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## **pwcompare** — Pairwise comparisons

Syntax Menu Description Options
Remarks and examples Stored results Methods and formulas References

Also see

# **Syntax**

options

```
pwcompare marginlist [, options]
```

where *marginlist* is a list of factor variables or interactions that appear in the current estimation results or <u>\_eqns</u> to reference equations. The variables may be typed with or without the i. prefix, and you may use any factor-variable syntax:

. pwcompare i.sex i.group i.sex#i.group

Description

- . pwcompare sex group sex#group
- . pwcompare sex##group

r	
Main	
mcompare(method)	adjust for multiple comparisons; default is mcompare(noadjust)
<u>asobs</u> erved	treat all factor variables as observed
Equations	
$\underline{\mathtt{eq}}\mathtt{uation}(\mathit{eqspec})$	perform comparisons within equation eqspec
$\underline{\underline{\text{ateq}}}$ uations	perform comparisons within each equation
Advanced	
<pre>emptycells(empspec)</pre>	treatment of empty cells for balanced factors
noestimcheck	suppress estimability checks
Reporting	
<u>l</u> evel(#)	confidence level; default is level(95)
<u>ci</u> effects	show effects table with confidence intervals; the default
pveffects	show effects table with p-values
<u>eff</u> ects	show effects table with confidence intervals and p-values
<u>cim</u> argins	show table of margins and confidence intervals
groups	show table of margins and group codes
sort	sort the margins or contrasts within each term
post	post margins and their VCEs as estimation results
display_options	control column formats, row spacing, line width, and factor-variable labeling
eform_option	report exponentiated contrasts
df(#)	use $t$ distribution with # degrees of freedom for computing $p$ -values and confidence intervals

df (#) does not appear in the dialog box.

method	Description
noadjust	do not adjust for multiple comparisons; the default
bonferroni adjustall	Bonferroni's method; adjust across all terms
sidak [adjustall]	Šidák's method; adjust across all terms
<u>sch</u> effe	Scheffé's method
* <u>tuk</u> ey	Tukey's method
* snk	Student-Newman-Keuls' method
* <u>dunc</u> an	Duncan's method
* <u>dunn</u> ett	Dunnett's method

<sup>\*</sup> tukey, snk, duncan, and dunnett are only allowed with results from anova, manova, regress, and mvreg. tukey, snk, duncan, and dunnett are not allowed with results from svy.

Time-series operators are allowed if they were used in the estimation.

### Menu

Statistics > Postestimation > Pairwise comparisons

## **Description**

pwcompare performs pairwise comparisons across the levels of factor variables from the most recently fit model. pwcompare can compare estimated cell means, marginal means, intercepts, marginal intercepts, slopes, or marginal slopes—collectively called margins. pwcompare reports the comparisons as contrasts (differences) of margins along with significance tests or confidence intervals for the contrasts. The tests and confidence intervals can be adjusted for multiple comparisons.

pwcompare can be used with svy estimation results; see [SVY] svy postestimation.

See [R] margins, pwcompare for performing pairwise comparisons of margins of linear and nonlinear predictions.

# **Options**

Main

mcompare(*method*) specifies the method for computing *p*-values and confidence intervals that account for multiple comparisons within a factor-variable term.

Most methods adjust the comparisonwise error rate,  $\alpha_c$ , to achieve a prespecified experimentwise error rate,  $\alpha_e$ .

mcompare(noadjust) is the default; it specifies no adjustment.

$$\alpha_c = \alpha_e$$

mcompare(bonferroni) adjusts the comparisonwise error rate based on the upper limit of the Bonferroni inequality:

$$\alpha_e \leq m\alpha_c$$

where m is the number of comparisons within the term.

The adjusted comparisonwise error rate is

$$\alpha_c = \alpha_e/m$$

mcompare(sidak) adjusts the comparisonwise error rate based on the upper limit of the probability inequality

$$\alpha_e \leq 1 - (1 - \alpha_c)^m$$

where m is the number of comparisons within the term.

The adjusted comparisonwise error rate is

$$\alpha_c = 1 - (1 - \alpha_e)^{1/m}$$

This adjustment is exact when the m comparisons are independent.

mcompare(scheffe) controls the experimentwise error rate using the F (or  $\chi^2$ ) distribution with degrees of freedom equal to the rank of the term.

For results from anova, regress, manova, and mvreg (see [R] anova, [R] regress, [MV] manova, and [MV] mvreg), pwcompare allows the following additional methods. These methods are not allowed with results that used vce(robust) or vce(cluster clustvar).

mcompare(tukey) uses what is commonly referred to as Tukey's honestly significant difference. This method uses the Studentized range distribution instead of the t distribution.

mcompare(snk) is a variation on mcompare(tukey) that counts only the number of margins in the range for a given comparison instead of the full number of margins.

mcompare(duncan) is a variation on mcompare(snk) with additional adjustment to the significance probabilities.

mcompare(dunnett) uses Dunnett's method for making comparisons with a reference category.

mcompare(method adjustall) specifies that the multiple-comparison adjustments count all comparisons across all terms rather than performing multiple comparisons term by term. This leads to more conservative adjustments when multiple variables or terms are specified in marginlist. This option is compatible only with the bonferroni and sidak methods.

asobserved specifies that factor covariates be evaluated using the cell frequencies observed when the model was fit. The default is to treat all factor covariates as though there were an equal number of observations at each level.

Equations

equation(eqspec) specifies the equation from which margins are to be computed. The default is to compute margins from the first equation.

atequations specifies that the margins be computed within each equation.

Advanced

emptycells(*empspec*) specifies how empty cells are handled in interactions involving factor variables that are being treated as balanced.

emptycells(strict) is the default; it specifies that margins involving empty cells be treated as not estimable.

emptycells(reweight) specifies that the effects of the observed cells be increased to accommodate any missing cells. This makes the margins estimable but changes their interpretation.

noestimcheck specifies that pwcompare not check for estimability. By default, the requested margins are checked and those found not estimable are reported as such. Nonestimability is usually caused by empty cells. If noestimcheck is specified, estimates are computed in the usual way and reported even though the resulting estimates are manipulable, which is to say they can differ across equivalent models having different parameterizations.

Reporting

- level(#) specifies the confidence level, as a percentage, for confidence intervals. The default is
  level(95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.
  The significance level used by the groups option is 100 #, expressed as a percentage.
- cieffects specifies that a table of the pairwise comparisons with their standard errors and confidence intervals be reported. This is the default.
- pveffects specifies that a table of the pairwise comparisons with their standard errors, test statistics, and p-values be reported.
- effects specifies that a table of the pairwise comparisons with their standard errors, test statistics, p-values, and confidence intervals be reported.
- cimargins specifies that a table of the margins with their standard errors and confidence intervals be reported.
- groups specifies that a table of the margins with their standard errors and group codes be reported. Margins with the same letter in the group code are not significantly different at the specified significance level.
- sort specifies that the reported tables be sorted on the margins or differences in each term.
- post causes pwcompare to behave like a Stata estimation (e-class) command. pwcompare posts the vector of estimated margins along with the estimated variance—covariance matrix to e(), so you can treat the estimated margins just as you would results from any other estimation command. For example, you could use test to perform simultaneous tests of hypotheses on the margins, or you could use lincom to create linear combinations.
- display\_options: vsquish, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt),
  pformat(%fmt), sformat(%fmt), and nolstretch.
  - vsquish specifies that the blank space separating factor-variable terms or time-series-operated variables from other variables in the model be suppressed.
  - nofvlabel displays factor-variable level values rather than attached value labels. This option overrides the fvlabel setting; see [R] set showbaselevels.
  - fvwrap(#) specifies how many lines to allow when long value labels must be wrapped. Labels
    requiring more than # lines are truncated. This option overrides the fvwrap setting; see [R] set
    showbaselevels.
  - fvwrapon(style) specifies whether value labels that wrap will break at word boundaries or break based on available space.
    - fvwrapon(word), the default, specifies that value labels break at word boundaries.
    - fvwrapon(width) specifies that value labels break based on available space.
    - This option overrides the further setting; see [R] set showbaselevels.
  - cformat(%fmt) specifies how to format contrasts or margins, standard errors, and confidence limits in the table of pairwise comparisons.
  - pformat (% fint) specifies how to format p-values in the table of pairwise comparisons.
  - sformat (% fmt) specifies how to format test statistics in the table of pairwise comparisons.
  - nolstretch specifies that the width of the table of pairwise comparisons not be automatically widened to accommodate longer variable names. The default, lstretch, is to automatically widen the table of pairwise comparisons up to the width of the Results window. To change the default, use set lstretch off. nolstretch is not shown in the dialog box.

