xttobit — Random-effects tobit model

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Description

xttobit fits a random-effects tobit model for panel data where the outcome variable is censored. Censoring limits may be fixed for all observations or vary across observations. The user can request that a likelihood-ratio test comparing the panel tobit model with the pooled tobit model be conducted at estimation time.

Quick start

Tobit model of y on x where y is censored at a lower limit of 5 using xtset data xttobit y x, 11(5)

Add indicators for levels of categorical variable a

xttobit y x i.a, ll(5)

Same as above, but specify that censoring occurs at 5 and 25

xttobit y x i.a, 11(5) u1(25)

Same as above, but where lower and upper are variables containing the censoring limits

xttobit y x i.a, ll(lower) ul(upper)

Add likelihood-ratio test comparing the random-effects model with the pooled model

xttobit y x i.a, ll(lower) ul(upper) tobit

Menu

Statistics > Longitudinal/panel data > Censored outcomes > Tobit regression (RE)

```
xttobit depvar [indepvars] [if] [in] [weight] [, options]
```

options	Description
Model	
<u>nocons</u> tant	suppress constant term
11[(<i>varname</i> #)]	left-censoring variable or limit
ul[(<i>varname</i> #)]	right-censoring variable or limit
offset(varname)	include varname in model with coefficient constrained to 1
<pre>constraints(constraints)</pre>	apply specified linear constraints
SE	
vce(vcetype)	<pre>vcetype may be oim, bootstrap, or jackknife</pre>
Reporting	
<u>l</u> evel(#)	set confidence level; default is level(95)
tobit	perform likelihood-ratio test comparing against pooled tobit model
lrmodel	perform the likelihood-ratio model test instead of the default Wald test
<u>nocnsr</u> eport	do not display constraints
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Integration	
<pre>intmethod(intmethod)</pre>	integration method; <i>intmethod</i> may be <u>mv</u> aghermite (the default) or ghermite
$\underline{\mathtt{intp}}\mathtt{oints}(\#)$	use # quadrature points; default is intpoints (12)
Maximization	
maximize_options	control the maximization process; seldom used
<u>col</u> linear	keep collinear variables
<u>coefl</u> egend	display legend instead of statistics

A panel variable must be specified; use xtset; see [XT] xtset.

indepvars may contain factor variables; see [U] 11.4.3 Factor variables.

depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.

by, collect, fp, and statsby are allowed; see [U] 11.1.10 Prefix commands.

iweights are allowed; see [U] 11.1.6 weight. Weights must be constant within panel.

collinear and coeflegend do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

Model

noconstant; see [R] Estimation options.

11[(varname | #)] and u1[(varname | #)] indicate the lower and upper limits for censoring, respectively. Observations with $depvar \le 11$ () are left-censored; observations with $depvar \ge u1$ () are right-censored; and remaining observations are not censored. You do not have to specify the censoring values. If you specify 11, the lower limit is the minimum of depvar. If you specify u1, the upper limit is the maximum of depvar.

offset(varname), constraints(constraints); see [R] Estimation options.

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim) and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] vce_options.

Reporting

level(#); see [R] Estimation options.

tobit specifies that a likelihood-ratio test comparing the random-effects model with the pooled (tobit) model be included in the output.

1rmodel, nocnsreport; see [R] Estimation options.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(% fmt), pformat(% fmt), sformat(% fmt), and nolstretch; see [R] Estimation options.

Integration

intmethod(intmethod), intpoints(#); see [R] Estimation options.

Maximization

difficult, technique(algorithm_spec), iterate(#), [no]log, trace, maximize_options: gradient, showstep, hessian, showtolerance, tolerance(#), ltolerance(#), nrtolerance(#), nonrtolerance, and from(init_specs); see [R] Maximize. These options are seldom used.

The following options are available with xttobit but are not shown in the dialog box:

collinear, coeflegend; see [R] Estimation options.

