

**xtrc** — Random-coefficients model

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## Description

`xtrc` fits the [Swamy \(1970\)](#) random-coefficients linear regression model, which does not impose the assumption of constant parameters across panels. Average coefficient estimates are reported by default, but panel-specific coefficients may be requested.

## Quick start

Random-coefficients regression of `y` on `x1` and `x2` using `xtset` data

```
xtrc y x1 x2
```

As above, but report panel-specific best linear predictors

```
xtrc y x1 x2, betas
```

Multiple-imputation estimates of random-coefficients regression using `mi xtset` data

```
mi estimate: xtrc y x
```

## Menu

Statistics > Longitudinal/panel data > Random-coefficients regression by GLS

## Syntax

```
xtrc depvar indepvars [if] [in] [, options]
```

<i>options</i>	Description
Main	
<code>noconstant</code>	suppress constant term
<code>offset(<i>varname</i>)</code>	include <i>varname</i> in model with coefficient constrained to 1
SE	
<code>vce(<i>vcetype</i>)</code>	<i>vcetype</i> may be <code>conventional</code> , <code>bootstrap</code> , or <code>jackknife</code>
Reporting	
<code>level(#)</code>	set confidence level; default is <code>level(95)</code>
<code>betas</code>	display group-specific best linear predictors
<code>display_options</code>	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
<code>coeflegend</code>	display legend instead of statistics

A panel variable must be specified; use `xtset`; see [XT] `xtset`.

*indepvars* may contain factor variables; see [U] 11.4.3 [Factor variables](#).

`by`, `collect`, `mi estimate`, and `statsby` are allowed; see [U] 11.1.10 [Prefix commands](#).

`vce(bootstrap)` and `vce(jackknife)` are not allowed with the `mi estimate` prefix; see [MI] [mi estimate](#).

`coeflegend` does not appear in the dialog box.

See [U] 20 [Estimation and postestimation commands](#) for more capabilities of estimation commands.

## Options

### Main

`noconstant`, `offset(varname)`; see [R] [Estimation options](#)

### SE

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`conventional`) and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [XT] [vce\\_options](#).

`vce(conventional)`, the default, uses the conventionally derived variance estimator for generalized least-squares regression.

### Reporting

`level(#)`; see [R] [Estimation options](#).

`betas` requests that the group-specific best linear predictors also be displayed.

`display_options`: `noci`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [Estimation options](#).

The following option is available with `xtrc` but is not shown in the dialog box:

`coeflegend`; see [R] [Estimation options](#).

## Remarks and examples

[stata.com](https://www.stata.com)

In random-coefficients models, we wish to treat the parameter vector as a realization (in each panel) of a stochastic process. `xtrc` fits the [Swamy \(1970\)](#) random-coefficients model, which is suitable for linear regression of panel data. See [Greene \(2012, chap. 11\)](#) and [Poi \(2003\)](#) for more information about this and other panel-data models.

### ► Example 1

[Greene \(2012, 1112\)](#) reprints data from a classic study of investment demand by [Grunfeld and Griliches \(1960\)](#). In [XT] `xtgls`, we use this dataset to illustrate many of the possible models that may be fit with the `xtgls` command. Although the models included in the `xtgls` command offer considerable flexibility, they all assume that there is no parameter variation across firms (the cross-sectional units).

To take a first look at the assumption of parameter constancy, we should `reshape` our data so that we may fit a simultaneous-equation model with `sureg`; see [R] [sureg](#). Because there are only five panels here, this is not too difficult.

```
. use https://www.stata-press.com/data/r17/invest2
. reshape wide invest market stock, i(time) j(company)
(j = 1 2 3 4 5)
```

Data	Long	->	Wide
Number of observations	100	->	20
Number of variables	5	->	16
j variable (5 values)	company	->	(dropped)
<i>x<sub>ij</sub></i> variables:			
	invest	->	invest1 invest2 ... invest5
	market	->	market1 market2 ... market5
	stock	->	stock1 stock2 ... stock5