eform\_option specifies that the contrasts table be displayed in exponentiated form.  $e^{\text{contrast}}$  is displayed rather than contrast. Standard errors and confidence intervals are also transformed. See [R] eform\_option for the list of available options.

The following option is available with pwcompare but is not shown in the dialog box:

df(#) specifies that the t distribution with # degrees of freedom be used for computing p-values and confidence intervals. The default is to use  $e(df_r)$  degrees of freedom or the standard normal distribution if  $e(df_r)$  is missing.

# Remarks and examples

stata.com

pwcompare performs pairwise comparisons of margins across the levels of factor variables from the most recently fit model. The margins can be estimated cell means, marginal means, intercepts, marginal intercepts, slopes, or marginal slopes. With the exception of slopes, we can also consider these margins to be marginal linear predictions.

The margins are calculated as linear combinations of the coefficients. Let k be the number of levels for a factor term in our model; then there are k margins for that term, and

$$m = \binom{k}{2} = \frac{k(k-1)}{2}$$

unique pairwise comparisons of those margins.

The confidence intervals and *p*-values for these pairwise comparisons can be adjusted to account for multiple comparisons. Bonferroni's, Šidák's, and Scheffé's adjustments can be made for multiple comparisons after fitting any type of model. In addition, Tukey's, Student–Newman–Keuls', Duncan's, and Dunnett's adjustments are available when fitting ANOVA, linear regression, MANOVA, or multivariate regression models.

Remarks are presented under the following headings:

Pairwise comparisons of means

Marginal means

All pairwise comparisons

Overview of multiple-comparison methods

Fisher's protected least-significant difference (LSD)

Bonferroni's adjustment

Šidák's adjustment

Scheffé's adjustment

Tukey's HSD adjustment

Student-Newman-Keuls' adjustment

Duncan's adjustment

Dunnett's adjustment

Example adjustments using one-way models

Fisher's protected LSD

Tukey's HSD

Dunnett's method for comparisons to a control

Two-way models

Pairwise comparisons of slopes

Nonlinear models

Multiple-equation models

Unbalanced data

Empty cells

#### F

## Pairwise comparisons of means

Suppose we are interested in the effects of five different fertilizers on wheat yield. We could estimate the following linear regression model to determine the effect of each type of fertilizer on the yield.

- . use http://www.stata-press.com/data/r13/yield
  (Artificial wheat yield dataset)
- . regress yield i.fertilizer

Source	SS	df		MS		Number of obs F( 4, 195)		200 5.33
Model Residual	1078.84207 9859.55334	4 195		710517 561812		Prob > F R-squared	=	0.0004 0.0986 0.0801
Total	10938.3954	199	54.9	668111		Adj R-squared Root MSE	=	7.1107
yield	Coef.	Std.	Err.	t	P> t	[95% Conf.	In	terval]
fertilizer								
10-08-22	3.62272	1.589	997	2.28	0.024	.4869212	6	.758518
16-04-08	.4906299	1.589	997	0.31	0.758	-2.645169	3	.626428
18-24-06	4.922803	1.589	997	3.10	0.002	1.787005	8	.058602
29-03-04	-1.238328	1.589	997	-0.78	0.437	-4.374127		1.89747
_cons	41.36243	1.124	298	36.79	0.000	39.14509	4	3.57977

In this simple case, the coefficients for fertilizers 10-08-22, 16-04-08, 18-24-06, and 29-03-04 indicate the difference in the mean yield for that fertilizer versus the mean yield for fertilizer 10-10-10. That the standard errors of all four coefficients are identical results from having perfectly balanced data.

### Marginal means

We can use pwcompare with the cimargins option to compute the mean yield for each of the fertilizers.

. pwcompare fertilizer, cimargins

Pairwise comparisons of marginal linear predictions

Margins : asbalanced

			Unadjusted
	Margin	Std. Err.	[95% Conf. Interval]
fertilizer			
10-10-10	41.36243	1.124298	39.14509 43.57977
10-08-22	44.98515	1.124298	42.7678 47.20249
16-04-08	41.85306	1.124298	39.63571 44.0704
18-24-06	46.28523	1.124298	44.06789 48.50258
29-03-04	40.1241	1.124298	37.90676 42.34145

Looking at the confidence intervals for fertilizers 10-10-10 and 10-08-22 in the table above, we might be tempted to conclude that these means are not significantly different because the intervals overlap. However, as discussed in *Interaction plots* of [R] **marginsplot**, we cannot draw conclusions about the differences in means by looking at confidence intervals for the means themselves. Instead, we would need to look at confidence intervals for the difference in means.

### All pairwise comparisons

By default, pwcompare calculates all pairwise differences of the margins, in this case pairwise differences of the mean yields.

. pwcompare fertilizer

Pairwise comparisons of marginal linear predictions

Margins : asbalanced

			Unadj	usted
	Contrast	Std. Err.	[95% Conf.	Interval]
fertilizer				
10-08-22 vs 10-10-10	3.62272	1.589997	.4869212	6.758518
16-04-08 vs 10-10-10	.4906299	1.589997	-2.645169	3.626428
18-24-06 vs 10-10-10	4.922803	1.589997	1.787005	8.058602
29-03-04 vs 10-10-10	-1.238328	1.589997	-4.374127	1.89747
16-04-08 vs 10-08-22	-3.13209	1.589997	-6.267889	.0037086
18-24-06 vs 10-08-22	1.300083	1.589997	-1.835715	4.435882
29-03-04 vs 10-08-22	-4.861048	1.589997	-7.996847	-1.725249
18-24-06 vs 16-04-08	4.432173	1.589997	1.296375	7.567972
29-03-04 vs 16-04-08	-1.728958	1.589997	-4.864757	1.406841
29-03-04 vs 18-24-06	-6.161132	1.589997	-9.29693	-3.025333

If a confidence interval does not include zero, the means for the compared fertilizers are significantly different. Therefore, at the 5% significance level, we would reject the hypothesis that the means for fertilizers 10-10-10 and 10-08-22 are equivalent—as we would do for 18-24-06 vs 10-10-10, 29-03-04 vs 10-08-22, 18-24-06 vs 16-04-08, and 29-03-04 vs 18-24-06.

We may prefer to see the p-values instead of looking at confidence intervals to determine whether the pairwise differences are significantly different from zero. We could use the pveffects option to see the differences with standard errors and p-values, or we could use the effects option to see both p-values and confidence intervals in the same table. Here we specify effects as well as the sort option so that the differences are sorted from smallest to largest.

