hessenbergd() — Hessenberg decomposition

DescriptionSyntaxRemarks and examplesConformabilityDiagnosticsAlso see

Description

hessenbergd(A, H, Q) calculates the Hessenberg decomposition of a square, numeric matrix, A, returning the upper Hessenberg form matrix in H and the orthogonal (unitary) matrix in Q. Q is orthogonal if A is real and unitary if A is complex.

_hessenbergd(A, Q) does the same as hessenbergd() except that it returns H in A.

_hessenbergd_la() is the interface into the LAPACK routines used to implement the above function; see [M-1] LAPACK. Its direct use is not recommended.

Syntax

void hessenbergd(numeric matrix A, H, Q)
void _hessenbergd(numeric matrix A, Q)

Remarks and examples

The Hessenberg decomposition of a matrix, A, can be written as

$$\mathbf{Q}' \times \mathbf{A} \times \mathbf{Q} = \mathbf{H}$$

where \mathbf{H} is upper Hessenberg; \mathbf{Q} is orthogonal if \mathbf{A} is real or unitary if \mathbf{A} is complex.

A matrix **H** is in upper Hessenberg form if all entries below its first subdiagonal are zero. For example, a 5×5 upper Hessenberg matrix looks like

	1	2	3	4	5
1	x	x	x	x	x
2	x	х	х	х	x
3	0	x	x	х	x
4	0	0	x	х	х
5	0	0	0	x	x

For instance,

: A	1	2	3	4	5	
1	3	2	1	-2	-5	
2	4	2	1	0	3	
3	4	4	0	1	-1	
4	5	6	7	-2	4	
5	6	7	1	2	-1	
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: hessenbergd(A, H=., Q=.)

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: H					
	1	2	3	4	5
1	3	2.903464745	552977683	-4.78764119	-1.530555451
2	-9.643650761	7.806451613	2.878001755	5.1085876	5.580422694
3	0	-3.454023879	-6.119229633	2347200215	1.467932097
4	0	0	1.404136249	-1.715823624	9870601994
5	0	0	0	-2.668128952	971398356
: Q	1	2	3	4	5
1		0	0	0	0
2	1	- /1/7806779	- 0368006164	- 4047768558	- 81/0007/88
2	0	- 11/7806779	- /871230/8/	- 5602300155	5163752637
1	0	- 518/758/7/	809613560/	- 07/8//0106	2647771074
5	0	6221710168	3253949238	.7117092805	0221645995

Many algorithms use a Hessenberg decomposition in the process of finding another decomposition with more structure.

Conformability

hessenbergd(A, H, Q): input: A: $n \times n$ output: H: $n \times n$ 0: $n \times n$ $_$ hessenbergd(A, Q): input: A: $n \times n$ output: A: $n \times n$ Q: $n \times n$

Diagnostics

_hessenbergd() aborts with error if A is a view.

hessenbergd() and _hessenbergd() return missing results if A contains missing values.

Karl Adolf Hessenberg (1904–1959) was born in Frankfurt am Main, Germany. He was an electrical engineer and gained degrees from the Technische Hochschule Darmstadt. His doctoral dissertation, approved in 1942, was on computation of the eigenvalues and eigensolutions of linear systems of equations. In concurrent work, he introduced what are now called Hessenberg matrices. The mathematician Gerhard Hessenberg was a near relative.

Also see

- [M-1] LAPACK The LAPACK linear-algebra routines
- [M-5] **schurd**() Schur decomposition
- [M-4] matrix Matrix functions