but we could ask for other statistics.

We can also select one of the stored estimates to be made active, making it as if we had just fit the model:

```
. estimates restore r_alt
```

```
(results r_alt are active now)
```

```
. regress
```

. 1081000							
Source	SS	df	MS	Numb	er of obs	s =	74
				- F(3,	70)	=	45.88
Model	1619.71935	3	539.906448	3 Prob	) > F	=	0.0000
Residual	823.740114	70	11.7677159	R-sc	uared	=	0.6629
				- Adi	R-squared	1 =	0,6484
Total	2443.45946	73	33.4720474		MSE	=	3.4304
mpg	Coef.	Std. Err.	t	P> t	[95% (	Conf.	Interval]
weight	0067745	.0011665	-5.81	0.000	00910	011	0044479
displacement	.0019286	.0100701	0.19	0.849	01815	556	.0220129
foreign	-1.600631	1.113648	-1.44	0.155	-3.8217	732	.6204699
_cons	41.84795	2.350704	17.80	0.000	37.159		46.53628
	•						

You can do a lot more with estimates; see [R] estimates. In particular, estimates makes it easy to perform cross-model tests, such as the Hausman specification test.

4

# 20.5 Saving estimation results

estimates can also save estimation results into a file.

. estimates save alt file alt.ster saved

That saved the active estimation results, meaning the ones we just estimated or, in our case, the ones we just restored. Later, even in another Stata session, we could reload our estimates:

estimates	use	alt

. regress

Source	SS	df	MS	Number of ol		74
Model Residual	1619.71935 823.740114	3 70	539.906448 11.7677159		= = = bo	45.88 0.0000 0.6629 0.6484
Total	2443.45946	73	33.4720474		=	3.4304
mpg	Coef.	Std. Err.	t	P> t  [95%	Conf.	Interval]
weight displacement foreign _cons	0067745 .0019286 -1.600631 41.84795	.0011665 .0100701 1.113648 2.350704	-5.81 0.19 -1.44 17.80	0.000009 0.849018 0.155 -3.82 0.000 37.1	1556 1732	0044479 .0220129 .6204699 46.53628

There is one important difference between storing results in memory and saving them in a file: e(sample) is lost. We have not discussed e(sample) yet, but it allows us to identify the observations among those currently in memory that were used in the estimation. For instance, after estimation, we could type

. summarize mpg weight displ foreign if e(sample)

and see the summary statistics of the relevant data. We could do that after estimates restore, too. But we cannot do it after estimates use. Part of the reason is that we might not even have the relevant data in memory. Even if we do, however, here is what will happen:

. summarize mj	og weight displ f	oreign if	e(sample)		
Variable	Obs	Mean	Std. Dev.	Min	Max
mpg weight displacement foreign	0 0 0 0				

Stata will just assume that none of the data in memory played a role in obtaining the estimation results.

There is more worth knowing. You could, for instance, type estimates describe to see the command line that produced the estimates. See [R] estimates.

# 20.6 Specifying the estimation subsample

You specify the estimation subsample—the sample to be used in estimation—by specifying the if *exp* and in *range* qualifiers with the estimation command.

Once an estimation command has been run or previous estimates restored, Stata remembers the estimation subsample, and you can use the qualifier if e(sample) on the end of other Stata commands. The term estimation subsample refers to the set of observations used to produce the active estimation results. That might turn out to be all the observations (as it was in the above example) or only some of the observations:

Source	SS	df	MS	Number o	f obs	=	21
		ui		- F(2, 18)		=	10.21
Model	423.317154	2	211.658577	/ Prob > F	•	=	0.0011
Residual	372.96856	18	20.7204756	8 R-square	d	=	0.5316
				- Adj R-sq	uared	=	0.4796
Total	796.285714	20	39.8142857	7 Root MSE		=	4.552
mpg	Coef.	Std. Err.	t	P> t  [	95% Conf		Interval]
weight	0131402	.0029684	-4.43	0.000	0193765		0069038
rep78							
Excellent	5.052676	2.13492	2.37	0.029 .	5673764		9.537977
		6.540147	8.08	0.000 3	9.12054		66.60122

. regress mpg weight 5.rep78 if foreign

. summarize mpg weight 5.rep78 if e(sample)

1	0 0	1	1		
Variable	Obs	Mean	Std. Dev.	Min	Max
mpg weight	21 21	25.28571 2263.333	6.309856 364.7099	17 1760	41 3170
rep78 Excellent	21	.4285714	.5070926	0	1

Twenty-one observations were used in the above regression, and we subsequently obtained the means for those same 21 observations by typing summarize ... if e(sample). Observations were dropped for two reasons: we specified if foreign when we ran the regression, and there were observations for which 5.rep78 was missing. The reason does not matter; e(sample) is true if the observation was used and is false otherwise.

You can use if e(sample) on the end of any Stata command that allows if exp.

Here, Stata has a shorthand command that produces the same results as summarize ... if e(sample):

21

. estat summarize, label Estimation sample regress Number of obs =								
Variable	Mean	Std. Dev.	Min	Max	Label			
mpg weight rep78 Excellent	25.28571 2263.333 .4285714	6.309856 364.7099 .5070926	17 1760 0	41 3170 1	Mileage (mpg) Weight (lbs.) Repair Record 1978			

See [R] estat summarize.

# 20.7 Specifying the width of confidence intervals

You can specify the width of the confidence intervals for the coefficients by using the level() option at estimation or when you play back the results.

### Example 4

To obtain narrower, 90% confidence intervals when we fit the model, we type

. regress mpg weight displ, level(90) Source SS MS Number of obs 74 df = F(2, 71) = 66.79 Model 1595.40969 2 797.704846 Prob > F= 0.0000 848.049768 Residual 71 11.9443629 R-squared = 0.6529 Adj R-squared = 0.6432 Total 2443.45946 73 33.4720474 Root MSE = 3.4561 mpg Coef. Std. Err. t P>|t| [90% Conf. Interval] -5.63 0.000 -.0065671.0011662 -.0085108 -.0046234weight displacement .0052808 .0098696 0.54 0.594 -.0111679 .0217294 \_cons 40.08452 2.02011 19.84 0.000 36.71781 43.45124

If we subsequently typed regress without arguments, 95% confidence intervals would be reported because that is the default. If we initially fit the model with 95% confidence intervals, we could later type regress, level(90) to redisplay results with 90% confidence intervals.

Also, we could type set level 90 to make 90% intervals our default; see [R] level.

Stata allows noninteger confidence intervals between 10.00 and 99.99, with a maximum of two digits following the decimal point. For instance, we could type

. regress mpg	weight dispi,	Tever(92.	5)			
Source	SS	df	MS	Number of	obs =	74
				F(2, 71)	=	66.79
Model	1595.40969	2	797.704846	Prob > F	=	0.0000
Residual	848.049768	71	11.9443629	R-squared	=	0.6529
				Adj R-squ	ared =	0.6432
Total	2443.45946	73	33.4720474	Root MSE	=	3.4561
	r					
mpg	Coef.	Std. Err.	t	P> t  [92.	5% Conf.	Interval]
weight	0065671	.0011662	-5.63	0.0000	086745	0044597
displacement	.0052808	.0098696	0.54	0.5940	125535	.023115
_cons	40.08452	2.02011	19.84	0.000 36	.43419	43.73485
	•					

regress mpg weight displ, level(92.5)

4

# 20.8 Formatting the coefficient table

You can change the formatting of the coefficient table with the sformat(), pformat(), and cformat() options. The sformat() option changes the output format of test statistics; pformat() changes p-values; and cformat() changes coefficients, standard errors, and confidence limits. We can reduce the number of decimal places by specifying %f fixed-width formats:

. regress mpg	weight dispi,	CI OI Mat ( //	J.OI / BIOIMC	re(%4.11) bio	mac(%+	. 21 /
Source	SS	df	MS	Number of d	obs =	74
				F(2, 71)	=	66.79
Model	1595.40969	2	797.704846	Prob > F	=	0.0000
Residual	848.049768	71	11.9443629	R-squared	=	0.6529
				Adj R-squar	red =	0.6432
Total	2443.45946	73	33.4720474	Root MSE	=	3.4561
	I					
mpg	Coef.	Std. Err.	t F	?> t  [95%	Conf.	Intervall
weight	-0.007	0.001	-5.6	0.00 -0	0.009	-0.004
0			0 5	0 50		0 005
displacement	0.005	0.010	0.5	0.59 -(	0.014	0.025
displacement _cons	0.005 40.085	0.010 2.020	0.5 19.8		5.014 5.057	44.113

. regress mpg weight displ, cformat(%6.3f) sformat(%4.1f) pformat(%4.2f)

The option cformat(%6.3f), for example, fixes a width of six characters with three digits to the right of the decimal point. For more information on formats, see [U] 12.5.1 Numeric formats.

The formatting options may also be specified when replaying results, so you can try different formats without refitting the model:

. regress, cic	ormat(%7.41)						
Source	SS	df	MS	Number	r of obs	=	74
				- F(2, <sup>·</sup>	71)	=	66.79
Model	1595.40969	2	797.704846	B Prob 2	> F	=	0.0000
Residual	848.049768	71	11.9443629	) R-squa	ared	=	0.6529
				- AdjR	-squared	=	0.6432
Total	2443.45946	73	33.4720474	l Root I	MSE	=	3.4561
mpg	Coef.	Std. Err.	t	P> t	[95% Cor	nf.	Interval]
weight	-0.0066	0.0012	-5.63	0.000	-0.0089	9	-0.0042
displacement	0.0053	0.0099	0.54	0.594	-0.0144	1	0.0250
_cons	40.0845	2.0201	19.84	0.000	36.0565	5	44.1125

#### . regress, cformat(%7.4f)

# 20.9 Obtaining the variance–covariance matrix

Typing estat vce displays the variance-covariance matrix of the estimators in active memory.

### Example 5

In example 2, we typed regress mpg weight displacement. The full variance-covariance matrix of the estimators can be displayed at any time after estimation:

. estat vce Covariance matrix of coefficients of regress model e(V) weight displace~t \_cons weight 1.360e-06 displacement -.0000103 .00009741 \_cons -.00207455 .01188356 4.0808455

Typing estat vce with the corr option presents this matrix as a correlation matrix:

#### See [R] estat vce.

Also, Stata's matrix commands understand that e(V) refers to the matrix:

```
. matrix list e(V)
symmetric e(V)[3,3]
                    weight displacement
                                                 _cons
               1.360e-06
      weight
displacement
                -.0000103
                               .00009741
       _cons
              -.00207455
                              .01188356
                                           4.0808455
. matrix Vinv = invsym(e(V))
. matrix list Vinv
symmetric Vinv[3,3]
                    weight displacement
                                                 _cons
      weight
                  60175851
displacement
                4081161.2
                               292709.46
                18706.732
                               1222.3339
                                             6.1953911
       _cons
```

See [U] 14.5 Accessing matrices created by Stata commands.

4

# 20.10 Obtaining predicted values

Our discussion below, although cast in terms of predicted values, applies equally to the other statistics generated by predict; see [R] predict.

When Stata fits a model, whether it is regression or anything else, it internally stores the results, including the estimated coefficients and the variable names. The predict command allows you to use that information.

### Example 6

Let's perform a linear regression of mpg on weight and the square of weight:

Source	SS	df	MS	Number of ob	s =	74
Model Residual	1642.52197 800.937487	2 71	821.260986 11.2808097		= = =	72.80 0.0000 0.6722 0.6630
Total	2443.45946	73	33.4720474	<b>U</b> 1	=	3.3587
mpg	Coef.	Std. Err.	t	P> t  [95%	Conf.	Interval]
weight	0141581	.0038835	-3.65	0.0010219	016	0064145
c.weight# c.weight	1.32e-06	6.26e-07	2.12	0.038 7.67e	-08	2.57e-06
_cons	51.18308	5.767884	8.87	0.000 39.68	225	62.68392

. regress mpg weight c.weight#c.weight

After the regression, predict is defined to be

 $-0.0141581 \texttt{weight} + 1.32 \times 10^{-6} \texttt{weight}^2 + 51.18308$ 

(Actually, it is more precise because the coefficients are internally stored at much higher precision than shown in the output.) Thus, we can create a new variable—let's call it fitted—equal to the prediction by typing predict fitted and then use scatter to display the fitted and actual values separately for domestic and foreign automobiles:

```
. predict fitted
(option xb assumed; fitted values)
. scatter mpg fitted weight, by(foreign, total) c(. 1) m(o i) sort
```





predict can calculate much more than just predicted values. For predict after linear regression, predict can calculate residuals, standardized residuals, Studentized residuals, influence statistics, and more. In any case, we specify what is to be calculated via an option, so if we wanted the residuals stored in new variable r, we would type

. predict r, resid

#### 16 [U] 20 Estimation and postestimation commands

The options that may be specified following predict vary according to the estimation command previously used; the predict options are documented along with the estimation command. For instance, to discover all the things predict can do following regress, see [R] regress.

4

## 20.10.1 Using predict

The use of predict is not limited to linear regression. predict can be used after any estimation command.

#### Example 7

You fit a logistic regression model of whether a car is manufactured outside the United States on the basis of its weight and mileage rating using either the logistic or the logit command; see [R] logistic and [R] logit. We will use logit.

. use http://w	www.stata-pres	ss.com/data/1	r14/auto2	, clear			
. logit foreig	gn weight mpg						
Iteration 0:	log likeliho	d = -45.03	3321				
Iteration 1:	log likelihood = -29.238536						
Iteration 2:	log likeliho	log likelihood = -27.244139					
Iteration 3:	log likeliho	pod = -27.175	5277				
Iteration 4:	log likeliho	d = -27.175	5156				
Iteration 5:	log likeliho	pod = -27.175	5156				
Logistic regre	ession			Number (	of obs	=	74
				LR chi2	(2)	=	35.72
				Prob > 0	chi2	=	0.0000
Log likelihood	d = -27.175156	3		Pseudo I	32	=	0.3966
	r						
foreign	Coef.	Std. Err.	z	P> z	[95%	Conf.	Interval]
weight	0039067	.0010116	-3.86	0.000	0058	3894	001924
0	1685869	.0919175	-1.83	0.067	3487	7418	.011568
_cons	13.70837	4.518709	3.03	0.002	4.851	L859	22.56487
weight mpg	0039067 1685869	.0010116	-3.86 -1.83	0.000	0058	3894 7418	001924 .011568

After logit, predict without options calculates the probability of a positive outcome (we learned that by looking at [R] logit). To obtain the predicted probabilities that each car is manufactured outside the United States, we type

```
. predict probhat
(option pr assumed; Pr(foreign))
. summarize probhat
    Variable
                       Obs
                                            Std. Dev.
                                   Mean
                                                             Min
                                                                         Max
     probhat
                        74
                               .2972973
                                                                     .8980594
                                            .3052979
                                                         .000729
. list make mpg weight foreign probhat in 1/5
       make
                        mpg
                               weight
                                          foreign
                                                      probhat
       AMC Concord
                          22
                                2,930
 1.
                                         Domestic
                                                     .1904363
 2.
       AMC Pacer
                          17
                                3,350
                                         Domestic
                                                     .0957767
                                2,640
 з.
       AMC Spirit
                          22
                                         Domestic
                                                     .4220815
 4.
                          20
                                3,250
                                                     .0862625
       Buick Century
                                         Domestic
                                4,080
                                                     .0084948
 5.
       Buick Electra
                          15
                                         Domestic
```

#### 20.10.2 Making in-sample predictions

predict does not retrieve a vector of prerecorded values—it calculates the predictions on the basis of the recorded coefficients and the data currently in memory. In the above examples, when we typed things like

. predict probhat

predict filled in the prediction everywhere that it could be calculated.