Remarks and examples

xttobit fits a random-effects tobit model. There is no command for a fixed-effects model, because there does not exist a sufficient statistic allowing the fixed effects to be conditioned out of the likelihood.

$$y_{it} = \mathbf{x}_{it}\boldsymbol{\beta} + \nu_i + \epsilon_{it}$$

for $i=1,\ldots,n$ panels, where $t=1,\ldots,n_i$. The random effects, ν_i , are i.i.d., $N(0,\sigma_\nu^2)$, and ϵ_{it} are i.i.d. $N(0,\sigma_\epsilon^2)$ independently of ν_i .

The observed data, y^o_{it} , represent possibly censored versions of y_{it} . If they are left-censored, all that is known is that $y_{it} \leq y^o_{it}$. If they are right-censored, all that is known is that $y_{it} \geq y^o_{it}$. If they are left-censored, y^o_{it} is determined by 11(). If they are right-censored, y^o_{it} is determined by ul(). If they are uncensored, y^o_{it} is determined by depvar.

Example 1: Random-effects tobit regression

Using the nlswork data described in [XT] xt, we fit a random-effects tobit model of adjusted (log) wages. We use the ul() option to impose an upper limit on the recorded log of wages.

. use https://www.stata-press.com/data/r19/nlswork3 (National Longitudinal Survey of Young Women, 14-24 years old in 1968)

. xttobit ln_wage i.union age grade not_smsa south##c.year, ul(1.9) tobit
(output omitted)

Random-effects tobit regression	Number of obs Uncensored		. ,
Limits: Lower = -inf Upper = 1.90	Left-censored Right-censored		0 6,890
Group variable: idcode Random effects u_i ~ Gaussian	Number of groups Obs per group:	=	4,148
	min avg max	=	1 4.6 12
Integration method: mvaghermite	Integration pts.	=	12
Log likelihood = -6814.4606			2925.68 0.0000

ln_wage	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
1.union	.1430527	.0069718	20.52	0.000	.1293883	.1567171
age	.0099132	.0017516	5.66	0.000	.0064801	.0133464
grade	.0784855	.0022764	34.48	0.000	.0740239	.0829472
not_smsa	1339978	.009206	-14.56	0.000	1520413	1159544
1.south	3507188	.0695554	-5.04	0.000	4870449	2143928
year	0008285	.0018371	-0.45	0.652	0044292	.0027721
south#c.year	.0031938	.0008606	3.71	0.000	.0015071	.0048805
_cons	.5101956	.1006646	5.07	0.000	.3128966	.7074946
/sigma_u	.3045992	.0048344	63.01	0.000	.2951239	.3140745
/sigma_e	.2488678	.0018254	136.34	0.000	. 24529	. 2524455
rho	.5996844	.0084095			.583118	.6160734

LR test of sigma_u=0: chibar2(01) = 6650.63

Prob >= chibar2 = 0.000

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The results from a tobit regression can be interpreted as we would those from a linear regression. Because the dependent variable is log transformed, the coefficients can be interpreted in terms of a percentage change. We see, for example, that on average, union members make 14.3% more than nonunion members.

The output also includes the overall and panel-level variance components (labeled sigma_e and sigma_u, respectively) together with ρ (labeled rho)

$$\rho = \frac{\sigma_{\nu}^2}{\sigma_{\epsilon}^2 + \sigma_{\nu}^2}$$

which is the percent contribution to the total variance of the panel-level variance component.

When rho is zero, the panel-level variance component is unimportant, and the panel estimator is not different from the pooled estimator. A likelihood-ratio test of this is included at the bottom of the output. This test formally compares the pooled estimator (tobit) with the panel estimator. In this case, we reject the null hypothesis that there are no panel-level effects.

□ Technical note

The random-effects model is calculated using quadrature, which is an approximation whose accuracy depends partially on the number of integration points used. We can use the quadchk command to see if changing the number of integration points affects the results. If the results change, the quadrature approximation is not accurate given the number of integration points. Try increasing the number of integration points using the intpoints() option and run quadchk again. Do not attempt to interpret the results of estimates when the coefficients reported by quadchk differ substantially. See [XT] quadchk for details and [XT] **xtprobit** for an example.

Because the xttobit likelihood function is calculated by Gauss-Hermite quadrature, on large problems the computations can be slow. Computation time is roughly proportional to the number of points used for the quadrature.

