Title

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factor postestimation — Postestimation tools for factor and factormat

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Postestimation commands

The following postestimation commands are of special interest after `factor` and `factormat`:

Command	Description
`estat anti`	anti-image correlation and covariance matrices
`estat common`	correlation matrix of the common factors
`estat factors`	AIC and BIC model-selection criteria for different numbers of factors
`estat kmo`	Kaiser–Meyer–Olkin measure of sampling adequacy
`estat residuals`	matrix of correlation residuals
`estat rotatecompare`	compare rotated and unrotated loadings
`estat smc`	squared multiple correlations between each variable and the rest
`estat structure`	correlations between variables and common factors
* `estat summarize`	estimation sample summary
`loadingplot`	plot factor loadings
`rotate`	rotate factor loadings
`scoreplot`	plot score variables
`screeplot`	plot eigenvalues

* `estat summarize` is not available after `factormat`.

The following standard postestimation commands are also available:

Command	Description
* `estimates`	cataloging estimation results; see [R] **estimates**
† `predict`	predict regression or Bartlett scores

* `estimates table` is not allowed, and `estimates stats` is allowed only with the `ml` factor method.

† `predict` after `factormat` works only if you have variables in memory that match the names specified in `factormat`. `predict` assumes mean zero and standard deviation one unless the `means()` and `sds()` options of `factormat` were provided.

predict

Description for predict

`predict` creates new variables containing predictions such as factors scored by the regression method or by the Bartlett method.

Menu for predict

Statistics > Postestimation

Syntax for predict

`predict` [*type*] {*stub*`*` | *newvarlist*} [*if*] [*in*] [`,` *statistic options*]

statistic	Description
Main	
`regression`	regression scoring method; the default
`bartlett`	Bartlett scoring method

options	Description
Main	
`norotated`	use unrotated results, even when rotated results are available
`notable`	suppress table of scoring coefficients
`format(`*%fmt*`)`	format for displaying the scoring coefficients

Options for predict

Main

`regression` produces factors scored by the regression method. This is the default.

`bartlett` produces factors scored by the method suggested by Bartlett (1937, 1938). This method produces unbiased factors, but they may be less accurate than those produced by the default regression method suggested by Thomson (1951). Regression-scored factors have the smallest mean squared error from the true factors but may be biased.

`norotated` specifies that unrotated factors be scored even when you have previously issued a `rotate` command. The default is to use rotated factors if they are available and unrotated factors otherwise.

`notable` suppresses the table of scoring coefficients.

`format(`*%fmt*`)` specifies the display format for scoring coefficients.

estat

Description for estat

`estat anti` displays the anti-image correlation and anti-image covariance matrices. These are minus the partial covariance and minus the partial correlation matrices of all pairs of variables, holding all other variables constant.

`estat common` displays the correlation matrix of the common factors. For orthogonal factor loadings, the common factors are uncorrelated, and hence an identity matrix is shown. `estat common` is of more interest after oblique rotations.

`estat factors` displays model-selection criteria (AIC and BIC) for models with 1, 2, ..., # factors. Each model is estimated using maximum likelihood (that is, using the `ml` option of `factor`).

`estat kmo` specifies that the Kaiser–Meyer–Olkin (KMO) measure of sampling adequacy be displayed. KMO takes values between 0 and 1, with small values meaning that overall the variables have too little in common to warrant a factor analysis. Historically, the following labels are given to values of KMO (Kaiser 1974):

0.00 to 0.49	unacceptable
0.50 to 0.59	miserable
0.60 to 0.69	mediocre
0.70 to 0.79	middling
0.80 to 0.89	meritorious
0.90 to 1.00	marvelous

`estat residuals` displays the raw or standardized residuals of the observed correlations with respect to the fitted (reproduced) correlation matrix.

`estat rotatecompare` displays the unrotated factor loadings and the most recent rotated factor loadings.

`estat smc` displays the squared multiple correlations between each variable and all other variables. SMC is a theoretical lower bound for communality, so it is an upper bound for uniqueness. The `pf` factor method estimates the communalities by `smc`.

`estat structure` displays the factor structure, that is, the correlations between the variables and the common factors.

`estat summarize` displays summary statistics of the variables in the factor analysis over the estimation sample. This subcommand is, of course, not available after `factormat`.

Menu for estat

Statistics > Postestimation

Syntax for estat

Anti-image correlation/covariance matrices

```
estat anti [, nocorr nocov format(%fmt)]
```

Correlation of common factors

```
estat common [, norotated format(%fmt)]
```

Model-selection criteria

```
estat factors [, factors(#) detail]
```

Sample adequacy measures

```
estat kmo [, novar format(%fmt)]
```

Residuals of correlation matrix

```
estat residuals [, fitted obs sresiduals format(%fmt)]
```

Comparison of rotated and unrotated loadings

```
estat rotatecompare [, format(%fmt)]
```

Squared multiple correlations

```
estat smc [, format(%fmt)]
```

Correlations between variables and common factors

```
estat structure [, norotated format(%fmt)]
```

Summarize variables for estimation sample

```
estat summarize [, labels noheader noweights]
```

Options for estat

Main

`nocorr`, an option used with `estat anti`, suppresses the display of the anti-image correlation matrix.

`nocov`, an option used with `estat anti`, suppresses the display of the anti-image covariance matrix.

`format(%fmt)` specifies the display format. The defaults differ between the subcommands.

`norotated`, an option used with `estat common` and `estat structure`, requests that the displayed and returned results be based on the unrotated original factor solution rather than on the last rotation (orthogonal or oblique).

`factors(#)`, an option used with `estat factors`, specifies the maximum number of factors to include in the summary table.

`detail`, an option used with `estat factors`, presents the output from each run of `factor` (or `factormat`) used in the computations of the AIC and BIC values.

`novar`, an option used with `estat kmo`, suppresses the KMO measures of sampling adequacy for the variables in the factor analysis, displaying the overall KMO measure only.

`fitted`, an option used with `estat residuals`, displays the fitted (reconstructed) correlation matrix on the basis of the retained factors.

`obs`, an option used with `estat residuals`, displays the observed correlation matrix.

`sresiduals`, an option used with `estat residuals`, displays the matrix of standardized residuals of the correlations. Be careful when interpreting these residuals; see Jöreskog and Sörbom (1988).

`labels`, `noheader`, and `noweights` are the same as for the generic `estat summarize` command; see [R] **estat summarize**.

Remarks and examples

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Remarks are presented under the following headings:

Postestimation statistics
Plots of eigenvalues, factor loadings, and scores
Rotating the factor loadings
Factor scores

Postestimation statistics

Many postestimation statistics are available after `factor` and `factormat`.