We sometimes have more data in memory than were used by the estimation command, either because we explicitly ignored some of the observations by specifying an if exp with the estimation command or because there are missing values. In such cases, if we want to restrict the calculation to the estimation subsample, we would do that in the usual way by adding if e(sample) to the end of the command:

```
. predict probhat if e(sample)
```

### 20.10.3 Making out-of-sample predictions

Because predict makes its calculations on the basis of the recorded coefficients and the data in memory, predict can do more than calculate predicted values for the data on which the estimation took place—it can make out-of-sample predictions, as well.

If you fit your model on a subset of the observations, you could then predict the outcome for all the observations:

```
. logit foreign weight mpg if rep78 > 3
```

. predict pall

If you do not specify if e(sample) at the end of the predict command, predict calculates the predictions for all observations possible.

In fact, because predict works from the active estimation results, you can use predict with any dataset that contains the necessary variables.

#### Example 8

Continuing with our previous logit example, assume that we have a second dataset containing the mpg and weight of a different sample of cars. We have just fit your model and now continue:

. use otherdat (Different car					
. predict prob (option <b>pr</b> ass		oreign))	Stata remembe.	rs the previous	s model
. summarize pr	cobhat fore	ign			
Variable	Obs	Mean	Std. Dev.	Min	Max
probhat	12	.2505068	.3187104	.0084948	.8920776
foreign	12	.1666667	.3892495	0	1

4

#### Example 9

We can obtain out-of-sample predictions in many ways. Above, we estimated on one dataset and then used another. If our first dataset had contained both sets of cars, marked, say, by the variable difcars being 0 if from the first sample and 1 if from the second, we could type

```
logit foreign weight mpg if difcars==0
same output as above appears
predict probhat
(option pr assumed; Pr(foreign))
summarize probhat foreign if difcars==1
same output as directly above appears
```

If we just had a few additional cars, we could even input them after estimation. Assume that our data once again contain only the first sample of cars, and assume that we are interested in an additional sample of only two cars; we could type

```
. use http://www.stata-press.com/data/r14/auto2
(1978 Automobile Data)
. keep make mpg weight foreign
. logit foreign weight mpg
same output as above appears
. input
                                                  foreign
                    make
                                        weight
                                mpg
75. "Merc. Zephyr" 20 2830 0
                                              we type in our new data
76. "VW Dasher" 23 2160 1
77. end
. predict probhat
                                              obtain all the predictions
(option pr assumed; Pr(foreign))
. list in -2/1
       make
                              weight
                                         foreign
                                                     probhat
                        mpg
 75.
       Merc. Zephyr
                         20
                               2,830
                                        Domestic
                                                     .3275397
       VW Dasher
                         23
 76.
                               2,160
                                                     .8009743
                                         Foreign
```

4

### 20.10.4 Obtaining standard errors, tests, and confidence intervals for predictions

When you use predict, you create, for each observation in the prediction sample, a statistic that is a function of the data and the estimated model parameters. You also could have generated your own customized predictions by using generate. In either case, to get standard errors, Wald tests, and confidence intervals for your predictions, use predictnl. For example, if we want the standard errors for our predicted probabilities, we could type

```
. drop probhat
```

```
. predictnl probhat = predict(), se(phat_se)
```

. list in 1/5

	make	mpg	weight	foreign	probhat	phat_se
1.	AMC Concord	22	2,930	Domestic	.1904363	.0658387
2.	AMC Pacer	17	3,350	Domestic	.0957767	.0536297
з.	AMC Spirit	22	2,640	Domestic	.4220815	.0892845
4.	Buick Century	20	3,250	Domestic	.0862625	.0461928
5.	Buick Electra	15	4,080	Domestic	.0084948	.0093079

4

Comparing this output to our previous listing of the first five predicted probabilities, you will notice that the output is identical except that we now have an additional variable, phat\_se, which contains the estimated standard error for each predicted probability.

We first had to drop probhat because predictnl will regenerate it. Note also the use of predict() within predictnl—it specified that we wanted to generate a point estimate (and standard error) for the default prediction after logit; see [R] predictnl for more details.

# 20.11 Accessing estimated coefficients

You can access coefficients and standard errors after estimation by referring to \_b[*name*] and \_se[*name*]; see [U] **13.5** Accessing coefficients and standard errors.

# Example 10

Let's return to linear regression. We are doing a study of earnings of men and women at a particular company. In addition to each person's earnings, we have information on their educational attainment and tenure with the company. We type the following:

. regress lnearn ed tenure i.female female#(c.ed c.tenure)
 (output omitted)

If you are not familiar with the # notation, see [U] 11.4.3 Factor variables.

We now wish to predict everyone's income as if they were male and then compare these as-if earnings with the actual earnings:

. generate asif = \_b[\_cons] + \_b[ed]\*ed + \_b[tenure]\*tenure

# ▷ Example 11

We are analyzing the mileage of automobiles and are using a slightly more sophisticated model than any we have used so far. As we have previously, we will fit a linear regression model of mpg on weight and the square of weight, but we also add the interaction of foreign with weight, the car's gear ratio (gear\_ratio), and foreign interacted with gear\_ratio. We will use factor-variable notation to create the squared term and the interactions; see [U] 11.4.3 Factor variables.

. use http://www.stata-press.com/data/r14/auto2, clear

(1978 Automobile Data)

```
. regress mpg weight c.weight#c.weight i.foreign#c.weight gear_ratio
```

> i.foreign#c.gear\_ratio

Source	SS	df	MS		er of obs = 68) =	
Model Residual	1737.05293 706.406534	5 68	347.410588 10.3883314	5 Prob 4 R-sq	o > F = uared = R-squared =	= 0.0000 = 0.7109
Total	2443.45946	73	33.4720474	0	MSE =	
mpg	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
weight	0118517	.0045136	-2.63	0.011	0208584	002845
c.weight# c.weight	9.81e-07	7.04e-07	1.39	0.168	-4.25e-07	2.39e-06
foreign# c.weight Foreign	0032241	.0015577	-2.07	0.042	0063326	0001157
gear_ratio	1.159741	1.553418	0.75	0.458	-1.940057	4.259539
foreign# c.gear_ratio						
Foreign	1.597462	1.205313	1.33	0.189	8077036	4.002627
_cons	44.61644	8.387943	5.32	0.000	27.87856	61.35432

If you are not experienced in both regression technology and automobile technology, you may find it difficult to interpret this regression. Putting aside issues of statistical significance, we find that mileage decreases with a car's weight but increases with the square of weight; decreases even more rapidly with weight for foreign cars; increases with higher gear ratio; and increases even more rapidly with higher gear ratio in foreign cars.

Thus, do foreign cars yield better or worse gas mileage? Results are mixed. As the foreign cars' weight increases, they do more poorly in relation to domestic cars, but they do better at higher gear ratios. One way to compare the results is to predict what mileage foreign cars would have if they were manufactured domestically. The regression provides all the information necessary for making that calculation. Mileage for domestic cars is estimated to be

 $-0.012\,\texttt{weight}+9.81\times10^{-7}\,\texttt{weight}^2+1.160\,\texttt{gear\_ratio}+44.6$ 

We can use that equation to predict the mileage of foreign cars and then compare it with the true outcome. The  $_b[]$  function simplifies reference to the estimated coefficients. We can type

```
. generate asif=_b[weight]*weight + _b[c.weight#c.weight]*c.weight#c.weight +
```

```
> _b[gear_ratio]*gear_ratio + _b[_cons]
```

\_b[weight] refers to the estimated coefficient on weight, \_b[c.weight#c.weight] to the estimated coefficient on c.weight#c.weight, and so on.

We might now ask how the actual mileage of a Honda compares with the asif prediction:

. list make asif mpg if strpos(make, "Honda")

	make	asif	mpg
61.	Honda /	 26.52597	25
62.	Honda (	30.62202	28

Notice the way we constructed our if clause to select Hondas. strpos() is the string function that returns the location in the first string where the second string is found or, if the second string does not occur in the first, returns 0. Thus any recorded make that contains the string "Honda" anywhere in it would be listed; see [FN] String functions.

We find that both Honda models yield slightly lower gas mileage than the asif domestic car-based prediction. (We do not endorse this model as a complete model of the determinants of mileage, nor do we single out Honda for any special scorn. In fact, please note that the observed values are within the root mean squared error of the average prediction.)

We might wish to compare the overall average mpg and the asif prediction over all foreign cars in the data:

. summarize m	pg asif if fo	oreign			
Variable	Obs	Mean	Std. Dev.	Min	Max
mpg asif	22 22	24.77273 26.67124	6.611187 3.142912	14 19.70466	41 30.62202

We find that, on average, foreign cars yield slightly lower mileage than our asif prediction. This might lead us to ask if any foreign cars do better than the asif prediction:

	make	asif	mpg
55.	BMW 320i	24.31697	25
57.	Datsun 210	28.96818	35
63.	Mazda GLC	29.32015	30
66.	Subaru	28.85993	35
68.	Toyota Corolla	27.01144	31
71.	VW Diesel	28.90355	41

. list make asif mpg if foreign & mpg>asif, sep(0)

We find six such automobiles.

# 20.12 Performing hypothesis tests on the coefficients

## 20.12.1 Linear tests

After estimation, test is used to perform tests of linear hypotheses on the basis of the variance– covariance matrix of the estimators (Wald tests).

#### Example 12

Using the automobile data, we perform the following regression:

```
. use http://www.stata-press.com/data/r14/auto2, clear (1978 Automobile Data)
```

- . generate weightsq=weight^2
- . regress mpg weight weightsq foreign

Source	SS	df	MS	Number of F(3, 70)	obs = =	74 52.25
Model Residual	1689.15372 754.30574	3 70	563.05124 10.7757963	Prob > F	=	0.0000
Total	2443.45946	73	33.4720474		=	3.2827
mpg	Coef.	Std. Err.	t	P> t  [95	% Conf.	Interval]
weight weightsq foreign _cons	0165729 1.59e-06 -2.2035 56.53884	.0039692 6.25e-07 1.059246 6.197383	2.55 -2.08	0.013 3.4 0.041 -4	44892 5e-07 .3161 17855	0086567 2.84e-06 0909002 68.89913

(Note: test has many syntaxes and features, so do not use this example as an excuse for not reading [R] test.) We can use the test command to calculate the joint significance of weight and weightsq:

```
. test weight weightsq
( 1) weight = 0
( 2) weightsq = 0
F( 2, 70) = 60.83
Prob > F = 0.0000
```

We are not limited to testing whether the coefficients are 0. We can test whether the coefficient on foreign is -2 by typing

```
. test foreign = -2
( 1) foreign = -2
F( 1, 70) = 0.04
Prob > F = 0.8482
```

We can even test more complicated hypotheses because test can perform basic algebra. Here is an absurd hypothesis:

```
. test 2*(weight+weightsq)=-3*(foreign-(weight-weightsq))
( 1) - weight + 5*weightsq + 3*foreign = 0
F( 1, 70) = 4.31
Prob > F = 0.0416
```

test simplified the algebra of our hypothesis and then presented the test results. We can also use test's accumulate option to combine this test with another test:

```
. test foreign+weight=0, accum
( 1) - weight + 5*weightsq + 3*foreign = 0
( 2) weight + foreign = 0
F( 2, 70) = 9.12
Prob > F = 0.0003
```

There are limitations. test can test only linear hypotheses. If we attempt to test a nonlinear hypothesis, test will tell us that it is not possible:

```
. test weight/foreign=0
not possible with test
r(131);
```

Testing nonlinear hypotheses is discussed in [U] 20.12.4 Nonlinear Wald tests below.

4

### 20.12.2 Using test

test bases its results on the estimated variance-covariance matrix of the estimators (that is, it performs a Wald test), so it can be used after any estimation command. For maximum likelihood estimation, test's results for a single variable are generally equivalent to the asymptotic z statistic presented in the coefficient table for that variable because test bases its results on the information matrix.

### Example 13

Let's examine the repair records of the cars in our automobile data as rated by Consumer Reports:

· · · · · · · · · · · ·			
Repair Record 1978	Car Domestic	type Foreign	Total
Poor	2	0	2
Fair	8	0	8
Average	27	3	30
Good	9	9	18
Excellent	2	9	11
Total	48	21	69

```
. tabulate rep78 foreign
```

The values are coded 1-5, corresponding to Poor, Fair, Average, Good, and Excellent. We will fit this variable by using a maximum-likelihood ordered logit model (the nolog option suppresses the iteration log, saving some space):

. ologit rep78	3 price foreig	gn weight we	ightsq di	.spl, nol	og		
Ordered logist	tic regression	1		Number LR chi2 Prob >	(5)	= = =	69 33.12 0.0000
Log likelihood	d = -77.133082	2		Pseudo	R2	=	0.1767
rep78	Coef.	Std. Err.	Z	P> z	[95%	Conf.	Interval]
price foreign weight weightsq displacement	000034 2.685647 0037447 7.87e-07 0108919	.0001188 .9320404 .0025609 4.50e-07 .0076805	-0.29 2.88 -1.46 1.75 -1.42	0.775 0.004 0.144 0.080 0.156	0002 .8588 0087 -9.43e 0259	8817 7639 9-08	.000199 4.512413 .0012745 1.67e-06 .0041617
/cut1 /cut2 /cut3 /cut4	-9.417196 -7.581864 -4.82209 -2.793441	4.298202 4.234091 4.14768 4.156221			-17.84 -15.88 -12.95 -10.93	8053 5139	992874 .7168028 3.307214 5.352602

We now wonder whether all our variables other than foreign are jointly significant. We test the hypothesis just as we would after linear regression:

You will have to decide whether you want to perform tests on the basis of the information matrix instead of constraining the equation, reestimating it, and then calculating the likelihood-ratio test. To compare this with the results performed by a likelihood-ratio test, see [U] **20.12.3 Likelihood-ratio** tests below. Results will differ little.

