Title

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vec intro — Introduction to vector error-correction models

Description   Remarks and examples   References   Also see

Description

Stata has a suite of commands for fitting, forecasting, interpreting, and performing inference on vector error-correction models (VECMs) with cointegrating variables. After fitting a VECM, the `irf` commands can be used to obtain impulse–response functions (IRFs) and forecast-error variance decompositions (FEVDs). The table below describes the available commands.

Fitting a VECM

`vec`	[TS] **vec**	Fit vector error-correction models

Model diagnostics and inference

`vecrank`	[TS] **vecrank**	Estimate the cointegrating rank of a VECM
`veclmar`	[TS] **veclmar**	Perform LM test for residual autocorrelation after `vec`
`vecnorm`	[TS] **vecnorm**	Test for normally distributed disturbances after `vec`
`vecstable`	[TS] **vecstable**	Check the stability condition of VECM estimates
`varsoc`	[TS] **varsoc**	Obtain lag-order selection statistics for VARs and VECMs

Forecasting from a VECM

`fcast compute`	[TS] **fcast compute**	Compute dynamic forecasts after `var`, `svar`, or `vec`
`fcast graph`	[TS] **fcast graph**	Graph forecasts after `fcast compute`

Working with IRFs and FEVDs

`irf`	[TS] **irf**	Create and analyze IRFs and FEVDs

This manual entry provides an overview of the commands for VECMs; provides an introduction to integration, cointegration, estimation, inference, and interpretation of VECM models; and gives an example of how to use Stata's `vec` commands.

Remarks and examples

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`vec` estimates the parameters of cointegrating VECMs. You may specify any of the five trend specifications in Johansen (1995, sec. 5.7). By default, identification is obtained via the Johansen normalization, but `vec` allows you to obtain identification by placing your own constraints on the parameters of the cointegrating vectors. You may also put more restrictions on the adjustment coefficients.

`vecrank` is the command for determining the number of cointegrating equations. `vecrank` implements Johansen's multiple trace test procedure, the maximum eigenvalue test, and a method based on minimizing either of two different information criteria.

Because Nielsen (2001) has shown that the methods implemented in `varsoc` can be used to choose the order of the autoregressive process, no separate `vec` command is needed; you can simply use `varsoc`. `veclmar` tests that the residuals have no serial correlation, and `vecnorm` tests that they are normally distributed.

All the `irf` routines described in [TS] **irf** are available for estimating, interpreting, and managing estimated IRFs and FEVDs for VECMs.

Remarks are presented under the following headings:

Introduction to cointegrating VECMs
What is cointegration?
The multivariate VECM specification
Trends in the Johansen VECM framework
VECM estimation in Stata
Selecting the number of lags
Testing for cointegration
Fitting a VECM
Fitting VECMs with Johansen's normalization
Postestimation specification testing
Impulse–response functions for VECMs
Forecasting with VECMs

Introduction to cointegrating VECMs

This section provides a brief introduction to integration, cointegration, and cointegrated vector error-correction models. For more details about these topics, see Hamilton (1994), Johansen (1995), Lütkepohl (2005), Watson (1994), and Becketti (2013).

What is cointegration?

Standard regression techniques, such as ordinary least squares (OLS), require that the variables be covariance stationary. A variable is covariance stationary if its mean and all its autocovariances are finite and do not change over time. Cointegration analysis provides a framework for estimation, inference, and interpretation when the variables are not covariance stationary.

Instead of being covariance stationary, many economic time series appear to be "first-difference stationary". This means that the level of a time series is not stationary but its first difference is. First-difference stationary processes are also known as integrated processes of order 1, or I(1) processes. Covariance-stationary processes are I(0). In general, a process whose dth difference is stationary is an integrated process of order d, or I(d).

The canonical example of a first-difference stationary process is the random walk. This is a variable x_t that can be written as

$$x_t = x_{t-1} + \epsilon_t \tag{1}$$

where the ϵ_t are independently and identically distributed (i.i.d.) with mean zero and a finite variance σ^2. Although $E[x_t] = 0$ for all t, $\text{Var}[x_t] = T\sigma^2$ is not time invariant, so x_t is not covariance stationary. Because $\Delta x_t = x_t - x_{t-1} = \epsilon_t$ and ϵ_t is covariance stationary, x_t is first-difference stationary.

These concepts are important because, although conventional estimators are well behaved when applied to covariance-stationary data, they have nonstandard asymptotic distributions and different rates of convergence when applied to I(1) processes. To illustrate, consider several variants of the model

$$y_t = a + bx_t + e_t \tag{2}$$

Throughout the discussion, we maintain the assumption that $E[e_t] = 0$.

If both y_t and x_t are covariance-stationary processes, e_t must also be covariance stationary. As long as $E[x_t e_t] = 0$, we can consistently estimate the parameters a and b by using OLS. Furthermore, the distribution of the OLS estimator converges to a normal distribution centered at the true value as the sample size grows.

If y_t and x_t are independent random walks and $b = 0$, there is no relationship between y_t and x_t, and (2) is called a spurious regression. Granger and Newbold (1974) performed Monte Carlo experiments and showed that the usual t statistics from OLS regression provide spurious results: given a large enough dataset, we can almost always reject the null hypothesis of the test that $b = 0$ even though b is in fact zero. Here the OLS estimator does not converge to any well-defined population parameter.

Phillips (1986) later provided the asymptotic theory that explained the Granger and Newbold (1974) results. He showed that the random walks y_t and x_t are first-difference stationary processes and that the OLS estimator does not have its usual asymptotic properties when the variables are first-difference stationary.

Because Δy_t and Δx_t are covariance stationary, a simple regression of Δy_t on Δx_t appears to be a viable alternative. However, if y_t and x_t cointegrate, as defined below, the simple regression of Δy_t on Δx_t is misspecified.

If y_t and x_t are I(1) and $b \neq 0$, e_t could be either I(0) or I(1). Phillips and Durlauf (1986) have derived the asymptotic theory for the OLS estimator when e_t is I(1), though it has not been widely used in applied work. More interesting is the case in which $e_t = y_t - a - bx_t$ is I(0). y_t and x_t are then said to be cointegrated. Two variables are cointegrated if each is an I(1) process but a linear combination of them is an I(0) process.

It is not possible for y_t to be a random walk and x_t and e_t to be covariance stationary. As Granger (1981) pointed out, because a random walk cannot be equal to a covariance-stationary process, the equation does not "balance". An equation balances when the processes on each side of the equal sign are of the same order of integration. Before attacking any applied problem with integrated variables, make sure that the equation balances before proceeding.