▷ Example 1: Squared multiple correlations

After `factor` and `factormat` there are several "classical" methods for assessing whether the variables have enough in common to have warranted the use of a factor model. One method is to examine the squared multiple correlations of each variable with all other variables—this is usually an upper bound to *communality* and thus a lower bound to 1 − *uniqueness*(= *communality*) of the variables.

```
. use http://www.stata-press.com/data/r14/bg2
(Physician-cost data)
. quietly factor bg2cost1-bg2cost6, factors(2) ml
. estat smc
Squared multiple correlations of variables with all other variables
```

Variable	smc
bg2cost1	0.1054
bg2cost2	0.1370
bg2cost3	0.1637
bg2cost4	0.0866
bg2cost5	0.1671
bg2cost6	0.1683

Other diagnostic tools, such as examining the anti-image correlation and anti-image covariance matrices (`estat anti`) and the Kaiser–Meyer–Olkin measure of sampling adequacy (`estat kmo`), are also available. See [MV] **pca postestimation** for an illustration of their use.

◁

▷ Example 2: Model-selection criteria

Another set of postestimation tools help in determining the number of factors that should be retained. Later we will show the use of `screeplot` for producing a scree plot—a plot of the explained variance by the common factors. This is often used as a visual guide for selecting the number of factors to retain.

Some authors advocate the standard model information criteria AIC and BIC for determining the number of factors (Schwarz 1978; Akaike 1987). This presupposes that the factors are extracted by maximum likelihood. `estat factors` provides these measures.

```
. estat factors
Factor analysis with different numbers of factors (maximum likelihood)
```

#factors	loglik	df_m	df_r	AIC	BIC
1	-60.53727	6	9	133.0745	159.1273
2	-6.842448	11	4	35.6849	83.44823
3	-3.34e-12	15	0	30	95.13182

```
no Heywood cases encountered
```

The table shows the AIC and BIC statistics for the models with 1, 2, and 3 factors. The three-factor model is saturated, with 0 degrees of freedom. In this trivial case, and excluding the saturated case, both criteria select the two-factor model.

◁

▷ Example 3: Structure matrix and observed correlations

Two `estat` subcommands display statistics that help in interpreting the model and the results—in particular after an oblique rotation. `estat structure` displays the *structure* matrix containing the correlations between the (manifest) variables and the common factors.

```
. estat structure
Structure matrix: correlations between variables and common factors
```

Variable	Factor1	Factor2
bg2cost1	-0.1371	0.4235
bg2cost2	0.4140	0.1994
bg2cost3	0.6199	0.3692
bg2cost4	0.3577	0.0909
bg2cost5	-0.3752	0.4355
bg2cost6	-0.4295	0.4395

This matrix of correlations coincides with the pattern matrix, that is, the matrix with factor loadings. This holds true for the unrotated factor solution as well as after an orthogonal rotation, such as a varimax rotation. It does not hold true after an oblique rotation. After an oblique rotation, the common factors are correlated. This correlation between the common factors also influences the correlation between the common factors and the manifest variables. The correlation matrix of the common factors is displayed by the `common` subcommand of `estat`. Because we have not yet rotated, we would see only an identity matrix. Later we show `estat common` output after an oblique rotation.

To assess the quality of a factor model, we may compare the observed correlation matrix $\mathbf{C}$ with the fitted ("reconstructed") matrix $\widehat{\mathbf{\Sigma}} = \widehat{\mathbf{\Lambda}}\widehat{\mathbf{\Phi}}\widehat{\mathbf{\Lambda}}' + \widehat{\mathbf{\Psi}}$ by examining the raw residuals $\mathbf{C} - \widehat{\mathbf{\Sigma}}$.

```
. estat residuals, obs fit
Observed correlations
```

Variable	bg2co~1	bg2co~2	bg2co~3	bg2co~4	bg2co~5	bg2co~6
bg2cost1	1.0000					
bg2cost2	0.0920	1.0000				
bg2cost3	0.0540	0.3282	1.0000			
bg2cost4	-0.0380	0.1420	0.2676	1.0000		
bg2cost5	0.2380	-0.1394	-0.0550	-0.0567	1.0000	
bg2cost6	0.2431	-0.0671	-0.1075	-0.1329	0.3524	1.0000

```
Fitted ("reconstructed") values for correlations
```

Variable	bg2co~1	bg2co~2	bg2co~3	bg2co~4	bg2co~5	bg2co~6
bg2cost1	1.0000					
bg2cost2	0.0277	1.0000				
bg2cost3	0.0714	0.3303	0.9999			
bg2cost4	-0.0106	0.1662	0.2553	1.0000		
bg2cost5	0.2359	-0.0685	-0.0718	-0.0946	1.0000	
bg2cost6	0.2450	-0.0902	-0.1040	-0.1137	0.3525	1.0000

```
Raw residuals of correlations (observed-fitted)
```

Variable	bg2co~1	bg2co~2	bg2co~3	bg2co~4	bg2co~5	bg2co~6
bg2cost1	-0.0000					
bg2cost2	0.0643	-0.0000				
bg2cost3	-0.0174	-0.0021	0.0001			
bg2cost4	-0.0274	-0.0242	0.0124	-0.0000		
bg2cost5	0.0021	-0.0709	0.0168	0.0379	0.0000	
bg2cost6	-0.0019	0.0231	-0.0035	-0.0193	-0.0002	-0.0000

To gauge the size of the residuals, `estat residuals` can also display the standardized residuals.

```
. estat residuals, sres
Standardized residuals of correlations
```

Variable	bg2co~1	bg2co~2	bg2co~3	bg2co~4	bg2co~5	bg2co~6
bg2cost1	-0.0001					
bg2cost2	1.5324	-0.0003				
bg2cost3	-0.4140	-0.0480	0.0011			
bg2cost4	-0.6538	-0.5693	0.2859	-0.0000		
bg2cost5	0.0484	-1.6848	0.3993	0.9003	0.0001	
bg2cost6	-0.0434	0.5480	-0.0836	-0.4560	-0.0037	-0.0000

Be careful when interpreting these standardized residuals, as they tend to be smaller than normalized residuals; that is, these residuals tend to have a smaller variance than 1 if the model is true (see Bollen [1989]).

◁

Plots of eigenvalues, factor loadings, and scores

Scree plots, factor loading plots, and score plots are easily obtained after `factor` and `factormat`.

