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ttest — t tests (mean-comparison tests)

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Syntax

One-sample t test

ttest
$$varname == \# [if] [in] [, \underline{1}evel(\#)]$$

Two-sample t test using groups

ttest
$$varname [if] [in]$$
, by $(groupvar) [options_1]$

Two-sample t test using variables

ttest
$$varname_1 == varname_2 [if] [in]$$
, unpaired [unequal welch level(#)]

Paired t test

ttest
$$varname_1 == varname_2 [if] [in] [, level(#)]$$

Immediate form of one-sample t test

ttesti
$$\#_{obs} \#_{mean} \#_{sd} \#_{val} [, \underline{l}evel(\#)]$$

Immediate form of two-sample t test

ttesti
$$\#_{\mathrm{obs}1}$$
 $\#_{\mathrm{mean}1}$ $\#_{\mathrm{sd}1}$ $\#_{\mathrm{obs}2}$ $\#_{\mathrm{mean}2}$ $\#_{\mathrm{sd}2}$ $\left[$, options $_{2}$ $\right]$

options₁ Description

Main

options

* by (*groupvar*) variable defining the groups

unequal unpaired data have unequal variances

welch use Welch's approximation

level(#) set confidence level; default is level(95)

Description

^{*}by(groupvar) is required.

opilons ₂	2 computer	
Main		
<u>une</u> qual	unpaired data have unequal variances	
<u>w</u> elch	use Welch's approximation	
level(#)	set confidence level; default is level(95)	

by is allowed with ttest; see [D] by.

Menu

ttest

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ttesti

Statistics > Summaries, tables, and tests > Classical tests of hypotheses > t test calculator

Description

ttest performs t tests on the equality of means. In the first form, ttest tests that varname has a mean of #. In the second form, ttest tests that varname has the same mean within the two groups defined by groupvar. In the third form, ttest tests that $varname_1$ and $varname_2$ have the same mean, assuming unpaired data. In the fourth form, ttest tests that $varname_1$ and $varname_2$ have the same mean, assuming paired data.

ttesti is the immediate form of ttest; see [U] 19 Immediate commands.

For the equivalent of a two-sample t test with sampling weights (pweights), use the svy: mean command with the over() option, and then use lincom; see [R] mean and [SVY] svy postestimation.

Options

Main

by (groupvar) specifies the groupvar that defines the two groups that ttest will use to test the hypothesis that their means are equal. Specifying by (groupvar) implies an unpaired (two sample) t test. Do not confuse the by () option with the by prefix; you can specify both.

unpaired specifies that the data be treated as unpaired. The unpaired option is used when the two sets of values to be compared are in different variables.

unequal specifies that the unpaired data not be assumed to have equal variances.

welch specifies that the approximate degrees of freedom for the test be obtained from Welch's formula (1947) rather than from Satterthwaite's approximation formula (1946), which is the default when unequal is specified. Specifying welch implies unequal.

level(#) specifies the confidence level, as a percentage, for confidence intervals. The default is level(95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.

Remarks and examples

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Remarks are presented under the following headings:

One-sample t test
Two-sample t test
Paired t test
Two-sample t test compared with one-way ANOVA
Immediate form
Video examples

One-sample t test

▶ Example 1

In the first form, ttest tests whether the mean of the sample is equal to a known constant under the assumption of unknown variance. Assume that we have a sample of 74 automobiles. We know each automobile's average mileage rating and wish to test whether the overall average for the sample is 20 miles per gallon.

- . use http://www.stata-press.com/data/r13/auto (1978 Automobile Data)
- . ttest mpg==20

One-sample t test

Variable	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf.	Interval]
mpg	74	21.2973	.6725511	5.785503	19.9569	22.63769
mean =	= mean(mpg) = 20			degrees	t :	= 1.9289 = 73
	ean < 20) = 0.9712		Ha: mean != T > t) =			ean > 20) = 0.0288

The test indicates that the underlying mean is not 20 with a significance level of 5.8%.

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Two-sample t test

Example 2: Two-sample t test using groups

We are testing the effectiveness of a new fuel additive. We run an experiment in which 12 cars are given the fuel treatment and 12 cars are not. The results of the experiment are as follows:

treated	mpg
0	20
0	23
0	21
0	25
0	18
0	17
0	18
0	24
0	20
0	24
0	23
0	19
1	24
1	25
1	21
1	22
1	23
1	18
1	17
1	28
1	24
1	27
1	21
1	23

The treated variable is coded as 1 if the car received the fuel treatment and 0 otherwise.