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```
. sureg (invest1 market1 stock1) (invest2 market2 stock2)
> (invest3 market3 stock3) (invest4 market4 stock4) (invest5 market5 stock5)
Seemingly unrelated regression
```

Equation	Obs	Params	RMSE	"R-squared"	chi2	P>chi2
invest1	20	2	84.94729	0.9207	261.32	0.0000
invest2	20	2	12.36322	0.9119	207.21	0.0000
invest3	20	2	26.46612	0.6876	46.88	0.0000
invest4	20	2	9.742303	0.7264	59.15	0.0000
invest5	20	2	95.85484	0.4220	14.97	0.0006

	Coefficient	Std. err.	z	P> z	[95% conf. interval]
invest1					
market1	.120493	.0216291	5.57	0.000	.0781007 .1628853
stock1	.3827462	.032768	11.68	0.000	.318522 .4469703
_cons	-162.3641	89.45922	-1.81	0.070	-337.7009 12.97279
invest2					
market2	.0695456	.0168975	4.12	0.000	.0364271 .1026641
stock2	.3085445	.0258635	11.93	0.000	.2578529 .3592362
_cons	.5043112	11.51283	0.04	0.965	-22.06042 23.06904
invest3					
market3	.0372914	.0122631	3.04	0.002	.0132561 .0613268
stock3	.130783	.0220497	5.93	0.000	.0875663 .1739997
_cons	-22.43892	25.51859	-0.88	0.379	-72.45443 27.57659
invest4					
market4	.0570091	.0113623	5.02	0.000	.0347395 .0792788
stock4	.0415065	.0412016	1.01	0.314	-.0392472 .1222602
_cons	1.088878	6.258805	0.17	0.862	-11.17815 13.35591
invest5					
market5	.1014782	.0547837	1.85	0.064	-.0058958 .2088523
stock5	.3999914	.1277946	3.13	0.002	.1495186 .6504642
_cons	85.42324	111.8774	0.76	0.445	-133.8525 304.6989

Here we instead fit a random-coefficients model:

