**xtrc** — Random-coefficients model



## Description

xtrc fits the Swamy (1970) random-coefficients linear regression model, which does not impose the assumption of constant parameters across panels. Average coefficient estimates are reported by default, but panel-specific coefficients may be requested.

## Quick start

Random-coefficients regression of y on x1 and x2 using xtset data

```
xtrc y x1 x2
```

Same as above, but report panel-specific best linear predictors

```
xtrc y x1 x2, betas
```

Multiple-imputation estimates of random-coefficients regression using mi xtset data

```
mi estimate: xtrc y x
```

## Menu

Statistics > Longitudinal/panel data > Random-coefficients regression by GLS

# Syntax

xtrc *depvar indepvars* [*if*] [*in*] [, *options*]

| *options* | Description |
|---|---|
| Main | |
| noconstant | suppress constant term |
| offset(*varname*) | include *varname* in model with coefficient constrained to 1 |
| SE | |
| vce(*vcetype*) | *vcetype* may be conventional, bootstrap, or jackknife |
| Reporting | |
| level(#) | set confidence level; default is level(95) |
| betas | display group-specific best linear predictors |
| *display_options* | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |
| coeflegend | display legend instead of statistics |

A panel variable must be specified; use xtset; see [XT] **xtset**.

*indepvars* may contain factor variables; see [U] **11.4.3 Factor variables**.

by, collect, mi estimate, and statsby are allowed; see [U] **11.1.10 Prefix commands**.

vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] **mi estimate**.

coeflegend does not appear in the dialog box.

See [U] **20 Estimation and postestimation commands** for more capabilities of estimation commands.

# Options

### Main

noconstant, offset(*varname*); see [R] **Estimation options**

### SE

vce(*vcetype*) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional) and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] ***vce_options***.

vce(conventional), the default, uses the conventionally derived variance estimator for generalized least-squares regression.

### Reporting

level(#); see [R] **Estimation options**.

betas requests that the group-specific best linear predictors also be displayed.

*display_options*: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(*style*), cformat(%*fmt*), pformat(%*fmt*), sformat(%*fmt*), and nolstretch; see [R] **Estimation options**.

The following option is available with xtrc but is not shown in the dialog box:

coeflegend; see [R] **Estimation options**.

# Remarks and examples

In random-coefficients models, we wish to treat the parameter vector as a realization (in each panel) of a stochastic process. xtrc fits the Swamy (1970) random-coefficients model, which is suitable for linear regression of panel data. See Greene (2012, chap. 11) and Poi (2003) for more information about this and other panel-data models.

## ▷ Example 1

Greene (2012, 1112) reprints data from a classic study of investment demand by Grunfeld and Griliches (1960). In [XT] **xtgls**, we use this dataset to illustrate many of the possible models that may be fit with the xtgls command. Although the models included in the xtgls command offer considerable flexibility, they all assume that there is no parameter variation across firms (the cross-sectional units).

To take a first look at the assumption of parameter constancy, we should reshape our data so that we may fit a simultaneous-equation model with sureg; see [R] **sureg**. Because there are only five panels here, this is not too difficult.

```
. use https://www.stata-press.com/data/r19/invest2

. reshape wide invest market stock, i(time) j(company)
(j = 1 2 3 4 5)

Data                                   Long   ->   Wide
-----------------------------------------------------------------------------
Number of observations                  100   ->   20
Number of variables                       5   ->   16
j variable (5 values)               company   ->   (dropped)
xij variables:
                                     invest   ->   invest1 invest2 ... invest5
                                     market   ->   market1 market2 ... market5
                                      stock   ->   stock1 stock2 ... stock5
-----------------------------------------------------------------------------
```

```
. sureg (invest1 market1 stock1) (invest2 market2 stock2)
> (invest3 market3 stock3) (invest4 market4 stock4) (invest5 market5 stock5)
Seemingly unrelated regression
```