We can test the equality of means of the treated and untreated group by typing

- . use http://www.stata-press.com/data/r13/fuel3
- . ttest mpg, by(treated)

Two-sample t test with equal variances

Group	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf.	Interval]
0	12 12	21 22.75	.7881701 .9384465	2.730301 3.250874	19.26525 20.68449	22.73475 24.81551
combined	24	21.875	.6264476	3.068954	20.57909	23.17091
diff		-1.75	1.225518		-4.291568	.7915684
diff = ====(0) =====(1)					+	_ 1 4000

We do not find a statistically significant difference in the means.

If we were not willing to assume that the variances were equal and wanted to use Welch's formula, we could type

. ttest mpg, by(treated) welch

Two-sample t test with unequal variances

Group	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf.	Interval]
0	12 12	21 22.75	.7881701 .9384465	2.730301 3.250874	19.26525 20.68449	22.73475 24.81551
combined	24	21.875	.6264476	3.068954	20.57909	23.17091
diff		-1.75	1.225518		-4.28369	.7836902
diff :	= mean(0) - = 0	mean(1)	Wel	ch's degrees	t of freedom	= -1.4280 = 23.2465

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□ Technical note

In two-sample using groups randomized designs, subjects will sometimes refuse the assigned treatment but still be measured for an outcome. In this case, take care to specify the group properly. You might be tempted to let *varname* contain missing where the subject refused and thus let ttest drop such observations from the analysis. Zelen (1979) argues that it would be better to specify that the subject belongs to the group in which he or she was randomized, even though such inclusion will dilute the measured effect.

Example 3: Two-sample t test using variables

There is a second, inferior way to organize the data in the preceding example. We ran a test on 24 cars, 12 without the additive and 12 with. We now create two new variables, mpg1 and mpg2.

mpg2
24
25
21
22
23
18
17
28
24
27
21
23

This method is inferior because it suggests a connection that is not there. There is no link between the car with 20 mpg and the car with 24 mpg in the first row of the data. Each column of data could be arranged in any order. Nevertheless, if our data are organized like this, ttest can accommodate us.

- . use http://www.stata-press.com/data/r13/fuel
- . ttest mpg1==mpg2, unpaired

Two-sample t test with equal variances

Variable	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf.	Interval]
mpg1 mpg2	12 12	21 22.75	.7881701 .9384465	2.730301 3.250874	19.26525 20.68449	22.73475 24.81551
combined	24	21.875	.6264476	3.068954	20.57909	23.17091
diff		-1.75	1.225518		-4.291568	.7915684
diff =	= mean(mpg1) = 0	- mean(mpg	(2)	degrees	t = of freedom =	= -1.4280 = 22

Paired t test

Example 4

Suppose that the preceding data were actually collected by running a test on 12 cars. Each car was run once with the fuel additive and once without. Our data are stored in the same manner as in example 3, but this time, there is most certainly a connection between the mpg values that appear in the same row. These come from the same car. The variables mpg1 and mpg2 represent mileage without and with the treatment, respectively.

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- . use http://www.stata-press.com/data/r13/fuel
- . ttest mpg1==mpg2

Paired t test

Variable	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf.	Interval]
mpg1 mpg2	12 12	21 22.75	.7881701 .9384465	2.730301 3.250874	19.26525 20.68449	22.73475 24.81551
diff	12	-1.75	.7797144	2.70101	-3.46614	0338602

We find that the means are statistically different from each other at any level greater than 4.6%.

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Two-sample t test compared with one-way ANOVA

Example 5

In example 2, we saw that ttest can be used to test the equality of a pair of means; see [R] **oneway** for an extension that allows testing the equality of more than two means.

Suppose that we have data on the 50 states. The dataset contains the median age of the population (medage) and the region of the country (region) for each state. Region 1 refers to the Northeast, region 2 to the North Central, region 3 to the South, and region 4 to the West. Using oneway, we can test the equality of all four means.

- . use http://www.stata-press.com/data/r13/census
 (1980 Census data by state)
- . oneway medage region

Analysis of Variance								
Source	SS	df	MS	F	Prob > F			
Between groups	46.3961903	3	15.4653968	7.56	0.0003			
Within groups	94.1237947	46	2.04616945					
Total	140.519985	49	2.8677548					

Bartlett's test for equal variances: chi2(3) = 10.5757 Prob>chi2 = 0.014

We find that the means are different, but we are interested only in testing whether the means for the Northeast (region==1) and West (region==4) are different. We could use oneway:

. oneway medage region if region==1 | region==4 Analysis of Variance 22 df Prob > F Source Between groups 46.241247 1 46.241247 20.02 0.0002 Within groups 46.1969169 20 2.30984584 Total 92.4381638 21 4.40181733

Bartlett's test for equal variances: chi2(1) = 2.4679 Prob>chi2 = 0.116

Pr(T > t) = 0.0001

We could also use ttest:

. ttest medage if region==1 | region==4, by(region)

Two-sample t test with equal variances

Group	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf.	Interval]
NE West	9 13	31.23333 28.28462	.3411581 .4923577	1.023474 1.775221	30.44662 27.21186	32.02005 29.35737
combined	22	29.49091	.4473059	2.098051	28.56069	30.42113
diff		2.948718	.6590372		1.57399	4.323445
diff =		- mean(West)		degrees	t of freedom	21 21 20
Ha· d-	iff < 0		Har diff !=	0	Ha∙ d	iff > 0

Pr(|T| > |t|) = 0.0002

The significance levels of both tests are the same.