Stored results

xttobit stores the following in e():

```
Scalars
                               number of observations
    e(N)
                               number of groups
    e(N_g)
    e(N_unc)
                               number of uncensored observations
                               number of left-censored observations
    e(N_lc)
    e(N_rc)
                               number of right-censored observations
    e(N_cd)
                               number of completely determined observations
    e(k)
                               number of parameters
    e(k_eq)
                               number of equations in e(b)
    e(k_eq_model)
                               number of equations in overall model test
    e(k_dv)
                               number of dependent variables
    e(df_m)
                               model degrees of freedom
    e(11)
                               log likelihood
    e(11_0)
                               log likelihood, constant-only model
                               log likelihood, comparison model
    e(11_c)
    e(chi2)
                               \chi^2 for comparison test
    e(chi2_c)
```

```
e(rho)
                               ρ
    e(sigma_u)
                               panel-level standard deviation
                               standard deviation of \epsilon_{it}
    e(sigma_e)
    e(n_quad)
                               number of quadrature points
    e(g_min)
                               smallest group size
                               average group size
    e(g_avg)
                               largest group size
    e(g_max)
                               p-value for model test
    e(p)
    e(rank)
                               rank of e(V)
                               rank of e(V) for constant-only model
    e(rank0)
    e(ic)
                               number of iterations
                               return code
    e(rc)
    e(converged)
                               1 if converged, 0 otherwise
Macros
    e(cmd)
                               xttobit
    e(cmdline)
                               command as typed
                               names of dependent variables
    e(depvar)
    e(ivar)
                               variable denoting groups
    e(llopt)
                               contents of 11()
                               contents of ul()
    e(ulopt)
    e(k_aux)
                               number of auxiliary parameters
    e(wtype)
                               weight type
    e(wexp)
                               weight expression
                               title in estimation output
    e(title)
    e(offset1)
                               offset
                               Wald or LR; type of model \chi^2 test
    e(chi2type)
                               Wald or LR; type of model \chi^2 test corresponding to e(chi2_c)
    e(chi2_ct)
    e(vce)
                               vcetype specified in vce()
    e(intmethod)
                               integration method
                               Gaussian; the distribution of the random effect
    e(distrib)
    e(opt)
                               type of optimization
                               max or min; whether optimizer is to perform maximization or minimization
    e(which)
    e(ml_method)
                               type of ml method
    e(user)
                               name of likelihood-evaluator program
    e(technique)
                               maximization technique
    e(properties)
    e(predict)
                               program used to implement predict
    e(marginsok)
                               predictions allowed by margins
    e(asbalanced)
                               factor variables fyset as asbalanced
    e(asobserved)
                                factor variables fyset as asobserved
Matrices
    e(b)
                               coefficient vector
                               constraints matrix
    e(Cns)
    e(ilog)
                               iteration log
    e(gradient)
                               gradient vector
    e(V)
                               variance-covariance matrix of the estimators
Functions
    e(sample)
                               marks estimation sample
```

In addition to the above, the following is stored in r():

```
Matrices
```

```
matrix containing the coefficients with their standard errors, test statistics, p-values, and
r(table)
                                 confidence intervals
```

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

Methods and formulas

Assuming a normal distribution, $N(0, \sigma_{\nu}^2)$, for the random effects ν_i , we have the joint (unconditional of ν_i) density of the observed data from the ith panel

$$f(y_{i1}^o,\dots,y_{in_i}^o|\mathbf{x}_{i1},\dots,\mathbf{x}_{in_i}) = \int_{-\infty}^{\infty} \frac{e^{-\nu_i^2/2\sigma_{\nu}^2}}{\sqrt{2\pi}\sigma_{\nu}} \left\{ \prod_{t=1}^{n_i} F(y_{it}^o,\mathbf{x}_{it}\boldsymbol{\beta} + \nu_i) \right\} d\nu_i$$