. pwcompare fertilizer, effects sort

Pairwise comparisons of marginal linear predictions

Margins : asbalanced

		Unadj		justed	Unadj	usted
	Contrast	Std. Err.	t	P> t	[95% Conf.	Interval]
fertilizer 29-03-04						
vs 18-24-06 29-03-04	-6.161132	1.589997	-3.87	0.000	-9.29693	-3.025333
vs 10-08-22 16-04-08	-4.861048	1.589997	-3.06	0.003	-7.996847	-1.725249
vs 10-08-22 29-03-04	-3.13209	1.589997	-1.97	0.050	-6.267889	.0037086
vs 16-04-08 29-03-04	-1.728958	1.589997	-1.09	0.278	-4.864757	1.406841
vs 10-10-10 16-04-08	-1.238328	1.589997	-0.78	0.437	-4.374127	1.89747
vs 10-10-10 18-24-06	.4906299	1.589997	0.31	0.758	-2.645169	3.626428
vs 10-08-22 10-08-22	1.300083	1.589997	0.82	0.415	-1.835715	4.435882
vs 10-10-10 18-24-06	3.62272	1.589997	2.28	0.024	.4869212	6.758518
vs 16-04-08 18-24-06	4.432173	1.589997	2.79	0.006	1.296375	7.567972
vs 10-10-10	4.922803	1.589997	3.10	0.002	1.787005	8.058602

We find that 5 of the 10 pairs of means are significantly different at the 5% significance level.

We can use the groups option to obtain a table that identifies groups whose means are not significantly different by assigning them the same letter.

. pwcompare fertilizer, groups sort

Pairwise comparisons of marginal linear predictions

Margins : asbalanced

	Margin	Std. Err.	Unadjusted Groups
fertilizer			
29-03-04	40.1241	1.124298	A
10-10-10	41.36243	1.124298	A
16-04-08	41.85306	1.124298	AB
10-08-22	44.98515	1.124298	BC
18-24-06	46.28523	1.124298	C

Note: Margins sharing a letter in the group label are not significantly different at the 5% level.

The letter A that is assigned to fertilizers 29-03-04, 10-10-10, and 16-04-08 designates that the mean yields for these fertilizers are not different at the 5% level.

## Overview of multiple-comparison methods

For a single test, if we choose a 5% significance level, we would have a 5% chance of concluding that two margins are different when the population values are actually equal. This is known as making a type I error. When we perform m = k(k-1)/2 pairwise comparisons of the k margins, we have m opportunities to make a type I error.

pwcompare with the mcompare() option allows us to adjust the confidence intervals and p-values for each comparison to account for the increased probability of making a type I error when making multiple comparisons. Bonferroni's adjustment, Šidák's adjustment, and Scheffé's adjustment can be used when making pairwise comparisons of the margins after any estimation command. Tukey's honestly significant difference, Student-Newman-Keuls' method, Duncan's method, and Dunnett's method are only available when fitting linear models after anova, manova, regress, or mvreg.

## Fisher's protected least-significant difference (LSD)

pwcompare does not offer an mcompare() option specifically for Fisher's protected least-significant difference (LSD). In this methodology, no adjustment is made to the confidence intervals or *p*-values. However, it is protected in the sense that no pairwise comparisons are tested unless the joint test for the corresponding term in the model is significant. Therefore, the default mcompare(noadjust) corresponds to Fisher's protected LSD assuming that the corresponding joint test was performed before using pwcompare.

Milliken and Johnson (2009) recommend using this methodology for planned comparisons, assuming the corresponding joint test is significant.

## Bonferroni's adjustment

mcompare(bonferroni) adjusts significance levels based on the Bonferroni inequality, which, in the case of multiple testing, tells us that the maximum error rate for all comparisons is the sum of the error rates for the individual comparisons. Assuming that we are using the same significance level for all tests, the experimentwise error rate is the error rate for a single test multiplied by the

number of comparisons. Therefore, a p-value for each comparison can be computed by multiplying the unadjusted p-value by the total number of comparisons. If the adjusted p-value is greater than 1, then pwcompare will report a p-value of 1.

Bonferroni's adjustment is popular because it is easy to compute manually and because it can be applied to any set of tests, not only the pairwise comparisons available in pwcompare. In addition, this method does not require equal sample sizes.

Because Bonferroni's adjustment is so general, it is more conservative than many of the other adjustments. It is especially conservative when a large number of tests is being performed.

## Šidák's adjustment

mcompare(sidak) performs an adjustment using Šidák's method. This adjustment, like Bonferroni's adjustment, is derived from an inequality. However, in this case, the inequality is based on the probability of not making a type I error. For a single test, the probability that we do not make a type I error is  $1-\alpha$ . For two independent tests, both using  $\alpha$  as a significance level, the probability is  $(1-\alpha)(1-\alpha)$ . Likewise, for m independent tests, the probability of not making a type I error is  $(1-\alpha)^m$ . Therefore, the probability of making one or more type I errors is  $1-(1-\alpha)^m$ . When tests are not independent, the probability of making at least one error is less than  $1-(1-\alpha)^m$ . Therefore, we can compute an adjusted p-value as  $1-(1-up)^m$ , where up is the unadjusted p-value for a single comparison.

Šidák's method is also conservative although slightly less so than Bonferroni's method. Like Bonferroni's method, this method does not require equal sample sizes.

### Scheffé's adjustment

Scheffé's adjustment is used when mcompare(scheffe) is specified. This adjustment is derived from the joint F test and its correspondence to the maximum normalized comparison. To adjust for multiple comparisons, the absolute value of the t statistic for a particular comparison can be compared with a critical value of  $\sqrt{(k-1)F_{k-1,\nu}}$ , where  $\nu$  is the residual degrees of freedom.  $F_{k-1,\nu}$  is the distribution of the joint F test for the corresponding term in a one-way ANOVA model. Winer, Brown, and Michels (1991, 191–195) discuss this in detail. For estimation commands that report z statistics instead of t statistics for the tests on coefficients, a  $\chi^2$  distribution is used instead of an F distribution.

Scheffé's method allows for making all possible comparisons of the k margins, not just the pairwise comparisons. Unlike the methods described above, it does not take into account the number of comparisons that are currently being made. Therefore, this method is even more conservative than the others. Because this method adjusts for all possible comparisons of the levels of the term, Milliken and Johnson (2009) recommend using this procedure when making unplanned contrasts that are suggested by the data. As Winer, Brown, and Michels (1991, 191) put it, this method is often used to adjust for "unfettered data snooping". When using this adjustment, a contrast will never be significant if the joint F or  $\chi^2$  test for the term is not also significant.

This is another method that does not require equal sample sizes.

### Tukey's HSD adjustment

Tukey's adjustment is also referred to as Tukey's honestly significant difference (HSD) and is used when mcompare(tukey) is specified. It is often applied to all pairwise comparisons of means. Tukey's HSD is commonly used as a post hoc test although this is not a requirement.

To adjust for multiple comparisons, Tukey's method compares the absolute value of the t statistic from the individual comparison with a critical value based on a Studentized range distribution with parameter equal to the number of levels in the term. When applied to pairwise comparisons of means,

$$q = \frac{\text{mean}_{\text{max}} - \text{mean}_{\text{min}}}{s}$$

follows a Studentized range distribution with parameter k and  $\nu$  degrees of freedom. Here mean $_{\max}$  and mean $_{\min}$  are the largest and smallest marginal means, and s is an estimate of the standard error of the means.