Pr(T < t) = 0.9999

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Immediate form

Example 6

ttesti is like ttest, except that we specify summary statistics rather than variables as arguments. For instance, we are reading an article that reports the mean number of sunspots per month as 62.6 with a standard deviation of 15.8. There are 24 months of data. We wish to test whether the mean is 75:

. ttesti 24 62.6 15.8 75

One-sample t test

	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf.	Interval]
х	24	62.6	3.225161	15.8	55.92825	69.27175
mean =	= mean(x) = 75			degrees	t of freedom	= -3.8448 = 23
	ean < 75) = 0.0004	Pr(Ha: mean != T > t) =			nean > 75 () = 0.9996

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▶ Example 7

There is no immediate form of ttest with paired data because the test is also a function of the covariance, a number unlikely to be reported in any published source. For nonpaired data, however, we might type

. ttesti 20 20 5 32 15 4

Two-sample t test with equal variances

	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf.	Interval]
х	20 32	20 15	1.118034 .7071068	5 4	17.65993 13.55785	22.34007 16.44215
combined	52	16.92308	.6943785	5.007235	15.52905	18.3171
diff		5	1.256135		2.476979	7.523021
<pre>diff = mean(x) - mean(y) Ho: diff = 0 Ha: diff < 0</pre>			Ha: diff !=	<u> </u>	t of freedom	0.0000

If we had typed ttesti 20 20 5 32 15 4, unequal, the test would have assumed unequal variances.

Pr(|T| > |t|) = 0.0002

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Pr(T > t) = 0.0001

Video examples

One-sample t test in Stata

t test for two independent samples in Stata

t test for two paired samples in Stata

Pr(T < t) = 0.9999

Immediate commands in Stata: One-sample t test from summary data

Immediate commands in Stata: Two-sample t test from summary data

Stored results

ttest and ttesti store the following in r():

Scalars			
r(N_1)	sample size n_1	$r(sd_1)$	standard deviation for first variable
r(N_2)	sample size n_2	r(sd_2)	standard deviation for second variable
r(p_1)	lower one-sided p-value	r(sd)	combined standard deviation
r(p_u)	upper one-sided p-value	r(mu_1)	\bar{x}_1 mean for population 1
r(p)	two-sided p-value	r(mu_2)	\bar{x}_2 mean for population 2
r(se)	estimate of standard error	$r(df_t)$	degrees of freedom
r(t)	t statistic	r(level)	confidence level

Methods and formulas

See, for instance, Hoel (1984, 140–161) or Dixon and Massey (1983, 121–130) for an introduction and explanation of the calculation of these tests. Acock (2014, 162–173) and Hamilton (2013, 145–150) describe t tests using applications in Stata.

The test for $\mu = \mu_0$ for unknown σ is given by

$$t = \frac{(\overline{x} - \mu_0)\sqrt{n}}{s}$$

The statistic is distributed as Student's t with n-1 degrees of freedom (Gosset [Student, pseud.] 1908).

The test for $\mu_x = \mu_y$ when σ_x and σ_y are unknown but $\sigma_x = \sigma_y$ is given by

$$t = \frac{\overline{x} - \overline{y}}{\left\{\frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2}\right\}^{1/2} \left(\frac{1}{n_x} + \frac{1}{n_y}\right)^{1/2}}$$

The result is distributed as Student's t with $n_x + n_y - 2$ degrees of freedom.

You could perform ttest (without the unequal option) in a regression setting given that regression assumes a homoskedastic error model. To compare with the ttest command, denote the underlying observations on x and y by x_j , $j = 1, \ldots, n_x$, and y_j , $j = 1, \ldots, n_y$. In a regression framework, typing ttest without the unequal option is equivalent to

- 1. creating a new variable z_j that represents the stacked observations on x and y (so that $z_j = x_j$ for $j = 1, \ldots, n_x$ and $z_{n_x + j} = y_j$ for $j = 1, \ldots, n_y$)
- 2. and then estimating the equation $z_j=\beta_0+\beta_1d_j+\epsilon_j$, where $d_j=0$ for $j=1,\ldots,n_x$ and $d_j=1$ for $j=n_x+1,\ldots,n_x+n_y$ (that is, $d_j=0$ when the z observations represent x, and $d_j=1$ when the z observations represent y).