where

$$F(y_{it}^o, \Delta_{it}) = \begin{cases} \left(\sqrt{2\pi}\sigma_{\epsilon}\right)^{-1} e^{-(y_{it}^o - \Delta_{it})^2/(2\sigma_{\epsilon}^2)} & \text{if } y_{it}^o \in C \\ \Phi\left(\frac{y_{it}^o - \Delta_{it}}{\sigma_{\epsilon}}\right) & \text{if } y_{it}^o \in L \\ 1 - \Phi\left(\frac{y_{it}^o - \Delta_{it}}{\sigma_{\epsilon}}\right) & \text{if } y_{it}^o \in R \end{cases}$$

where C is the set of noncensored observations, L is the set of left-censored observations, R is the set of right-censored observations, and $\Phi()$ is the cumulative normal distribution.

The panel level likelihood l_i is given by

$$\begin{split} l_i &= \int_{-\infty}^{\infty} \frac{e^{-\nu_i^2/2\sigma_{\nu}^2}}{\sqrt{2\pi}\sigma_{\nu}} \left\{ \prod_{t=1}^{n_i} F(y_{it}^o, \mathbf{x}_{it} \boldsymbol{\beta} + \nu_i) \right\} d\nu_i \\ &\equiv \int_{-\infty}^{\infty} g(y_{it}^o, x_{it}, \nu_i) d\nu_i \end{split}$$

This integral can be approximated with M-point Gauss-Hermite quadrature

$$\int_{-\infty}^{\infty} e^{-x^2} h(x) dx \approx \sum_{m=1}^{M} w_m^* h(a_m^*)$$

This is equivalent to

$$\int_{-\infty}^{\infty} f(x) dx \approx \sum_{m=1}^{M} w_m^* \exp\left\{(a_m^*)^2\right\} f(a_m^*)$$

where the w_m^* denote the quadrature weights and the a_m^* denote the quadrature abscissas. The log likelihood, L, is the sum of the logs of the panel level likelihoods l_i .

The default approximation of the log likelihood is by adaptive Gauss-Hermite quadrature, which approximates the panel level likelihood with

$$l_i \approx \sqrt{2} \hat{\sigma}_i \sum_{m=1}^M w_m^* \exp\left\{(a_m^*)^2\right\} g(y_{it}^o, x_{it}, \sqrt{2} \hat{\sigma}_i a_m^* + \hat{\mu}_i)$$

where $\hat{\sigma}_i$ and $\hat{\mu}_i$ are the adaptive parameters for panel i. Therefore, with the definition of $g(y^o_{it}, x_{it}, \nu_i)$, the total log likelihood is approximated by

$$\begin{split} L \approx \sum_{i=1}^{n} w_{i} \log & \bigg[\sqrt{2} \hat{\sigma}_{i} \sum_{m=1}^{M} w_{m}^{*} \exp\{(a_{m}^{*})^{2}\} \frac{\exp\{-(\sqrt{2} \hat{\sigma}_{i} a_{m}^{*} + \hat{\mu}_{i})^{2}/2\sigma_{\nu}^{2}\}}{\sqrt{2\pi}\sigma_{\nu}} \\ & \prod_{t=1}^{n_{i}} F(y_{it}^{o}, x_{it}\beta + \sqrt{2} \hat{\sigma}_{i} a_{m}^{*} + \hat{\mu}_{i}) \bigg] \end{split} \tag{1}$$

where w_i is the user-specified weight for panel i; if no weights are specified, $w_i = 1$.

The default method of adaptive Gauss-Hermite quadrature is to calculate the posterior mean and variance and use those parameters for $\hat{\mu}_i$ and $\hat{\sigma}_i$ by following the method of Naylor and Smith (1982), further discussed in Skrondal and Rabe-Hesketh (2004). We start with $\hat{\sigma}_{i,0}=1$ and $\hat{\mu}_{i,0}=0$, and the posterior means and variances are updated in the kth iteration. That is, at the kth iteration of the optimization for l_i we use

$$l_{i,k} \approx \sum_{m=1}^{M} \sqrt{2} \hat{\sigma}_{i,k-1} w_m^* \exp\{a_m^*)^2\} g(y_{it}^o, x_{it}, \sqrt{2} \hat{\sigma}_{i,k-1} a_m^* + \hat{\mu}_{i,k-1})$$