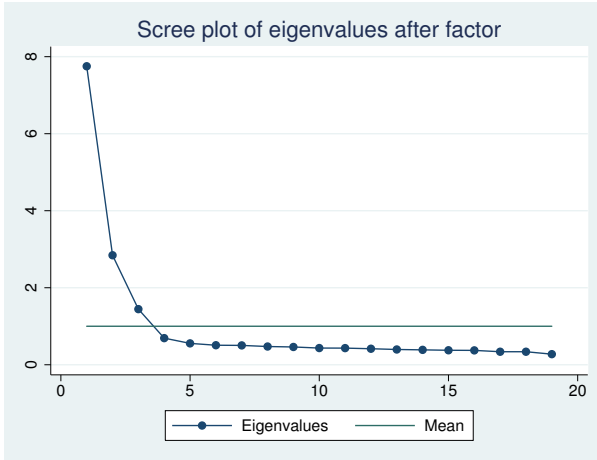
▷ Example 4: The scree plot

The scree plot is a popular tool for determining the number of factors to be retained. A scree plot is a plot of the eigenvalues shown in decreasing order (Cattell 1966). We fit a factor model, extracting factors with the principal factor method.

```
. use http://www.stata-press.com/data/r14/sp2
. factor ghp31-ghp05, pcf
```
(*output omitted*)

How many factors should we retain? We issue the `screeplot` command with the `mean` option, specifying that a horizontal line be plotted at the mean of the eigenvalues (a height of 1 because we are dealing with the eigenvalues of a correlation matrix).

```
. screeplot, mean
```



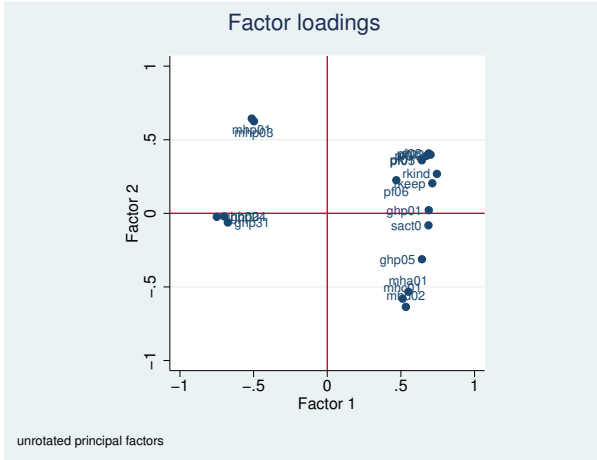
The plot suggests that we retain three factors, both because of the shape of the scree plot and because of Kaiser's well-known criterion suggesting that we retain factors with eigenvalue larger than 1. We may specify the option `mineigen(1)` during estimation to enforce this criterion. Here there is no need—`mineigen(1)` is the default with `pcf`.

◁

▷ Example 5: Factor loadings plot

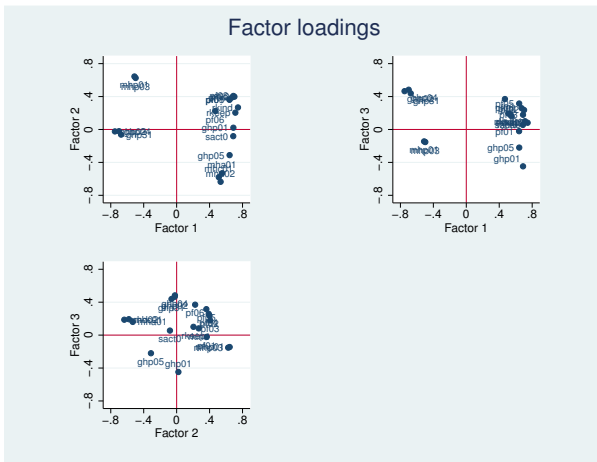
A second plot that is sometimes useful is the factor loadings plot. We display the plot with the loadings of the leading two factors.

```
. loadingplot, xline(0) yline(0) aspect(1) note(unrotated principal factors)
```



The plot makes it relatively easy to identify clusters of variables with similar loadings. With more than two factors, we can choose to see the multiple plots in a matrix style or a combined-graph style. The default is matrix style, but the combined style allows better control over various graph options—for instance, the addition of `xline(0)` and `yline(0)`. Here is a combined style graph.

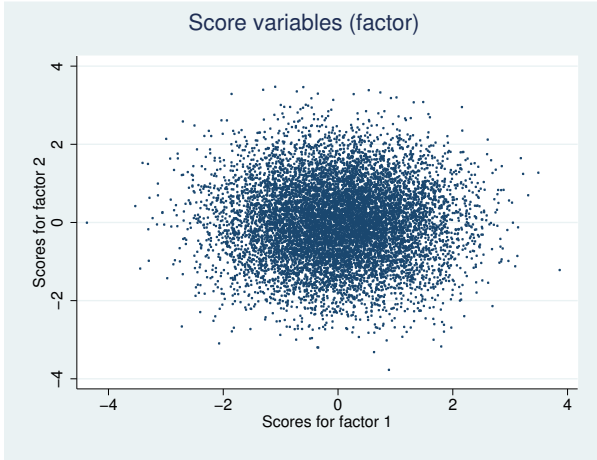
```
. loadingplot, factors(3) combined xline(0) yline(0) aspect(1)
> xlabel(-0.8(0.4)0.8) ylabel(-0.8(0.4)0.8)
```



Example 6: Score variables plot

Common factor scores can also be plotted for the observations by using the `scoreplot` command. (See the discussion of `predict` to see how you can produce score variables.)

```
. scoreplot, msymbol(smcircle) msize(tiny)
```



With so many observations, the plot's main purpose is to identify extreme cases. With smaller datasets with meaningful descriptions of the observations (for example, country names, brands), the score plot is good for visually clustering observations with similar loadings.

◁

See [MV] **scoreplot** for more examples of `loadingplot` and `scoreplot`.

❑ Technical note

The loading plots and score plots we have shown were for the original unrotated factor solution. After rotating (which we will discuss next), these plots display the most recent rotated solution. Specify option `norotated` to refer to the unrotated result. To display the plots of rotated and unrotated results at the same time, you may use either of the following two approaches. First, you may display them in different Graph windows.

```
. plotcmd, norotated name(name1)
. plotcmd, name(name2)
```

Alternatively, you may save the plots and create a combined graph

```
. plotcmd, norotated saving(name1)
. plotcmd, saving(name2)
. graph combine name1.gph name2.gph
```

See [G-2] **graph combine** for details.

❑

Rotating the factor loadings

Rotation is an attempt to describe the information in several factors by reexpressing them so that loadings on a few variables are as large as possible, and loadings on the rest of the variables are as small as possible. We have this freedom to reexpress because of the indeterminant nature of the factor model. For example, if you find that $\mathbf{z}_1$ and $\mathbf{z}_2$ are two factors, then $\mathbf{z}_1 + \mathbf{z}_2$ and $\mathbf{z}_1 - \mathbf{z}_2$ are equally valid solutions.

❑ Technical note

Said more technically: we are trying to find a set of f factor variables such that the observed variables can be best explained by regressing them on the f factor variables. Usually, f is a small number such as 1 or 2. If $f \geq 2$, there is an inherent indeterminacy in the construction of the factors because any linear combination of the calculated factors serves equally well as a set of regressors. Rotation capitalizes on this indeterminacy to create a set of variables that looks as much like the original variables as possible.