| Equation | Obs | Params | RMSE | "R-squared" | chi2 | P>chi2 |
|---|---|---|---|---|---|---|
| invest1 | 20 | 2 | 84.94729 | 0.9207 | 261.32 | 0.0000 |
| invest2 | 20 | 2 | 12.36322 | 0.9119 | 207.21 | 0.0000 |
| invest3 | 20 | 2 | 26.46612 | 0.6876 | 46.88 | 0.0000 |
| invest4 | 20 | 2 | 9.742303 | 0.7264 | 59.15 | 0.0000 |
| invest5 | 20 | 2 | 95.85484 | 0.4220 | 14.97 | 0.0006 |

| | Coefficient | Std. err. | z | P>\|z\| | [95% conf. | interval] |
|---|---|---|---|---|---|---|
| invest1 | | | | | | |
| market1 | .120493 | .0216291 | 5.57 | 0.000 | .0781007 | .1628853 |
| stock1 | .3827462 | .032768 | 11.68 | 0.000 | .318522 | .4469703 |
| _cons | -162.3641 | 89.45922 | -1.81 | 0.070 | -337.7009 | 12.97279 |
| invest2 | | | | | | |
| market2 | .0695456 | .0168975 | 4.12 | 0.000 | .0364271 | .1026641 |
| stock2 | .3085445 | .0258635 | 11.93 | 0.000 | .2578529 | .3592362 |
| _cons | .5043112 | 11.51283 | 0.04 | 0.965 | -22.06042 | 23.06904 |
| invest3 | | | | | | |
| market3 | .0372914 | .0122631 | 3.04 | 0.002 | .0132561 | .0613268 |
| stock3 | .130783 | .0220497 | 5.93 | 0.000 | .0875663 | .1739997 |
| _cons | -22.43892 | 25.51859 | -0.88 | 0.379 | -72.45443 | 27.57659 |
| invest4 | | | | | | |
| market4 | .0570091 | .0113623 | 5.02 | 0.000 | .0347395 | .0792788 |
| stock4 | .0415065 | .0412016 | 1.01 | 0.314 | -.0392472 | .1222602 |
| _cons | 1.088878 | 6.258805 | 0.17 | 0.862 | -11.17815 | 13.35591 |
| invest5 | | | | | | |
| market5 | .1014782 | .0547837 | 1.85 | 0.064 | -.0058958 | .2088523 |
| stock5 | .3999914 | .1277946 | 3.13 | 0.002 | .1495186 | .6504642 |
| _cons | 85.42324 | 111.8774 | 0.76 | 0.445 | -133.8525 | 304.6989 |

Here we instead fit a random-coefficients model:

```
. use https://www.stata-press.com/data/r19/invest2, clear
. xtrc invest market stock
Random-coefficients regression                     Number of obs     =     100
Group variable: company                            Number of groups  =       5

Time variable: time                                Obs per group:
                                                                 min =      20
                                                                 avg =    20.0
                                                                 max =      20

                                                   Wald chi2(2)      =   17.55
                                                   Prob > chi2       =  0.0002

------------------------------------------------------------------------------
      invest | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
      market |   .0807646   .0250829     3.22   0.001     .0316031    .1299261
       stock |   .2839885   .0677899     4.19   0.000     .1511229    .4168542
       _cons |  -23.58361   34.55547    -0.68   0.495    -91.31108    44.14386
------------------------------------------------------------------------------
Test of parameter constancy: chi2(12) = 603.99          Prob > chi2 = 0.0000
```

Just as the results of our simultaneous-equation model do not support the assumption of parameter constancy, the test included with the random-coefficients model also indicates that the assumption is not valid for these data. With large panel datasets, we would not want to take the time to look at a simultaneous-equations model (aside from the fact that our doing so was subjective).