Letting

$$\tau_{i.m.k-1} = \sqrt{2}\hat{\sigma}_{i.k-1}a_m^* + \hat{\mu}_{i.k-1}$$

$$\hat{\mu}_{i,k} = \sum_{m=1}^{M} (\tau_{i,m,k-1}) \frac{\sqrt{2} \hat{\sigma}_{i,k-1} w_m^* \exp\{(a_m^*)^2\} g(y_{it}^o, x_{it}, \tau_{i,m,k-1})}{l_{i,k}}$$

and

$$\hat{\sigma}_{i,k} = \sum_{m=1}^{M} (\tau_{i,m,k-1})^2 \frac{\sqrt{2} \hat{\sigma}_{i,k-1} w_m^* \exp\{(a_m^*)^2\} g(y_{it}^o, x_{it}, \tau_{i,m,k-1})}{l_{i,k}} - \left(\hat{\mu}_{i,k}\right)^2$$

and this is repeated until $\hat{\mu}_{i,k}$ and $\hat{\sigma}_{i,k}$ have converged for this iteration of the maximization algorithm. This adaptation is applied on every iteration until the log-likelihood change from the preceding iteration is less than a relative difference of 1e-6; after this, the quadrature parameters are fixed.

The log likelihood can also be calculated by nonadaptive Gauss-Hermite quadrature if the intmethod(ghermite) option is specified. For nonadaptive Gauss-Hermite quadrature, the following formula for the log likelihood is used in place of (1).

$$\begin{split} L &= \sum_{i=1}^n w_i \log \Big\{ \Pr(y_{i1}, \dots, y_{in_i} | \mathbf{x}_{i1}, \dots, \mathbf{x}_{in_i}) \Big\} \\ &\approx \sum_{i=1}^n w_i \log \left[\frac{1}{\sqrt{\pi}} \sum_{m=1}^M w_m^* \prod_{t=1}^{n_i} F \Big\{ y_{it}^o, \mathbf{x}_{it} \mathbf{\beta} + \sqrt{2} \sigma_{\nu} a_m^* \Big\} \right] \end{split}$$

Both quadrature formulas require that the integrated function be well approximated by a polynomial of degree equal to the number of quadrature points. Panel size can affect whether

$$\prod_{t=1}^{n_i} F(y^o_{it}, \mathbf{x}_{it} \boldsymbol{\beta} + \nu_i)$$

is well approximated by a polynomial. As panel size and ρ increase, the quadrature approximation can become less accurate. For large ρ , the random-effects model can also become unidentified. Adaptive quadrature gives better results for correlated data and large panels than nonadaptive quadrature; however, we recommend that you use the quadchk command (see [XT] quadchk) to verify the quadrature approximation used in this command, whichever approximation you choose.

References

- Naylor, J. C., and A. F. M. Smith. 1982. Applications of a method for the efficient computation of posterior distributions. Journal of the Royal Statistical Society, C ser., 31: 214–225. https://doi.org/10.2307/2347995.
- Pendergast, J. F., S. J. Gange, M. A. Newton, M. J. Lindstrom, M. Palta, and M. R. Fisher. 1996. A survey of methods for analyzing clustered binary response data. International Statistical Review 64: 89–118. https://doi.org/10.2307/1403425.
- Skrondal, A., and S. Rabe-Hesketh. 2004. Generalized Latent Variable Modeling: Multilevel, Longitudinal, and Structural Equation Models. Boca Raton, FL: Chapman and Hall/CRC.

Also see

- [XT] **xttobit postestimation** Postestimation tools for xttobit
- [XT] quadchk Check sensitivity of quadrature approximation
- [XT] **xteintreg** Extended random-effects interval regression
- [XT] **xtintreg** Random-effects interval-data regression model
- [XT] **xtreg** Linear models for panel data
- [XT] **xtset** Declare data to be panel data
- [ME] metobit Multilevel mixed-effects tobit regression
- [R] **tobit** Tobit regression
- [U] 20 Estimation and postestimation commands

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