Now for the comparison of the smallest and largest means, we can say that the probability of not making a type I error is

$$\Pr\left(\frac{\text{mean}_{\max} - \text{mean}_{\min}}{s} \le q_{k,\nu}\right) = 1 - \alpha$$

Then the following inquality holds for all pairs of means simultaneously:

$$\Pr\left(\frac{|\mathsf{mean}_i - \mathsf{mean}_j|}{s} \le q_{k,\nu}\right) \ge 1 - \alpha$$

Based on this procedure, Tukey's HSD computes the *p*-value for each of the individual comparisons using the Studentized range distribution. However, because the equality holds only for the difference in the largest and smallest means, this procedure produces conservative tests for the remaining comparisons. Winer, Brown, and Michels (1991, 172–182) discuss this in further detail.

With unequal sample sizes, mcompare(tukey) produces the Tukey-Kramer adjustment (Tukey 1953; Kramer 1956).

#### Student-Newman-Keuls' adjustment

The Student-Newman-Keuls (SNK) method is used when mcompare(snk) is specified. It is a modification to Tukey's method and is less conservative. In this procedure, we first order the means. We then test the difference in the smallest and largest means using a critical value from the Studentized range distribution with parameter k, where k is the number of levels in the term. This step uses the same methodology as in Tukey's procedure. However, in the next step, we will then test for differences in the two sets of means that are the endpoints of the two ranges including k-1 means. Specifically, we test the difference in the smallest mean and the second-largest mean using a critical value from the Studentized range distribution with parameter k-1. We would also test the difference in the second-smallest mean and the largest mean using this critical value. Likewise, the means that are the endpoints of ranges including k-2 means when ordered are tested using the Studentized range distribution with parameter k-2, and so on.

Equal sample sizes are required for this method.

### Duncan's adjustment

When mcompare (duncan) is specified, tests are adjusted for multiple comparisons using Duncan's method, which is sometimes referred to as Duncan's new multiple range method. This adjustment produces tests that are less conservative than both Tukey's HSD and SNK. This procedure is performed in the same manner as SNK except that the p-values for the individual comparisons are adjusted as  $1-(1-{\rm snk}p_i)^{1/(r+1)}$ , where  ${\rm snk}p$  is the p-value computed using the SNK method and r represents the number of means that, when ordered, fall between the two that are being compared.

Again equal sample sizes are required for this adjustment.

### **Dunnett's adjustment**

Dunnett's adjustment is obtained by specifying mcompare(dunnett). It is used when one of the levels of a factor can be considered a control or reference level with which each of the other levels is being compared. When Dunnett's adjustment is requested, k-1 instead of k(k-1)/2 pairwise comparisons are made. Dunnett (1955, 1964) developed tables of critical values for what Miller (1981, 76) refers to as the "many-one t statistic". The t statistics for individual comparisons are compared with these critical values when making many comparisons to a single reference level.

This method also requires equal sample sizes.

### Example adjustments using one-way models

#### Fisher's protected LSD

Fisher's protected LSD requires that we first verify that the joint test for a term in our model is significant before proceeding with pairwise comparisons. Using our previous example, we could have first used the contrast command to obtain a joint test for the effects of fertilizer.

. contrast fertilizer

Contrasts of marginal linear predictions

Margins : asbalanced

	df	F	P>F
fertilizer	4	5.33	0.0004
Denominator	195		

This test for the effects of fertilizer is highly significant. Now we can say we are using Fisher's protected LSD when looking at the unadjusted p-values that were obtained from our previous command,

. pwcompare fertilizer, effects sort

### Tukey's HSD

Because we fit a linear regression model and are interested in all pairwise comparisons of the marginal means, we may instead choose to use Tukey's HSD.

. pwcompare fertilizer, effects sort mcompare(tukey)
Pairwise comparisons of marginal linear predictions
Margins : asbalanced

Number of Comparisons

			Tu	key	Tul	ĸev
	Contrast	Std. Err.	t	P> t		. Interval]
fertilizer 29-03-04						
vs 18-24-06 29-03-04	-6.161132	1.589997	-3.87	0.001	-10.53914	-1.78312
vs 10-08-22 16-04-08	-4.861048	1.589997	-3.06	0.021	-9.239059	4830368
vs 10-08-22 29-03-04	-3.13209	1.589997	-1.97	0.285	-7.510101	1.245921
vs 16-04-08 29-03-04	-1.728958	1.589997	-1.09	0.813	-6.106969	2.649053
vs 10-10-10 16-04-08	-1.238328	1.589997	-0.78	0.936	-5.616339	3.139683
vs 10-10-10 18-24-06	.4906299	1.589997	0.31	0.998	-3.887381	4.868641
vs 10-08-22 10-08-22	1.300083	1.589997	0.82	0.925	-3.077928	5.678095
vs 10-10-10 18-24-06	3.62272	1.589997	2.28	0.156	7552913	8.000731
vs 16-04-08 18-24-06	4.432173	1.589997	2.79	0.046	.0541623	8.810185
vs 10-10-10	4.922803	1.589997	3.10	0.019	.5447922	9.300815

This time, our p-values have been modified, and we find that only four of the pairwise differences are considered significantly different from zero at the 5% level.

If we only are interested in performing pairwise comparisons of a subset of our means, we can use factor-variable operators to select the levels of the factor that we want to compare. Here we exclude all comparisons involving fertilizer 10-10-10.

. pwcompare i(2/5).fertilizer, effects sort mcompare(tukey)

Pairwise comparisons of marginal linear predictions

Margins : asbalanced

	Number of Comparisons
fertilizer	6

	Contrast	Std. Err.	Tul t	key P> t	Tuk [95% Conf.	xey Interval]
fertilizer 29-03-04						
vs 18-24-06 29-03-04	-6.161132	1.589997	-3.87	0.001	-10.28133	-2.040937
vs 10-08-22 16-04-08	-4.861048	1.589997	-3.06	0.013	-8.981242	7408538
vs 10-08-22 29-03-04	-3.13209	1.589997	-1.97	0.203	-7.252284	.9881042
vs 16-04-08 18-24-06	-1.728958	1.589997	-1.09	0.698	-5.849152	2.391236
vs 10-08-22 18-24-06	1.300083	1.589997	0.82	0.846	-2.820111	5.420278
vs 16-04-08	4.432173	1.589997	2.79	0.030	.3119792	8.552368

The adjusted p-values and confidence intervals differ from those in the previous output because Tukey's adjustment takes into account the total number of comparisons being made when determining the appropriate degrees of freedom to use for the Studentized range distribution.

## Dunnett's method for comparisons to a control

If one of our five fertilizer groups represents fields where no fertilizer was applied, we may want to use Dunnett's method to compare each of the four fertilizers with the control group. In this case, we make only k-1 comparisons for k groups.

. pwcompare fertilizer, effects mcompare(dunnett)
Pairwise comparisons of marginal linear predictions

Margins : asbalanced

	Number of Comparisons
fertilizer	4

			Dun	nett	Dunn	ett
	Contrast	Std. Err.	t	P> t	[95% Conf.	Interval]
fertilizer 10-08-22						
10-10-10 16-04-08	3.62272	1.589997	2.28	0.079	2918331	7.537273
10-10-10 18-24-06 vs	.4906299	1.589997	0.31	0.994	-3.423923	4.405183
10-10-10 29-03-04 vs	4.922803	1.589997	3.10	0.008	1.00825	8.837356
10-10-10	-1.238328	1.589997	-0.78	0.852	-5.152881	2.676225

In our previous regress command, fertilizer 10-10-10 was treated as the base. Therefore, by default, it was treated as the control when using Dunnett's adjustment, and the pairwise comparisons are equivalent to the coefficients reported by regress. Based on our regress output, we would conclude that fertilizers 10-08-22 and 18-24-06 are different from fertilizer 10-10-10 at the 5% level. However, using Dunnett's adjustment, we find only fertilizer 18-24-06 to be different from fertilizer 10-10-10 at this same significance level.