```
. use https://www.stata-press.com/data/r17/invest2
. xtrc invest market stock
Random-coefficients regression
Group variable: company
Time variable: time
Number of obs = 100
Number of groups = 5
Obs per group:
    min = 20
    avg = 20.0
    max = 20
Wald chi2(2) = 17.55
Prob > chi2 = 0.0002
```

invest	Coefficient	Std. err.	z	P> z	[95% conf. interval]
market	.0807646	.0250829	3.22	0.001	.0316031 .1299261
stock	.2839885	.0677899	4.19	0.000	.1511229 .4168542
_cons	-23.58361	34.55547	-0.68	0.495	-91.31108 44.14386

Test of parameter constancy: chi2(12) = 603.99      Prob > chi2 = 0.0000

Just as the results of our simultaneous-equation model do not support the assumption of parameter constancy, the test included with the random-coefficients model also indicates that the assumption is not valid for these data. With large panel datasets, we would not want to take the time to look at a simultaneous-equations model (aside from the fact that our doing so was subjective).

◀

## Stored results

`xtrc` stores the following in `e()`:

### Scalars

<code>e(N)</code>	number of observations
<code>e(N_g)</code>	number of groups
<code>e(df_m)</code>	model degrees of freedom
<code>e(chi2)</code>	$\chi^2$
<code>e(chi2_c)</code>	$\chi^2$ for comparison test
<code>e(df_chi2c)</code>	degrees of freedom for comparison $\chi^2$ test
<code>e(g_min)</code>	smallest group size
<code>e(g_avg)</code>	average group size
<code>e(g_max)</code>	largest group size
<code>e(rank)</code>	rank of <code>e(V)</code>

### Macros

<code>e(cmd)</code>	<code>xtrc</code>
<code>e(cmdline)</code>	command as typed
<code>e(depvar)</code>	name of dependent variable
<code>e(ivar)</code>	variable denoting groups
<code>e(tvar)</code>	variable denoting time within groups
<code>e(title)</code>	title in estimation output
<code>e(offset)</code>	linear offset variable
<code>e(chi2type)</code>	Wald; type of model $\chi^2$ test
<code>e(vce)</code>	<i>vce</i> type specified in <code>vce()</code>
<code>e(properties)</code>	<code>b V</code>
<code>e(predict)</code>	program used to implement <code>predict</code>
<code>e(marginsnotok)</code>	predictions disallowed by <code>margins</code>
<code>e(asbalanced)</code>	factor variables <code>fvset</code> as <code>asbalanced</code>
<code>e(asobserved)</code>	factor variables <code>fvset</code> as <code>asobserved</code>

### Matrices

<code>e(b)</code>	coefficient vector
<code>e(Sigma)</code>	$\hat{\Sigma}$ matrix
<code>e(beta_ps)</code>	matrix of best linear predictors
<code>e(V)</code>	variance-covariance matrix of the estimators
<code>e(V_ps)</code>	matrix of variances for the best linear predictors; row <i>i</i> contains vec of variance matrix for group <i>i</i> predictor

### Functions

<code>e(sample)</code>	marks estimation sample
------------------------	-------------------------

In addition to the above, the following is stored in `r()`:

### Matrices

<code>r(table)</code>	matrix containing the coefficients with their standard errors, test statistics, <i>p</i> -values, and confidence intervals
-----------------------	--

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r`-class command is run after the estimation command.

## Methods and formulas

In a random-coefficients model, the parameter heterogeneity is treated as stochastic variation. Assume that we write

$$\mathbf{y}_i = \mathbf{X}_i \boldsymbol{\beta}_i + \boldsymbol{\epsilon}_i$$

where  $i = 1, \dots, m$ , and  $\boldsymbol{\beta}_i$  is the coefficient vector ( $k \times 1$ ) for the  $i$ th cross-sectional unit, such that

$$\boldsymbol{\beta}_i = \boldsymbol{\beta} + \boldsymbol{\nu}_i \quad E(\boldsymbol{\nu}_i) = \mathbf{0} \quad E(\boldsymbol{\nu}_i \boldsymbol{\nu}_i') = \boldsymbol{\Sigma}$$

Our goal is to find  $\widehat{\boldsymbol{\beta}}$  and  $\widehat{\boldsymbol{\Sigma}}$ .

The derivation of the estimator assumes that the cross-sectional specific coefficient vector  $\boldsymbol{\beta}_i$  is the outcome of a random process with mean vector  $\boldsymbol{\beta}$  and covariance matrix  $\boldsymbol{\Sigma}$ ,

$$\mathbf{y}_i = \mathbf{X}_i \boldsymbol{\beta}_i + \boldsymbol{\epsilon}_i = \mathbf{X}_i (\boldsymbol{\beta} + \boldsymbol{\nu}_i) + \boldsymbol{\epsilon}_i = \mathbf{X}_i \boldsymbol{\beta} + (\mathbf{X}_i \boldsymbol{\nu}_i + \boldsymbol{\epsilon}_i) = \mathbf{X}_i \boldsymbol{\beta} + \boldsymbol{\omega}_i$$

where  $E(\boldsymbol{\omega}_i) = \mathbf{0}$  and

$$E(\boldsymbol{\omega}_i \boldsymbol{\omega}_i') = E\left\{(\mathbf{X}_i \boldsymbol{\nu}_i + \boldsymbol{\epsilon}_i)(\mathbf{X}_i \boldsymbol{\nu}_i + \boldsymbol{\epsilon}_i)'\right\} = E(\boldsymbol{\epsilon}_i \boldsymbol{\epsilon}_i') + \mathbf{X}_i E(\boldsymbol{\nu}_i \boldsymbol{\nu}_i') \mathbf{X}_i' = \sigma_i^2 \mathbf{I} + \mathbf{X}_i \boldsymbol{\Sigma} \mathbf{X}_i' = \boldsymbol{\Pi}_i$$

