Title

sktest — Skewness and kurtosis test for normality

Syntax

`sktest` *varlist* [*if*] [*in*] [*weight*] [, noadjust]

`aweight`s and `fweight`s are allowed; see [U] **11.1.6 weight**.

Menu

Statistics > Summaries, tables, and tests > Distributional plots and tests > Skewness and kurtosis normality test

Description

For each variable in *varlist*, `sktest` presents a test for normality based on skewness and another based on kurtosis and then combines the two tests into an overall test statistic. `sktest` requires a minimum of 8 observations to make its calculations. See [MV] **mvtest normality** for multivariate tests of normality.

Option

Main

`noadjust` suppresses the empirical adjustment made by Royston (1991c) to the overall χ^2 and its significance level and presents the unaltered test as described by D'Agostino, Belanger, and D'Agostino (1990).

Remarks and examples

Also see [R] **swilk** for the Shapiro–Wilk and Shapiro–Francia tests for normality. Those tests are, in general, preferred for nonaggregated data (Gould and Rogers 1991; Gould 1992; Royston 1991c). Moreover, a normal quantile plot should be used with any test for normality; see [R] **diagnostic plots** for more information.

▷ Example 1

Using our automobile dataset, we will test whether the variables `mpg` and `trunk` are normally distributed:

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. sktest mpg trunk
                    Skewness/Kurtosis tests for Normality
                                                       ------ joint ------
    Variable |    Obs   Pr(Skewness)   Pr(Kurtosis)  adj chi2(2)    Prob>chi2
-------------+---------------------------------------------------------------
         mpg |     74      0.0015         0.0804        10.95         0.0042
       trunk |     74      0.9115         0.0445         4.19         0.1228
```

We can reject the hypothesis that `mpg` is normally distributed, but we cannot reject the hypothesis that `trunk` is normally distributed, at least at the 12% level. The kurtosis for `trunk` is 2.19, as can be verified by issuing the command