An example from Engle and Granger (1987) provides more intuition. Redefine y_t and x_t to be

$$y_t + \beta x_t = \epsilon_t, \qquad \epsilon_t = \epsilon_{t-1} + \xi_t \tag{3}$$

$$y_t + \alpha x_t = \nu_t, \qquad \nu_t = \rho\nu_{t-1} + \zeta_t, \quad |\rho| < 1 \tag{4}$$

where ξ_t and ζ_t are i.i.d. disturbances over time that are correlated with each other. Because ϵ_t is I(1), (3) and (4) imply that both x_t and y_t are I(1). The condition that $|\rho| < 1$ implies that ν_t and $y_t + \alpha x_t$ are I(0). Thus y_t and x_t cointegrate, and $(1, \alpha)$ is the cointegrating vector.

Using a bit of algebra, we can rewrite (3) and (4) as

$$\Delta y_t = \beta\delta z_{t-1} + \eta_{1t} \tag{5}$$

$$\Delta x_t = -\delta z_{t-1} + \eta_{2t} \tag{6}$$

where $\delta = (1-\rho)/(\alpha-\beta)$, $z_t = y_t + \alpha x_t$, and η_{1t} and η_{2t} are distinct, stationary, linear combinations of ξ_t and ζ_t. This representation is known as the vector error-correction model (VECM). One can think of $z_t = 0$ as being the point at which y_t and x_t are in equilibrium. The coefficients on z_{t-1} describe how y_t and x_t adjust to z_{t-1} being nonzero, or out of equilibrium. z_t is the "error" in the system, and (5) and (6) describe how system adjusts or corrects back to the equilibrium. As ρ goes to 1, the system degenerates into a pair of correlated random walks. The VECM parameterization highlights this point, because $\delta \rightarrow 0$ as $\rho \rightarrow 1$.

If we knew α, we would know z_t, and we could work with the stationary system of (5) and (6). Although knowing α seems silly, we can conduct much of the analysis as if we knew α because there is an estimator for the cointegrating parameter α that converges to its true value at a faster rate than the estimator for the adjustment parameters β and δ.

The definition of a bivariate cointegrating relation requires simply that there exist a linear combination of the I(1) variables that is I(0). If y_t and x_t are I(1) and there are two finite real numbers $a \neq 0$ and $b \neq 0$, such that $ay_t + bx_t$ is I(0), then y_t and x_t are cointegrated. Although there are two parameters, a and b, only one will be identifiable because if $ay_t + bx_t$ is I(0), so is $cay_t + cbx_t$ for any finite, nonzero, real number c. Obtaining identification in the bivariate case is relatively simple. The coefficient on y_t in (4) is unity. This natural construction of the model placed the necessary identification restriction on the cointegrating vector. As we discuss below, identification in the multivariate case is more involved.

If $\mathbf{y}_t$ is a $K \times 1$ vector of I(1) variables and there exists a vector $\boldsymbol{\beta}$, such that $\boldsymbol{\beta}\mathbf{y}_t$ is a vector of I(0) variables, then $\mathbf{y}_t$ is said to be cointegrating of order (1,0) with cointegrating vector $\boldsymbol{\beta}$. We say that the parameters in $\boldsymbol{\beta}$ are the parameters in the cointegrating equation. For a vector of length K, there may be at most $K-1$ distinct cointegrating vectors. Engle and Granger (1987) provide a more general definition of cointegration, but this one is sufficient for our purposes.

The multivariate VECM specification

In practice, most empirical applications analyze multivariate systems, so the rest of our discussion focuses on that case. Consider a VAR with p lags

$$\mathbf{y}_t = \mathbf{v} + \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t-2} + \cdots + \mathbf{A}_p\mathbf{y}_{t-p} + \boldsymbol{\epsilon}_t \tag{7}$$

where $\mathbf{y}_t$ is a $K \times 1$ vector of variables, $\mathbf{v}$ is a $K \times 1$ vector of parameters, $\mathbf{A}_1$–$\mathbf{A}_p$ are $K \times K$ matrices of parameters, and $\boldsymbol{\epsilon}_t$ is a $K \times 1$ vector of disturbances. $\boldsymbol{\epsilon}_t$ has mean $\mathbf{0}$, has covariance matrix $\boldsymbol{\Sigma}$, and is i.i.d. normal over time. Any VAR(p) can be rewritten as a VECM. Using some algebra, we can rewrite (7) in VECM form as

$$\Delta\mathbf{y}_t = \mathbf{v} + \boldsymbol{\Pi}\mathbf{y}_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta\mathbf{y}_{t-i} + \boldsymbol{\epsilon}_t \tag{8}$$

where $\boldsymbol{\Pi} = \sum_{j=1}^{j=p} \mathbf{A}_j - \mathbf{I}_k$ and $\boldsymbol{\Gamma}_i = -\sum_{j=i+1}^{j=p} \mathbf{A}_j$. The $\mathbf{v}$ and $\boldsymbol{\epsilon}_t$ in (7) and (8) are identical.

Engle and Granger (1987) show that if the variables $\mathbf{y}_t$ are I(1) the matrix $\boldsymbol{\Pi}$ in (8) has rank $0 \leq\ r < K$, where r is the number of linearly independent cointegrating vectors. If the variables cointegrate, $0 < r < K$ and (8) shows that a VAR in first differences is misspecified because it omits the lagged level term $\boldsymbol{\Pi}\mathbf{y}_{t-1}$.

Assume that $\boldsymbol{\Pi}$ has reduced rank $0 < r < K$ so that it can be expressed as $\boldsymbol{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$, where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are both $r \times K$ matrices of rank r. Without further restrictions, the cointegrating vectors are not identified: the parameters $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ are indistinguishable from the parameters $(\boldsymbol{\alpha}\mathbf{Q}, \boldsymbol{\beta}\mathbf{Q}^{-1\prime})$ for any $r \times r$ nonsingular matrix $\mathbf{Q}$. Because only the rank of $\boldsymbol{\Pi}$ is identified, the VECM is said to identify the rank of the cointegrating space, or equivalently, the number of cointegrating vectors. In practice, the estimation of the parameters of a VECM requires at least r^2 identification restrictions. Stata's `vec` command can apply the conventional Johansen restrictions discussed below or use constraints that the user supplies.

The VECM in (8) also nests two important special cases. If the variables in $\mathbf{y}_t$ are I(1) but not cointegrated, $\boldsymbol{\Pi}$ is a matrix of zeros and thus has rank 0. If all the variables are I(0), $\boldsymbol{\Pi}$ has full rank K.