If the model is fit without a base level for a factor variable, then pwcompare will choose the first level as the reference level. If we want to make comparisons with a different level than the one mcompare(dunnett) chooses by default, we can use the b. operator to override the default. Here we use fertilizer 5 (29-03-04) as the reference level.

. pwcompare b5.fertilizer, effects sort mcompare(dunnett)

Pairwise comparisons of marginal linear predictions

Margins : asbalanced

	Number of Comparisons
fertilizer	4

			Dun	nett	Dunn	ett
	Contrast	Std. Err.	t	P> t	[95% Conf.	Interval]
fertilizer						
vs 29-03-04 16-04-08	1.238328	1.589997	0.78	0.852	-2.676225	5.152881
vs 29-03-04 10-08-22	1.728958	1.589997	1.09	0.649	-2.185595	5.643511
vs 29-03-04 18-24-06	4.861048	1.589997	3.06	0.009	.9464951	8.775601
vs 29-03-04	6.161132	1.589997	3.87	0.001	2.246579	10.07568

## Two-way models

In the previous examples, we have performed pairwise comparisons after fitting a model with a single factor. Now we include two factors and their interaction in our model.

. regress yield fertilizer##irrigation

Source	SS	df		MS		Number of obs		200
						F( 9, 190)	=	27.63
Model	6200.81605	9		979561		Prob > F	=	0.0000
Residual	4737.57936	190	24.9	346282		R-squared	=	0.5669
						Adj R-squared	=	0.5464
Total	10938.3954	199	54.9	668111		Root MSE	=	4.9935
yield	Coef.	Std. 1	Err.	t	P> t	[95% Conf.	In	terval]
fertilizer								
10-08-22	1.882256	1.579	907	1.19	0.235	-1.232505	4	.997016
16-04-08	5687418	1.579	907	-0.36	0.719	-3.683502	2	.546019
18-24-06	4.904999	1.579	907	3.11	0.002	1.790239		8.01976
29-03-04	-1.217496	1.57	907	-0.77	0.442	-4.332257	1	.897264
1.irrigation	8.899721	1.579	907	5.64	0.000	5.784961	1	2.01448
fertilizer#								
irrigation								
10-08-22#1	3.480928	2.233	143	1.56	0.121	9240084	7	.885865
16-04-08#1	2.118743	2.233	143	0.95	0.344	-2.286193		6.52368
18-24-06#1	.0356082	2.233	143	0.02	0.987	-4.369328	4	.440545
29-03-04#1	0416636	2.233	143	-0.02	0.985	-4.4466	4	.363273
_cons	36.91257	1.116	571	33.06	0.000	34.7101	3	9.11504

We can perform pairwise comparisons of the cell means defined by the fertilizer and irrigation interaction.

. pwcompare fertilizer#irrigation, sort groups mcompare(tukey)

Pairwise comparisons of marginal linear predictions

Margins : asbalanced

	Number of Comparisons
fertilizer#irrigation	45

	Margin	Std. Err.	Tukey Groups
fertilizer#irrigation			
29-03-04#0	35.69507	1.116571	A
16-04-08#0	36.34383	1.116571	A
10-10-10#0	36.91257	1.116571	AB
10-08-22#0	38.79482	1.116571	AB
18-24-06#0	41.81757	1.116571	BC
29-03-04#1	44.55313	1.116571	CD
10-10-10#1	45.81229	1.116571	CDE
16-04-08#1	47.36229	1.116571	DEF
18-24-06#1	50.7529	1.116571	EF
10-08-22#1	51.17547	1.116571	F
	ı		

Note: Margins sharing a letter in the group label are not significantly different at the 5% level.

Based on Tukey's HSD and a 5% significance level, we would conclude that the mean yield for fertilizer 29-03-04 without irrigation is not significantly different from the mean yields for fertilizers 10-10-10, 10-08-22, and 16-04-08 when used without irrigation but is significantly different from the remaining means.

Up to this point, most of the pairwise comparisons that we have performed could have also been obtained with pwmean (see [R] pwmean) if we had not been interested in examining the results from the estimation command before making pairwise comparisons of the means. For instance, we could reproduce the results from the above pwcompare command by typing

. pwmean yield, over(fertilizer irrigation) sort group mcompare(tukey)

However, pwcompare extends the capabilities of pwmean in many ways. For instance, pwmean only allows for pairwise comparisons of the cell means determined by the highest level interaction of the variables specified in the over() option. However, pwcompare allows us to fit a single model, such as the two-way model that we fit above,

. regress yield fertilizer##irrigation

and compute pairwise comparisons of the marginal means for only one of the variables in the model:

. pwcompare fertilizer, sort effects mcompare(tukey)

Pairwise comparisons of marginal linear predictions

Margins : asbalanced

	Number of Comparisons
fertilizer	10

	T					
			Tul	key	Tuk	
	Contrast	Std. Err.	t	P> t	[95% Conf.	Interval]
fertilizer 29-03-04						
vs 18-24-06 29-03-04	-6.161132	1.116571	-5.52	0.000	-9.236338	-3.085925
vs 10-08-22 16-04-08	-4.861048	1.116571	-4.35	0.000	-7.936255	-1.785841
vs 10-08-22 29-03-04	-3.13209	1.116571	-2.81	0.044	-6.207297	0568832
vs 16-04-08 29-03-04	-1.728958	1.116571	-1.55	0.532	-4.804165	1.346249
vs 10-10-10 16-04-08	-1.238328	1.116571	-1.11	0.802	-4.313535	1.836879
vs 10-10-10 18-24-06	.4906299	1.116571	0.44	0.992	-2.584577	3.565837
vs 10-08-22 10-08-22	1.300083	1.116571	1.16	0.772	-1.775123	4.37529
vs 10-10-10 18-24-06	3.62272	1.116571	3.24	0.012	.5475131	6.697927
vs 16-04-08 18-24-06	4.432173	1.116571	3.97	0.001	1.356967	7.50738
vs 10-10-10	4.922803	1.116571	4.41	0.000	1.847597	7.99801

Here the standard errors for the differences in marginal means and the residual degrees of freedom are based on the full model. Therefore, the results will differ from those obtained from pwcompare after fitting the one-way model with only fertilizer (or equivalently using pwmean).

## Pairwise comparisons of slopes

If we fit a model with a factor variable that is interacted with a continuous variable, pwcompare will even allow us to make pairwise comparisons of the slopes of the continuous variable for the levels of the factor variable.

In this case, we have a continuous variable, NO3\_N, indicating the amount of nitrate nitrogen already existing in the soil, based on a sample taken from each field.