❑

The `rotate` command modifies the results of the last `factor` or `factormat` command to create a set of loadings that are more interpretable than those produced by `factor` or `factormat`. You may perform one factor analysis followed by several `rotate` commands, thus experimenting with different types of rotation. If you retain too few factors, the variables for several distinct concepts may be merged, as in our example below. If you retain too many factors, several factors may attempt to measure the same concept, causing the factors to get in each other's way, suggesting too many distinct concepts after rotation.

❑ Technical note

It is possible to restrict rotation to a number of leading factors. For instance, if you extracted three factors, you may specify the option `factors(2)` to `rotate` to exclude the third factor from being rotated. The new two leading factors are combinations of the initial two leading factors and are not affected by the fixed factor.

❑

Example 7: Orthogonal varimax rotation

We return to our physician-cost example in [MV] **factor** and perform a factor analysis using the principal-component factor method, retaining two factors. We then tell `rotate` to apply the default orthogonal varimax rotation (Kaiser 1958).

```
. use http://www.stata-press.com/data/r14/bg2, clear
(Physician-cost data)
. quietly factor bg2cost1-bg2cost6, pcf factors(2)
. rotate
Factor analysis/correlation                      Number of obs    =        568
    Method: principal-component factors          Retained factors =          2
    Rotation: orthogonal varimax (Kaiser off)    Number of params =         11
```

Factor	Variance	Difference	Proportion	Cumulative
Factor1	1.57170	0.03430	0.2619	0.2619
Factor2	1.53740	.	0.2562	0.5182

```
    LR test: independent vs. saturated:  chi2(15) =  269.07 Prob>chi2 = 0.0000
Rotated factor loadings (pattern matrix) and unique variances
```

Variable	Factor1	Factor2	Uniqueness
bg2cost1	0.6853	0.2300	0.4775
bg2cost2	-0.0126	0.7142	0.4898
bg2cost3	-0.0161	0.7818	0.3886
bg2cost4	-0.1502	0.5703	0.6521
bg2cost5	0.7292	-0.1198	0.4539
bg2cost6	0.7398	-0.1537	0.4290

```
Factor rotation matrix
```

	Factor1	Factor2
Factor1	0.7460	-0.6659
Factor2	0.6659	0.7460

Here the factors are rotated so that the three "negative" items are grouped together and the three "positive" items are grouped.

Look at the uniqueness column. *Uniqueness* is the percentage of variance for the variable that is not explained by the common factors; we may also think of it as the variances of the specific factors for the variables. We stress that rotation involves the "common factors", so the *uniqueness* is not affected by the rotation. As we noted in [MV] **factor**, the uniqueness is relatively high in this example, placing doubt on the usefulness of the factor model here.

◁

Example 8: More orthogonal varimax rotation

Here we examine 19 variables describing various aspects of health. These variables were collected from a random selection of 9,999 visitors to doctors' offices by Tarlov et al. (1989). Factor analysis yields three clear factors. We then examine several rotations of these three factors.

```
. use http://www.stata-press.com/data/r14/sp2
. describe
Contains data from http://www.stata-press.com/data/r14/sp2.dta
  obs:         9,999
 vars:            20                          26 Jan 2014 09:26
 size:       779,922                          (_dta has notes)
```

variable name	storage type	display format	value label	variable label
patid	int	%9.0g		Case ID
ghp31	float	%9.0g		Health excellent, very good, good, fair, poor
pf01	float	%9.0g		How long limit vigorous activity
pf02	float	%9.0g		How long limit moderate activity
pf03	float	%9.0g		How long limit walk/climb
pf04	float	%9.0g		How long limit bend/stoop
pf05	float	%9.0g		How long limit walk 1 block
pf06	float	%9.0g		How long limit eat/dress/bath
rkeep	float	%9.0g		Does health keep work-job-hse
rkind	float	%9.0g		Can't do kind/amount of work
sact0	float	%9.0g		Last month limit activities
mha01	float	%9.0g		Last month very nervous
mhp03	float	%9.0g		Last month calm/peaceful
mhd02	float	%9.0g		Last month downhearted/blue
mhp01	float	%9.0g		Last month a happy person
mhc01	float	%9.0g		Last month down in the dumps
ghp01	float	%9.0g		Somewhat ill
ghp04	float	%9.0g		Healthy as anybody I know
ghp02	float	%9.0g		Health is excellent
ghp05	float	%9.0g		Feel bad lately

```
Sorted by: patid
```

We now perform our factorization, requesting that three factors be retained.

```
. factor ghp31-ghp05, factors(3)
(obs=9,999)
```

```
Factor analysis/correlation                    Number of obs    =      9,999
    Method: principal factors                  Retained factors =          3
    Rotation: (unrotated)                      Number of params =         54
```

Factor	Eigenvalue	Difference	Proportion	Cumulative
Factor1	7.27086	4.90563	0.7534	0.7534
Factor2	2.36523	1.38826	0.2451	0.9985
Factor3	0.97697	1.00351	0.1012	1.0997
Factor4	-0.02654	0.00538	-0.0027	1.0970
Factor5	-0.03191	0.00378	-0.0033	1.0937
Factor6	-0.03569	0.00353	-0.0037	1.0900
Factor7	-0.03922	0.00271	-0.0041	1.0859
Factor8	-0.04193	0.00662	-0.0043	1.0815
Factor9	-0.04855	0.01015	-0.0050	1.0765
Factor10	-0.05870	0.00250	-0.0061	1.0704
Factor11	-0.06120	0.00224	-0.0063	1.0641
Factor12	-0.06344	0.00376	-0.0066	1.0575
Factor13	-0.06720	0.00345	-0.0070	1.0506
Factor14	-0.07065	0.00185	-0.0073	1.0432
Factor15	-0.07250	0.00033	-0.0075	1.0357
Factor16	-0.07283	0.00772	-0.0075	1.0282
Factor17	-0.08055	0.01190	-0.0083	1.0198
Factor18	-0.09245	0.00649	-0.0096	1.0103
Factor19	-0.09894	.	-0.0103	1.0000

```
LR test: independent vs. saturated: chi2(171) = 1.0e+05 Prob>chi2 = 0.0000
```

```
Factor loadings (pattern matrix) and unique variances
```