Stacking the  $m$  equations, we have

$$\mathbf{y} = \mathbf{X} \boldsymbol{\beta} + \boldsymbol{\omega}$$

where  $\boldsymbol{\Pi} \equiv E(\boldsymbol{\omega} \boldsymbol{\omega}')$  is a block diagonal matrix with  $\boldsymbol{\Pi}_i$ ,  $i = 1 \dots m$ , along the main diagonal and zeros elsewhere. The GLS estimator of  $\widehat{\boldsymbol{\beta}}$  is then

$$\widehat{\boldsymbol{\beta}} = \left( \sum_i \mathbf{X}_i' \boldsymbol{\Pi}_i^{-1} \mathbf{X}_i \right)^{-1} \sum_i \mathbf{X}_i' \boldsymbol{\Pi}_i^{-1} \mathbf{y}_i = \sum_{i=1}^m \mathbf{W}_i \mathbf{b}_i$$

where

$$\mathbf{W}_i = \left\{ \sum_{i=1}^m (\boldsymbol{\Sigma} + \mathbf{V}_i)^{-1} \right\}^{-1} (\boldsymbol{\Sigma} + \mathbf{V}_i)^{-1}$$

$\mathbf{b}_i = (\mathbf{X}_i' \mathbf{X}_i)^{-1} \mathbf{X}_i' \mathbf{y}_i$  and  $\mathbf{V}_i = \sigma_i^2 (\mathbf{X}_i' \mathbf{X}_i)^{-1}$ , showing that the resulting GLS estimator is a matrix-weighted average of the panel-specific OLS estimators. The variance of  $\widehat{\boldsymbol{\beta}}$  is

$$\text{Var}(\widehat{\boldsymbol{\beta}}) = \sum_{i=1}^m (\boldsymbol{\Sigma} + \mathbf{V}_i)^{-1}$$

To calculate the above estimator  $\hat{\beta}$  for the unknown  $\Sigma$  and  $\mathbf{V}_i$  parameters, we use the two-step approach suggested by [Swamy \(1970\)](#):

$$\begin{aligned} \mathbf{b}_i &= \text{OLS panel-specific estimator} \\ \hat{\sigma}_i^2 &= \frac{\hat{\boldsymbol{\epsilon}}_i' \hat{\boldsymbol{\epsilon}}_i}{n_i - k} \\ \hat{\mathbf{V}}_i &= \hat{\sigma}_i^2 (\mathbf{X}'_i \mathbf{X}_i)^{-1} \\ \bar{\mathbf{b}} &= \frac{1}{m} \sum_{i=1}^m \mathbf{b}_i \\ \hat{\Sigma} &= \frac{1}{m-1} \left( \sum_{i=1}^m \mathbf{b}_i \mathbf{b}'_i - m \bar{\mathbf{b}} \bar{\mathbf{b}}' \right) - \frac{1}{m} \sum_{i=1}^m \hat{\mathbf{V}}_i \end{aligned}$$

The two-step procedure begins with the usual OLS estimates of  $\beta_i$ . With those estimates, we may proceed by obtaining estimates of  $\hat{\mathbf{V}}_i$  and  $\hat{\Sigma}$  (and thus  $\hat{\mathbf{W}}_i$ ) and then obtain an estimate of  $\beta$ .

[Swamy \(1970\)](#) further points out that the matrix  $\hat{\Sigma}$  may not be positive definite and that because the second term is of order  $1/(mT)$ , it is negligible in large samples. A simple and asymptotically expedient solution is simply to drop this second term and instead use

$$\hat{\Sigma} = \frac{1}{m-1} \left( \sum_{i=1}^m \mathbf{b}_i \mathbf{b}'_i - m \bar{\mathbf{b}} \bar{\mathbf{b}}' \right)$$

As discussed by [Judge et al. \(1985, 541\)](#), the feasible best linear predictor of  $\beta_i$  is given by

$$\begin{aligned} \hat{\beta}_i &= \hat{\beta} + \hat{\Sigma} \mathbf{X}'_i \left( \mathbf{X}_i \hat{\Sigma} \mathbf{X}'_i + \hat{\sigma}_i^2 \mathbf{I} \right)^{-1} \left( \mathbf{y}_i - \mathbf{X}_i \hat{\beta} \right) \\ &= \left( \hat{\Sigma}^{-1} + \hat{\mathbf{V}}_i^{-1} \right)^{-1} \left( \hat{\Sigma}^{-1} \hat{\beta} + \hat{\mathbf{V}}_i^{-1} \mathbf{b}_i \right) \end{aligned}$$

The conventional variance of  $\hat{\beta}_i$  is given by

$$\text{Var}(\hat{\beta}_i) = \text{Var}(\hat{\beta}) + (\mathbf{I} - \mathbf{A}_i) \left\{ \hat{\mathbf{V}}_i - \text{Var}(\hat{\beta}) \right\} (\mathbf{I} - \mathbf{A}_i)'$$

where

$$\mathbf{A}_i = \left( \hat{\Sigma}^{-1} + \hat{\mathbf{V}}_i^{-1} \right)^{-1} \hat{\Sigma}^{-1}$$

To test the model, we may look at the difference between the OLS estimate of  $\beta$ , ignoring the panel structure of the data and the matrix-weighted average of the panel-specific OLS estimators. The test statistic suggested by [Swamy \(1970\)](#) is given by

$$\chi^2_{k(m-1)} = \sum_{i=1}^m (\mathbf{b}_i - \bar{\beta}^*)' \hat{\mathbf{V}}_i^{-1} (\mathbf{b}_i - \bar{\beta}^*) \quad \text{where} \quad \bar{\beta}^* = \left( \sum_{i=1}^m \hat{\mathbf{V}}_i^{-1} \right)^{-1} \sum_{i=1}^m \hat{\mathbf{V}}_i^{-1} \mathbf{b}_i$$

Johnston and DiNardo (1997) have shown that the test is algebraically equivalent to testing

$$H_0 : \beta_1 = \beta_2 = \cdots = \beta_m$$

in the generalized (groupwise heteroskedastic) `xtgls` model, where  $\mathbf{V}$  is block diagonal with  $i$ th diagonal element  $\mathbf{\Pi}_i$ .

## References

- Eberhardt, M. 2012. [Estimating panel time-series models with heterogeneous slopes](#). *Stata Journal* 12: 61–71.
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## Also see

- [XT] [xtrc postestimation](#) — Postestimation tools for `xtrc`
- [XT] [xtreg](#) — Fixed-, between-, and random-effects and population-averaged linear models
- [XT] [xtset](#) — Declare data to be panel data
- [ME] [mixed](#) — Multilevel mixed-effects linear regression
- [MI] [Estimation](#) — Estimation commands for use with `mi estimate`
- [U] [20 Estimation and postestimation commands](#)