```
. summarize trunk, detail
  (output omitted )
```

and the p-value of 0.0445 shown in the table above indicates that it is significantly different from the kurtosis of a normal distribution at the 5% significance level. However, on the basis of skewness alone, we cannot reject the hypothesis that `trunk` is normally distributed.

◁

❑ Technical note

`sktest` implements the test as described by D'Agostino, Belanger, and D'Agostino (1990) but with the adjustment made by Royston (1991c). In the above example, if we had specified the `noadjust` option, the χ^2 values would have been 13.13 for `mpg` and 4.05 for `trunk`. With the adjustment, the χ^2 value might show as '`.`'. This result should be interpreted as an absurdly large number; the data are most certainly not normal.

❑

Stored results

`sktest` stores the following in `r()`:

Scalars

`r(chi2)`	χ^2
`r(P_skew)`	Pr(skewness)
`r(P_kurt)`	Pr(kurtosis)
`r(P_chi2)`	Prob $>$ chi2

Matrices

`r(N)`	matrix of observations
`r(Utest)`	matrix of test results, one row per variable

Methods and formulas

sktest implements the test described by D'Agostino, Belanger, and D'Agostino (1990) with the empirical correction developed by Royston (1991c).

Let g_1 denote the coefficient of skewness and b_2 denote the coefficient of kurtosis as calculated by summarize, and let n denote the sample size. If weights are specified, then g_1, b_2, and n denote the weighted coefficients of skewness and kurtosis and weighted sample size, respectively. See [R] **summarize** for the formulas for skewness and kurtosis.

To perform the test of skewness, we compute

$$Y = g_1 \left\{ \frac{(n+1)(n+3)}{6(n-2)} \right\}^{1/2}$$

$$\beta_2(g_1) = \frac{3(n^2+27n-70)(n+1)(n+3)}{(n-2)(n+5)(n+7)(n+9)}$$

$$W^2 = -1 + [2\{\beta_2(g_1)-1\}]^{1/2}$$

and

$$\alpha = \left\{2/(W^2-1)\right\}^{1/2}$$

Then the distribution of the test statistic

$$Z_1 = \frac{1}{\sqrt{\ln W}} \ln \left[Y/\alpha + \left\{ (Y/\alpha)^2 + 1 \right\}^{1/2} \right]$$

is approximately standard normal under the null hypothesis that the data are distributed normally.

To perform the test of kurtosis, we compute

$$E(b_2) = \frac{3(n-1)}{n+1}$$

$$\text{var}(b_2) = \frac{24n(n-2)(n-3)}{(n+1)^2(n+3)(n+5)}$$

$$X = \{b_2 - E(b_2)\} / \sqrt{\text{var}(b_2)}$$

$$\sqrt{\beta_1(b_2)} = \frac{6(n^2-5n+2)}{(n+7)(n+9)} \left\{ \frac{6(n+3)(n+5)}{n(n-2)(n-3)} \right\}^{1/2}$$

and

$$A = 6 + \frac{8}{\sqrt{\beta_1(b_2)}} \left[\frac{2}{\sqrt{\beta_1(b_2)}} + \left\{ 1 + \frac{4}{\beta_1(b_2)} \right\}^{1/2} \right]$$

Then the distribution of the test statistic

$$Z_2 = \frac{1}{\sqrt{2/(9A)}} \left[\left(1 - \frac{2}{9A}\right) - \left\{ \frac{1-2/A}{1+X\sqrt{2/(A-4)}} \right\}^{1/3} \right]$$

is approximately standard normal under the null hypothesis that the data are distributed normally.

D'Agostino, Balanger, and D'Agostino Jr.'s omnibus test of normality uses the statistic

$$K^2 = Z_1^2 + Z_2^2$$

which has approximately a χ^2 distribution with 2 degrees of freedom under the null of normality.

Royston (1991c) proposed the following adjustment to the test of normality, which sktest uses by default. Let $\Phi(x)$ denote the cumulative standard normal distribution function for x, and let $\Phi^{-1}(p)$ denote the inverse cumulative standard normal function [that is, $x = \Phi^{-1}\{\Phi(x)\}$]. Define the following terms:

$$\begin{aligned} Z_c &= -\Phi^{-1}\left\{\exp\left(-\frac{1}{2}K^2\right)\right\} \\ Z_t &= 0.55n^{0.2} - 0.21 \\ a_1 &= (-5 + 3.46\ln n)\exp(-1.37\ln n) \\ b_1 &= 1 + (0.854 - 0.148\ln n)\exp(-0.55\ln n) \\ a_2 &= a_1 - \{2.13/(1 - 2.37\ln n)\}\, Z_t \end{aligned}$$

and

$$b_2 = 2.13/(1 - 2.37\ln n) + b1$$

If $Z_c < -1$ set $Z = Z_c$; else if $Z_c < Z_t$ set $Z = a_1 + b_1 Z_c$; else set $Z = a_2 + b_2 Z_c$. Define $P = 1 - \Phi(Z)$. Then $K^2 = -2\ln P$ is approximately distributed χ^2 with 2 degrees of freedom.

The relative merits of the skewness and kurtosis test versus the Shapiro–Wilk and Shapiro–Francia tests have been a subject of debate. The interested reader is directed to the articles in the *Stata Technical Bulletin*. Our recommendation is to use the Shapiro–Francia test whenever possible, that is, whenever dealing with nonaggregated or ungrouped data (Gould and Rogers 1991; Gould 1992); see [R] **swilk**. If normality is rejected, use sktest to determine the source of the problem.

As both D'Agostino, Belanger, and D'Agostino (1990) and Royston (1991d) mention, researchers should also examine the normal quantile plot to determine normality rather than blindly relying on a few test statistics. See the qnorm command documented in [R] **diagnostic plots** for more information on normal quantile plots.

sktest is similar in spirit to the Jarque–Bera (1987) test of normality. The Jarque–Bera test statistic is also calculated from the sample skewness and kurtosis, though it is based on asymptotic standard errors with no corrections for sample size. In effect, sktest offers two adjustments for sample size, that of Royston (1991c) and that of D'Agostino, Belanger, and D'Agostino (1990).

Acknowledgments

sktest has benefited greatly by the comments and work of Patrick Royston of the MRC Clinical Trials Unit, London, and coauthor of the Stata Press book *Flexible Parametric Survival Analysis Using Stata: Beyond the Cox Model*. At this point, the program should be viewed as due as much to Royston as to us, except, of course, for any errors. We are also indebted to Nicholas J. Cox of the Department of Geography at Durham University, UK, and coeditor of the *Stata Journal* for his helpful comments.

References

D'Agostino, R. B., A. J. Belanger, and R. B. D'Agostino, Jr. 1990. A suggestion for using powerful and informative tests of normality. *American Statistician* 44: 316–321.

——. 1991. sg3.3: Comment on tests of normality. *Stata Technical Bulletin* 3: 20. Reprinted in *Stata Technical Bulletin Reprints*, vol. 1, pp. 105–106. College Station, TX: Stata Press.

Gould, W. W. 1991. sg3: Skewness and kurtosis tests of normality. *Stata Technical Bulletin* 1: 20–21. Reprinted in *Stata Technical Bulletin Reprints*, vol. 1, pp. 99–101. College Station, TX: Stata Press.

——. 1992. sg11.1: Quantile regression with bootstrapped standard errors. *Stata Technical Bulletin* 9: 19–21. Reprinted in *Stata Technical Bulletin Reprints*, vol. 2, pp. 137–139. College Station, TX: Stata Press.

Gould, W. W., and W. H. Rogers. 1991. sg3.4: Summary of tests of normality. *Stata Technical Bulletin* 3: 20–23. Reprinted in *Stata Technical Bulletin Reprints*, vol. 1, pp. 106–110. College Station, TX: Stata Press.

Jarque, C. M., and A. K. Bera. 1987. A test for normality of observations and regression residuals. *International Statistical Review* 2: 163–172.

Marchenko, Y. V., and M. G. Genton. 2010. A suite of commands for fitting the skew-normal and skew-t models. *Stata Journal* 10: 507–539.

Royston, P. 1991a. sg3.1: Tests for departure from normality. *Stata Technical Bulletin* 2: 16–17. Reprinted in *Stata Technical Bulletin Reprints*, vol. 1, pp. 101–104. College Station, TX: Stata Press.

——. 1991b. sg3.2: Shapiro–Wilk and Shapiro–Francia tests. *Stata Technical Bulletin* 3: 19. Reprinted in *Stata Technical Bulletin Reprints*, vol. 1, p. 105. College Station, TX: Stata Press.

——. 1991c. sg3.5: Comment on sg3.4 and an improved D'Agostino test. *Stata Technical Bulletin* 3: 23–24. Reprinted in *Stata Technical Bulletin Reprints*, vol. 1, pp. 110–112. College Station, TX: Stata Press.

——. 1991d. sg3.6: A response to sg3.3: Comment on tests of normality. *Stata Technical Bulletin* 4: 8–9. Reprinted in *Stata Technical Bulletin Reprints*, vol. 1, pp. 112–114. College Station, TX: Stata Press.

Also see

[R] **diagnostic plots** — Distributional diagnostic plots

[R] **ladder** — Ladder of powers

[R] **lv** — Letter-value displays

[R] **swilk** — Shapiro–Wilk and Shapiro–Francia tests for normality

[MV] **mvtest normality** — Multivariate normality tests