. regress yie	ld fertilizer#	#c.NO3_	N			
Source	SS	df	MS		Number of obs	
	7005 40000		770 444005		F( 9, 190)	
Model	7005.69932	-	778.411035		Prob > F	= 0.0000
Residual	3932.69609	190	20.6984005		R-squared Adj R-squared	= 0.6405 = 0.6234
Total	10938.3954	199	54.9668111		Root MSE	= 4.5495
yield	Coef.	Std. E	rr. t	P> t	[95% Conf.	Interval]
fertilizer						
10-08-22	18.65019	8.4520	61 2.21	0.029	1.97826	35.32212
16-04-08	-13.34076	10.075	95 -1.32	0.187	-33.21585	6.534327
18-24-06	24.35061	9.9114	63 2.46	0.015	4.799973	43.90125
29-03-04	17.58529	8.4467	36 2.08	0.039	.9238646	34.24671
NO3_N	4.915653	.79835	09 6.16	0.000	3.340884	6.490423
fertilizer#						
c.N03_N						
10-08-22	-1.282039	.89534	19 -1.43	0.154	-3.048126	.4840487
16-04-08	-1.00571	.90258	62 -1.11	0.267	-2.786087	.7746662
18-24-06	-2.97627	.91363		0.001	-4.778438	-1.174102
29-03-04	-3.275947	.82473	85 -3.97	0.000	-4.902767	-1.649127
_cons	-5.459168	7.6382	41 -0.71	0.476	-20.52581	9.607477

These are the pairwise differences of the slopes of NO3\_N for each pair of fertilizers:

. pwcompare fertilizer#c.NO3\_N, pveffects sort mcompare(scheffe)

Pairwise comparisons of marginal linear predictions

Margins : asbalanced

	Number of Comparisons
fertilizer#c.NO3_N	10

			Sch	effe
	Contrast	Std. Err.	t	P> t
fertilizer#c.NO3_N				
29-03-04 vs 10-10-10	-3.275947	.8247385	-3.97	0.004
18-24-06 vs 10-10-10	-2.97627	.9136338	-3.26	0.034
29-03-04 vs 16-04-08	-2.270237	.4691771	-4.84	0.000
29-03-04 vs 10-08-22	-1.993909	.4550851	-4.38	0.001
18-24-06 vs 16-04-08	-1.97056	.612095	-3.22	0.038
18-24-06 vs 10-08-22	-1.694232	.6013615	-2.82	0.099
10-08-22 vs 10-10-10	-1.282039	.8953419	-1.43	0.727
16-04-08 vs 10-10-10	-1.00571	.9025862	-1.11	0.871
29-03-04 vs 18-24-06	2996772	.4900939	-0.61	0.984
16-04-08 vs 10-08-22	.276328	.5844405	0.47	0.994

Using Scheffé's adjustment, we find that five of the pairs have significantly different slopes at the 5% level.

#### Nonlinear models

pwcompare can also perform pairwise comparisons of the marginal linear predictions after fitting a nonlinear model. For instance, we can use the dataset from Beyond linear models in [R] contrast and fit the following logistic regression model of patient satisfaction on hospital:

. use http://www.stata-press.com/data/r13/hospital (Artificial hospital satisfaction data)

. logit satisfied i.hospital

Iteration 0:  $log\ likelihood = -393.72216$ Iteration 1:  $log\ likelihood = -387.55736$ Iteration 2: log likelihood = -387.4768Iteration 3: log likelihood = -387.47679

Logistic regression

Number of obs 802 LR chi2(2) 12.49 Prob > chi2 0.0019 Pseudo R2 0.0159

Log likelihood = -387.47679

satisfied	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]
hospital 2 3	.5348129 .7354519	.2136021 .2221929	2.50 3.31	0.012 0.001	.1161604 .2999618	.9534654 1.170942
_cons	1.034708	.1391469	7.44	0.000	.7619855	1.307431

For this model, the marginal linear predictions are the predicted log odds for each hospital and can be obtained with the cimargins option:

. pwcompare hospital, cimargins

Pairwise comparisons of marginal linear predictions

Margins : asbalanced

	Margin	Std. Err.	Unadju [95% Conf.	
hospital				
1	1.034708	.1391469	.7619855	1.307431
2	1.569521	.1620618	1.251886	1.887157
3	1.77016	.1732277	1.43064	2.10968

The pairwise comparisons are, therefore, differences in the log odds. We can specify mcompare (bonferroni) and effects to request Bonferroni-adjusted p-values and confidence intervals.

. pwcompare hospital, effects mcompare(bonferroni)

Pairwise comparisons of marginal linear predictions

Margins : asbalanced

	Number of Comparisons
satisfied hospital	3

	Contrast	Std. Err.	Bonfe z	erroni P> z	Bonfe [95% Conf.	
satisfied hospital 2 vs 1 3 vs 1 3 vs 2	.5348129	.2136021	2.50	0.037	.0234537	1.046172
	.7354519	.2221929	3.31	0.003	.2035265	1.267377
	.200639	.2372169	0.85	1.000	3672535	.7685314

For nonlinear models, only Bonferroni's adjustment, Šidák's adjustment, and Scheffé's adjustment are available.

If we want pairwise comparisons reported as odds ratios, we can specify the or option.

. pwcompare hospital, effects mcompare(bonferroni) or Pairwise comparisons of marginal linear predictions

Margins : asbalanced

	Number of Comparisons
satisfied hospital	3

	Odds Ratio	Std. Err.	Bonfe z	erroni P> z		erroni Interval]
satisfied hospital 2 vs 1 3 vs 1 3 vs 2	1.707129	.3646464	2.50	0.037	1.023731	2.846733
	2.086425	.4635888	3.31	0.003	1.225718	3.551525
	1.222183	.2899226	0.85	1.000	.6926341	2.156597

Notice that these tests are still performed on the marginal linear predictions. The odds ratios reported here are the exponentiated versions of the pairwise differences of log odds in the previous output. For further discussion, see [R] **contrast**.

## Multiple-equation models

pwcompare works with models containing multiple equations. Commands such as intreg and gnbreg allow their ancillary parameters to be modeled as a function of independent variables, and pwcompare can compare the margins within these equations. The equation() option can be used to specify the equation for which pairwise comparisons of the margins should be made. The atequations option specifies that pairwise comparisons be computed for each equation. In addition, pwcompare allows a special pseudofactor for equation—called \_eqns—when working with results from manova, mvreg, mlogit, and mprobit.

Here we use the jaw fracture dataset described in example 4 of [MV] manova. We fit a multivariate regression model including one independent factor variable, fracture.

. use http://www.stata-press.com/data/r13/jaw

(Table 4.6 Two-Way Unbalanced Data for Fractures of the Jaw -- Rencher (1998))

. mvreg y1 y2 y3 = i.fracture

Equa	ation	Obs	Par	ms	RMS	E	"R-sq"		F		P
y1 y2 y3		27 27 27		3 3 3	10.4236 6.32539 5.97697	8	0.2966 0.1341 0.1024	1	.060804 .858342 .368879	0.01 0.17 0.27	77
		Co	ef.	Sto	d. Err.	t	P>	t	[95%	Conf.	Interval]
	fracture compo simpl	-8.833	333		957441 394759	-1.7 1.1		087 277	-19.00 -5.13		1.398322 17.13423
	_cons		37	3.9	939775	9.3	9 0.	000	28.	8687	45.1313
y2	fracture										
	compo simpl	-5.761 -3.053			008327 273705	-1.9 -0.9		067 360	-11.9° -9.81		.446977 3.703023
	_cons	38.42	857	2.3	390776	16.0	7 0.	000	33.4	9425	43.36289
у3	fracture										
	compo simpl	4.261 .9285			342618 093377	1.5 0.3		147 767	-1.66 -5.45		10.12878 7.312989
	_cons	58.57	143	2.2	259083	25.9	3 0.	000	53.9	0891	63.23395

pwcompare performs pairwise comparisons of the margins using the coefficients from the first equation by default:

. pwcompare fracture, mcompare(bonferroni)