4

### 20.12.3 Likelihood-ratio tests

After maximum likelihood estimation, you can obtain likelihood-ratio tests by fitting both the unconstrained and the constrained models, storing the results using estimates store, and then running lrtest. See [R] lrtest for the full details.

Example 14

In [U] **20.12.2 Using test** above, we fit an ordered logit on rep78 and then tested the significance of all the explanatory variables except foreign.

To obtain the likelihood-ratio test, sometime after fitting the full model, we type estimates store *full\_model\_name*, where *full\_model\_name* is just a label that we assign to these results.

. ologit rep78 price foreign weight weightsq displ (output omitted)

. estimates store myfullmodel

This command saves the current model results with the name myfullmodel.

4

Next, we fit the constrained model. After that, typing 'lrtest myfullmodel .' compares the current model with the model we saved:

. ologit rep78	3 foreign					
Iteration 0:	log likeliho	ood = -93.692	2061			
Iteration 1:	log likeliho	pod = -79.696	6089			
Iteration 2:	0	pod = -79.034				
Iteration 3:	0	pod = -79.029				
Iteration 4:	log likeliho	pod = -79.029	9243			
Ordered logist	cic regression	1		Number	of obs =	69
-	-			LR chi2	(1) =	29.33
				Prob >	chi2 =	0.0000
Log likelihood	1 = -79.029243	3		Pseudo 1	R2 =	0.1565
rep78	Coef.	Std. Err.	Z	P> z	[95% Conf.	Interval]
foreign	2.98155	.6203644	4.81	0.000	1.765658	4.197442
/cut1	-3.158382	.7224269			-4.574313	-1.742452
/cut2	-1.362642	.3557343			-2.059868	6654154
/cut3	1.232161	.3431227			.5596532	1.90467
/cut4	3.246209	.5556657			2.157124	4.335293
. lrtest myful	lmodel .					
Likelihood-rat	io test			L	R chi2(4) =	3.79
(Assumption:	nested in my	fullmodel)		P	rob > chi2 =	0.4348

When we tested the same constraint with test (which performed a Wald test), we obtained a  $\chi^2$  of 3.63 and a significance level of 0.4590. We used . (the dot) to specify the results in active memory, although we could have stored them with estimates store and referred to them by name instead. Also, the order in which you specify the two models to lrtest doesn't matter; lrtest is smart enough to know the full model from the constrained model.

Two other postestimation commands work in the same way as lrtest, meaning that they accept names of stored estimation results as their input: hausman for performing Hausman specification tests and suest for seemingly unrelated estimation. We do not cover these commands here; see [R] hausman and [R] suest for more details.

### 20.12.4 Nonlinear Wald tests

testnl can be used to test nonlinear hypotheses about the parameters of the active estimation results. testnl, like test, bases its results on the variance-covariance matrix of the estimators (that is, it performs a Wald test), so it can be used after any estimation command; see [R] testnl.

Example 15

We fit the model

. regress price mpg weight foreign (output omitted)

and then type

We performed this test on linear regression estimates, but tests of this type could be performed after any estimation command.

4

4

# 20.13 Obtaining linear combinations of coefficients

lincom computes point estimates, standard errors, t or z statistics, p-values, and confidence intervals for a linear combination of coefficients after any estimation command. Results can optionally be displayed as odds ratios, incidence-rate ratios, or relative-risk ratios.

## Example 16

We fit a linear regression:

```
. use http://www.stata-press.com/data/r14/regress, clear
```

. regress y xi	1 x2 x3					
Source	SS	df	MS	Number of obs	=	148
				- F(3, 144)	=	96.12
Model	3259.3561	3	1086.45203	B Prob > F	=	0.0000
Residual	1627.56282	144	11.3025196	R-squared	=	0.6670
				- Adj R-squared	=	0.6600
Total	4886.91892	147	33.2443464	Root MSE	=	3.3619
У	Coef.	Std. Err.	t	P> t  [95% C	onf.	Interval]
x1	1.457113	1.07461	1.36	0.1776669	34	3.581161
x2	2.221682	.8610358	2.58	0.011 .51977	97	3.923583
xЗ	006139	.0005543	-11.08	0.00000723	45	0050435
_cons	36.10135	4.382693	8.24	0.000 27.438	63	44.76407

Suppose that we want to see the difference of the coefficients of x2 and x1. We type

. lincom x2 - x1 ( 1) - x1 + x2 = 0

У	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
(1)	.7645682	.9950282	0.77	0.444	-1.20218	2.731316

lincom is handy for computing the odds ratio of one covariate group relative to another.

# ▷ Example 17

We estimate the parameters of a logistic model of low birthweight:

. use http://w (Hosmer & Leme	www.stata-pres eshow data)	r14/lbw3					
. logit low ag	ge lwd i.race	smoke ptd h	t ui				
Iteration 0: log likelihood = -117.336 Iteration 1: log likelihood = -99.3982 Iteration 2: log likelihood = -98.780418 Iteration 3: log likelihood = -98.777998 Iteration 4: log likelihood = -98.777998							
Logistic regre	ession				of obs =	189	
				LR chi2		37.12	
		_		Prob >		0.0000	
Log likelihood	1 = -98.777993	8		Pseudo	R2 =	0.1582	
low	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]	
age							
	0464796	.0373888	-1.24	0.214	1197603	.0268011	
lwd	0464796 .8420615	.0373888	-1.24 2.08	0.214 0.038	1197603 .0472299	.0268011 1.636893	
lwd race	.8420615	.4055338	2.08	0.038	.0472299	1.636893	
lwd race black	.8420615 1.073456	.4055338	2.08 2.08	0.038	.0472299	1.636893 2.082985	
lwd race	.8420615	.4055338	2.08	0.038	.0472299	1.636893	
lwd race black	.8420615 1.073456	.4055338	2.08 2.08	0.038	.0472299	1.636893 2.082985	
lwd race black other	.8420615 1.073456 .815367	.4055338 .5150753 .4452979	2.08 2.08 1.83	0.038 0.037 0.067	.0472299 .0639273 0574008	1.636893 2.082985 1.688135	
lwd race black other smoke	.8420615 1.073456 .815367 .8071996	.4055338 .5150753 .4452979 .404446	2.08 2.08 1.83 2.00	0.038 0.037 0.067 0.046	.0472299 .0639273 0574008 .0145001	1.636893 2.082985 1.688135 1.599899	
lwd race black other smoke ptd	.8420615 1.073456 .815367 .8071996 1.281678	.4055338 .5150753 .4452979 .404446 .4621157	2.08 2.08 1.83 2.00 2.77	0.038 0.037 0.067 0.046 0.006	.0472299 .0639273 0574008 .0145001 .3759478 .1646414	1.636893 2.082985 1.688135 1.599899 2.187408	

Level 1 of race designates white, level 2 designates black, and level 3 designates other.

If we want to obtain the odds ratio for black smokers relative to white nonsmokers (the reference group), we type

```
. lincom 2.race + smoke, or
 (1)
       [low]2.race + [low]smoke = 0
         low
               Odds Ratio
                             Std. Err.
                                                   P>|z|
                                                              [95% Conf. Interval]
                                             z
         (1)
                  6.557805
                             4.744692
                                           2.60
                                                   0.009
                                                              1.588176
                                                                          27.07811
```

lincom computed  $\exp(\beta_{2.\text{race}} + \beta_{\text{smoke}}) = 6.56.$ 

4

# 20.14 Obtaining nonlinear combinations of coefficients

lincom is limited to estimating linear combinations of coefficients, for example, 2.race + smoke, or exponentiated linear combinations, as in the above. For general nonlinear combinations, use nlcom.

#### Example 18

Continuing our previous example, suppose that we want the ratio of the coefficients (and standard errors, Wald test, confidence interval, etc.) of blacks and races other than white and black:

```
. nlcom _b[2.race]/_b[3.race]
_nl_1: _b[2.race]/_b[3.race]
```

low	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]
_nl_1	1.316531	.7359262	1.79	0.074	1258574	2.75892

The Wald test given is that of the null hypothesis that the nonlinear combination is 0 versus the two-sided alternative—this is probably not informative for a ratio. If we would instead like to test whether this ratio is 1, we can rerun nlcom, this time subtracting 1 from our ratio estimate.

```
. nlcom _b[2.race]/_b[3.race] - 1
_nl_1: _b[2.race]/_b[3.race] - 1
```

low	Coef.	Std. Err.	Z	P> z	[95% Conf.	Interval]
_nl_1	.3165314	.7359262	0.43	0.667	-1.125857	1.75892

We can interpret this as not much evidence that the ratio minus 1 is different from 0, meaning that we cannot reject the null hypothesis that the ratio equals 1.

When using nlcom, we needed to refer to the model coefficients by their "proper" names, for example, \_b[2.race], and not by the shorthand 2.race, such as when using lincom. If we had typed

. nlcom 2.race/3.race

Stata would have reported an error.

If you have difficulty determining what to type for a coefficient when using lincom or nlcom, replay your results by using the coeflegend option. Here are the results for our current estimates:

. logit, coef	legend				
0 0	Logistic regression Log likelihood = -98.777998		Number of obs LR chi2(8) Prob > chi2 Pseudo R2	= = =	189 37.12 0.0000 0.1582
low	Coef.	Legend			
age lwd	0464796 .8420615	- 0			
race black other		_b[2.race] _b[3.race]			
smoke ptd ht ui _cons	1.281678 1.435227 .6576256				

# 20.15 Obtaining marginal means, adjusted predictions, and predictive margins

predict uses the current estimation results (the coefficients and the VCE) to estimate the value of statistics for observations in the data. lincom and nlcom use the current estimation results to estimate a specific linear or nonlinear expression of the coefficients. The margins command combines aspects of both and estimates margins of responses.

margins answers the question "What does my model have to say about such-and-such", where such-and-such might be

- my estimation sample or another sample
- a sample with the values of some covariates fixed
- a sample evaluated at each level of a treatment
- a population represented by a complex survey sample
- someone who looks like the fifth person in my sample
- someone who looks like the mean of the covariates in my sample
- someone who looks like the median of the covariates in my sample
- someone who looks like the 25th percentile of the covariates in my sample
- someone who looks like some other function of the covariates in my sample
- a standardized population
- a balanced experimental design
- any combination of the above
- any comparison of the above

margins answers these questions either conditionally on fixed values of all covariates or averaged over the observations in a sample. It answers these questions about almost any predictions or any other response that you can calculate as a function of your estimated parameters—linear responses, probabilities, hazards, survival times, odds ratios, risk differences, etc. You can even make multiple predictions at the same time when appropriate. For example, you may want the predicted probabilities and the linear prediction after logit.

margins answers these questions in terms of the response given covariate levels, or in terms of the change in the response for a change in levels (also known as marginal effects). It answers these questions providing standard errors, test statistics, and confidence intervals; and those statistics can take the covariates as given or adjust for sampling, also known as predictive margins and survey statistics.

A margin is a statistic based on a response for a fitted model calculated over a dataset in which some of or all the covariates are fixed at values different from what they really are.

Margins go by different names in different fields, and they can estimate many interesting statistics related to a fitted model. We discuss some common uses below; see [R] margins for more applications.

# 20.15.1 Obtaining estimated marginal means

A classic application of margins is to estimate the expected marginal means from a linear estimator as though the design for the covariates were balanced—assuming an equal number of observations for each unique combination of levels for the factor-variable covariates. These means have a long history in the study of ANOVA and MANOVA but are of limited use with nonexperimental data. For a discussion, see Obtaining margins as though the data were balanced in [R] margins and example 4 in [R] anova.

Estimated marginal means are also called least-squares means.

Consider an analysis of variance of the change in systolic blood pressure as determined by one of four drug treatments and adjusting for the patient's disease (Afifi and Azen 1979).

```
. use http://www.stata-press.com/data/r14/systolic (Systolic Blood Pressure Data)
```

```
. tabulate drug disease
```

	Pat	ient's Dise	ase	
Drug Used	1	2	3	Total
1	6	4	5	15
2	5	4	6	15
3	3	5	4	12
4	5	6	5	16
Total	19	19	20	58

. anova systolic drug##disease

	Number of obs = Root MSE =	58 10.5096		ed = quared =	0.4560 0.3259
Source	Partial SS	df	MS	F	Prob>F
Model	4259.3385	11	387.21259	3.51	0.0013
drug disease drug#disease	2997.4719 415.87305 707.26626	3 2 6	999.15729 207.93652 117.87771	9.05 1.88 1.07	0.0001 0.1637 0.3958
Residual	5080.8167	46	110.45254		
Total	9340.1552	57	163.86237		

Despite having randomized on drug, we see in the tabulation that our data are not balanced—for example, 12 patients were administered drug 3, whereas 16 were administered drug 4. The diseases are also not balanced across drugs. To estimate the marginal mean for each level of drug while treating the design as though it were balanced, we type

. margins drug	g, asbalanced					
Adjusted pred	ictions			Number	of obs =	58
-	: Linear pred: : drug disease	iction, predio (asbalan (asbalan	nced)			
	I Margin	Delta-method Std. Err.	t	P> t	[95% Conf	. Interval]
drug						
1	25.99444	2.751008	9.45	0.000	20.45695	31.53194
2	26.55556	2.751008	9.65	0.000	21.01806	32.09305
3	9.744444	3.100558	3.14	0.003	3.503344	15.98554
4	13.54444	2.637123	5.14	0.000	8.236191	18.8527

Assuming everyone in the sample were treated with drug 4 and that the diseases were equally distributed across the drug treatments, the expected mean change in pressure resulting from treatment with drug 4 is 13.54. Because we are treating the data as balanced, we could also say that 13.54 is

the expected mean change resulting from drug 4 for any sample where an equal number of patients has each of the three diseases.

If we want an estimate of the mean that uses the distribution of diseases observed in the sample, we would remove the asbalanced option:

. margins drug	5					
Predictive man	rgins			Number c	of obs	= 58
Expression	: Linear pred:	iction, predi	ct()			
	I	Delta-method				
	Margin	Std. Err.	t	P> t	[95% Cor	f. Interval]
drug						
1	25.89799	2.750533	9.42	0.000	20.36145	31.43452
2	26.41092	2.742762	9.63	0.000	20.89003	31.93181
3	9.722989	3.099185	3.14	0.003	3.484652	15.96132
4	13.55575	2.640602	5.13	0.000	8.24049	18.871

We can now say that a pressure change of 13.56 is expected if everyone in the sample is given drug 4 and the distribution of diseases is as observed in the sample.

The second set of margins are not usually called estimated marginal means because they do not impose a balanced design when estimating the mean. They are adjusted predictions that just happen to be means because the response is linear.