There are several different frameworks for estimation and inference in cointegrating systems. Although the methods in Stata are based on the maximum likelihood (ML) methods developed by Johansen (1988, 1991, 1995), other useful frameworks have been developed by Park and Phillips (1988, 1989); Sims, Stock, and Watson (1990); Stock (1987); and Stock and Watson (1988); among others. The ML framework developed by Johansen was independently developed by Ahn and Reinsel (1990). Maddala and Kim (1998) and Watson (1994) survey all these methods. The cointegration methods in Stata are based on Johansen's maximum likelihood framework because it has been found to be particularly useful in several comparative studies, including Gonzalo (1994) and Hubrich, Lütkepohl, and Saikkonen (2001).

Trends in the Johansen VECM framework

Deterministic trends in a cointegrating VECM can stem from two distinct sources; the mean of the cointegrating relationship and the mean of the differenced series. Allowing for a constant and a linear trend and assuming that there are r cointegrating relations, we can rewrite the VECM in (8) as

$$\Delta \mathbf{y}_t = \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{y}_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta \mathbf{y}_{t-i} + \mathbf{v} + \boldsymbol{\delta} t + \epsilon_t \tag{9}$$

where $\boldsymbol{\delta}$ is a $K \times 1$ vector of parameters. Because (9) models the differences of the data, the constant implies a linear time trend in the levels, and the time trend $\boldsymbol{\delta} t$ implies a quadratic time trend in the levels of the data. Often we may want to include a constant or a linear time trend for the differences without allowing for the higher-order trend that is implied for the levels of the data. VECMs exploit the properties of the matrix $\boldsymbol{\alpha}$ to achieve this flexibility.

Because $\boldsymbol{\alpha}$ is a $K \times r$ rank matrix, we can rewrite the deterministic components in (9) as

$$\mathbf{v} = \boldsymbol{\alpha}\boldsymbol{\mu} + \boldsymbol{\gamma} \tag{10a}$$

$$\boldsymbol{\delta} t = \boldsymbol{\alpha}\boldsymbol{\rho} t + \boldsymbol{\tau} t \tag{10b}$$

where $\boldsymbol{\mu}$ and $\boldsymbol{\rho}$ are $r \times 1$ vectors of parameters and $\boldsymbol{\gamma}$ and $\boldsymbol{\tau}$ are $K \times 1$ vectors of parameters. $\boldsymbol{\gamma}$ is orthogonal to $\boldsymbol{\alpha}\boldsymbol{\mu}$, and $\boldsymbol{\tau}$ is orthogonal to $\boldsymbol{\alpha}\boldsymbol{\rho}$; that is, $\boldsymbol{\gamma}'\boldsymbol{\alpha}\boldsymbol{\mu} = \mathbf{0}$ and $\boldsymbol{\tau}'\boldsymbol{\alpha}\boldsymbol{\rho} = \mathbf{0}$, allowing us to rewrite (9) as

$$\Delta \mathbf{y}_t = \boldsymbol{\alpha}(\boldsymbol{\beta}'\mathbf{y}_{t-1} + \boldsymbol{\mu} + \boldsymbol{\rho} t) + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta \mathbf{y}_{t-i} + \boldsymbol{\gamma} + \boldsymbol{\tau} t + \epsilon_t \tag{11}$$

Placing restrictions on the trend terms in (11) yields five cases.

CASE 1: Unrestricted trend

If no restrictions are placed on the trend parameters, (11) implies that there are quadratic trends in the levels of the variables and that the cointegrating equations are stationary around time trends (trend stationary).

CASE 2: Restricted trend, $\boldsymbol{\tau} = \mathbf{0}$

By setting $\boldsymbol{\tau} = \mathbf{0}$, we assume that the trends in the levels of the data are linear but not quadratic. This specification allows the cointegrating equations to be trend stationary.

CASE 3: Unrestricted constant, $\boldsymbol{\tau} = \mathbf{0}$ and $\boldsymbol{\rho} = \mathbf{0}$

By setting $\boldsymbol{\tau} = \mathbf{0}$ and $\boldsymbol{\rho} = \mathbf{0}$, we exclude the possibility that the levels of the data have quadratic trends, and we restrict the cointegrating equations to be stationary around constant means. Because $\boldsymbol{\gamma}$ is not restricted to zero, this specification still puts a linear time trend in the levels of the data.

CASE 4: Restricted constant, $\boldsymbol{\tau}=\mathbf{0}$, $\boldsymbol{\rho}=\mathbf{0}$, and $\gamma=\mathbf{0}$

By adding the restriction that $\gamma=\mathbf{0}$, we assume there are no linear time trends in the levels of the data. This specification allows the cointegrating equations to be stationary around a constant mean, but it allows no other trends or constant terms.

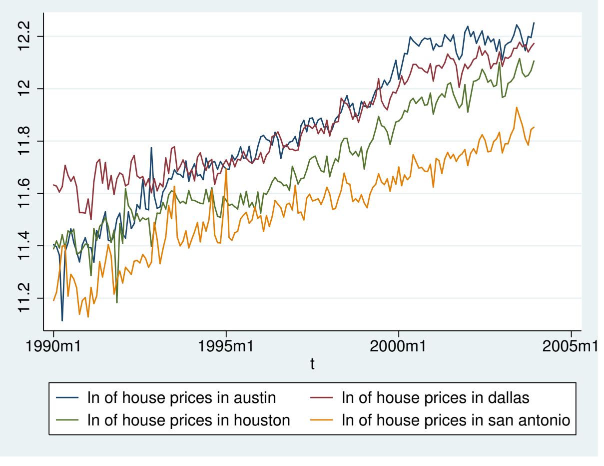
CASE 5: No trend, $\boldsymbol{\tau}=\mathbf{0}$, $\boldsymbol{\rho}=\mathbf{0}$, $\gamma=\mathbf{0}$, and $\boldsymbol{\mu}=\mathbf{0}$

This specification assumes that there are no nonzero means or trends. It also assumes that the cointegrating equations are stationary with means of zero and that the differences and the levels of the data have means of zero.

This flexibility does come at a price. Below we discuss testing procedures for determining the number of cointegrating equations. The asymptotic distribution of the LR for hypotheses about r changes with the trend specification, so we must first specify a trend specification. A combination of theory and graphical analysis will aid in specifying the trend before proceeding with the analysis.