Variable	Factor1	Factor2	Factor3	Uniqueness
ghp31	-0.6519	-0.0562	0.3440	0.4535
pf01	0.6150	0.3226	-0.0072	0.5177
pf02	0.6867	0.3737	0.2175	0.3415
pf03	0.6712	0.3774	0.1621	0.3807
pf04	0.6540	0.3588	0.2268	0.3921
pf05	0.6209	0.3258	0.2631	0.4392
pf06	0.4370	0.1803	0.2241	0.7263
rkeep	0.6868	0.1820	0.0870	0.4876
rkind	0.7244	0.2464	0.0780	0.4085
sact0	0.6556	-0.0719	0.0461	0.5628
mha01	0.5297	-0.4773	0.1268	0.4755
mhp03	-0.4810	0.5691	-0.1238	0.4294
mhd02	0.5208	-0.5949	0.1623	0.3485
mhp01	-0.4980	0.5955	-0.1225	0.3824
mhc01	0.4927	-0.5215	0.1531	0.4618
ghp01	0.6686	0.0194	-0.3621	0.4215
ghp04	-0.6833	-0.0195	0.4089	0.3656
ghp02	-0.7398	-0.0227	0.4212	0.2748
ghp05	0.6163	-0.2760	-0.1626	0.5175

The first factor is a general health factor. (To understand that claim, compare the factor loadings with the description of the variables as shown by `describe` above. Also, just as with the physician-cost data, the sense of some of the coded responses is reversed.) The second factor loads most highly on the five "mental health" items. The third factor loads most highly on "general health perception" items—those with names having the letters `ghp` in them. The other items describe "physical health".

These designations are based primarily on the wording of the questions, which is summarized in the variable labels.

```
. rotate, varimax
Factor analysis/correlation                      Number of obs    =      9,999
    Method: principal factors                    Retained factors =          3
    Rotation: orthogonal varimax (Kaiser off)    Number of params =         54
```

Factor	Variance	Difference	Proportion	Cumulative
Factor1	4.20556	0.83302	0.4358	0.4358
Factor2	3.37253	0.33756	0.3495	0.7852
Factor3	3.03497	.	0.3145	1.0997

```
    LR test: independent vs. saturated: chi2(171) = 1.0e+05 Prob>chi2 = 0.0000
```

Rotated factor loadings (pattern matrix) and unique variances

Variable	Factor1	Factor2	Factor3	Uniqueness
ghp31	-0.2968	-0.1647	-0.6567	0.4535
pf01	0.5872	0.0263	0.3699	0.5177
pf02	0.7740	0.0848	0.2287	0.3415
pf03	0.7386	0.0580	0.2654	0.3807
pf04	0.7484	0.0842	0.2018	0.3921
pf05	0.7256	0.1063	0.1518	0.4392
pf06	0.5023	0.1268	0.0730	0.7263
rkeep	0.6023	0.2048	0.3282	0.4876
rkind	0.6590	0.1669	0.3597	0.4085
sact0	0.4187	0.3875	0.3342	0.5628
mha01	0.1467	0.6859	0.1803	0.4755
mhp03	-0.0613	-0.7375	-0.1514	0.4294
mhd02	0.0921	0.7893	0.1416	0.3485
mhp01	-0.0570	-0.7671	-0.1612	0.3824
mhc01	0.1102	0.7124	0.1359	0.4618
ghp01	0.2783	0.1977	0.6797	0.4215
ghp04	-0.2652	-0.1908	-0.7264	0.3656
ghp02	-0.2986	-0.2116	-0.7690	0.2748
ghp05	0.1755	0.4756	0.4748	0.5175

Factor rotation matrix

	Factor1	Factor2	Factor3
Factor1	0.6658	0.4796	0.5715
Factor2	0.5620	-0.8263	0.0387
Factor3	0.4908	0.2954	-0.8197

With rotation, the structure of the data becomes much clearer. The first rotated factor is physical health, the second is mental health, and the third is general health perception. The a priori designation of the items is confirmed.

After rotation, physical health is the first factor. `rotate` has ordered the factors by explained variance. Still, we warn that the importance of any factor must be gauged against the number of variables that purportedly measure it. Here we included nine variables that measured physical health, five that measured mental health, and five that measured general health perception. Had we started with only one mental health item, it would have had a high uniqueness, but we would not want to conclude that it was, therefore, largely noise.

◁

❑ Technical note

Some people prefer specifying the option `normalize` to apply a Kaiser normalization (Horst 1965), which places equal weight on all rows of the matrix to be rotated.

❑

▷ Example 9: Oblique oblimin rotation

The literature suggests that physical health and mental health are related. Also, general health perception may be largely a combination of the two. For these reasons, an oblique rotation of a two-factor solution is worth trying. We try the `oblique oblimin` rotation (Harman 1976).

```
. factor ghp31-ghp05, factors(2)
(obs=9,999)
Factor analysis/correlation                       Number of obs    =      9,999
    Method: principal factors                     Retained factors =          2
    Rotation: (unrotated)                         Number of params =         37
```

Factor	Eigenvalue	Difference	Proportion	Cumulative
Factor1	7.27086	4.90563	0.7534	0.7534
Factor2	2.36523	1.38826	0.2451	0.9985
Factor3	0.97697	1.00351	0.1012	1.0997
Factor4	-0.02654	0.00538	-0.0027	1.0970
Factor5	-0.03191	0.00378	-0.0033	1.0937
Factor6	-0.03569	0.00353	-0.0037	1.0900
Factor7	-0.03922	0.00271	-0.0041	1.0859
Factor8	-0.04193	0.00662	-0.0043	1.0815
Factor9	-0.04855	0.01015	-0.0050	1.0765
Factor10	-0.05870	0.00250	-0.0061	1.0704
Factor11	-0.06120	0.00224	-0.0063	1.0641
Factor12	-0.06344	0.00376	-0.0066	1.0575
Factor13	-0.06720	0.00345	-0.0070	1.0506
Factor14	-0.07065	0.00185	-0.0073	1.0432
Factor15	-0.07250	0.00033	-0.0075	1.0357
Factor16	-0.07283	0.00772	-0.0075	1.0282
Factor17	-0.08055	0.01190	-0.0083	1.0198
Factor18	-0.09245	0.00649	-0.0096	1.0103
Factor19	-0.09894	.	-0.0103	1.0000

```
LR test: independent vs. saturated: chi2(171) = 1.0e+05 Prob>chi2 = 0.0000
```

Factor loadings (pattern matrix) and unique variances

Variable	Factor1	Factor2	Uniqueness
ghp31	-0.6519	-0.0562	0.5718
pf01	0.6150	0.3226	0.5178
pf02	0.6867	0.3737	0.3888
pf03	0.6712	0.3774	0.4070
pf04	0.6540	0.3588	0.4435
pf05	0.6209	0.3258	0.5084
pf06	0.4370	0.1803	0.7765
rkeep	0.6868	0.1820	0.4952
rkind	0.7244	0.2464	0.4145
sact0	0.6556	-0.0719	0.5650
mha01	0.5297	-0.4773	0.4916
mhp03	-0.4810	0.5691	0.4448
mhd02	0.5208	-0.5949	0.3748
mhp01	-0.4980	0.5955	0.3974
mhc01	0.4927	-0.5215	0.4853
ghp01	0.6686	0.0194	0.5526
ghp04	-0.6833	-0.0195	0.5327
ghp02	-0.7398	-0.0227	0.4522
ghp05	0.6163	-0.2760	0.5439

```
. rotate, oblimin oblique
```