Neither of these values is the average pressure change for those taking drug 4 in our sample because margins treats everyone in the sample as having taken drug 4. Treating everyone as though they have taken each drug is what makes the means comparable. We are essentially standardizing on the values of all the other covariates in our model (in this example, just disease).

To obtain the observed mean for those taking drug 4, we must tell margins to treat drug 4 as its sample, which we do with the over() option.

. summarize systolic if drug==4

Variable	Obs	Mean	Std.	Dev.	Min	I	Max
systolic	16	13.5	9.323	805	-5		27
. margins, ove	er(drug)						
Predictive man	rgins			Number	of obs	=	58
-	: Linear pred: : drug	iction, pred	ict()				
	I Margin	Delta-method Std. Err.	t	P> t	[95%	Conf.	Interval]
drug							
1	26.06667	2.713577	9.61	0.000	20.6	0452	31.52881
2	25.53333	2.713577	9.41	0.000	20.0	7119	30.99548
3	8.75	3.033872	2.88	0.006	2.64	3133	14.85687
4	13.5	2.62741	5.14	0.000	8.21	1298	18.7887

The margin in the last line of the table matches the mean from summarize.

For many questions, we prefer one of the first two estimates of margins to the last one. If we compare drugs 3 and 4 from the last results, the 8.75 and 13.5 include both the effect from the drug and the differing distribution of diseases among patients taking drug 3 and drug 4 in our sample.

Our first set of margins, those from margins drug, asbalanced, assumed that for both drug 3 and drug 4, we had an equal number of patients with each disease. Our second set of margins, those from margins drug, assumed that for both drug 3 and drug 4, we wanted the observed distribution of patients from the whole sample. By assuming a common distribution of diseases across the drugs, our first two sets of margins remove the effect of disease when we compare across drugs.

# 20.15.2 Obtaining adjusted predictions

We will use the term adjusted predictions to refer to margins that are evaluated at fixed values for all covariates. The margins command has a great deal of flexibility in letting you choose what those fixed values are. Consider a model of high blood pressure as a function of sex, age group, and body mass index (BMI, a common measure of weight relative to height; variable bmi). We will allow the effect of age to differ for males and females by interacting the age group and sex variables. We will also allow the effect of BMI to differ across all combinations of age group and sex by specifying a full factorial model. . use http://www.stata-press.com/data/r14/nhanes2

```
. logistic highbp sex##agegrp##c.bmi
```

Logistic regres	sion	-		Number of LR chi2(2		10,351 2521.83
				Prob > ch	-	0.0000
Log likelihood	= -5789.851			Pseudo R2		0.1788
Log IIkeIIIlood	- 5765.651					0.1700
highbp	Odds Ratio	Std. Err.	Z	P> z	[95% Conf.	Interval]
sex						
Female	.4012124	.2695666	-1.36	0.174	.107515	1.497199
agegrp						
30-39	.8124869	.6162489	-0.27	0.784	.1837399	3.592768
40-49	1.346976	1.101181	0.36	0.716	.2713222	6.687051
50-59	5.415758	4.254136	2.15	0.032	1.161532	25.2515
60-69	16.31623	10.09529	4.51	0.000	4.852423	54.86321
70+	161.2491	130.7332	6.27	0.000	32.9142	789.9717
sex#agegrp						
Female#30-39	1.441256	1.44721	0.36	0.716	.2013834	10.31475
Female#40-49	6.29497	6.575021	1.76	0.078	.8126879	48.75998
Female#50-59	4.377185	4.43183	1.46	0.145	.6016818	31.84366
Female#60-69	1.790026	1.502447	0.69	0.488	.3454684	9.27492
Female#70+	.1958758	.2165763	-1.47	0.140	.0224297	1.710562
bmi	1.18539	.0221872	9.09	0.000	1.142692	1.229684
sex#c.bmi						
Female	.9809543	.0250973	-0.75	0.452	.9329775	1.031398
agegrp#c.bmi						
30-39	1.021812	.0299468	0.74	0.462	.9647712	1.082225
40-49	1.00982	.0315328	0.31	0.754	.9498702	1.073554
50-59	.979291	.0298836	-0.69	0.493	.9224373	1.039649
60-69	.9413883	.0228342	-2.49	0.013	.8976813	.9872234
70+	.8738056	.0278416	-4.23	0.000	.8209061	.930114
sex#agegrp# c.bmi						
Female#30-39	1.000676	.0377954	0.02	0.986	.9292736	1.077564
Female#40-49	.9702656	.0382854	-0.76	0.300	.8980559	1.048281
Female#40-49 Female#50-59	.9852929	.0380345	-0.38	0.444	.9134969	1.040201
Female#60-69	1.028896	.0330473	0.89	0.375	.9661212	1.002732
Female#70+	1.12236	.0480541	2.70	0.007	1.032019	1.220609
I CHAIC#/UT	1.12230		2.10	0.001	1.002019	1.220009
_cons	.0052191	.0024787	-11.07	0.000	.0020575	.0132388

We can evaluate the probability of having high blood pressure for each age group while holding the proportion of males and females and the value of bmi to its average by specifying the covariate agegrp to margins and including the option atmeans:

. margins age	grp, atmeans						
Adjusted pred: Model VCE	ictions : OIM			Number	of	obs =	10,351
-	: Pr(highbp), : 1.sex 2.sex 1.agegrp 2.agegrp 3.agegrp 4.agegrp 5.agegrp 6.agegrp bmi		1748333 5251667 2241329 1566998 1228867 1247222 2763018 0952565 25.5376	(mean) (mean) (mean) (mean) (mean) (mean) (mean)			
	1	Delta-method	1				
	Margin	Std. Err.	z	P> z		[95% Conf.	Interval]
agegrp 20-29 30-39 40-49 50-59 60-69 70+	.1611491 .2487466 .3679695 .5204507 .5714605 .6637982	.0091135 .0121649 .0144456 .0146489 .0095866 .0154566	17.68 20.45 25.47 35.53 59.61 42.95	0.000 0.000 0.000 0.000		.1432869 .2249038 .3396567 .4917394 .5526711 .6335038	.1790113 .2725893 .3962823 .549162 .5902499 .6940927

The header of the table showed us the mean values of each covariate. These are the values at which the probabilities were evaluated. The mean values for the levels of agegrp appear in the header even though they were not used. agegrp assumed the values 1, 2, 3, 4, 5, and 6, as shown in the table. The means of the levels of agegrp are shown because we might have asked for more margins in the table, for example, margins sex agegrp.

The modeled probability is just below 25% for those under 40 years of age, and it then increases fairly rapidly to 52% in the 50–59 age group. Above age 59, the probability remains under 67%. It is often easier for nonstatisticians to interpret the statistics computed by margins versus the coefficients of a fitted model.

### 20.15.3 Obtaining predictive margins

Rather than evaluate the probability of having high blood pressure at one fixed point (the means), as we did above, we can evaluate the probability at the covariate values for each observation in our data and average those probabilities. Here is the modeled probability averaged over our sample:

. margins							
Predictive man Model VCE	rgins : OIM			Number	of obs	=	10,351
Expression	: Pr(highbp),	<pre>predict()</pre>					
	]	Delta-method					
	Margin	Std. Err.	Z	P> z	[95%	Conf.	Interval]
_cons	.4227611	.0042939	98.46	0.000	.4143	3451	.4311771

If we fix the level of agegrp to 1, compute the probability for each observation, and then average those probabilities, the result is the predictive margin for level 1 of agegrp. margins, by default, computes these margins for each level of each variable specified on the command line. Let's compute the predictive margins for agegrp:

. margins ageg	grp							
Predictive man Model VCE	rgins OIM			Number o	f obs =	10,351		
Expression : Pr(highbp), predict()								
	I Margin	Delta-method Std. Err.	Z	P> z	[95% Conf	Intervall		
			Z	17   2				
agegrp								
20-29	.2030932	.0087166	23.30	0.000	.1860089	.2201774		
30-39	.2829091	.010318	27.42	0.000	.2626862	.3031319		
40-49	.3769536	.0128744	29.28	0.000	.3517202	.4021871		
50-59	.5153439	.0136201	37.84	0.000	.4886491	.5420387		
60-69	.5641065	.009136	61.75	0.000	.5462003	.5820127		
70+	.6535679	.0151371	43.18	0.000	.6238997	.683236		

One way of looking at predictive margins is that they answer the question "What would the average response (probability) be in my sample if everyone were in one age group?" Another way of looking at predictive margins is that they standardize the effect of being in an age group with the distribution of other covariate values in our sample. The margins above are comparable because only the level of agegrp is changing across the margins. They represent our sample because all the other covariates take on their values in the sample when the margins are evaluated.

The predictive margins in this table differ from the adjusted predictions we estimated in [U] **20.15.2 Obtaining adjusted predictions** because the probability is a nonlinear function of the coefficients in a logistic model; see *Example 3: Average response versus response at average* in [R] **margins** for details.

Our analysis so far has been a bit naïve. The dataset we are using is from the Second National Health and Nutrition Examination Survey (NHANES II). It has weights to make it representative of the population from which it was drawn as well as other survey characteristics—strata and primary sampling units. The data have already been svyset; see [SVY] svyset. We should take note of these characteristics and use the svy prefix when fitting our model.

. svy: logistic highbp sex##agegrp##c.bmi
(output omitted)

If we were to repeat the command margins agegrp, we would see that our point estimates differ only a little, but our standard errors are generally larger.

We are not restricted to margining over a single factor variable. Let's see if the pattern of high blood pressure over age groups differs for men and women. We do that by specifying the interaction of sex and agegrp to margins. We add the vce(unconditional) option to accommodate the survey design.

. margins sex#agegrp, vce(unconditional)

Predictive margins

Number of obs = 10,351

Expression : Pr(highbp), predict()

		-				
	Margin	Linearized Std. Err.	t	P> t	[95% Conf.	Interval]
sex#agegrp Male#20-29 Male#30-39 Male#40-49 Male#50-59 Male#60-69 Male#70+ Female#20-29 Female#30-39 Female#30-49 Female#50-59 Female#60-69	. 2931664 . 3664032 . 3945619 . 5376423 . 5780997 . 6507023 . 1069761 . 1898006 . 3250246 . 4855339 . 5441773	.0204899 .0241677 .0240343 .0295377 .0224681 .0209322 .0135978 .0143975 .0236775 .03364 .0186243	14.31 15.16 16.42 18.20 25.73 31.09 7.87 13.18 13.73 14.43 29.22	0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000	.251377 .3171128 .3455435 .4773997 .5322756 .6080109 .0792432 .1604367 .276734 .4169247 .5061928	. 3349557 .4156936 .4435802 .5978849 .6239237 .6933938 .1347091 .2191646 .3733152 .5541431 .5821618
Female#70+	.6195342	.0275568	22.48	0.000	.5633317	.6757367

Each line in the table corresponds to holding both sex and agegrp to fixed values while using the observed level of bmi to evaluate the probability and then averaging over the observations in the sample. To calculate the results in the first line of the table, margins fixed sex = 1 and agegrp = 1, evaluated the probability for each observation, and then averaged the probabilities. All of these margins reflect the observed distribution of bmi in the sample.

The first six lines represent males, and the second six lines represent females. Comparing males with females for the same age groups, males are almost three times as likely to have high blood pressure in the first age group (0.293/0.107 = 2.7); they are almost twice as likely in the second age group; and while the relative gap narrows, it is not until above age 70 that the probability for males drops below the probability for females.

Can the pattern of probabilities be affected by controlling one's bmi? Let's reevaluate the probabilities while holding bmi to two levels—20 (which is well within the normal range) and 30 (which is at the boundary between overweight and obese). We add the option at(bmi=(20 30)) to set bmi first to 20 and then to 30.
Adjusted pre					Number of	obs =	10,351
Expression	: Pr(	(highbp),	<pre>predict()</pre>				
1at	: bmi	Ĺ	=	20			
2at	: bmi	i	=	30			
			Linearized				
		Margin	Std. Err.	t	P> t	[95% Conf.	Interval]
_at#se	x#						
agegr		1200252	0017200	6 41	0.000	004011	1025504
1#Male#20-29		.1392353	.0217328	6.41	0.000	.094911 .1222249	.1835596
1#Male#30-39		.1714727 .1914061	.0241469	7.10	0.000		.2207205
1#Male#40-49			.0366133	5.23	0.000	.1167329	.2660794
1#Male#50-59		.3380778	.0380474	8.89	0.000	.2604797	.4156759
1#Male#60-69		.4311378	.0371582	11.60	0.000	.3553532	.5069225
1#Male#70+		.6131166	.0521657	11.75	0.000	.506724	.7195092
	#						
Female		0420044	0001000	7 44	0 000	0040000	05000
20-29		.0439911	.0061833	7.11	0.000	.0313802	.056602
	#						
Female		075000	0404774			0400400	400000
30-39		.075806	.0134771	5.62	0.000	.0483193	.1032926
	#						
Female							
40-49		.1941274	.0231872	8.37	0.000	.1468367	.2414181
	#						
Female							
50-59		.3493224	.0405082	8.62	0.000	.2667055	.4319394
	#						
Female	#						
60-69		.3897998	.0226443	17.21	0.000	.3436165	.4359831
1#Female#70+		.4599175	.0338926	13.57	0.000	.3907931	.5290419
2#Male#20-29		.4506376	.0370654	12.16	0.000	.3750422	.526233
2#Male#30-39		.569466	.04663	12.21	0.000	.4743635	.6645686
2#Male#40-49		.6042078	.039777	15.19	0.000	.5230821	.6853334
2#Male#50-59		.7268547	.0339618	21.40	0.000	.657589	.7961203
2#Male#60-69		.7131811	.0271145	26.30	0.000	.6578807	.7684816
2#Male#70+		.6843337	.0357432	19.15	0.000	.611435	.7572323
2	#						
Female	#						
20-29		.1638185	.024609	6.66	0.000	.1136282	.2140088
2	#						
Female	#						
30-39		.3038899	.0271211	11.20	0.000	.2485761	.3592037
2	#						
Female	#						
40-49		.4523337	.0364949	12.39	0.000	.3779019	.5267655
	#						
Female							
50-59		.6132219	.0376898	16.27	0.000	.536353	.6900908
	#			10.21	5.000		
Female							
60-69		.68786	.0274712	25.04	0.000	.631832	.7438879
2#Female#70+		.7643662	.0343399	25.04 22.26	0.000	.6943296	
Z#I CHIAIC#/UT		.1040002	.0040059	22.20	0.000	.0343230	.0044028

That is a lot of margins, but they are in sets of six age groups. The first six margins are men with a BMI of 20, the second six are women with a BMI of 20, the third six are men with a BMI of 30, and the last six are women with a BMI of 30. These margins tell a more complete story. The

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probability of high blood pressure is much lower for both men and women who maintain a BMI of 20. More interesting is that the relationship between men and women differs depending on BMI. While young men who maintain a BMI of 20 are still twice as likely as young women to have high blood pressure (0.139/0.044) and youngish men are over 50% more likely (0.171/0.076), the gap narrows substantially for men in the four older groups. The story is worse for those with a BMI of 30. Both men and women with a high BMI have a substantially increased risk of high blood pressure, with men ages 50–69 almost 10 percentage points higher than women. Before you dismiss these differences as caused by the usual attenuation of the logistic curve in the tails, recall that when we fit the model, we allowed the effect of bmi to be different for each combination of sex and agegrp.