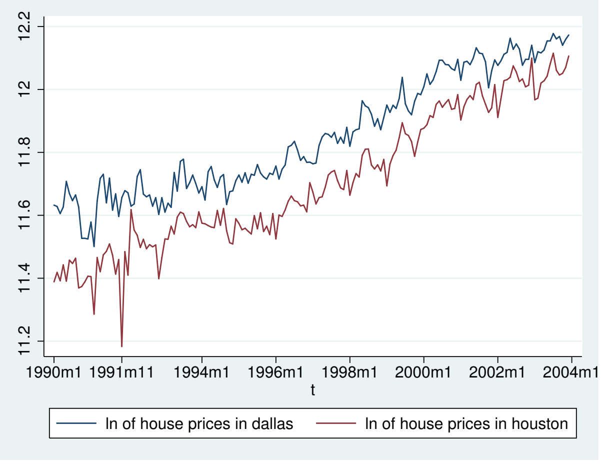
VECM estimation in Stata

We provide an overview of the `vec` commands in Stata through an extended example. We have monthly data on the average selling prices of houses in four cities in Texas: Austin, Dallas, Houston, and San Antonio. In the dataset, these average housing prices are contained in the variables `austin`, `dallas`, `houston`, and `sa`. The series begin in January of 1990 and go through December 2003, for a total of 168 observations. The following graph depicts our data.



The plots on the graph indicate that all the series are trending and potential I(1) processes. In a competitive market, the current and past prices contain all the information available, so tomorrow's price will be a random walk from today's price. Some researchers may opt to use [TS] **dfgls** to investigate the presence of a unit root in each series, but the test for cointegration we use includes the case in which all the variables are stationary, so we defer formal testing until we test for cointegration. The time trends in the data appear to be approximately linear, so we will specify `trend(constant)` when modeling these series, which is the default with `vec`.

The next graph shows just Dallas' and Houston's data, so we can more carefully examine their relationship.



Except for the crash at the end of 1991, housing prices in Dallas and Houston appear closely related. Although average prices in the two cities will differ because of resource variations and other factors, if the housing markets become too dissimilar, people and businesses will migrate, bringing the average housing prices back toward each other. We therefore expect the series of average housing prices in Houston to be cointegrated with the series of average housing prices in Dallas.

Selecting the number of lags

To test for cointegration or fit cointegrating VECMs, we must specify how many lags to include. Building on the work of Tsay (1984) and Paulsen (1984), Nielsen (2001) has shown that the methods implemented in `varsoc` can be used to determine the lag order for a VAR model with I(1) variables. As can be seen from (9), the order of the corresponding VECM is always one less than the VAR. `vec` makes this adjustment automatically, so we will always refer to the order of the underlying VAR. The output below uses `varsoc` to determine the lag order of the VAR of the average housing prices in Dallas and Houston.

```
. use http://www.stata-press.com/data/r13/txhprice
. varsoc dallas houston
   Selection-order criteria
   Sample:  1990m5 - 2003m12                   Number of obs      =       164
```

lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	299.525				.000091	-3.62835	-3.61301	-3.59055
1	577.483	555.92	4	0.000	3.2e-06	-6.9693	-6.92326	-6.85589
2	590.978	26.991*	4	0.000	2.9e-06*	-7.0851*	-7.00837*	-6.89608*
3	593.437	4.918	4	0.296	2.9e-06	-7.06631	-6.95888	-6.80168
4	596.364	5.8532	4	0.210	3.0e-06	-7.05322	-6.9151	-6.71299

```
   Endogenous:  dallas houston
    Exogenous:  _cons
```

We will use two lags for this bivariate model because the Hannan–Quinn information criterion (HQIC) method, Schwarz Bayesian information criterion (SBIC) method, and sequential likelihood-ratio (LR) test all chose two lags, as indicated by the “*” in the output.

The reader can verify that when all four cities’ data are used, the LR test selects three lags, the HQIC method selects two lags, and the SBIC method selects one lag. We will use three lags in our four-variable model.

Testing for cointegration

The tests for cointegration implemented in vecrank are based on Johansen's method. If the log likelihood of the unconstrained model that includes the cointegrating equations is significantly different from the log likelihood of the constrained model that does not include the cointegrating equations, we reject the null hypothesis of no cointegration.

Here we use vecrank to determine the number of cointegrating equations:

```
. vecrank dallas houston
                       Johansen tests for cointegration
Trend: constant                                         Number of obs =     166
Sample:  1990m3 - 2003m12                                        Lags =       2
-------------------------------------------------------------------------------
                                                         5%
maximum                                      trace    critical
  rank    parms       LL       eigenvalue  statistic    value
    0      6      576.26444         .       46.8252     15.41
    1      9      599.58781     0.24498       0.1785*    3.76
    2      10     599.67706     0.00107
-------------------------------------------------------------------------------
```

Besides presenting information about the sample size and time span, the header indicates that test statistics are based on a model with two lags and a constant trend. The body of the table presents test statistics and their critical values of the null hypotheses of no cointegration (line 1) and one or fewer cointegrating equations (line 2). The eigenvalue shown on the last line is used to compute the trace statistic in the line above it. Johansen's testing procedure starts with the test for zero cointegrating equations (a maximum rank of zero) and then accepts the first null hypothesis that is not rejected.

In the output above, we strongly reject the null hypothesis of no cointegration and fail to reject the null hypothesis of at most one cointegrating equation. Thus we accept the null hypothesis that there is one cointegrating equation in the bivariate model.

Using all four series and a model with three lags, we find that there are two cointegrating relationships.

```
. vecrank austin dallas houston sa, lag(3)
                       Johansen tests for cointegration
Trend: constant                                         Number of obs =     165
Sample:  1990m4 - 2003m12                                        Lags =       3
-------------------------------------------------------------------------------
                                                         5%
maximum                                      trace    critical
  rank    parms       LL       eigenvalue  statistic    value
    0      36     1107.7833         .       101.6070    47.21
    1      43     1137.7484      0.30456     41.6768    29.68
    2      48     1153.6435      0.17524      9.8865*   15.41
    3      51     1158.4191      0.05624      0.3354     3.76
    4      52     1158.5868      0.00203
-------------------------------------------------------------------------------
```

Fitting a VECM

vec estimates the parameters of cointegrating VECMs. There are four types of parameters of interest:

1. The parameters in the cointegrating equations β
2. The adjustment coefficients α
3. The short-run coefficients
4. Some standard functions of β and α that have useful interpretations

Although all four types are discussed in [TS] **vec**, here we discuss only types 1–3 and how they appear in the output of vec.