Pairwise comparisons of marginal linear predictions

Margins : asbalanced

		Number of Comparisons
у1	fracture	3

			Bonfe	rroni
	Contrast	Std. Err.	[95% Conf.	<pre>Interval]</pre>
y1				
fracture				
two compound fractures				
vs				
one compound fracture	-8.833333	4.957441	-21.59201	3.925341
one simple fracture				
vs				
one compound fracture	6	5.394759	-7.884173	19.88417
one simple fracture				
vs				
two compound fractures	14.83333	4.75773	2.588644	27.07802

We can use the equation() option to get pwcompare to perform comparisons in the y2 equation:

. pwcompare fracture, equation(y2) mcompare(bonferroni)

Pairwise comparisons of marginal linear predictions

Margins : asbalanced

		Number of Comparisons
у2	fracture	3

			Bonfe	rroni
	Contrast	Std. Err.	[95% Conf.	Interval]
y2				
fracture				
two compound fractures				
vs				
one compound fracture one simple fracture	-5.761905	3.008327	-13.50426	1.980449
vs				
one compound fracture one simple fracture	-3.053571	3.273705	-11.47891	5.371769
two compound fractures	2.708333	2.887136	-4.722119	10.13879

Because we are working with mvreg results, we can use the \_eqns pseudofactor to compare the margins between the three dependent variables. The levels of \_eqns index the equations: 1 for the first equation, 2 for the second, and 3 for the third.

. pwcompare \_eqns, mcompare(bonferroni)

Pairwise comparisons of marginal linear predictions

Margins : asbalanced

	Number of Comparisons
_eqns	3

			Bonfe	rroni
	Contrast	Std. Err.	[95% Conf.	Interval]
_eqns				
2 vs 1	5654762	2.545923	-7.117768	5.986815
3 vs 1	24.24603	2.320677	18.27344	30.21862
3 vs 2	24.81151	2.368188	18.71664	30.90637

For the previous command, the only methods available are mcompare(bonferroni), mcompare(sidak), or mcompare(scheffe). Methods that use the Studentized range are not appropriate for making comparisons across equations.

### Unbalanced data

pwcompare treats all factors as balanced when it computes the marginal means. By "balanced", we mean that the number of observations in each combination of factor levels (in each cell mean) is equal. We can alternatively specify the asobserved option when we have unbalanced data to obtain marginal means that are based on the observed cell frequencies from the model fit. For more details on the difference in these two types of marginal means and a discussion of when each may be appropriate, see [R] margins and [R] contrast.

In addition, when our data are not balanced, some of the multiple-comparison adjustments are no longer appropriate. Student-Newman-Keuls' method, Duncan's method, and Dunnett's method assume equal numbers of observations per group.

Here we use an unbalanced dataset and fit a two-way ANOVA model for cholesterol levels on race and age group. Then we perform pairwise comparisons of the mean cholesterol levels for each race, requesting Šidák's adjustment as well as marginal means that are computed using the observed cell frequencies.

. use http://www.stata-press.com/data/r13/cholesterol3
(Artificial cholesterol data, unbalanced)

. anova chol race##agegrp

	Number of obs Root MSE			quared R-squared	=	0.8179 0.7689
Source	Partial SS	df	MS	F	Pı	rob > F
Model	16379.9926	14	1169.99947	16.68		0.0000
race agegrp race#agegrp	230.754396 13857.9877 857.815209	2 4 8	115.377198 3464.49693 107.226901	1.64 49.39 1.53		0.2029 0.0000 0.1701
Residual	3647.2774	52	70.13995			
Total	20027.27	66	303.443485			

<sup>.</sup> pwcompare race, asobserved mcompare(sidak)

Pairwise comparisons of marginal linear predictions

Margins : asobserved

	Number of Comparisons
race	3

	Contrast	Std. Err.		Sidak [95% Conf. Interval]		
race white vs black other vs black other vs white	-7.232433	2.686089	-13.85924	6056277		
	-5.231198	2.651203	-11.77194	1.309541		
	2.001235	2.414964	-3.956682	7.959152		

## **Empty cells**

An empty cell is a combination of the levels of factor variables that is not observed in the estimation sample. When we have empty cells in our data, the marginal means involving those empty cells are not estimable as described in [R] margins. In addition, all pairwise comparisons involving a marginal mean that is not estimable are themselves not estimable. Here we use a dataset where we do not have any observations for white individuals in the 20–29 age group. We can use the emptycells(reweight) option to reweight the nonempty cells so that we can estimate the marginal mean for whites and compute pairwise comparisons involving that marginal mean.

. use http://www.stata-press.com/data/r13/cholesterol2 (Artificial cholesterol data, empty cells)

. tabulate race agegrp

race	10-19	20-29	agegrp 30-39	40-59	60-79	Total
black white other	5 5 5	5 0 5	5 5 5	5 5 5	5 5 5	25 20 25
Total	15	10	15	15	15	70

. anova chol race##agegrp

	Number of obs Root MSE			quared R-squared	= 0.7582 = 0.7021
Source	Partial SS	df	MS	F	Prob > F
Model	15751.6113	13	1211.66241	13.51	0.0000
race	305.49046	2	152.74523	1.70	0.1914
agegrp	14387.8559	4	3596.96397	40.10	0.0000
race#agegrp	795.807574	7	113.686796	1.27	0.2831
Residual	5022.71559	56	89.6913498		
Total	20774.3269	69	301.077201		

<sup>.</sup> pwcompare race, emptycells(reweight)

Pairwise comparisons of marginal linear predictions

Margins : asbalanced Empty cells : reweight

	Contrast	Std. Err.		Unadjusted [95% Conf. Interval]	
race	0.000740	0.044400	0 740740	0.011000	
white vs black other vs black	2.922769 -4.12621	2.841166 2.678677	-2.768769 -9.492244	8.614308 1.239824	
other vs white	-7.048979	2.841166	-12.74052	-1.35744	

For further details on the emptycells(reweight) option, see [R] margins and [R] contrast.

## Stored results

pwcompare stores the following in r():

```
Scalars
                                variance degrees of freedom
    r(df_r)
    r(k_terms)
                                number of terms in marginlist
    r(level)
                                confidence level of confidence intervals
    r(balanced)
                                1 if fully balanced data; 0 otherwise
Macros
    r(cmd)
                                pwcompare
    r(cmdline)
                                command as typed
    r(est_cmd)
                                e(cmd) from original estimation results
    r(est_cmdline)
                                e(cmdline) from original estimation results
    r(title)
                                title in output
                                empspec from emptycells()
    r(emptycells)
    r(groups#)
                                group codes for the #th margin in r(b)
    r(mcmethod_vs)
                                method from mcompare()
    r(mctitle_vs)
                                title for method from mcompare()
    r(mcadjustall_vs)
                                adjustall or empty
    r(margin_method)
                                asbalanced or asobserved
                                vcetype specified in vce() in original estimation command
    r(vce)
Matrices
    r(b)
                                margin estimates
    r(V)
                                variance-covariance matrix of the margin estimates
    r(error)
                                margin estimability codes;
                                  0 means estimable.
                                  8 means not estimable
                                matrix containing the margins with their standard errors, test statistics, p-values,
    r(table)
                                    and confidence intervals
    r(M)
                                matrix that produces the margins from the model coefficients
    r(b_vs)
                                margin difference estimates
    r(V_vs)
                                variance-covariance matrix of the margin difference estimates
                                margin difference estimability codes;
    r(error_vs)
                                  0 means estimable,
                                  8 means not estimable
    r(table_vs)
                                matrix containing the margin differences with their standard errors, test statistics,
                                    p-values, and confidence intervals
    r(I.)
                                matrix that produces the margin differences from the model coefficients
    r(k_groups)
                                number of significance groups for each term
```