You may have noticed that the header of the prior results says "Adjusted predictions" rather than "Predictive margins". That is because our model has only three covariates, and we have fixed the values of each. margins is no longer averaging over the data, but is instead evaluating the margins at fixed points that we have requested. It lets us know that by changing the header.

We could post the results of margins and form linear combinations or perform tests about any of the assertions above; see *Example 10: Testing margins*—contrasts of margins in [R] margins.

There is much more to know about margins and the margins command. Consider the headings for the *Remarks and examples* section of [R] margins:

Introduction
Obtaining margins of responses
Example 1: A simple case after regress
Example 2: A simple case after logistic
Example 3: Average response versus response at average
Example 4: Multiple margins from one command
Example 5: Margins with interaction terms
Example 6: Margins with continuous variables
Example 7: Margins of continuous variables
Example 8: Margins of interactions
Example 9: Decomposing margins
Example 10: Testing margins—contrasts of margins
Example 11: Margins of a specified prediction
Example 12: Margins of a specified expression
Example 13: Margins with multiple outcomes (responses)
Example 14: Margins with multiple equations
Example 15: Margins evaluated out of sample
Obtaining margins of derivatives of responses (a.k.a. marginal effects)
Use at() freely, especially with continuous variables
Expressing derivatives as elasticities
Derivatives versus discrete differences
Example 16: Average marginal effect (partial effects)
Example 17: Average marginal effect of all covariates
Example 18: Evaluating marginal effects over the response surface
Obtaining margins with survey data and representative samples
Example 19: Inferences for populations, margins of response
Example 20: Inferences for populations, marginal effects
Example 21: Inferences for populations with svyset data
Standardizing margins
Obtaining margins as though the data were balanced
Balancing using asbalanced
Balancing by standardization
Balancing nonlinear responses
Treating a subset of covariates as balanced
Using fvset design
Balancing in the presence of empty cells
Obtaining margins with nested designs
Introduction
Margins with nested designs as though the data were balanced
Coding of nested designs

Special topics Requirements for model specification Estimability of margins Manipulability of tests Using margins after the estimates use command Syntax of at() Estimation commands that may be used with margins Video examples Glossary

# 20.16 Obtaining conditional and average marginal effects

Marginal effects measure the change in a response given a change in a covariate, which is to say that marginal effects are derivatives. As used here, marginal effects can also be the discrete change in a response as an indicator goes from 0 to 1. Some authors reserve the term marginal effect for the continuous change and use the term partial effect for the discrete change. We will not make that distinction. Regardless, marginal effects are most often used to make it easier to interpret how changes in covariates affect a nonlinear response from a fitted model—a probability, a censored dependent variable, a survival time, a hazard, etc.

Marginal effects can either be evaluated at a specified point for all the covariates in our model (conditional marginal effects) or be evaluated at the observed values of the covariates in a dataset and then averaged (average marginal effects).

To Stata, marginal effects are just margins whose response happens to be the derivative of another response. Those interested in marginal effects will be interested in all or most of [R] margins.

### 20.16.1 Obtaining conditional marginal effects

We call a marginal effect conditional when we fix the values of all the covariates and then take the derivative of the response with respect to a covariate. The mean of all covariates is often used as the fixed point, and this is sometimes called the marginal effect at the means.

Consider a simple probit model of union membership for women as a function of having graduated from college (collgrad), living in the South (south), tenure on the job (tenure), and the interaction of south and tenure. We are interested in how being in the South affects union membership. We fit the model by using an extract from 1988 of the U.S. National Longitudinal Survey of Labor Market Experience (see [XT] xt).

. use http://w (NLSW, 1988 ex	-	ss.com/data/	r14/nlsw8	38b, clea	ır		
. probit unior	n i.collgrad i	i.south tenu	ire south#	c.tenure	•		
Iteration 0: Iteration 1: Iteration 2: Iteration 3:	log likeliho log likeliho	log likelihood = -1042.6816 log likelihood = -997.71809 log likelihood = -997.60984 log likelihood = -997.60983					
Probit regress	sion				of obs		1,000
				LR chi2	2(4) chi2	=	00.11
Log likelihood	1 = -997.60983	3		Pseudo		=	0.0432
union	Coef.	Std. Err.	Z	P> z	[95%	Conf.	Interval]
collgrad							
not grad	.2783278	.0726167	3.83	0.000	.1360	018	.4206539
1.south	2534964		-2.41				0475921
tenure	.0362944	.0068205	5.32	0.000	.0229	264	.0496624
south# c.tenure							
1	0239785	.0119533	-2.01	0.045	0474	065	0005504
_cons	8497418	.0664524	-12.79	0.000	9799	862	7194974

Clearly, being located in the South decreases union membership. Using the dydx() and atmeans options of margins, we can ask how much it decreases membership by evaluating the marginal effect of being southern at the means of all covariates:

```
. margins, dydx(south) atmeans
Conditional marginal effects
                                                  Number of obs
                                                                            1,868
Model VCE
             : OIM
Expression
             : Pr(union), predict()
dy/dx w.r.t. : 1.south
at
              : 0.collgrad
                                      .7521413 (mean)
                                =
                1.collgrad
                                =
                                      .2478587 (mean)
               0.south
                                =
                                     .5744111 (mean)
               1.south
                                =
                                      .4255889 (mean)
               tenure
                                =
                                     6.571065 (mean)
                           Delta-method
                     dy/dx
                             Std. Err.
                                                             [95% Conf. Interval]
                                             z
                                                  P>|z|
                              .019431
     1.south
                 -.1236055
                                          -6.36
                                                  0.000
                                                            -.1616896
                                                                        -.0855215
```

Note: dy/dx for factor levels is the discrete change from the base level.

At the means of all the covariates, southern women are 12 percentage points less likely to be members of a union. This marginal effect includes both the direct effect of *i.south* and the interaction south#c.tenure.

As margins reports below the table, this change in the response is for the discrete change of going from not southern (0) to southern (1).

The header of margins tells us where the marginal effect was estimated. This margin fixes tenure to be 6.6 years. There is nothing special about this point. We could also evaluate the marginal effect at the median of tenure:

. margins, dyd	lx(south) atme	ans at((medi	ans) _	continuous	3)		
Conditional ma Model VCE	arginal effect : OIM	S		Number	of obs	=	1,868
Expression : dy/dx w.r.t. : at :	: 1.south : 0.collgrad 1.collgrad 0.south	= .75 = .24 = .57 = .42	78587 44111 55889	(mean) (mean)			
	I dy/dx	elta-method Std. Err.	Z	P> z	[95% (	Conf.	Interval]
1.south	1061338	.0201722	-5.26	0.000	1456	706	066597

Note: dy/dx for factor levels is the discrete change from the base level.

With tenure at its median of 4.67, the marginal effect is about 2 percentage points less than it was at the mean of 6.6.

When examining conditional marginal effects, it is often useful to evaluate them at a range of values for the covariates. We can do that by asking both for values of the indicator covariate collgrad and for a range of values for tenure:

. margins coll	Lgrad, dydx(s	outh) at(tenu	re=(0(5)	)25))			
Conditional ma Model VCE	arginal effec : OIM	ts		Number	of obs	=	1,868
Expression : dy/dx w.r.t. :	Pr(union), 1.south	<pre>predict()</pre>					
1at :	: tenure	=	0				
2at :	: tenure	=	5				
3at :	: tenure	=	10				
4at :	: tenure	=	15				
5at :	: tenure	=	20				
6at :	: tenure	=	25				
		Delta-method					
	dy/dx	Std. Err.	Z	P> z	[95%	Conf.	Interval]
1.south							
_at#collgrad							
1#grad	0627725	.0254161	-2.47	0.014	112	587	0129579
1#not grad	0791483	.0321151	-2.46	0.014	1420		0162038
2#grad	1031957	.0189184	-5.45	0.000	140		0661164
2#not grad	1256566	.0232385	-5.41	0.000	1712		0801101
3#grad	1496772	.022226	-6.73	0.000	1932		1061151
3#not grad	1760137	.0266874	-6.60	0.000	2283		1237073
4#grad	2008801	.036154	-5.56	0.000	2717		1300196
4#not grad	2282	.0419237	-5.44	0.000	310		146031
5#grad	2549707	.0546355	-4.67	0.000	3620	543	1478872
5#not grad	2799495	.0613127	-4.57	0.000	4001	201	1597789
6#grad	3097656	.0747494	-4.14	0.000	4562		1632594
6#not grad	3289702	.0816342	-4.03	0.000	4889	703	1689701

Note: dy/dx for factor levels is the discrete change from the base level.

We now have a more complete picture of the effect that being in the South has on union participation. For those with no tenure and without a college degree (the first line in the table), being in the South

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decreases union participation by only 6 percentage points. For those with 25 years of tenure and with a college degree (the last line in the table), being in the South decreases participation by almost 33 percentage points. We can read the effect for any combination of tenure and college graduation status from the other lines in the table.

#### 20.16.2 Obtaining average marginal effects

To compute average marginal effects, the marginal effect is first computed for each observation in the dataset and then averaged. If the sample over which we compute the average marginal effect represents a population, then we have estimated the marginal effect for the population.

We continue with our example of labor union participation.

```
. use http://www.stata-press.com/data/r14/nlsw88b
(NLSW, 1988 extract)
. probit union i.collgrad i.south tenure south#c.tenure
(output omitted)
```

To estimate the average marginal effect for each of our regressors, we type

```
. margins, dydx(*)
Average marginal effects Number of obs = 1,868
Model VCE : OIM
Expression : Pr(union), predict()
dy/dx w.r.t. : 1.collgrad 1.south tenure
```

	dy/dx	Delta-method Std. Err.	z	P> z	[95% Conf	. Interval]
collgrad not grad 1.south tenure	.0878847 126164 .0083571	.0238065 .0191504 .0016521	3.69 -6.59 5.06	0.000 0.000 0.000	.0412248 1636981 .005119	.1345447 0886299 .0115951

Note: dy/dx for factor levels is the discrete change from the base level.

For this sample, the average marginal effect is very close to the marginal effect at the mean that we computed earlier. That is not always true; it depends on the distribution of the other covariates. The results also tell us that on average, for populations like the one from which our sample was drawn, union participation increases 0.8 percentage points for every year of tenure on the job. College graduates are, on average, 8.8 percentage points more likely to participate.

In the examples above, we treated the covariates in the sample as fixed and known. We could have accounted for the fact that this sample was drawn from a population and the covariates represent just one sample from that population. We do that by adding the vce(robust) or vce(cluster *clustvar*) option when fitting the model and the vce(unconditional) option when estimating the margins; see *Obtaining margins with survey data and representative samples* in [R] **margins**. It makes little difference in the examples above.

# 20.17 Obtaining pairwise comparisons

pwcompare performs pairwise comparisons across the levels of factor variables. pwcompare can compare estimated cell means, marginal means, intercepts, marginal intercepts, slopes, or marginal slopes—collectively called margins. pwcompare reports comparisons as contrasts (differences) of margins along with significance tests or confidence intervals for the contrasts. The tests and confidence intervals can be adjusted for multiple comparisons. pwcompare is for use after an estimation command in which you have used factor variables in specifying the model. You could not use pwcompare after typing

. regress yield fertilizer1-fertilizer5

but you could use pwcompare after typing

. regress yield i.fertilizer

Below, we fit a linear regression of wheat yield on type of fertilizer, and then we compare the mean yields for each pair of fertilizers and obtain *p*-values and confidence intervals adjusted for multiple comparisons by using Tukey's honestly significant difference.

```
. use http://www.stata-press.com/data/r14/yield (Artificial wheat yield dataset)
```

. regress yiel	ld i.fertilize:	r					
Source	SS	df	MS		per of obs	=	200
				F(4,	195)	=	5.33
Model	1078.84207	4	269.710517	Prob	> F	=	0.0004
Residual	9859.55334	195	50.561812	R-sc	uared	=	0.0986
				Adj	R-squared	=	0.0801
Total	10938.3954	199	54.9668111	Root	MSE	=	7.1107
yield	Coef.	Std. Err.	t	P> t	[95% C	onf.	Interval]
fertilizer							
10-08-22	3.62272	1.589997	2.28	0.024	.48692	12	6.758518
16-04-08	.4906299	1.589997	0.31	0.758	-2.6451	69	3.626428
18-24-06	4.922803	1.589997	3.10	0.002	1.7870	05	8.058602
29-03-04	-1.238328	1.589997	-0.78	0.437	-4.3741	27	1.89747
_cons	41.36243	1.124298	36.79	0.000	39.145	09	43.57977

. pwcompare fertilizer, effects mcompare(tukey)
Pairwise comparisons of marginal linear predictions
Margins : asbalanced

	Number of Comparisons
fertilizer	10

			Tul	key		key
	Contrast	Std. Err.	t	P> t	[95% Conf	. Interval]
fertilizer 10-08-22						
vs 10-10-10 16-04-08	3.62272	1.589997	2.28	0.156	7552913	8.000731
vs 10-10-10 18-24-06	.4906299	1.589997	0.31	0.998	-3.887381	4.868641
vs 10-10-10 29-03-04	4.922803	1.589997	3.10	0.019	.5447922	9.300815
vs 10-10-10 16-04-08	-1.238328	1.589997	-0.78	0.936	-5.616339	3.139683
vs 10-08-22 18-24-06	-3.13209	1.589997	-1.97	0.285	-7.510101	1.245921
vs 10-08-22 29-03-04	1.300083	1.589997	0.82	0.925	-3.077928	5.678095
vs 10-08-22 18-24-06	-4.861048	1.589997	-3.06	0.021	-9.239059	4830368
vs 16-04-08 29-03-04	4.432173	1.589997	2.79	0.046	.0541623	8.810185
vs 16-04-08 29-03-04	-1.728958	1.589997	-1.09	0.813	-6.106969	2.649053
vs 18-24-06	-6.161132	1.589997	-3.87	0.001	-10.53914	-1.78312

See [R] pwcompare and [R] margins, pwcompare.