Having determined that there is a cointegrating equation between the Dallas and Houston series, we now want to estimate the parameters of a bivariate cointegrating VECM for these two series by using vec.

```
. vec dallas houston

Vector error-correction model

Sample:  1990m3 - 2003m12                          No. of obs      =        166
                                                   AIC             = -7.115516
Log likelihood =  599.5878                         HQIC            =  -7.04703
Det(Sigma_ml)  =  2.50e-06                         SBIC            = -6.946794

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
D_dallas              4     .038546   0.1692   32.98959   0.0000
D_houston             4     .045348   0.3737   96.66399   0.0000
----------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
D_dallas     |
        _ce1 |
         L1. |  -.3038799   .0908504    -3.34   0.001    -.4819434   -.1258165
             |
      dallas |
         LD. |  -.1647304   .0879356    -1.87   0.061     -.337081    .0076202
             |
     houston |
         LD. |  -.0998368   .0650838    -1.53   0.125    -.2273988    .0277251
             |
       _cons |   .0056128   .0030341     1.85   0.064    -.0003339    .0115595
-------------+----------------------------------------------------------------
D_houston    |
        _ce1 |
         L1. |   .5027143   .1068838     4.70   0.000     .2932258    .7122028
             |
      dallas |
         LD. |  -.0619653   .1034547    -0.60   0.549    -.2647327    .1408022
             |
     houston |
         LD. |  -.3328437     .07657    -4.35   0.000    -.4829181   -.1827693
             |
       _cons |   .0033928   .0035695     0.95   0.342    -.0036034     .010389
------------------------------------------------------------------------------

Cointegrating equations

Equation           Parms    chi2     P>chi2
-------------------------------------------
_ce1                  1   1640.088   0.0000
-------------------------------------------

Identification:  beta is exactly identified
                 Johansen normalization restriction imposed
------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
_ce1         |
      dallas |          1          .        .       .            .           .
     houston |  -.8675936   .0214231   -40.50   0.000    -.9095821    -.825605
       _cons |  -1.688897          .        .       .            .           .
------------------------------------------------------------------------------
```

The header contains information about the sample, the fit of each equation, and overall model fit statistics. The first estimation table contains the estimates of the short-run parameters, along with their standard errors, z statistics, and confidence intervals. The two coefficients on L._ce1 are the parameters in the adjustment matrix $\boldsymbol{\alpha}$ for this model. The second estimation table contains the estimated parameters of the cointegrating vector for this model, along with their standard errors, z statistics, and confidence intervals.

Using our previous notation, we have estimated

$$\widehat{\boldsymbol{\alpha}} = (-0.304, 0.503) \qquad \widehat{\boldsymbol{\beta}} = (1, -0.868) \qquad \widehat{\mathbf{v}} = (0.0056, 0.0034)$$

and

$$\widehat{\boldsymbol{\Gamma}} = \begin{pmatrix} -0.165 & -0.0998 \\ -0.062 & -0.333 \end{pmatrix}$$

Overall, the output indicates that the model fits well. The coefficient on houston in the cointegrating equation is statistically significant, as are the adjustment parameters. The adjustment parameters in this bivariate example are easy to interpret, and we can see that the estimates have the correct signs and imply rapid adjustment toward equilibrium. When the predictions from the cointegrating equation are positive, dallas is above its equilibrium value because the coefficient on dallas in the cointegrating equation is positive. The estimate of the coefficient [D_dallas]L._ce1 is $-.3$. Thus when the average housing price in Dallas is too high, it quickly falls back toward the Houston level. The estimated coefficient [D_houston]L._ce1 of .5 implies that when the average housing price in Dallas is too high, the average price in Houston quickly adjusts toward the Dallas level at the same time that the Dallas prices are adjusting.

Fitting VECMs with Johansen's normalization

As discussed by Johansen (1995), if there are r cointegrating equations, then at least r^2 restrictions are required to identify the free parameters in $\boldsymbol{\beta}$. Johansen proposed a default identification scheme that has become the conventional method of identifying models in the absence of theoretically justified restrictions. Johansen's identification scheme is

$$\boldsymbol{\beta}' = (\mathbf{I}_r, \widetilde{\boldsymbol{\beta}}')$$

where $\mathbf{I}_r$ is the $r \times r$ identity matrix and $\widetilde{\boldsymbol{\beta}}$ is an $(K - r) \times r$ matrix of identified parameters. vec applies Johansen's normalization by default.

To illustrate, we fit a VECM with two cointegrating equations and three lags on all four series. We are interested only in the estimates of the parameters in the cointegrating equations, so we can specify the noetable option to suppress the estimation table for the adjustment and short-run parameters.

```
. vec austin dallas houston sa, lags(3) rank(2) noetable

Vector error-correction model

Sample:  1990m4 - 2003m12                          No. of obs      =       165
                                                   AIC             = -13.40174
Log likelihood =  1153.644                         HQIC            = -13.03496
Det(Sigma_ml)  =  9.93e-12                         SBIC            = -12.49819

Cointegrating equations

Equation           Parms    chi2     P>chi2
-------------------------------------------
_ce1                  2   586.3044   0.0000
_ce2                  2   2169.826   0.0000
-------------------------------------------

Identification:  beta is exactly identified

                 Johansen normalization restrictions imposed
------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
_ce1         |
      austin |          1          .        .       .            .           .
      dallas |  -1.30e-17          .        .       .            .           .
     houston |  -.2623782   .1893625    -1.39   0.166    -.6335219    .1087655
          sa |  -1.241805    .229643    -5.41   0.000    -1.691897   -.7917128
       _cons |   5.577099          .        .       .            .           .
-------------+----------------------------------------------------------------
_ce2         |
      austin |  -1.41e-18          .        .       .            .           .
      dallas |          1          .        .       .            .           .
     houston |  -1.095652   .0669898   -16.36   0.000     -1.22695   -.9643545
          sa |   .2883986   .0812396     3.55   0.000     .1291718    .4476253
       _cons |  -2.351372          .        .       .            .           .
------------------------------------------------------------------------------
```

The Johansen identification scheme has placed four constraints on the parameters in β: [_ce1]austin=1, [_ce1]dallas=0, [_ce2]austin=0, and [_ce2]dallas=1. (The computational method used imposes zero restrictions that are numerical rather than exact. The values −3.48e–17 and −1.26e–17 are indistinguishable from zero.) We interpret the results of the first equation as indicating the existence of an equilibrium relationship between the average housing price in Austin and the average prices of houses in Houston and San Antonio.