◁

## Stored results

`xtrc` stores the following in `e()`:

Scalars

| | |
|---|---|
| `e(N)` | number of observations |
| `e(N_g)` | number of groups |
| `e(df_m)` | model degrees of freedom |
| `e(chi2)` | $\chi^2$ |
| `e(chi2_c)` | $\chi^2$ for comparison test |
| `e(df_chi2c)` | degrees of freedom for comparison $\chi^2$ test |
| `e(g_min)` | smallest group size |
| `e(g_avg)` | average group size |
| `e(g_max)` | largest group size |
| `e(rank)` | rank of `e(V)` |

Macros

| | |
|---|---|
| `e(cmd)` | `xtrc` |
| `e(cmdline)` | command as typed |
| `e(depvar)` | name of dependent variable |
| `e(ivar)` | variable denoting groups |
| `e(tvar)` | variable denoting time within groups |
| `e(title)` | title in estimation output |
| `e(offset)` | linear offset variable |
| `e(chi2type)` | `Wald`; type of model $\chi^2$ test |
| `e(vce)` | *vcetype* specified in `vce()` |
| `e(properties)` | `b V` |
| `e(predict)` | program used to implement `predict` |
| `e(marginsnotok)` | predictions disallowed by `margins` |
| `e(asbalanced)` | factor variables `fvset` as `asbalanced` |
| `e(asobserved)` | factor variables `fvset` as `asobserved` |

Matrices

| | |
|---|---|
| `e(b)` | coefficient vector |
| `e(Sigma)` | $\widehat{\mathbf{\Sigma}}$ matrix |
| `e(beta_ps)` | matrix of best linear predictors |
| `e(V)` | variance–covariance matrix of the estimators |
| `e(V_ps)` | matrix of variances for the best linear predictors; row *i* contains vec of variance matrix for group *i* predictor |

Functions

| | |
|---|---|
| `e(sample)` | marks estimation sample |

In addition to the above, the following is stored in `r()`:

Matrices

| | |
|---|---|
| `r(table)` | matrix containing the coefficients with their standard errors, test statistics, $p$-values, and confidence intervals |

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

# Methods and formulas

In a random-coefficients model, the parameter heterogeneity is treated as stochastic variation. Assume that we write

$$\mathbf{y}_i = \mathbf{X}_i \boldsymbol{\beta}_i + \boldsymbol{\epsilon}_i$$

where $i = 1, \ldots, m$, and $\boldsymbol{\beta}_i$ is the coefficient vector ($k \times 1$) for the $i$th cross-sectional unit, such that

$$\boldsymbol{\beta}_i = \boldsymbol{\beta} + \boldsymbol{\nu}_i \qquad E(\boldsymbol{\nu}_i) = \mathbf{0} \qquad E(\boldsymbol{\nu}_i \boldsymbol{\nu}_i') = \mathbf{\Sigma}$$

Our goal is to find $\widehat{\boldsymbol{\beta}}$ and $\widehat{\mathbf{\Sigma}}$.

The derivation of the estimator assumes that the cross-sectional specific coefficient vector $\boldsymbol{\beta}_i$ is the outcome of a random process with mean vector $\boldsymbol{\beta}$ and covariance matrix $\mathbf{\Sigma}$,

$$\mathbf{y}_i = \mathbf{X}_i \boldsymbol{\beta}_i + \boldsymbol{\epsilon}_i = \mathbf{X}_i(\boldsymbol{\beta} + \boldsymbol{\nu}_i) + \boldsymbol{\epsilon}_i = \mathbf{X}_i \boldsymbol{\beta} + (\mathbf{X}_i \boldsymbol{\nu}_i + \boldsymbol{\epsilon}_i) = \mathbf{X}_i \boldsymbol{\beta} + \boldsymbol{\omega}_i$$

where $E(\boldsymbol{\omega}_i) = \mathbf{0}$ and

$$E(\boldsymbol{\omega}_i \boldsymbol{\omega}_i') = E\Big\{(\mathbf{X}_i \boldsymbol{\nu}_i + \boldsymbol{\epsilon}_i)(\mathbf{X}_i \boldsymbol{\nu}_i + \boldsymbol{\epsilon}_i)'\Big\} = E(\boldsymbol{\epsilon}_i \boldsymbol{\epsilon}_i') + \mathbf{X}_i E(\boldsymbol{\nu}_i \boldsymbol{\nu}_i')\mathbf{X}_i' = \sigma_i^2 \mathbf{I} + \mathbf{X}_i \mathbf{\Sigma}\, \mathbf{X}_i' = \mathbf{\Pi}_i$$