## 20.18 Obtaining contrasts, tests of interactions, and main effects

contrast estimates and tests contrasts—comparisons of levels of factor variables. It also performs joint tests of these contrasts and can produce ANOVA-style tests of main effects, interaction effects, simple effects, and nested effects. It can be used after most estimation commands.

contrast provides a set of contrast operators such as r., ar., and p.. These operators are prefixed onto variable names—for example, r.varname—to specify the contrasts to be performed. The operators can be used with the contrast and margins commands.

Below, we fit a regression of cholesterol level on age group category.

. regress chol i.agegrp

The reported coefficients on *i.agegrp* will themselves be contrasts, namely, contrasts on the reference category. After estimation, if we wanted to compare the cell mean of each age group with that of the previous group, we would perform a reverse-adjacent contrast by typing

. contrast ar.agegrp

That is exactly what we will do:

. use http://www.stata-press.com/data/r14/cholesterol (Artificial cholesterol data)

. regress chol i.agegrp

Source	SS	df	MS		er of obs	=	75
Model Residual	14943.3997 7468.21971	4 70	3735.84993 106.688853	R-squ	> F mared	=	35.02 0.0000 0.6668 0.6477
Total	22411.6194	74	302.859722	0	l-squared MSE	=	10.329
chol	Coef.	Std. Err.	t	P> t	[95% Con	f.	Interval]
agegrp 20-29 30-39 40-59 60-79	8.203575 21.54105 30.15067 38.76221	3.771628 3.771628 3.771628 3.771628 3.771628	5.71 7.99	0.033 0.000 0.000 0.000	.6812991 14.01878 22.6284 31.23993		15.72585 29.06333 37.67295 46.28448
_cons	180.5198	2.666944	67.69	0.000	175.2007		185.8388

. contrast ar.agegrp

Contrasts of marginal linear predictions

Margins : asbalanced

	df	F	P>F	
agegrp				
(20-29 vs 10-19)	1	4.73	0.0330	
(30-39 vs 20-29)	1	12.51	0.0007	
(40-59 vs 30-39)	1	5.21	0.0255	
(60-79 vs 40-59)	1	5.21	0.0255	
Joint	4	35.02	0.0000	
Denominator	70			
	Contrast	Std. Err.	[95% Conf.	Interval]
agegrp				
(20-29 vs 10-19)	8.203575	3.771628	.6812991	15.72585
(30-39 vs 20-29)	13.33748	3.771628	5.815204	20.85976
(40-59 vs 30-39)	8.60962	3.771628	1.087345	16.1319
(60-79 vs 40-59)	8.611533	3.771628	1.089257	16.13381

We could use orthogonal polynomial contrasts to test whether there is a linear, quadratic, or even higher-order trend in the estimated cell means.

. contrast p.agegrp, noeffects

Contrasts of m	narginal linear	prediction	s
Margins	: asbalanced		
	df	F	P>F
agegrp			
(linear)	1	139.11	0.0000
(quadratic)	1	0.15	0.6962
- (cubic)	1	0.37	0.5448
(quartic)	1	0.43	0.5153
Joint	4	35.02	0.0000
Denominator	70		

You are not limited to using contrast in one-way models. Had we fit

. regress chol agegrp##race

we could contrast to obtain tests of the main effects and interaction effects.

. contrast agegrp##race

These results would be the same as would be reported by anova. We mention this because you can use contrast after any estimation command that allows factor variables and works with margins. You could type

. logistic highbp agegrp##race

. contrast agegrp##race

See [R] contrast and [R] margins, contrast.

# 20.19 Graphing margins, marginal effects, and contrasts

Using marginsplot, you can graph any of the results produced by margins, and because margins can replicate any of the results produced by pwcompare and contrast, you can graph any of the results produced by them, too.

In [U] **20.15.3 Obtaining predictive margins**, we did the following:

- . use http://www.stata-press.com/data/r14/nhanes2
- . svy: logistic highbp sex##agegrp##c.bmi
- . margins sex#agegrp, vce(unconditional)

We can now graph those results by typing

```
. marginsplot, xdimension(agegrp)
Variables that uniquely identify margins: sex agegrp
```



See [R] marginsplot. Mitchell (2012) shows how to make similar graphs for a variety of predictions and models.

## 20.20 Dynamic forecasts and simulations

The forecast suite of commands lets you obtain forecasts from forecast models, collections of equations that jointly determine the outcomes of one or more endogenous variables. You fit stochastic equations using estimation commands such as regress or var, and then you add those results to your forecast model. You can also specify identities that define variables in terms of other variables, and you can also specify exogenous variables whose values are already known or otherwise determined by factors outside your model. forecast then solves the resulting system of equations to obtain forecasts.

forecast works with time-series and panel datasets, and you can obtain either dynamic or static forecasts. Dynamic forecasts use previous periods' forecast values wherever lags appear in the model's equations and thus allow you to obtain forecasts for multiple periods in the future. Static forecasts use previous periods' actual values wherever lags appear in the model's equations, so if you use lags, you cannot make predictions much beyond the end of the time horizon in your dataset. However, static forecasts are useful during model development.

You can incorporate outside information into your forecasts, and you can specify a future path for some of the model's variables and obtain forecasts for the other variables conditional on that path. These features allow you to produce forecasts under different scenarios, and they allow you to explore how different policy interventions would affect your forecasts.

forecast also has the capability to produce confidence intervals around the forecasts. You can have forecast account for the sampling variance of the estimated parameters in the stochastic equations. There are two ways to account for an additive stochastic error term in the stochastic equations. You can request either that forecast assume the error terms are normally distributed and take draws from a random-number generator or that forecast take random samples from the pool of static-forecast residuals.

See [TS] forecast.

## 20.21 Obtaining robust variance estimates

Many Stata estimation commands provide robust and cluster-robust variance estimates. To obtain these estimates, you simply specify option vce(robust) to obtain robust standard errors or vce(cluster *clustvar*) to obtain cluster-robust standard errors. Below, we provide a general discussion of why you might specify one of these options, how to interpret standard errors with and without vce(robust) specified, and an overview of important concepts relating to cluster-robust standard errors.

Estimates of variance refer to estimated standard errors or, more completely, the estimated variance– covariance matrix of the estimators of which the standard errors are a subset, being the square root of the diagonal elements. Call this matrix the variance. All estimation commands produce an estimate of variance and, using that, produce confidence intervals and significance tests.

In addition to the conventional estimator of variance, there is another estimator that has been called by various names because it has been derived independently in different ways by different authors. Two popular names associated with the calculation are Huber and White, but it is also known as the sandwich estimator of variance (because of how the calculation formula physically appears) and the robust estimator of variance (because of claims made about it). Also, this estimator has an independent and long tradition in the survey literature.

The conventional estimator of variance is derived by starting with a model. Let's start with the regression model

$$y_i = \mathbf{x}_i \boldsymbol{\beta} + \epsilon_i, \qquad \epsilon_i \sim N(0, \sigma^2)$$

although it is not important for the discussion that we are using regression. Under the model-based approach, we assume that the model is true and thereby derive an estimator for  $\beta$  and its variance.

The estimator of the standard error of  $\hat{\beta}$  we develop is based on the assumption that the model is true in every detail.  $y_i$  is not exactly equal to  $\mathbf{x}_i \boldsymbol{\beta}$  (so that we would only need to solve an equation to obtain precisely that value of  $\boldsymbol{\beta}$ ) because the observed  $y_i$  has noise  $\epsilon_i$  added to it, the noise is Gaussian, and it has constant variance. That noise leads to the uncertainty about  $\boldsymbol{\beta}$ , and it is from the characteristics of that noise that we are able to calculate a sampling distribution for  $\hat{\boldsymbol{\beta}}$ .

The key thought here is that the standard error of  $\hat{\beta}$  arises because of  $\epsilon$  and is valid only because the model is absolutely, without question, true; we just do not happen to know the particular values of  $\beta$  and  $\sigma^2$  that make the model true. The implication is that, in an infinite-sized sample, the estimator  $\hat{\beta}$  for  $\beta$  would converge to the true value of  $\beta$  and that its variance would go to 0.

Now here is another interpretation of the estimation problem: We are going to fit the model

$$y_i = \mathbf{x}_i \mathbf{b} + e_i$$

and, to obtain estimates of b, we are going to use the calculation formula

$$\widehat{\mathbf{b}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$$

We have made no claims that the model is true or any claims about  $e_i$  or its distribution. We shifted our notation from  $\beta$  and  $\epsilon_i$  to **b** and  $e_i$  to emphasize this. All we have stated are the physical actions we intend to carry out on the data. Interestingly, it is possible to calculate a standard error for  $\hat{\mathbf{b}}$ here. At least, it is possible if you will agree with us on what the standard error measures are.

We are going to define the standard error as measuring the standard error of the calculated  $\mathbf{b}$  if we were to repeat the data collection followed by estimation over and over again.

This is a different concept of the standard error from the conventional, model-based ideas, but it is related. Both measure uncertainty about b (or  $\beta$ ). The regression model-based derivation states from where the variation arises and so can make grander statements about the applicability of the measured standard error. The weaker second interpretation makes fewer assumptions and so produces a standard error suitable for one purpose.

There is a subtle difference in interpretation of these identically calculated point estimates.  $\hat{\beta}$  is the estimate of  $\beta$  under the assumption that the model is true.  $\hat{\mathbf{b}}$  is the estimate of  $\mathbf{b}$ , which is merely what the estimator would converge to if we collected more and more data.

Is the estimate of **b** unbiased? If we mean, "Does  $\mathbf{b} = \beta$ ?" that depends on whether the model is true.  $\hat{\mathbf{b}}$  is, however, an unbiased estimate of **b**, which admittedly is not saying much.

What if x and e are correlated? Don't we have a problem then? We may have an interpretation problem—b may not measure what we want to measure, namely,  $\beta$ —but we measure  $\hat{\mathbf{b}}$  to be such-and-such and expect, if the experiment and estimation were repeated, that we would observe results in the range we have reported.

So, we have two different understandings of what the parameters mean and how the variance in their estimators arises. However, both interpretations must confront the issue of how to make valid statistical inference about the coefficient estimates when the data do not come from a simple random sample or the distribution of  $(\mathbf{x}_i, \epsilon_i)$  is not independent and identically distributed (i.i.d.). In essence, we need an estimator of the standard errors that is robust to this deviation from the standard case.

Hence, the name *the robust estimate of variance*; its associated authors are Huber (1967) and White (1980, 1982) (who developed it independently), although many others have extended its development, including Gail, Tan, and Piantadosi (1988); Kent (1982); Royall (1986); and Lin and Wei (1989). In the survey literature, this same estimator has been developed; see Kish and Frankel (1974), Fuller (1975), and Binder (1983). Most of Stata's estimation commands can produce this alternative estimate of variance and do so via the vce(robust) option.

### 20.21.1 Interpreting standard errors

Without vce(robust), we get one measure of variance:

```
. use http://www.stata-press.com/data/r14/auto7 (1978 Automobile Data)
```

. regress mpg weight foreign

Source	SS	df	MS	Number of - F(2, 71)	obs =	11
Model Residual	1619.2877 824.171761	2 71	809.643849 11.608053	Prob > F R-squared	=	0.0000 0.6627
Total	2443.45946	73	33.4720474	- Adj R-squ A Root MSE	ared =	
mpg	Coef.	Std. Err.	t	P> t  [9	5% Conf.	Interval]
weight foreign _cons	0065879 -1.650029 41.6797	.0006371 1.075994 2.165547		0.130 -	078583 3.7955 .36172	0053175 .4954422 45.99768

With vce(robust), we get another:

. regress mpg	weight foreig	gn, vce(robu	ist)			
Linear regression Number of obs					f obs =	74
_				F(2, 71)	=	73.81
				Prob > F	' =	0.0000
				R-square	ed =	0.6627
				Root MSE	=	3.4071
		Robust				
mpg	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
weight	0065879	.0005462	-12.06	0.000	007677	0054988
foreign	-1.650029	1.132566	-1.46	0.150	-3.908301	.6082424
_cons	41.6797	1.797553	23.19	0.000	38.09548	45.26392

Either way, the point estimates are the same. (See [R] regress for an example where specifying vce(robust) produces strikingly different standard errors.)

How do we interpret these results? Let's consider the model-based interpretation. Suppose that

$$y_i = \mathbf{x}_i \boldsymbol{\beta} + \boldsymbol{\epsilon}_i$$

where  $(\mathbf{x}_i, \epsilon_i)$  are i.i.d. with variance  $\sigma^2$ . For the model-based interpretation, we also must assume that  $\mathbf{x}_i$  and  $\epsilon_i$  are uncorrelated. With these assumptions and a few technical regularity conditions, our first regression gives us consistent parameter estimates and standard errors that we can use for valid statistical inference about the coefficients. Now suppose that we weaken our assumptions so that  $(\mathbf{x}_i, \epsilon_i)$  are independent and—but not necessarily—identically distributed. Our parameter estimates are still consistent, but the standard errors from the first regression can no longer be used to make valid inference. We need estimates of the standard errors that are robust to the fact that the error term is not identically distributed. The standard errors in our second regression are just what we need. We can use them to make valid statistical inference about our coefficients, even though our data are not identically distributed.

Now consider a non-model-based interpretation. If our data come from a survey design that ensures that  $(\mathbf{x}_i, e_i)$  are i.i.d., then we can use the nonrobust standard errors for valid statistical inference about the population parameters **b**. For this interpretation, we do not need to assume that  $\mathbf{x}_i$  and  $e_i$  are uncorrelated. If they are uncorrelated, the population parameters **b** and the model parameters  $\beta$  are the same. However, if they are correlated, then the population parameters **b** that we are estimating are not the same as the model-based  $\beta$ . So, what we are estimating is different, but we still need standard errors that allow us to make valid statistical inference. If the process that we used to collect the data caused  $(\mathbf{x}_i, e_i)$  to be independent but not identically distributed, then we need to use the robust standard errors to make valid statistical inference about the population parameters **b**.

### 20.21.2 Correlated errors: Cluster–robust standard errors

The robust estimator of variance has one feature that the conventional estimator does not have: the ability to relax the assumption of independence of the observations. That is, if you specify the vce(cluster *clustvar*) option, it can produce "correct" standard errors (in the measurement sense), even if the observations are correlated.

For the automobile data, it is difficult to believe that the models of the various manufacturers are truly independent. Manufacturers, after all, use common technology, engines, and drive trains across their model lines. The VW Dasher in the above regression has a measured residual of -2.80. Having

been told that, do you really believe that the residual for the VW Rabbit is as likely to be above 0 as below? (The residual is -2.32.) Similarly, the measured residual for the Chevrolet Malibu is 1.27. Does that provide information about the expected value of the residual of the Chevrolet Monte Carlo (which turns out to be 1.53)?