The Johansen normalization restricted the coefficient on dallas to be unity in the second cointegrating equation, but we could instead constrain the coefficient on houston. Both sets of restrictions define just-identified models, so fitting the model with the latter set of restrictions will yield the same maximized log likelihood. To impose the alternative set of constraints, we use the constraint command.

```
. constraint define 1 [_ce1]austin = 1
. constraint define 2 [_ce1]dallas = 0
. constraint define 3 [_ce2]austin = 0
. constraint define 4 [_ce2]houston = 1
```

```
. vec austin dallas houston sa, lags(3) rank(2) noetable bconstraints(1/4)
Iteration 1:     log likelihood = 1148.8745
  (output omitted)
Iteration 25:    log likelihood = 1153.6435
Vector error-correction model
Sample:  1990m4 - 2003m12                         No. of obs      =        165
                                                  AIC             = -13.40174
Log likelihood =  1153.644                        HQIC            = -13.03496
Det(Sigma_ml)  =  9.93e-12                        SBIC            = -12.49819
Cointegrating equations
Equation           Parms    chi2     P>chi2
-------------------------------------------
_ce1                 2   586.3392   0.0000
_ce2                 2   3455.469   0.0000
-------------------------------------------

Identification:  beta is exactly identified
 ( 1)  [_ce1]austin = 1
 ( 2)  [_ce1]dallas = 0
 ( 3)  [_ce2]austin = 0
 ( 4)  [_ce2]houston = 1
------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
_ce1         |
      austin |          1          .        .       .            .           .
      dallas |          0  (omitted)
     houston |  -.2623784   .1876727    -1.40   0.162    -.6302102    .1054534
          sa |  -1.241805   .2277537    -5.45   0.000    -1.688194   -.7954157
       _cons |   5.577099          .        .       .            .           .
-------------+----------------------------------------------------------------
_ce2         |
      austin |          0  (omitted)
      dallas |  -.9126985   .0595804   -15.32   0.000    -1.029474   -.7959231
     houston |          1          .        .       .            .           .
          sa |  -.2632209   .0628791    -4.19   0.000    -.3864617   -.1399802
       _cons |   2.146094          .        .       .            .           .
------------------------------------------------------------------------------
```

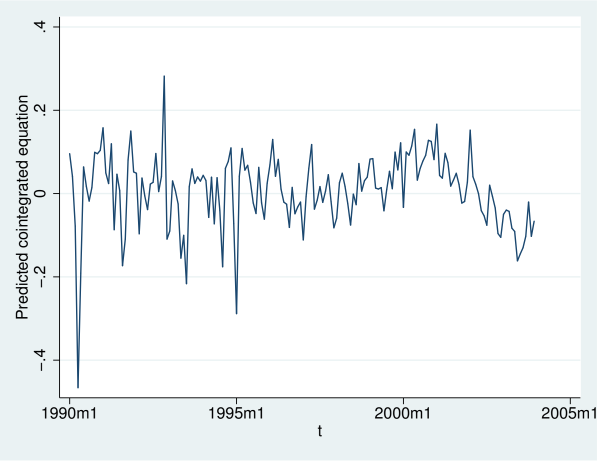
Only the estimates of the parameters in the second cointegrating equation have changed, and the new estimates are simply the old estimates divided by -1.095652 because the new constraints are just an alternative normalization of the same just-identified model. With the new normalization, we can interpret the estimates of the parameters in the second cointegrating equation as indicating an equilibrium relationship between the average house price in Houston and the average prices of houses in Dallas and San Antonio.

Postestimation specification testing

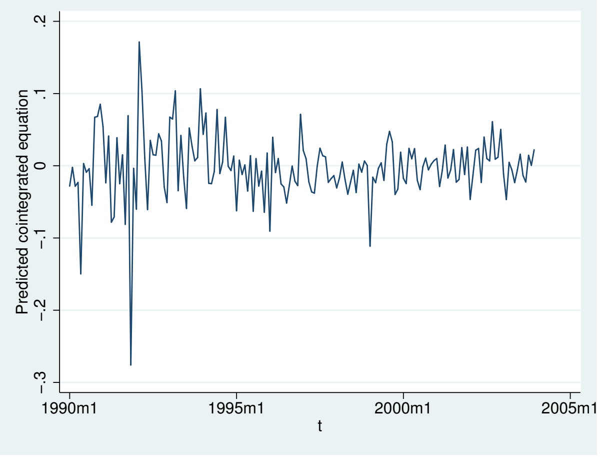
Inference on the parameters in α depends crucially on the stationarity of the cointegrating equations, so we should check the specification of the model. As a first check, we can predict the cointegrating equations and graph them over time.

```
. predict ce1, ce equ(#1)
. predict ce2, ce equ(#2)
```

```
. twoway line ce1 t
```




```
. twoway line ce2 t
```



Although the large shocks apparent in the graph of the levels have clear effects on the predictions from the cointegrating equations, our only concern is the negative trend in the first cointegrating equation since the end of 2000. The graph of the levels shows that something put a significant brake on the growth of housing prices after 2000 and that the growth of housing prices in San Antonio slowed during 2000 but then recuperated while Austin maintained slower growth. We suspect that this indicates that the end of the high-tech boom affected Austin more severely than San Antonio. This difference is what causes the trend in the first cointegrating equation. Although we could try to account for this effect with a more formal analysis, we will proceed as if the cointegrating equations are stationary.

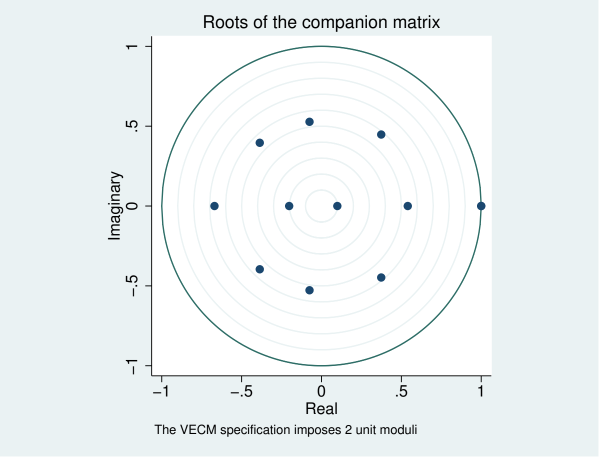
We can use `vecstable` to check whether we have correctly specified the number of cointegrating equations. As discussed in [TS] **vecstable**, the companion matrix of a VECM with K endogenous variables and r cointegrating equations has $K-r$ unit eigenvalues. If the process is stable, the moduli of the remaining r eigenvalues are strictly less than one. Because there is no general distribution theory for the moduli of the eigenvalues, ascertaining whether the moduli are too close to one can be difficult.

```
. vecstable, graph
```

Eigenvalue stability condition

Eigenvalue	Modulus
1	1
1	1
-.6698661	.669866
.3740191 + .4475996*i*	.583297
.3740191 - .4475996*i*	.583297
-.386377 + .395972*i*	.553246
-.386377 - .395972*i*	.553246
.540117	.540117
-.0749239 + .5274203*i*	.532715
-.0749239 - .5274203*i*	.532715
-.2023955	.202395
.09923966	.09924

The VECM specification imposes 2 unit moduli.