Factor analysis/correlation — Number of obs = 9,999
Method: principal factors — Retained factors = 2
Rotation: oblique oblimin (Kaiser off) — Number of params = 37

Factor	Variance	Proportion	Rotated factors are correlated
Factor1	6.58719	0.6826	
Factor2	4.65444	0.4823	

LR test: independent vs. saturated: chi2(171) = 1.0e+05 Prob>chi2 = 0.0000

Rotated factor loadings (pattern matrix) and unique variances

Variable	Factor1	Factor2	Uniqueness
ghp31	-0.5517	-0.2051	0.5718
pf01	0.7179	-0.0747	0.5178
pf02	0.8115	-0.0968	0.3888
pf03	0.8022	-0.1068	0.4070
pf04	0.7750	-0.0951	0.4435
pf05	0.7249	-0.0756	0.5084
pf06	0.4743	-0.0044	0.7765
rkeep	0.6712	0.0939	0.4952
rkind	0.7478	0.0449	0.4145
sact0	0.4608	0.3340	0.5650
mha01	0.0652	0.6869	0.4916
mhp03	0.0401	-0.7587	0.4448
mhd02	-0.0280	0.8003	0.3748
mhp01	0.0462	-0.7918	0.3974
mhc01	0.0039	0.7160	0.4853
ghp01	0.5378	0.2484	0.5526
ghp04	-0.5494	-0.2541	0.5327
ghp02	-0.5960	-0.2736	0.4522
ghp05	0.2805	0.5213	0.5439

```
Factor rotation matrix
```

	Factor1	Factor2
Factor1	0.9277	0.6831
Factor2	0.3733	-0.7303

The first factor is defined predominantly by physical health and the second by mental health. General health perception loads on both, but more on physical health than mental health. To compare the rotated and unrotated solution, looking at both in parallel form is often useful.

```
. estat rotatecompare
Rotation matrix — oblique oblimin (Kaiser off)
```

Variable	Factor1	Factor2
Factor1	0.9277	0.6831
Factor2	0.3733	-0.7303

```
Factor loadings
```

	Rotated		Unrotated	
Variable	Factor1	Factor2	Factor1	Factor2
ghp31	-0.5517	-0.2051	-0.6519	-0.0562
pf01	0.7179	-0.0747	0.6150	0.3226
pf02	0.8115	-0.0968	0.6867	0.3737
pf03	0.8022	-0.1068	0.6712	0.3774
pf04	0.7750	-0.0951	0.6540	0.3588
pf05	0.7249	-0.0756	0.6209	0.3258
pf06	0.4743	-0.0044	0.4370	0.1803
rkeep	0.6712	0.0939	0.6868	0.1820
rkind	0.7478	0.0449	0.7244	0.2464
sact0	0.4608	0.3340	0.6556	-0.0719
mha01	0.0652	0.6869	0.5297	-0.4773
mhp03	0.0401	-0.7587	-0.4810	0.5691
mhd02	-0.0280	0.8003	0.5208	-0.5949
mhp01	0.0462	-0.7918	-0.4980	0.5955
mhc01	0.0039	0.7160	0.4927	-0.5215
ghp01	0.5378	0.2484	0.6686	0.0194
ghp04	-0.5494	-0.2541	-0.6833	-0.0195
ghp02	-0.5960	-0.2736	-0.7398	-0.0227
ghp05	0.2805	0.5213	0.6163	-0.2760

Look again at the `factor` output. The variances of the first and second factor of the unrotated solution are 7.27 and 2.37, respectively. After an orthogonal rotation, the explained variance of $7.27 + 2.37$ is distributed differently over the two factors. For instance, after an orthogonal varimax rotation, the first factor has variance 5.75, and the second factor has 3.88—within rounding error $7.27 + 2.37 = 5.75 + 3.88$. The situation after an oblique rotation is different. The variances of the first and second factors are 6.59 and 4.65, which add up to more than in the orthogonal case. In the oblique case, the common factors are correlated and thus "partly explain the same variance". Therefore, the cumulative proportion of variance explained by the factors is not displayed here.

Most researchers would not be willing to accept a solution in which the common factors are highly correlated.

```
. estat common
Correlation matrix of the oblimin(0) rotated common factors
```

Factors	Factor1	Factor2
Factor1	1	
Factor2	.3611	1

The correlation of .36 seems acceptable, so we think that the oblique rotation was a success here. ◁

Factor scores

The `predict` command creates a set of new variables that are estimates of the first k common factors produced by `factor`, `factormat`, or `rotate`. Two types of scoring are available: regression or Thomson scoring and Bartlett scoring.

The number of variables may be less than the number of factors. If so, the first such factors will be used. If the number of variables is greater than the number of factors created or rotated, the unused factors will be filled with missing values.

▷ Example 10: Predicting scores

Using our automobile data, we wish to develop an index of roominess on the basis of a car's headroom, rear-seat leg room, and trunk space. We begin by extracting the factors of the three variables:

```
. use http://www.stata-press.com/data/r14/autofull
(Automobile Models)
. factor headroom rear_seat trunk
(obs=74)
Factor analysis/correlation                    Number of obs    =         74
    Method: principal factors                  Retained factors =          1
    Rotation: (unrotated)                      Number of params =          3
```

Factor	Eigenvalue	Difference	Proportion	Cumulative
Factor1	1.71426	1.79327	1.1799	1.1799
Factor2	-0.07901	0.10329	-0.0544	1.1255
Factor3	-0.18231	.	-0.1255	1.0000

```
    LR test: independent vs. saturated:  chi2(3)  =   82.93 Prob>chi2 = 0.0000
Factor loadings (pattern matrix) and unique variances
```

Variable	Factor1	Uniqueness
headroom	0.7280	0.4700
rear_seat	0.7144	0.4897
trunk	0.8209	0.3261

All the factor loadings are positive, so we have indeed obtained a "roominess" factor. The `predict` command will now create the one retained factor, which we will call `f1`:

```
. predict f1
(regression scoring assumed)
Scoring coefficients (method = regression)
```

Variable	Factor1
headroom	0.28323
rear_seat	0.26820
trunk	0.45964

The table with scoring coefficients informs us that the factor is obtained as a weighted sum of standardized versions of `headroom`, `rear_seat`, and `trunk` with weights 0.28, 0.27, and 0.46.