We need to be careful about picking examples from data; we have not told you about the Datsun 210 and 510 (residuals +8.28 and -1.01) or the Cadillac Eldorado and Seville (residuals -1.99 and +7.58), but you should at least question the assumption of independence. It may be believable that the measured mpg given the weight of one manufacturer's vehicles is independent of other manufacturers' vehicles, but it is at least questionable whether a manufacturer's vehicles are independent of one another.

In commands with the vce(robust) option, another option—vce(cluster *clustvar*)—relaxes the independence assumption and requires only that the observations be independent across the clusters:

-	-				_	
. regress mpg	weight foreig	gn, vce(clus	ter manu	facturer)		
Linear regress	Linear regression Number of obs = 7				74	
				F(2, 22	) =	90.93
				Prob >	F =	0.0000
				R-squar	ed =	0.6627
				Root MS	E =	3.4071
		(Std. Err. a	djusted	for 23 cl	usters in man	ufacturer)
mpg	Coef.	Robust Std. Err.	t	P> t	[95% Conf.	Interval]
weight foreign _cons	0065879 -1.650029 41.6797	.0005339 1.039033 1.844559	-12.34 -1.59 22.60	0.000 0.127 0.000	0076952 -3.804852 37.85432	0054806 .5047939 45.50508

It turns out that, in these data, whether or not we specify vce(cluster *clustvar*) makes little difference. The vW and Chevrolet examples above were not representative; had they been, the confidence intervals would have widened. (In the above, manuf is a variable that takes on values such as "Chev." or "VW", recording the manufacturer of the vehicle. This variable was created from variable make, which contains values such as "Chev. Malibu" or "VW Rabbit", by extracting the first word.)

As a demonstration of how well clustering can work, in [R] regress we fit a random-effects model with regress, vce(robust) and then compared the results with ordinary least squares and the generalized least squares (GLS) random-effects estimator. Here we will simply summarize the results.

We start with a dataset on 4,711 women aged 14–46 years. Subjects appear an average of 6.057 times in the data; there are a total of 28,534 observations. The model we use is log wage on age, age-squared, and job tenure. The focus of the example is the estimated coefficient on tenure. We obtain the following results:

Estimator	Point estimate	Confidence interval
(Inappropriate) least squares	0.039	[0.038, 0.041]
Robust clustered	0.039	[0.036, 0.042]
GLS random effects	0.026	[0.025, 0.027]

Notice how well the robust clustered estimate does compared with the GLS random-effects model. We then run a Hausman specification test, obtaining  $\chi^2(3) = 336.62$ , which casts grave doubt on the assumptions justifying the use of the GLS estimator and hence on the GLS results. At this point, we will simply quote our comments:

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Meanwhile, our robust regression results still stand, as long as we are careful about the interpretation. The correct interpretation is that if the data collection were repeated (on women sampled the same way as in the original sample) and if we were to refit the model, then 95% of the time we would expect the estimated coefficient on tenure to be in the range [0.036, 0.042].

Even with robust regression, we must be careful about going beyond that statement. Here the Hausman test is probably picking up something that differs within- and betweenperson, which would cast doubt on our robust regression model in terms of interpreting [0.036, 0.042] to contain the rate of return for keeping a job, economywide, for all women, without exception.

The formula for the robust estimator of variance is

$$\widehat{\mathcal{V}} = \widehat{\mathbf{V}} \Big( \sum_{j=1}^{N} \mathbf{u}_{j}' \mathbf{u}_{j} \Big) \widehat{\mathbf{V}}$$

where  $\widehat{\mathbf{V}} = (-\partial^2 \ln L/\partial\beta^2)^{-1}$  (the conventional estimator of variance) and  $\mathbf{u}_j$  (a row vector) is the contribution from the *j*th observation to  $\partial \ln L/\partial\beta$ .

In the example above, observations are assumed to be independent. Assume for a moment that the observations denoted by j are not independent but that they can be divided into M groups  $G_1$ ,  $G_2$ , ...,  $G_M$  that are independent. The robust estimator of variance is

$$\widehat{\mathcal{V}} = \widehat{\mathbf{V}} \Big( \sum_{k=1}^{M} \mathbf{u}_{k}^{(G)} \mathbf{u}_{k}^{(G)} \Big) \widehat{\mathbf{V}}$$

where  $\mathbf{u}_k^{(G)}$  is the contribution of the *k*th group to  $\partial \ln L/\partial \boldsymbol{\beta}$ . That is, application of the robust variance formula merely involves using a different decomposition of  $\partial \ln L/\partial \boldsymbol{\beta}$ , namely,  $\mathbf{u}_k^{(G)}$ ,  $k = 1, \ldots, M$ , rather than  $\mathbf{u}_j$ ,  $j = 1, \ldots, N$ . Moreover, if the log-likelihood function is additive in the observations denoted by j,

$$\ln L = \sum_{j=1}^{N} \ln L_j$$

then  $\mathbf{u}_i = \partial \ln L_i / \partial \boldsymbol{\beta}$ , so

$$\mathbf{u}_k^{(G)} = \sum_{j \in G_k} \mathbf{u}_j$$

That is what the vce(cluster clustvar) option does. (This point was first made in writing by Rogers [1993], although he considered the point an obvious generalization of Huber [1967] and the calculation—implemented by Rogers—had appeared in Stata a year earlier.)

### Technical note

What is written above is asymptotically correct but ignores a finite-sample adjustment to  $\widehat{\mathcal{V}}$ . For maximum likelihood estimators, when you specify vce(robust) but not vce(cluster *clustvar*), a better estimate of variance is  $\widehat{\mathcal{V}}^* = \{N/(N-1)\}\widehat{\mathcal{V}}$ . When you also specify the vce(cluster *clustvar*) option, this becomes  $\widehat{\mathcal{V}}^* = \{M/(M-1)\}\widehat{\mathcal{V}}$ .

For linear regression, the finite-sample adjustment is N/(N-k) without vce(cluster *clust-var*)—where k is the number of regressors—and is  $\{M/(M-1)\}\{(N-1)/(N-k)\}$  with vce(cluster *clustvar*). Also, two data-dependent modifications to the calculation for  $\hat{\mathcal{V}}^*$ , suggested by MacKinnon and White (1985), are provided by regress; see [R] regress. Angrist and Pischke (2009, chap. 8) is devoted to robust covariance matrix estimation and offers practical guidance on the use of vce(robust) and vce(cluster *clustvar*) in both cross-sectional and panel-data applications.

Halbert Lynn White Jr. (1950–2012) was born in Kansas City. After receiving economics degrees at Princeton and MIT, he taught and researched econometrics at the University of Rochester and, from 1979, at the University of California in San Diego. He also co-founded an economics and legal consulting firm known for its rigorous use of econometrics methods. His 1980 paper on heteroskedasticity introduced the use of robust covariance matrices to economists and passed 16,000 citations in Google Scholar in 2012. His 1982 paper on maximum likelihood estimation of misspecified models helped develop the now-common use of quasi–maximum likelihood estimation techniques. Later in his career, he explored the use of neural networks, nonparametric models, and time-series modeling of financial markets.

Among his many awards and distinctions, White was made a fellow of the American Academy of Arts and Sciences and the Econometric Society, and he won a fellowship from the John Simon Guggenheim Memorial Foundation. Had he not died prematurely, many scholars believe he would have eventually been awarded the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel.

Aside from his academic work, White was an avid jazz musician who played with well-known jazz trombonist and fellow University of California at San Diego teacher Jimmy Cheatam.

Peter Jost Huber (1934–) was born in Wohlen (Aargau, Switzerland). He gained mathematics degrees from ETH Zürich, including a PhD thesis on homotopy theory, and then studied statistics at Berkeley on postdoctoral fellowships. This visit yielded a celebrated 1964 paper on robust estimation, and Huber's later monographs on robust statistics were crucial in directing that field. Thereafter, his career took him back and forth across the Atlantic, with periods at Cornell, ETH Zürich, Harvard, MIT, and Bayreuth. His work has touched several other major parts of statistics, theoretical and applied, including regression, exploratory multivariate analysis, large datasets, and statistical computing. Huber also has a major long-standing interest in Babylonian astronomy.

## 20.22 Obtaining scores

Many of the estimation commands that provide the vce(robust) option also provide the ability to generate equation-level score variables via the predict command. With the score option, predict returns an important ingredient into the robust variance calculation that is sometimes useful in its own right. As explained above in [U] **20.21 Obtaining robust variance estimates**, ignoring the finite-sample corrections, the robust estimate of variance is

$$\widehat{\mathcal{V}} = \widehat{\mathbf{V}} \Big( \sum_{j=1}^{N} \mathbf{u}_j' \mathbf{u}_j \Big) \widehat{\mathbf{V}}$$

where  $\widehat{\mathbf{V}} = (-\partial^2 \ln L/\partial\beta^2)^{-1}$  is the conventional estimator of variance. If we consider likelihood functions that are additive in the observations

$$\ln L = \sum_{j=1}^{N} \ln L_j$$

then  $\mathbf{u}_j = \partial \ln L_j / \partial \boldsymbol{\beta}$ . In general, function  $L_j$  is a function of  $\mathbf{x}_j$  and  $\boldsymbol{\beta}$ ,  $L_j(\boldsymbol{\beta}; \mathbf{x}_j)$ . For many likelihood functions, however, it is only the linear form  $\mathbf{x}_j \boldsymbol{\beta}$  that enters the function. In those cases,

$$\frac{\partial \ln L_j(\mathbf{x}_j \boldsymbol{\beta})}{\partial \boldsymbol{\beta}} = \frac{\partial \ln L_j(\mathbf{x}_j \boldsymbol{\beta})}{\partial (\mathbf{x}_j \boldsymbol{\beta})} \frac{\partial (\mathbf{x}_j \boldsymbol{\beta})}{\partial \boldsymbol{\beta}} = \frac{\partial \ln L_j(\mathbf{x}_j \boldsymbol{\beta})}{\partial (\mathbf{x}_j \boldsymbol{\beta})} \mathbf{x}_j$$

By writing  $u_j = \partial \ln L_j(\mathbf{x}_j \boldsymbol{\beta}) / \partial(\mathbf{x}_j \boldsymbol{\beta})$ , this becomes simply  $u_j \mathbf{x}_j$ . Thus the formula for the robust estimate of variance can be rewritten as

$$\widehat{\mathcal{V}} = \widehat{\mathbf{V}} \Big( \sum_{j=1}^{N} u_j^2 \mathbf{x}_j' \mathbf{x}_j \Big) \widehat{\mathbf{V}}$$

We refer to  $u_j$  as the equation-level score (in the singular), and it is  $u_j$  that is returned when you use predict with the score option.  $u_j$  is like a residual in that

- 1.  $\sum_{i} u_{i} = 0$  and
- 2. correlation of  $u_j$  and  $\mathbf{x}_j$ , calculated over  $j = 1, \ldots, N$ , is 0.

In fact, for linear regression,  $u_i$  is the residual, normalized,

$$\frac{\partial \ln L_j}{\partial (\mathbf{x}_j \boldsymbol{\beta})} = \frac{\partial}{\partial (\mathbf{x}_j \boldsymbol{\beta})} \ln f \left\{ (y_j - \mathbf{x}_j \boldsymbol{\beta}) / \sigma \right\}$$
$$= (y_j - \mathbf{x}_j \boldsymbol{\beta}) / \sigma$$

where  $f(\cdot)$  is the standard normal density.

#### Example 19

probit provides the vce(robust) option and predict, score. Equation-level scores play an important role in calculating the robust estimate of variance, but we can use predict, score regardless of whether we specify vce(robust):

. use http://w	www.stata-pres	ss.com/data/	r14/auto	2			
. probit fore:	ign mpg weigh	5					
Iteration 0: Iteration 1: Iteration 2: Iteration 3: Iteration 4: Iteration 5:	log likelih log likelih log likelih log likelih log likelih log likelih	pod = -45.0 pod = -27.91 pod = -26.85 pod = -26.84 pod = -26.84	4626 8074 4197 4189				
Probit regress		)		Number LR chi2 Prob > Pseudo	2(2) chi2	= = =	74 36.38 0.0000 0.4039
foreign	Coef.	Std. Err.	Z	P> z	[95%	Conf.	Interval]
mpg weight _cons	1039503 0023355 8.275464	.0515689 .0005661 2.554142	-2.02 -4.13 3.24	0.044 0.000 0.001	2050 003 3.269	445	0028772 0012261 13.28149
. predict dou . summarize u Variable	ole u, score	Mean	Std. 1	Dev.	Min	I	Max
u . correlate u (obs=74)	74 mpg weight	-6.64e-14	. 5988	325 -1.6	355439	1.660	787
	u	mpg wei	ght				
u mpg weight		L.0000 ).8072 1.0	000				
. list make fo	oreign mpg we	ight u if ab	s(u)>1.6	5			
make	fore	ign mpg	weight		u		
24. Ford Fi 64. Peugeo			1,800 3,420	-1.65543			

The light, high-mileage Ford Fiesta is surprisingly domestic, whereas the heavy, low-mileage Peugeot 604 is surprisingly foreign.

#### Technical note

For some estimation commands, one score is not enough. Consider a likelihood that can be written as  $L_j(\mathbf{x}_j\beta_1, \mathbf{z}_j\beta_2)$ , a function of two linear forms (or linear equations). Then  $\partial \ln L_j/\partial \beta$  can be written as  $(\partial \ln L_j/\partial \beta_1, \partial \ln L_j/\partial \beta_2)$ . Each of the components can in turn be written as  $[\partial \ln L_j/\partial (\beta_1 \mathbf{x})]\mathbf{x} = u_1 \mathbf{x}$  and  $[\partial \ln L_j/\partial (\beta_2 \mathbf{z})]\mathbf{z} = u_2 \mathbf{z}$ . There are then two equation-level scores,  $u_1$  and  $u_2$ , and, in general, there could be more.

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Stata's streg, distribution(weibull) command is an example of this: it estimates  $\beta$  and a shape parameter,  $\ln p$ , the latter of which can be thought of as a degenerate linear form  $(\ln p)\mathbf{z}$  with  $\mathbf{z} = \mathbf{1}$ . After this command, predict, scores requires that you specify two new variable names, or you can specify *stub*\*, which will generate new variables *stub*1 and *stub*2; the first will be defined containing  $u_1$ —the score associated with  $\beta$ —and the second will be defined containing  $u_2$ —the score associated with  $\ln p$ .