Stacking the $m$ equations, we have

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\omega}$$

where $\mathbf{\Pi} \equiv E(\boldsymbol{\omega}\boldsymbol{\omega}')$ is a block diagonal matrix with $\mathbf{\Pi}_i$, $i = 1...m$, along the main diagonal and zeros elsewhere. The GLS estimator of $\widehat{\boldsymbol{\beta}}$ is then

$$\widehat{\boldsymbol{\beta}} = \left(\sum_i \mathbf{X}_i' \mathbf{\Pi}_i^{-1} \mathbf{X}_i\right)^{-1} \sum_i \mathbf{X}_i' \mathbf{\Pi}_i^{-1} \mathbf{y}_i = \sum_{i=1}^m \mathbf{W}_i \mathbf{b}_i$$

where

$$\mathbf{W}_i = \left\{\sum_{i=1}^m (\mathbf{\Sigma} + \mathbf{V}_i)^{-1}\right\}^{-1} (\mathbf{\Sigma} + \mathbf{V}_i)^{-1}$$

$\mathbf{b}_i = (\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{X}_i'\mathbf{y}_i$ and $\mathbf{V}_i = \sigma_i^2(\mathbf{X}_i'\mathbf{X}_i)^{-1}$, showing that the resulting GLS estimator is a matrix-weighted average of the panel-specific OLS estimators. The variance of $\widehat{\boldsymbol{\beta}}$ is

$$\text{Var}(\widehat{\boldsymbol{\beta}}) = \sum_{i=1}^{m}(\boldsymbol{\Sigma} + \mathbf{V}_i)^{-1}$$

To calculate the above estimator $\widehat{\boldsymbol{\beta}}$ for the unknown $\boldsymbol{\Sigma}$ and $\mathbf{V}_i$ parameters, we use the two-step approach suggested by Swamy (1970):

$$\begin{aligned}
\mathbf{b}_i &= \text{OLS panel-specific estimator} \\
\widehat{\sigma}_i^2 &= \frac{\widehat{\boldsymbol{\epsilon}}_i'\widehat{\boldsymbol{\epsilon}}_i}{n_i - k} \\
\widehat{\mathbf{V}}_i &= \widehat{\sigma}_i^2(\mathbf{X}_i'\mathbf{X}_i)^{-1} \\
\overline{\mathbf{b}} &= \frac{1}{m}\sum_{i=1}^{m}\mathbf{b}_i \\
\widehat{\boldsymbol{\Sigma}} &= \frac{1}{m-1}\left(\sum_{i=1}^{m}\mathbf{b}_i\mathbf{b}_i' - m\overline{\mathbf{b}}\,\overline{\mathbf{b}}'\right) - \frac{1}{m}\sum_{i=1}^{m}\widehat{\mathbf{V}}_i
\end{aligned}$$

The two-step procedure begins with the usual OLS estimates of $\boldsymbol{\beta}_i$. With those estimates, we may proceed by obtaining estimates of $\widehat{\mathbf{V}}_i$ and $\widehat{\boldsymbol{\Sigma}}$ (and thus $\widehat{\mathbf{W}}_i$) and then obtain an estimate of $\boldsymbol{\beta}$.

Swamy (1970) further points out that the matrix $\widehat{\boldsymbol{\Sigma}}$ may not be positive definite and that because the second term is of order $1/(mT)$, it is negligible in large samples. A simple and asymptotically expedient solution is simply to drop this second term and instead use

$$\widehat{\boldsymbol{\Sigma}} = \frac{1}{m-1}\left(\sum_{i=1}^{m}\mathbf{b}_i\mathbf{b}_i' - m\overline{\mathbf{b}}\,\overline{\mathbf{b}}'\right)$$

As discussed by Judge et al. (1985, 541), the feasible best linear predictor of $\boldsymbol{\beta}_i$ is given by

$$\begin{aligned}
\widehat{\boldsymbol{\beta}_i} &= \widehat{\boldsymbol{\beta}} + \widehat{\boldsymbol{\Sigma}}\mathbf{X}_i'\left(\mathbf{X}_i\widehat{\boldsymbol{\Sigma}}\mathbf{X}_i' + \widehat{\sigma}_i^2\mathbf{I}\right)^{-1}\left(\mathbf{y}_i - \mathbf{X}_i\widehat{\boldsymbol{\beta}}\right) \\
&= \left(\widehat{\boldsymbol{\Sigma}}^{-1} + \widehat{\mathbf{V}}_i^{-1}\right)^{-1}\left(\widehat{\boldsymbol{\Sigma}}^{-1}\widehat{\boldsymbol{\beta}} + \widehat{\mathbf{V}}_i^{-1}\mathbf{b}_i\right)
\end{aligned}$$