If `factor` had retained more than one factor, typing `predict f1` would still have added only the first factor to our data. Typing `predict f1 f2`, however, would have added the first two factors to our data. `f1` is now our "roominess" index, so we might compare the roominess of domestic and foreign cars:

```
. table foreign, c(mean f1 sd f1) row
```

Foreign	mean(f1)	sd(f1)
Domestic	.2022442	.9031404
Foreign	-.4780318	.6106609
Total	4.51e-09	.8804116

We find that domestic cars are, on average, roomier than foreign cars, at least in our data.

◁

❑ Technical note

Are common factors not supposed to be normalized to have mean 0 and standard deviation 1? In our example above, the mean is 4.5×10^{-9} and the standard deviation is 0.88. Why is that?

For the mean, the deviation from zero is due to numerical roundoff, which would diminish dramatically if we had typed `predict double f1` instead. The explanation for the standard deviation of 0.88, on the other hand, is not numerical roundoff. At a theoretical level, the factor is supposed to have standard deviation 1, but the estimation method almost never yields that result unless an exact solution to the factor model is found. This happens for the same reason that, when you regress y on x, you do not get the same equation as if you regress x on y, unless x and y are perfectly collinear.

By the way, if you had two factors, you would expect the correlation between the two factors to be zero because that is how they are theoretically defined. The matrix algebra, however, does not usually work out that way. It is somewhat analogous to the fact that if you regress y on x and the regression assumption that the errors are uncorrelated with the dependent variable is satisfied, then it automatically cannot be satisfied if you regress x on y.

The covariance matrix of the estimated factors is

$$E(\widehat{\mathbf{f}}\widehat{\mathbf{f}}') = \mathbf{I} - (\mathbf{I} + \mathbf{\Gamma})^{-1}$$

where

$$\mathbf{\Gamma} = \mathbf{\Lambda}'\mathbf{\Psi}^{-1}\mathbf{\Lambda}$$

The columns of Λ are orthogonal to each other, but the inclusion of Ψ in the middle of the equation destroys that relationship unless all the elements of Ψ are equal.

❑

▷ Example 11: Rescaling the scores

Let's pretend that we work for the K. E. Watt Company, a fictional industry group that generates statistics on automobiles. Our "roominess" index has mean 0 and standard deviation 0.88, but indexes we present to the public generally have mean 100 and standard deviation 10. First, we wish to rescale our index:

```
. generate roomidx = (f1/.88041161)*10 + 100
. table foreign, c(mean roomidx sd roomidx freq) row format(%9.2f)
```

Foreign	mean(roomidx)	sd(roomidx)	Freq.
Domestic	102.30	10.26	52.00
Foreign	94.57	6.94	22.00
Total	100.00	10.00	74.00

Now when we release our results, we can write, "The K. E. Watt index of roominess shows that domestic cars are, on average, roomier, with an index of 102 versus only 95 for foreign cars."

Now let's find the "roomiest" car in our data:

```
. sort roomidx
. list fullname roomidx in l
```

	fullname	roomidx
74.	Merc. Marquis	116.7469

We can also write, "K. E. Watt finds that the Mercury Marquis is the roomiest automobile among those surveyed, with a roominess index of 117 versus an average of 100."

◁

❑ Technical note

`predict` provides two methods of scoring: the default regression scoring, which we have used above, and the optional Bartlett method. An artificial example will best illustrate the use and meaning of the methods. We begin by creating a known-to-be-correct factor model in which the true loadings are 0.4, 0.6, and 0.8. The variances of the unique factors are $1 - 0.4^2 = 0.84$, $1 - 0.6^2 = 0.64$, and $1 - 0.8^2 = 0.36$, respectively. We make the sample size large enough so that random fluctuations are not important.

```
. drop _all
. set seed 12345
. set obs 10000
number of observations (_N) was 0, now 10,000
. generate ftrue = rnormal()
. generate x1 = .4*ftrue + sqrt(.84)*rnormal()
```

```
. generate x2 = .6*ftrue + sqrt(.64)*rnormal()
. generate x3 = .8*ftrue + sqrt(.36)*rnormal()
. summarize x1 x2 x3
```

Variable	Obs	Mean	Std. Dev.	Min	Max
x1	10,000	.0195519	1.011165	-3.778123	4.267452
x2	10,000	.0127835	1.001259	-3.828994	4.102375
x3	10,000	.0058335	1.002475	-3.595906	3.89754

Because we concocted our data, the iterated principal-factor method reproduces the true loadings most faithfully:

```
. factor x1 x2 x3, ipf factors(1)
(obs=10,000)
Factor analysis/correlation                    Number of obs    =     10,000
    Method: iterated principal factors         Retained factors =         1
    Rotation: (unrotated)                      Number of params =         3
```

Factor	Eigenvalue	Difference	Proportion	Cumulative
Factor1	1.16678	1.16662	1.0000	1.0000
Factor2	0.00016	0.00036	0.0001	1.0002
Factor3	-0.00020	.	-0.0002	1.0000

```
    LR test: independent vs. saturated:  chi2(3)  = 3887.29 Prob>chi2 = 0.0000
Factor loadings (pattern matrix) and unique variances
```

Variable	Factor1	Uniqueness
x1	0.4156	0.8273
x2	0.6046	0.6345
x3	0.7928	0.3715

Let us now compare regression and Bartlett scoring:

```
. predict freg
(regression scoring assumed)
Scoring coefficients (method = regression)
```

Variable	Factor1
x1	0.14449
x2	0.27410
x3	0.61377

```
. predict fbar, bartlett
Scoring coefficients (method = Bartlett)
```

Variable	Factor1
x1	0.20285
x2	0.38475
x3	0.86162

Comparing the two scoring vectors, we see that Bartlett scoring yields larger coefficients. The regression scoring method is biased insofar as $E(\texttt{freg}|\texttt{ftrue})$ is not `ftrue`, something we can reveal by regressing `freg` on `ftrue`:

```
. regress freg ftrue
      Source |       SS           df       MS      Number of obs   =    10,000
-------------+----------------------------------   F(1, 9998)      =  25339.34
       Model |  5107.57467         1  5107.57467   Prob > F        =    0.0000
    Residual |  2015.26671     9,998  .201566984   R-squared       =    0.7171
-------------+----------------------------------   Adj R-squared   =    0.7170
       Total |  7122.84138     9,999  .712355374   Root MSE        =    .44896

------------------------------------------------------------------------------
        freg |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       ftrue |   .7169557   .004504   159.18   0.000      .708127    .7257843
       _cons |  -.0088417    .00449    -1.97   0.049    -.0176429   -.0000404
------------------------------------------------------------------------------
```