Because we specified the **graph** option, **vecstable** plotted the eigenvalues of the companion matrix. The graph of the eigenvalues shows that none of the remaining eigenvalues appears close to the unit circle. The stability check does not indicate that our model is misspecified.

Here we use **veclmar** to test for serial correlation in the residuals.

```
. veclmar, mlag(4)
```

Lagrange-multiplier test

lag	chi2	df	Prob > chi2
1	56.8757	16	0.00000
2	31.1970	16	0.01270
3	30.6818	16	0.01477
4	14.6493	16	0.55046

H0: no autocorrelation at lag order

The results clearly indicate serial correlation in the residuals. The results in Gonzalo (1994) indicate that underspecifying the number of lags in a VECM can significantly increase the finite-sample bias in the parameter estimates and lead to serial correlation. For this reason, we refit the model with five lags instead of three.

```
. vec austin dallas houston sa, lags(5) rank(2) noetable bconstraints(1/4)
Iteration 1:     log likelihood = 1200.5402
  (output omitted )
Iteration 20:    log likelihood = 1203.9465
Vector error-correction model
Sample:  1990m6 - 2003m12                         No. of obs      =        163
                                                  AIC             = -13.79075
Log likelihood =  1203.946                        HQIC            =  -13.1743
Det(Sigma_ml)  =  4.51e-12                        SBIC            = -12.27235
Cointegrating equations
Equation           Parms    chi2     P>chi2
-------------------------------------------
_ce1                  2   498.4682   0.0000
_ce2                  2   4125.926   0.0000
-------------------------------------------
Identification:  beta is exactly identified
 ( 1)  [_ce1]austin = 1
 ( 2)  [_ce1]dallas = 0
 ( 3)  [_ce2]austin = 0
 ( 4)  [_ce2]houston = 1
------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
_ce1         |
      austin |          1          .        .       .            .           .
      dallas |          0  (omitted)
     houston |  -.6525574   .2047061    -3.19   0.001    -1.053774   -.2513407
          sa |  -.6960166   .2494167    -2.79   0.005    -1.184864   -.2071688
       _cons |   3.846275          .        .       .            .           .
-------------+----------------------------------------------------------------
_ce2         |
      austin |          0  (omitted)
      dallas |   -.932048   .0564332   -16.52   0.000    -1.042655   -.8214409
     houston |          1          .        .       .            .           .
          sa |  -.2363915   .0599348    -3.94   0.000    -.3538615   -.1189215
       _cons |   2.065719          .        .       .            .           .
------------------------------------------------------------------------------
```

Comparing these results with those from the previous model reveals that

1. there is now evidence that the coefficient `[_ce1]houston` is not equal to zero,
2. the two sets of estimated coefficients for the first cointegrating equation are different, and
3. the two sets of estimated coefficients for the second cointegrating equation are similar.

The assumption that the errors are independently, identically, and normally distributed with zero mean and finite variance allows us to derive the likelihood function. If the errors do not come from a normal distribution but are just independently and identically distributed with zero mean and finite variance, the parameter estimates are still consistent, but they are not efficient.

We use `vecnorm` to test the null hypothesis that the errors are normally distributed.

```
. qui vec austin dallas houston sa, lags(5) rank(2) bconstraints(1/4)
. vecnorm
```

Jarque-Bera test

Equation	chi2	df	Prob > chi2
D_austin	74.324	2	0.00000
D_dallas	3.501	2	0.17370
D_houston	245.032	2	0.00000
D_sa	8.426	2	0.01481
ALL	331.283	8	0.00000

Skewness test

Equation	Skewness	chi2	df	Prob > chi2
D_austin	.60265	9.867	1	0.00168
D_dallas	.09996	0.271	1	0.60236
D_houston	-1.0444	29.635	1	0.00000
D_sa	.38019	3.927	1	0.04752
ALL		43.699	4	0.00000

Kurtosis test

Equation	Kurtosis	chi2	df	Prob > chi2
D_austin	6.0807	64.458	1	0.00000
D_dallas	3.6896	3.229	1	0.07232
D_houston	8.6316	215.397	1	0.00000
D_sa	3.8139	4.499	1	0.03392
ALL		287.583	4	0.00000

The results indicate that we can strongly reject the null hypothesis of normally distributed errors. Most of the errors are both skewed and kurtotic.

Impulse–response functions for VECMs

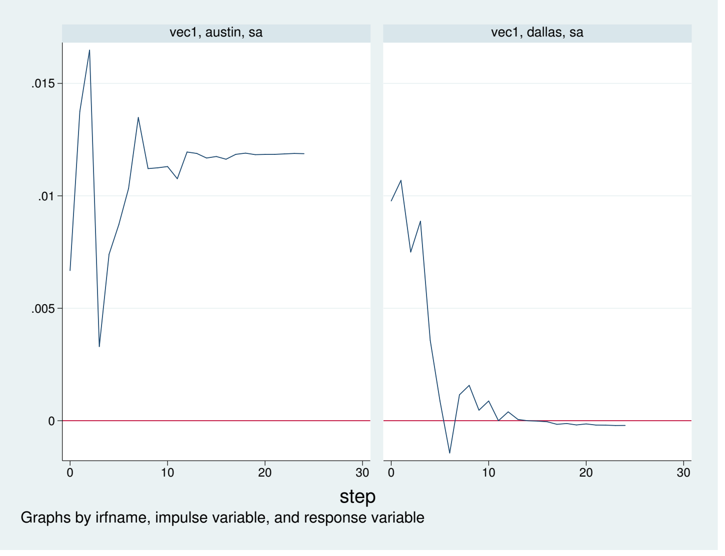
With a model that we now consider acceptably well specified, we can use the `irf` commands to estimate and interpret the IRFs. Whereas IRFs from a stationary VAR die out over time, IRFs from a cointegrating VECM do not always die out. Because each variable in a stationary VAR has a time-invariant mean and finite, time-invariant variance, the effect of a shock to any one of these variables must die out so that the variable can revert to its mean. In contrast, the I(1) variables modeled in a cointegrating VECM are not mean reverting, and the unit moduli in the companion matrix imply that the effects of some shocks will not die out over time.

These two possibilities gave rise to new terms. When the effect of a shock dies out over time, the shock is said to be transitory. When the effect of a shock does not die out over time, the shock is said to be permanent.