The conventional variance of $\widehat{\boldsymbol{\beta}_i}$ is given by

$$\text{Var}(\widehat{\boldsymbol{\beta}_i}) = \text{Var}(\widehat{\boldsymbol{\beta}}) + (\mathbf{I} - \mathbf{A}_i)\left\{\widehat{\mathbf{V}}_i - \text{Var}(\widehat{\boldsymbol{\beta}})\right\}(\mathbf{I} - \mathbf{A}_i)'$$

where

$$\mathbf{A}_i = \left(\widehat{\boldsymbol{\Sigma}}^{-1} + \widehat{\mathbf{V}}_i^{-1}\right)^{-1}\widehat{\boldsymbol{\Sigma}}^{-1}$$

To test the model, we may look at the difference between the OLS estimate of $\boldsymbol{\beta}$, ignoring the panel structure of the data and the matrix-weighted average of the panel-specific OLS estimators. The test statistic suggested by Swamy (1970) is given by

$$\chi^2_{k(m-1)} = \sum_{i=1}^{m}(\mathbf{b}_i - \overline{\boldsymbol{\beta}}^*)'\widehat{\mathbf{V}}_i^{-1}(\mathbf{b}_i - \overline{\boldsymbol{\beta}}^*) \quad \text{where} \quad \overline{\boldsymbol{\beta}}^* = \left(\sum_{i=1}^{m}\widehat{\mathbf{V}}_i^{-1}\right)^{-1}\sum_{i=1}^{m}\widehat{\mathbf{V}}_i^{-1}\mathbf{b}_i$$

Johnston and DiNardo (1997) have shown that the test is algebraically equivalent to testing

$$H_0 : \boldsymbol{\beta}_1 = \boldsymbol{\beta}_2 = \cdots = \boldsymbol{\beta}_m$$

in the generalized (groupwise heteroskedastic) `xtgls` model, where $\mathbf{V}$ is block diagonal with $i$th diagonal element $\boldsymbol{\Pi}_i$.

## References


Eberhardt, M. 2012. Estimating panel time-series models with heterogeneous slopes. *Stata Journal* 12: 61–71.

Greene, W. H. 2012. *Econometric Analysis*. 7th ed. Upper Saddle River, NJ: Prentice Hall.

Grunfeld, Y., and Z. Griliches. 1960. Is aggregation necessarily bad? *Review of Economics and Statistics* 42: 1–13. https://doi.org/10.2307/1926089.

Johnston, J., and J. DiNardo. 1997. *Econometric Methods*. 4th ed. New York: McGraw–Hill.

Judge, G. G., W. E. Griffiths, R. C. Hill, H. Lütkepohl, and T.-C. Lee. 1985. *The Theory and Practice of Econometrics*. 2nd ed. New York: Wiley.

Nichols, A. 2007. Causal inference with observational data. *Stata Journal* 7: 507–541.

Poi, B. P. 2003. From the help desk: Swamy's random-coefficients model. *Stata Journal* 3: 302–308.

Swamy, P. A. V. B. 1970. Efficient inference in a random coefficient regression model. *Econometrica* 38: 311–323. https://doi.org/10.2307/1913012.

———. 1971. *Statistical Inference in Random Coefficient Regression Models*. New York: Springer. https://doi.org/10.1007/978-3-642-80653-7.


## Also see


[XT] **xtrc postestimation** — Postestimation tools for xtrc

[XT] **xtreg** — Linear models for panel data

[XT] **xtset** — Declare data to be panel data

[ME] **mixed** — Multilevel mixed-effects linear regression

[MI] **Estimation** — Estimation commands for use with mi estimate

**[U] 20 Estimation and postestimation commands**