Note the coefficient on `ftrue` of $0.717 < 1$. The Bartlett scoring method, on the other hand, is unbiased:

```
. regress fbar ftrue
      Source |       SS           df       MS      Number of obs   =    10,000
-------------+----------------------------------   F(1, 9998)      =  25339.33
       Model |  10065.1734         1  10065.1734   Prob > F        =    0.0000
    Residual |  3971.35998     9,998  .397215441   R-squared       =    0.7171
-------------+----------------------------------   Adj R-squared   =    0.7170
       Total |  14036.5334     9,999  1.40379372   Root MSE        =    .63025

------------------------------------------------------------------------------
        fbar |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       ftrue |   1.006458   .0063226   159.18   0.000      .9940642    1.018851
       _cons |  -.0124119    .006303    -1.97   0.049     -.024767   -.0000568
------------------------------------------------------------------------------
```

The zero bias of the Bartlett method comes at the costs of less accuracy, for example, in terms of the mean squared error.

```
. generate dbar = (fbar - ftrue)^2
. generate dreg = (freg - ftrue)^2
. summarize ftrue fbar freg dbar dreg
    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
       ftrue |     10,000    .0123322     .996866  -4.196032   3.815439
        fbar |     10,000    2.08e-10    1.184818   -3.78561   4.550449
        freg |     10,000   -6.44e-10    .8440115  -2.696714   3.241498
        dbar |     10,000    .3973295    .5654751   1.31e-09   7.656609
        dreg |     10,000    .2812835    .4053233   9.68e-10   4.814044
```

Neither estimator follows the assumption that the scaled factor has unit variance. The regression estimator has a variance less than 1, and the Bartlett estimator has a variance greater than 1.

The difference between the two scoring methods is not as important as it might seem because the bias in the regression method is only a matter of scaling and shifting.

```
. correlate freg fbar ftrue
(obs=10,000)
             |     freg     fbar    ftrue
-------------+---------------------------
        freg |   1.0000
        fbar |   1.0000   1.0000
       ftrue |   0.8468   0.8468   1.0000
```

Therefore, the choice of which scoring method we apply is largely immaterial.

❑

Stored results

Let p be the number of variables and f, the number of factors.

`predict`, in addition to generating variables, also stores the following in `r()`:

Macros
- `r(method)` — `regression` or `Bartlett`

Matrices
- `r(scoef)` — $p \times f$ matrix of scoring coefficients

`estat anti` stores the following in `r()`:

Matrices
- `r(acov)` — $p \times p$ anti-image covariance matrix
- `r(acorr)` — $p \times p$ anti-image correlation matrix

`estat common` stores the following in `r()`:

Matrices
- `r(Phi)` — $f \times f$ correlation matrix of common factors

`estat factors` stores the following in `r()`:

Matrices
- `r(stats)` — $k \times 5$ matrix with log likelihood, degrees of freedom, AIC, and BIC for models with 1 to k factors estimated via maximum likelihood

`estat kmo` stores the following in `r()`:

Scalars
- `r(kmo)` — the Kaiser–Meyer–Olkin measure of sampling adequacy

Matrices
- `r(kmow)` — column vector of KMO measures for each variable

`estat residuals` stores the following in `r()`:

Matrices
- `r(fit)` — fitted matrix for the correlations, $\widehat{\mathbf{C}}=\widehat{\mathbf{\Lambda}}\widehat{\mathbf{\Phi}}\widehat{\mathbf{\Lambda}}'+\widehat{\mathbf{\Psi}}$
- `r(res)` — raw residual matrix $\mathbf{C}-\widehat{\mathbf{C}}$
- `r(SR)` — standardized residuals (`sresiduals` option only)

`estat smc` stores the following in `r()`:

Matrices
- `r(smc)` — vector of squared multiple correlations of variables with all other variables

`estat structure` stores the following in `r()`:

Matrices
- `r(st)` — $p \times f$ matrix of correlations between variables and common factors

See [R] **estat summarize** for the stored results of `estat summarize`.

rotate after factor and factormat add to the existing e():

Scalars
- e(r_f) number of factors in rotated solution
- e(r_fmin) rotation criterion value

Macros
- e(r_class) orthogonal or oblique
- e(r_criterion) rotation criterion
- e(r_ctitle) title for rotation
- e(r_normalization) kaiser or none

Matrices
- e(r_L) rotated loadings
- e(r_T) rotation
- e(r_Phi) correlations between common factors
- e(r_Ev) explained variance by common factors

The factors in the rotated solution are in decreasing order of e(r_Ev).

Methods and formulas

Methods and formulas are presented under the following headings:

estat
rotate
predict

estat

See *Methods and formulas* of [MV] **pca postestimation** for the formulas for estat anti, estat kmo, and estat smc.

estat residuals computes the standardized residuals $\widetilde{r}_{ij}$ as

$$\widetilde{r}_{ij} = \frac{\sqrt{N}(r_{ij} - f_{ij})}{\sqrt{f_{ij}^2 + f_{ii}f_{jj}}}$$

suggested by Jöreskog and Sörbom (1986), where N is the number of observations, r_{ij} is the observed correlation of variables i and j, and f_{ij} is the fitted correlation of variables i and j. Also see Bollen (1989). Caution is warranted in interpretation of these residuals; see Jöreskog and Sörbom (1988).

estat structure computes the correlations of the variables and the common factors as $\mathbf{\Lambda\Phi}$.

rotate

See *Methods and formulas* of [MV] **rotatemat** for the details of rotation.

The correlation of common factors after rotation is $\mathbf{T}'\mathbf{T}$, where $\mathbf{T}$ is the factor rotation matrix, satisfying $\mathbf{L}_{\text{rotated}} = \mathbf{L}_{\text{unrotated}}(\mathbf{T}')^{-1}$

predict

The formula for regression scoring (Thomson 1951) in the orthogonal case is

$$\widehat{\mathbf{f}} = \boldsymbol{\Lambda}'\boldsymbol{\Sigma}^{-1}\mathbf{x}$$

where $\boldsymbol{\Lambda}$ is the unrotated or orthogonally rotated loading matrix. For oblique rotation, the regression scoring is defined as

$$\widehat{\mathbf{f}} = \boldsymbol{\Phi}\boldsymbol{\Lambda}'\boldsymbol{\Sigma}^{-1}\mathbf{x}$$

where $\boldsymbol{\Phi}$ is the correlation matrix of the common factors.

The formula for Bartlett scoring (Bartlett 1937, 1938) is

$$\boldsymbol{\Gamma}^{-1}\boldsymbol{\Lambda}'\boldsymbol{\Psi}^{-1}\mathbf{x}$$

where

$$\boldsymbol{\Gamma} = \boldsymbol{\Lambda}'\boldsymbol{\Psi}^{-1}\boldsymbol{\Lambda}$$

See Harman (1976) and Lawley and Maxwell (1971).

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Also see *References* in [MV] **factor**.

Also see

[MV] **factor** — Factor analysis

[MV] **rotate** — Orthogonal and oblique rotations after factor and pca

[MV] **scoreplot** — Score and loading plots

[MV] **screeplot** — Scree plot

[U] 20 Estimation and postestimation commands