Below we use `irf create` to estimate the IRFs and `irf graph` to graph two of the orthogonalized IRFs.

```
. irf create vec1, set(vecintro, replace) step(24)
(file vecintro.irf created)
(file vecintro.irf now active)
(file vecintro.irf updated)

. irf graph oirf, impulse(austin dallas) response(sa) yline(0)
```



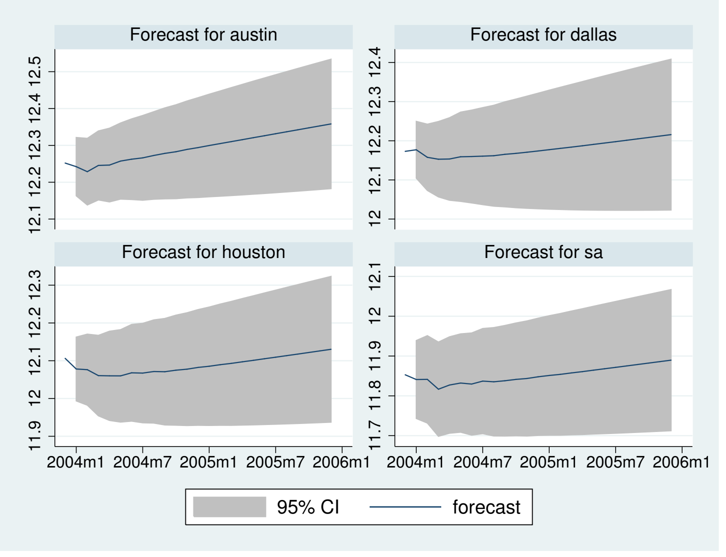
The graphs indicate that an orthogonalized shock to the average housing price in Austin has a permanent effect on the average housing price in San Antonio but that an orthogonalized shock to the average price of housing in Dallas has a transitory effect. According to this model, unexpected shocks that are local to the Austin housing market will have a permanent effect on the housing market in San Antonio, but unexpected shocks that are local to the Dallas housing market will have only a transitory effect on the housing market in San Antonio.

Forecasting with VECMs

Cointegrating VECMs are also used to produce forecasts of both the first-differenced variables and the levels of the variables. Comparing the variances of the forecast errors of stationary VARs with those from a cointegrating VECM reveals a fundamental difference between the two models. Whereas the variances of the forecast errors for a stationary VAR converge to a constant as the prediction horizon grows, the variances of the forecast errors for the levels of a cointegrating VECM diverge with the forecast horizon. (See sec. 6.5 of Lütkepohl [2005] for more about this result.) Because all the variables in the model for the first differences are stationary, the forecast errors for the dynamic forecasts of the first differences remain finite. In contrast, the forecast errors for the dynamic forecasts of the levels diverge to infinity.

We use `fcast compute` to obtain dynamic forecasts of the levels and `fcast graph` to graph these dynamic forecasts, along with their asymptotic confidence intervals.

```
. tsset
        time variable:  t, 1990m1 to 2003m12
                delta:  1 month
. fcast compute m1_, step(24)
. fcast graph m1_austin m1_dallas m1_houston m1_sa
```



As expected, the widths of the confidence intervals grow with the forecast horizon.

References

Ahn, S. K., and G. C. Reinsel. 1990. Estimation for partially nonstationary multivariate autoregressive models. *Journal of the American Statistical Association* 85: 813–823.

Becketti, S. 2013. *Introduction to Time Series Using Stata*. College Station, TX: Stata Press.

Engle, R. F., and C. W. J. Granger. 1987. Co-integration and error correction: Representation, estimation, and testing. *Econometrica* 55: 251–276.

Gonzalo, J. 1994. Five alternative methods of estimating long-run equilibrium relationships. *Journal of Econometrics* 60: 203–233.

Granger, C. W. J. 1981. Some properties of time series data and their use in econometric model specification. *Journal of Econometrics* 16: 121–130.

Granger, C. W. J., and P. Newbold. 1974. Spurious regressions in econometrics. *Journal of Econometrics* 2: 111–120.

Hamilton, J. D. 1994. *Time Series Analysis*. Princeton: Princeton University Press.

Hubrich, K., H. Lütkepohl, and P. Saikkonen. 2001. A review of systems cointegration tests. *Econometric Reviews* 20: 247–318.

Johansen, S. 1988. Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control* 12: 231–254.

——. 1991. Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models. *Econometrica* 59: 1551–1580.

——. 1995. *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford: Oxford University Press.

Lütkepohl, H. 2005. *New Introduction to Multiple Time Series Analysis*. New York: Springer.

Maddala, G. S., and I.-M. Kim. 1998. *Unit Roots, Cointegration, and Structural Change*. Cambridge: Cambridge University Press.

Nielsen, B. 2001. Order determination in general vector autoregressions. Working paper, Department of Economics, University of Oxford and Nuffield College. http://ideas.repec.org/p/nuf/econwp/0110.html.

Park, J. Y., and P. C. B. Phillips. 1988. Statistical inference in regressions with integrated processes: Part I. *Econometric Theory* 4: 468–497.

——. 1989. Statistical inference in regressions with integrated processes: Part II. *Econometric Theory* 5: 95–131.

Paulsen, J. 1984. Order determination of multivariate autoregressive time series with unit roots. *Journal of Time Series Analysis* 5: 115–127.

Phillips, P. C. B. 1986. Understanding spurious regressions in econometrics. *Journal of Econometrics* 33: 311–340.

Phillips, P. C. B., and S. N. Durlauf. 1986. Multiple time series regressions with integrated processes. *Review of Economic Studies* 53: 473–495.

Sims, C. A., J. H. Stock, and M. W. Watson. 1990. Inference in linear time series models with some unit roots. *Econometrica* 58: 113–144.

Stock, J. H. 1987. Asymptotic properties of least squares estimators of cointegrating vectors. *Econometrica* 55: 1035–1056.

Stock, J. H., and M. W. Watson. 1988. Testing for common trends. *Journal of the American Statistical Association* 83: 1097–1107.

Tsay, R. S. 1984. Order selection in nonstationary autoregressive models. *Annals of Statistics* 12: 1425–1433.

Watson, M. W. 1994. Vector autoregressions and cointegration. In Vol. 4 of *Handbook of Econometrics*, ed. R. F. Engle and D. L. McFadden. Amsterdam: Elsevier.

Also see

[TS] **vec** — Vector error-correction models

[TS] **irf** — Create and analyze IRFs, dynamic-multiplier functions, and FEVDs