28

pwcompare with the post option also stores the following in e():

```
Scalars
    e(df_r)
                                variance degrees of freedom
    e(k_terms)
                                number of terms in marginlist
    e(balanced)
                                1 if fully balanced data; 0 otherwise
Macros
    e(cmd)
                                pwcompare
    e(cmdline)
                                command as typed
    e(est_cmd)
                                e(cmd) from original estimation results
    e(est_cmdline)
                                e(cmdline) from original estimation results
    e(title)
                                title in output
    e(emptycells)
                                empspec from emptycells()
    e(margin_method)
                                asbalanced or asobserved
                                vcetype specified in vce() in original estimation command
    e(vce)
    e(properties)
Matrices
    e(b)
                                margin estimates
    e(V)
                                variance-covariance matrix of the margin estimates
    e(error)
                                margin estimability codes;
                                  0 means estimable,
                                  8 means not estimable
    e(M)
                                matrix that produces the margins from the model coefficients
    e(b_vs)
                                margin difference estimates
    e(V_vs)
                                variance-covariance matrix of the margin difference estimates
    e(error_vs)
                                margin difference estimability codes;
                                  0 means estimable.
                                  8 means not estimable
    e(L)
                                matrix that produces the margin differences from the model coefficients
    e(k_groups)
                                number of significance groups for each term
```

## Methods and formulas

Methods and formulas are presented under the following headings:

Notation
Unadjusted comparisons
Bonferroni's method
Šidák's method
Scheffé's method
Tukey's method
Student-Newman-Keuls' method
Duncan's method
Dunnett's method

### **Notation**

pwcompare performs comparisons of margins; see *Methods and formulas* in [R] contrast. If there are k margins for a given factor term, then there are

$$m = \binom{k}{2} = \frac{k(k-1)}{2}$$

unique pairwise comparisons. Let the ith pairwise comparison be denoted by

$$\hat{\delta}_i = l_i' \mathbf{b}$$

where **b** is a column vector of coefficients from the fitted model and  $l_i$  is a column vector that forms the corresponding linear combination. If  $\hat{\mathbf{V}}$  denotes the estimated variance matrix for **b**, then the standard error for  $\hat{\delta}_i$  is given by

$$\widehat{\operatorname{se}}(\widehat{\delta}_i) = \sqrt{l_i' \widehat{\mathbf{V}} l_i}$$

The corresponding test statistic is then

$$t_i = \frac{\widehat{\delta}_i}{\widehat{\operatorname{se}}(\widehat{\delta}_i)}$$

and the limits for a 100(1-lpha)% confidence interval for the expected value of  $\widehat{\delta}_i$  are

$$\hat{\delta}_i \pm c_i(\alpha) \, \widehat{\operatorname{se}}(\hat{\delta}_i)$$

where  $c_i(\alpha)$  is the critical value corresponding to the chosen multiple-comparison method.

## **Unadjusted comparisons**

pwcompare computes unadjusted p-values and confidence intervals by default. pwcompare uses the t distribution with  $\nu = e(df_r)$  degrees of freedom when  $e(df_r)$  is posted by the estimation command. The unadjusted two-sided p-value is

$$_{u}p_{i}=2\Pr(t_{\nu}>|t_{i}|)$$

and the unadjusted critical value  $uc_i(\alpha)$  satisfies the following probability statement:

$$\alpha = 2 \Pr \{t_{\nu} > {}_{\nu}c_{i}(\alpha)\}$$

pwcompare uses the standard normal distribution when e(df\_r) is not posted.

#### Bonferroni's method

For mcompare(bonferroni), the adjusted p-value is

$$_bp_i = \min(1, m_up_i)$$

and the adjusted critical value is

$$_{b}c_{i}(\alpha) = {}_{u}c_{i}(\alpha/m)$$

### Šidák's method

For mcompare(sidak), the adjusted p-value is

$$_{\rm si}p_i = 1 - (1 - _up_i)^m$$

and the adjusted critical value is

$$_{si}c_i(\alpha) = {}_{u}c_i \left\{ 1 - (1 - \alpha)^{1/m} \right\}$$

### Scheffé's method

For mcompare(scheffe), the adjusted p-value is

$$_{\mathrm{sc}}p_{i} = \Pr\left(F_{d,\nu} > t_{i}^{2}/d\right)$$

where  $F_{d,\nu}$  is distributed as an F with d numerator and  $\nu$  denominator degrees of freedom and d is the rank of the VCE for the term. The adjusted critical value satisfies the following probability statement:

$$\alpha = \Pr \left[ F_{d,\nu} > \left\{ {}_{sc}c_i(\alpha) \right\}^2 / d \right]$$

pwcompare uses the  $\chi^2$  distribution when e(df\_r) is not posted.

## Tukey's method

For mcompare(tukey), the adjusted p-value is

$$_{t}p_{i} = \Pr\left(q_{k,\nu} > |t_{i}|\sqrt{2}\right)$$

where  $q_{k,\nu}$  is distributed as the Studentized range statistic for k means and  $\nu$  residual degrees of freedom (Miller 1981). The adjusted critical value satisfies the following probability statement:

$$\alpha = \Pr\left\{q_{k,\nu} > {}_{t}c_{i}(\alpha)\sqrt{2}\right\}$$

## Student-Newman-Keuls' method

For mcompare(snk), suppose  $t_i$  is comparing two margins that have r other margins between them. Then the adjusted p-value is

$$_{\rm snk}p_i = \Pr\left(q_{r+2,\nu} > |t_i|\sqrt{2}\right)$$

where r ranges from 0 to k-2. The adjusted critical value  $\mathrm{snk}\,c_i(\alpha)$  satisfies the following probability statement:

$$\alpha = \Pr\left\{q_{r+2,\nu} > {}_{\rm snk}c_i(\alpha)\sqrt{2}\right\}$$

#### **Duncan's method**

For mcompare (duncan), the adjusted p-value is

$$dunc p_i = 1 - (1 - snk p_i)^{1/(r+1)}$$

and the adjusted critical value is

$$_{\mathrm{dunc}}c_i(\alpha) = _{\mathrm{snk}}c_i \{1 - (1 - \alpha)^{r+1}\}$$

#### Dunnett's method

For mcompare(dunnett), the margins are compared with a reference category, resulting in only k-1 pairwise comparisons. The adjusted p-value is

$$_{\mathrm{dunn}}p_i = \Pr(d_{k-1,\nu} > |t_i|)$$

where  $d_{k-1,\nu}$  is distributed as the many-one t statistic (Miller 1981, 76). The adjusted critical value  $\operatorname{dunn} c_i(\alpha)$  satisfies the following probability statement:

$$\alpha = \Pr \left\{ d_{k-1,\nu} > _{\operatorname{dunn}} c_i(\alpha) \right\}$$

The multiple-comparison methods for mcompare(tukey), mcompare(snk), mcompare(duncan), and mcompare(dunnett) assume the normal distribution with equal variance; thus these methods are allowed only with results from anova, regress, manova, and mvreg. mcompare(snk), mcompare(duncan), and mcompare(dunnett) assume equal sample size for each marginal mean. These options will cause pwcompare to report a footnote if unbalanced factors are detected.

## References

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## Also see

- [R] **pwcompare postestimation** Postestimation tools for pwcompare
- [R] contrast Contrasts and linear hypothesis tests after estimation
- [R] **lincom** Linear combinations of estimators
- [R] margins Marginal means, predictive margins, and marginal effects
- [R] margins, pwcompare Pairwise comparisons of margins
- [R] **pwmean** Pairwise comparisons of means
- [R] **test** Test linear hypotheses after estimation
- [U] 20 Estimation and postestimation commands