The estimated value of β_1 , b_1 , will equal $\overline{y} - \overline{x}$, and the reported t statistic will be the same t statistic as given by the formula above.

The test for $\mu_x = \mu_y$ when σ_x and σ_y are unknown and $\sigma_x \neq \sigma_y$ is given by

$$t = \frac{\overline{x} - \overline{y}}{\left(s_x^2/n_x + s_y^2/n_y\right)^{1/2}}$$

The result is distributed as Student's t with ν degrees of freedom, where ν is given by (with Satterthwaite's [1946] formula)

$$\frac{\left(s_x^2/n_x + s_y^2/n_y\right)^2}{\frac{\left(s_x^2/n_x\right)^2}{n_x - 1} + \frac{\left(s_y^2/n_y\right)^2}{n_y - 1}}$$

With Welch's formula (1947), the number of degrees of freedom is given by

$$-2 + \frac{\left(s_x^2/n_x + s_y^2/n_y\right)^2}{\frac{\left(s_x^2/n_x\right)^2}{n_x + 1} + \frac{\left(s_y^2/n_y\right)^2}{n_y + 1}}$$

The test for $\mu_x = \mu_y$ for matched observations (also known as paired observations, correlated pairs, or permanent components) is given by

$$t = \frac{\overline{d}\sqrt{n}}{s_d}$$

where \overline{d} represents the mean of $x_i - y_i$ and s_d represents the standard deviation. The test statistic t is distributed as Student's t with n-1 degrees of freedom.

You can also use ttest without the unpaired option in a regression setting because a paired comparison includes the assumption of constant variance. The ttest with an unequal variance assumption does not lend itself to an easy representation in regression settings and is not discussed here. $(x_j - y_j) = \beta_0 + \epsilon_j$.

William Sealy Gosset (1876–1937) was born in Canterbury, England. He studied chemistry and mathematics at Oxford and worked as a chemist with the brewers Guinness in Dublin. Gosset became interested in statistical problems, which he discussed with Karl Pearson and later with Fisher and Neyman. He published several important papers under the pseudonym "Student", and he lent that name to the *t* test he invented.

References

Acock, A. C. 2014. A Gentle Introduction to Stata. 4th ed. College Station, TX: Stata Press.

Boland, P. J. 2000. William Sealy Gosset—alias 'Student' 1876–1937. In Creators of Mathematics: The Irish Connection, ed. K. Houston, 105–112. Dublin: University College Dublin Press.

Dixon, W. J., and F. J. Massey, Jr. 1983. Introduction to Statistical Analysis. 4th ed. New York: McGraw-Hill.

Gleason, J. R. 1999. sg101: Pairwise comparisons of means, including the Tukey wsd method. Stata Technical Bulletin 47: 31–37. Reprinted in Stata Technical Bulletin Reprints, vol. 8, pp. 225–233. College Station, TX: Stata Press.

Gosset, W. S. 1943. "Student's" Collected Papers. London: Biometrika Office, University College.

Gosset [Student, pseud.], W. S. 1908. The probable error of a mean. Biometrika 6: 1-25.

Hamilton, L. C. 2013. Statistics with Stata: Updated for Version 12. 8th ed. Boston: Brooks/Cole.

Hoel, P. G. 1984. Introduction to Mathematical Statistics. 5th ed. New York: Wiley.

Pearson, E. S., R. L. Plackett, and G. A. Barnard. 1990. 'Student': A Statistical Biography of William Sealy Gosset. Oxford: Oxford University Press.

Preece, D. A. 1982. t is for trouble (and textbooks): A critique of some examples of the paired-samples t-test. *Statistician* 31: 169–195.

Satterthwaite, F. E. 1946. An approximate distribution of estimates of variance components. *Biometrics Bulletin* 2: 110–114.

Senn, S. J., and W. Richardson. 1994. The first t-test. Statistics in Medicine 13: 785-803.

Welch, B. L. 1947. The generalization of 'student's' problem when several different population variances are involved. Biometrika 34: 28–35.

Zelen, M. 1979. A new design for randomized clinical trials. New England Journal of Medicine 300: 1242-1245.

Also see

- [R] **bitest** Binomial probability test
- [R] ci Confidence intervals for means, proportions, and counts
- [R] esize Effect size based on mean comparison
- [R] mean Estimate means
- [R] **oneway** One-way analysis of variance
- [R] **prtest** Tests of proportions
- [R] **sdtest** Variance-comparison tests
- [MV] **hotelling** Hotelling's T-squared generalized means test