#### Technical note

Using Stata's matrix commands—see [P] **matrix**—we can make the robust variance calculation for ourselves and then compare it with that made by Stata.

```
. use http://www.stata-press.com/data/r14/auto2, clear
(1978 Automobile Data)
. quietly probit foreign mpg weight
. predict double u, score
. matrix accum S = mpg weight [iweight=u^2*74/73]
(obs=26.53642547)
. matrix rV = e(V) * S * e(V)
. matrix list rV
symmetric rV[3,3]
                                foreign:
                   foreign:
                                            foreign:
                                weight
                                              _cons
                       mpg
                 .00352299
   foreign:mpg
                .00002216 2.434e-07
foreign:weight
foreign:_cons -.14090346 -.00117031
                                          6.4474174
. quietly probit foreign mpg weight, vce(robust)
. matrix list e(V)
symmetric e(V)[3,3]
                   foreign:
                                foreign:
                                            foreign:
                                weight
                                              _cons
                       mpg
   foreign:mpg
                 .00352299
foreign:weight
                .00002216
                              2.434e-07
foreign:_cons -.14090346 -.00117031
                                          6.4474174
```

The results are the same.

There is an important lesson here for programmers. Given the scores, conventional variance estimates can be easily transformed to robust estimates. If we were writing a new estimation command, it would not be difficult to include a vce(robust) option.

It is, in fact, easy if we ignore clustering. With clustering, it is more work because the calculation involves forming sums within clusters. For programmers interested in implementing robust variance calculations, Stata provides a \_robust command to ease the task. This is documented in [P] \_robust.

To use <u>robust</u>, you first produce conventional results (a vector of coefficients and covariance matrix) along with a variable containing the scores  $u_j$  (or variables if the likelihood function has more than one stub). You then call <u>robust</u>, and it will transform your conventional variance estimate into the robust estimate. <u>\_robust</u> will handle the work associated with clustering and the details of the finite-sample adjustment, and it will even label your output so that the word *Robust* appears above the standard error when the results are displayed.

Of course, this is all even easier if you write your commands with Stata's ml maximum likelihood optimization, in which case you merely pass the vce(robust) option on to ml. Then, ml will call \_robust itself and do all the work for you.

### Technical note

For some estimation commands, predict, score computes parameter-level scores  $\partial L_j / \partial \beta$  instead of equation-level scores  $\partial L_j / \partial x_j \beta$ . Those commands are

```
asclogit
asmprobit
asroprobit
nlogit
stcox
stcrreg
mixed
```

These models share the characteristic that there are multiple observations per independent event.

In making the robust variance calculation, parameter-level scores  $\partial L_j/\partial\beta$  are really needed, and so you may be asking yourself why predict, score does not always produce parameter-level scores. In the usual case, we can obtain them from equation-level scores via the chain rule, and fewer variables are required if we adopt this approach. In the cases above, however, the likelihood is calculated at the group level and is not split into contributions from the individual observations. Thus, the chain rule cannot be used, and we must use the parameter level scores directly.

\_robust can be tricked into using them if each parameter appears to be in its own equation as a constant. This requires resetting the row and column stripes on the covariance matrix before \_robust is called. The equation names for each row and column must be unique, and the variable names must all be \_cons.

## 20.23 Weighted estimation

The syntax for weights was introduced in [U] **11.1.6 weight**. Stata provides four kinds of weights: fweights, or frequency weights; pweights, or sampling weights; aweights, or analytic weights; and iweights, or importance weights. The syntax for using each is the same. Type

. regress y x1 x2

and you obtain unweighted estimates; type

. regress y x1 x2 [pweight=pop]

and you obtain (in this example) pweighted estimates.

The sections below explain how each type of weight is used in estimation.

### 20.23.1 Frequency weights

Frequency weights—fweights—are integers and are nothing more than replication counts. The weight is statistically uninteresting, but from a data-processing perspective it is important. Consider the following data,

у	x1	x2
22	1	0
22	1	0
22	1	1
23	0	1
23	0	1
23	0	1

and the estimation command

. regress y x1 x2

Equivalent is the following, more compressed data,

У	x1	x2	pop
у 22	1	0	2
22	1	1	1
23	0	1	3

and the corresponding estimation command

. regress y x1 x2 [fweight=pop]

When you specify frequency weights, you are treating each observation as one or more real observations.

### □ Technical note

You might occasionally run across a command that does not allow weights at all, especially among user-written commands. You can use expand (see [D] expand) with such commands to obtain frequency-weighted results. The expand command duplicates observations so that the data become self-weighting. Suppose that you want to run the command usercmd, which does something or other, and you would like to type usercmd y x1 x2 [fw=pop]. Unfortunately, usercmd does not allow weights. Instead, you type

. expand pop. usercmd y x1 x2

to obtain your result. Moreover, there is an important principle here: the results of running any command with frequency weights should be the same as running the command on the unweighted, expanded data. Unweighted, duplicated data and frequency-weighted data are merely two ways of recording identical information.

#### 20.23.2 Analytic weights

Analytic weights—analytic is a term we made up—statistically arise in one particular problem: linear regression on data that are themselves observed means. That is, think of the model

$$y_i = \mathbf{x}_i \boldsymbol{\beta} + \epsilon_i, \qquad \epsilon_i \sim N(0, \sigma^2)$$

and now think about fitting this model on data  $(\overline{y}_j, \overline{x}_j)$  that are themselves observed averages. For instance, a piece of the underlying data for  $(y_i, x_i)$  might be (3, 1), (4, 2), and (2, 2), but you do not know that. Instead, you have one observation  $\{(3 + 4 + 2)/3, (1 + 2 + 2)/3\} = (3, 1.67)$  and know only that the (3, 1.67) arose as the average of three underlying observations. All your data are like that.

regress with aweights is the solution to that problem:

. regress y x [aweight=pop]

There is a history of misusing such weights. A researcher does not have cell-mean data but instead has a probability-weighted random sample. Long before Stata existed, some researchers were using aweights to produce estimates from such samples. We will come back to this point in [U] 20.23.3 Sampling weights below.

Anyway, the statistical problem that aweights resolve can be written as

$$y_i = \mathbf{x}_i \boldsymbol{\beta} + \epsilon_i, \qquad \epsilon_i \sim N(0, \sigma^2/w_i)$$

where the  $w_i$  are the analytic weights. The details of the solution are to make linear regression calculations using the weights as if they were fweights but to normalize them to sum to N before doing that.

Most commands that allow aweights handle them in this manner. That is, if you specify aweights, they are

- 1. normalized to sum to N and then
- 2. inserted in the calculation formulas in the same way as fweights.

#### 20.23.3 Sampling weights

Sampling weights—probability weights or pweights—refer to probability-weighted random samples. Actually, what you specify in [pweight=...] is a variable recording the number of subjects in the full population that the sampled observation in your data represents. That is, an observation that had probability 1/3 of being included in your sample has pweight 3.

Some researchers have used aweights with these kinds of data. If they do, they are probably making a mistake. Consider the regression model

$$y_i = \mathbf{x}_i \boldsymbol{\beta} + \epsilon_i, \qquad \epsilon_i \sim N(0, \sigma^2)$$

Begin by considering the exact nature of the problem of fitting this model on cell-mean data—for which aweights are the solution: heteroskedasticity arising from the grouping. The error term  $\epsilon_i$  is homoskedastic (meaning that it has constant variance  $\sigma^2$ ). Say that the first observation in the data is the mean of three underlying observations. Then,

$$y_1 = \mathbf{x}_1 \boldsymbol{\beta} + \epsilon_1, \qquad \epsilon_i \sim N(0, \sigma^2)$$
  

$$y_2 = \mathbf{x}_2 \boldsymbol{\beta} + \epsilon_2, \qquad \epsilon_i \sim N(0, \sigma^2)$$
  

$$y_3 = \mathbf{x}_3 \boldsymbol{\beta} + \epsilon_3, \qquad \epsilon_i \sim N(0, \sigma^2)$$

and taking the mean,

$$(y_1 + y_2 + y_3)/3 = \{(\mathbf{x}_1 + \mathbf{x}_2 + \mathbf{x}_3)/3\}\beta + (\epsilon_1 + \epsilon_2 + \epsilon_3)/3$$

For another observation in the data—which may be the result of summing a different number of observations—the variance will be different. Hence, the model for the data is

$$\overline{y}_i = \overline{x}_i \beta + \overline{\epsilon}_i, \qquad \overline{\epsilon}_i \sim N(0, \sigma^2/N_i)$$

This makes intuitive sense. Consider two observations, one recording means over 2 subjects and the other recording means over 100,000 subjects. You would expect the variance of the residual to be less in the 100,000-subject observation; that is, there is more information in the 100,000-subject observation.

Now instead say that you are fitting the same model,  $y_i = \mathbf{x}_i \boldsymbol{\beta} + \epsilon_i$ ,  $\epsilon_i \sim N(0, \sigma^2)$ , on probabilityweighted data. Each observation in your data is one subject, but the different subjects have different chances of being included in your sample. Therefore, for each subject in your data,

$$y_i = \mathbf{x}_i \boldsymbol{\beta} + \epsilon_i, \qquad \epsilon_i \sim N(0, \sigma^2)$$

That is, there is no heteroskedasticity problem. The use of the aweighted estimator cannot be justified on these grounds.

As a matter of fact, from the argument just given, you do not need to adjust for the weights at all, although the argument does not justify not making an adjustment. If you do not adjust, you are holding tightly to the assumed truth of your model. Two issues arise when considering adjustment for sampling weights:

- 1. the efficiency of the point estimate  $\hat{\beta}$  of  $\beta$  and
- 2. the reported standard errors (and, more generally, the variance matrix of  $\hat{\beta}$ ).

Efficiency argues in favor of adjustment, and that, by the way, is why many researchers have used aweights with pweighted data. The adjustment implied by pweights to the point estimates is the same as the adjustment implied by aweights.

With regard to the second issue, the use of aweights produces incorrect results because it interprets larger weights as designating more accurately measured points. For pweights, however, the point is no more accurately measured—it is still just one observation with one residual  $\epsilon_j$  and variance  $\sigma^2$ . In [U] **20.21 Obtaining robust variance estimates** above, we introduced another estimator of variance that measures the variation that would be observed if the data collection followed by the estimation were repeated. Those same formulas provide the solution to pweights, and they have the added advantage that they are not conditioned on the model being true. If we have any hopes of measuring the variation that would be observed were the data collection followed by estimation repeated, we must include the probability of the observations being sampled in the calculation.

In Stata, when you type

. regress y x1 x2 [pw=pop]

the results are the same as if you had typed

. regress y x1 x2 [pw=pop], vce(robust)

That is, specifying pweights implies the vce(robust) option and, hence, the robust variance calculation (but weighted). In this example, we use regress simply for illustration. The same is true of probit and all of Stata's estimation commands. Estimation commands that do not have a vce(robust) option (there are a few) do not allow pweights.

pweights are adequate for handling random samples where the probability of being sampled varies. pweights may be all you need. If, however, the observations are not sampled independently but are sampled in groups—called clusters in the jargon—you should specify the estimator's vce(cluster *clustvar*) option as well:

. regress y x1 x2 [pw=pop], vce(cluster block)

There are two ways of thinking about this:

- The robust estimator answers the question of which variation would be observed were the data collection followed by the estimation repeated; if that question is to be answered, the estimator must account for the clustered nature of how observations are selected. If observations 1 and 2 are in the same cluster, then you cannot select observation 1 without selecting observation 2 (and, by extension, you cannot select observations like 1 without selecting observations like 2).
- 2. If you prefer, you can think about potential correlations. Observations in the same cluster may not really be independent—that is an empirical question to be answered by the data. For instance, if the clusters are neighborhoods, it would not be surprising that the individual neighbors are similar in their incomes, their tastes, and their attitudes, and even more similar than two randomly drawn persons from the area at large with similar characteristics, such as age and sex.

Either way of thinking leads to the same (robust) estimator of variance.

Sampling weights usually arise from complex sampling designs, which often involve not only unequal probability sampling and cluster sampling but also stratified sampling. There is a family of commands in Stata designed to work with the features of complex survey data, and those are the commands that begin with svy. To fit a linear regression model with stratification, for example, you would use the svy: regress command.

Non-svy commands that allow pweights and clustering give essentially identical results to the svy commands. If the sampling design is simple enough that it can be accommodated by the non-svy command, that is a fine way to perform the analysis. The svy commands differ in that they have more features, and they do all the little details correctly for real survey data. See [SVY] survey for a brief discussion of some of the issues involved in the analysis of survey data and for a list of all the differences between the svy and non-svy commands.

Not all model estimation commands in Stata allow pweights. This is often because they are computationally or statistically difficult to implement.

#### 20.23.4 Importance weights

Stata's iweights—importance weights—are the emergency exit. These weights are for those who want to take control and create special effects. For example, programmers have used regress with iweights to compute iteratively reweighted least-squares solutions for various problems.

iweights are treated much like aweights, except that they are not normalized. Stata's iweight rule is that

- 1. the weights are not normalized and
- 2. they are generally inserted into calculation formulas in the same way as fweights. There are exceptions; see the *Methods and formulas* for the particular command.

iweights are used mostly by programmers who are often on the way to implementing one of the other kinds of weights.

## 20.24 A list of postestimation commands

The following commands can be used after estimation:

[R] contrast	contrasts and ANOVA-style joint tests of estimates
[R] estat ic	Akaike's and Schwarz's Bayesian information criteria (AIC and BIC)
[R] estat summarize	summary statistics for the estimation sample
[R] estat vce	variance-covariance matrix of the estimators (VCE)
[R] estimates	cataloging estimation results
[TS] forecast	dynamic forecasts and simulations
[R] hausman	Hausman specification test
[R] lincom	point estimates, standard errors, testing, and inference for linear combinations of coefficients
[R] linktest	specification link test for single-equation models
[R] Irtest	likelihood-ratio test
[R] margins	marginal means, predictive margins, and marginal effects
[R] marginsplot	graph the results from margins (profile plots, interaction plots, etc.)
[R] <b>nlcom</b>	point estimates, standard errors, testing, and inference for generalized predictions
[R] predict	predictions, residuals, influence statistics, and other diagnostic measures
[R] predictnl	point estimates, standard errors, testing, and inference for generalized predictions
[R] pwcompare	pairwise comparisons of estimates
[R] suest	seemingly unrelated estimation
[R] test	Wald tests of simple and composite linear hypotheses
[R] testnl	Wald tests of nonlinear hypotheses

Also see [U] **13.5** Accessing coefficients and standard errors for accessing coefficients and standard errors.

The commands above are general-purpose postestimation commands that can be used after almost all estimation commands. Many estimation commands provide other estimator-specific postestimation commands.

To see which postestimation commands are available, launch the Postestimation Selector by selecting **Statistics** > **Postestimation**. You will see a list of all postestimation features that are available for the active estimation results. This list is automatically updated when a new estimation command is run or estimates are restored from memory or disk. See [R] **postest** for more details.

You can also see the full list of postestimation commands available for an estimator by looking in the entry titled *estimator* **postestimation** that immediately follows each estimator's entry in the reference manuals.

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