

xtcointtest — Panel-data cointegration tests

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Description

`xtcointtest` performs the [Kao \(1999\)](#), [Pedroni \(1999, 2004\)](#), and [Westerlund \(2005\)](#) tests of cointegration on a panel dataset. Panel-specific means (fixed effects) and panel-specific time trends may be included in the cointegrating regression model.

All tests have a common null hypothesis of no cointegration. The alternative hypothesis of the Kao tests and the Pedroni tests is that the variables are cointegrated in all panels. In one version of the Westerlund test, the alternative hypothesis is that the variables are cointegrated in some of the panels. In another version of the Westerlund test, the alternative hypothesis is that the variables are cointegrated in all the panels.

Quick start

Kao test of no cointegration between y and x with the alternative hypothesis that they are cointegrated in all panels using `xtset` data

```
xtcointtest kao y x
```

Pedroni test of no cointegration using a panel-specific autoregressive (AR) term and panel-specific time trends with the alternative hypothesis of cointegration in all panels

```
xtcointtest pedroni y x, trend
```

As above, but use the same AR term in all panels

```
xtcointtest pedroni y x, trend ar(same)
```

Westerlund test of no cointegration with the alternative hypothesis that the variables are cointegrated in some of the panels

```
xtcointtest westerlund y x
```

Westerlund test of no cointegration with the alternative hypothesis of cointegration in all panels

```
xtcointtest westerlund y x, allpanels
```

Menu

Statistics > Longitudinal/panel data > Cointegrated data > Tests for cointegration

Syntax

Kao test

```
xtcointtest kao depvar varlist [if] [in] [, kao_options]
```

Pedroni test

```
xtcointtest pedroni depvar varlist [if] [in] [, pedroni_options]
```

Westerlund test

```
xtcointtest westerlund depvar varlist [if] [in] [, westerlund_options]
```

kao_options

Description

Main

<code><u>lags</u>(<i>lspec</i>)</code>	specify lag structure for augmented Dickey–Fuller regressions
<code><u>kernel</u>(<i>kspec</i>)</code>	specify method to estimate long-run variance
<code><u>demean</u></code>	subtract cross-sectional means

pedroni_options

Description

Main

<code><u>ar</u>(<u>panelspecific</u> <u>same</u>)</code>	specify autoregressive parameter as panel specific or as the same for all panels; <code>ar(panelspecific)</code> is the default
<code><u>trend</u></code>	include panel-specific time trends
<code><u>noconstant</u></code>	suppress panel-specific means
<code><u>lags</u>(<i>lspec</i>)</code>	specify lag structure for augmented Dickey–Fuller regressions
<code><u>kernel</u>(<i>kspec</i>)</code>	specify method to estimate long-run variance
<code><u>demean</u></code>	subtract cross-sectional means

westerlund_options

Description

Main

<code><u>somepanels</u></code>	use alternative hypothesis of cointegration in some panels; the default
<code><u>allpanels</u></code>	use alternative hypothesis of cointegration in all panels
<code><u>trend</u></code>	include panel-specific time trends
<code><u>demean</u></code>	subtract cross-sectional means

lspec is

<code>#</code>	number of lags of series; 1 is the default
<code>aic #</code>	Akaike information criterion (AIC) with up to <code>#</code> lags
<code>bic #</code>	Bayesian information criterion (BIC) with up to <code>#</code> lags
<code>hqic #</code>	Hannan–Quinn information criterion (HQIC) with up to <code>#</code> lags

kspec is

<code>bartlett nwest</code>	Bartlett kernel with Newey–West lags; the default
<code>bartlett #</code>	Bartlett kernel with up to <code>#</code> lags
<code>parzen nwest</code>	Parzen kernel with Newey–West lags
<code>parzen #</code>	Parzen kernel with up to <code>#</code> lags
<code>quadraticspectral nwest</code>	quadratic spectral kernel with Newey–West lags
<code>quadraticspectral #</code>	quadratic spectral kernel with up to <code>#</code> lags

Options

Options are presented under the following headings:

[Options for *xtcointtest* *ka*](#)
[Options for *xtcointtest* *pedroni*](#)
[Options for *xtcointtest* *westerlund*](#)

Options for *xtcointtest* *ka*

Main

`lags(lspec)` specifies the lag structure to use for the augmented Dickey–Fuller (ADF) regressions performed in computing the test statistic.

`lags(#)` specifies that `#` lags of the series be used in the ADF regressions. `#` must be a nonnegative integer. The default is `lags(1)`.

`lags(aic|bic|hqic #)` specifies that *xtcointtest* fit ADF regressions with 1 to `#` lags and choose the number of lags for which the AIC, BIC, or HQIC is minimized.

`kernel(kspec)` specifies the method used to estimate the long-run variance of each panel’s series. You may specify the kernel type and either `#`, the maximum number of lags as a positive integer, or `nwest`, the maximum number of lags selected by the bandwidth-selection algorithm given in [Newey and West \(1994\)](#). The kernel type may be `bartlett`, `parzen`, or `quadraticspectral`. The default is `kernel(bartlett nwest)`.

`demean` specifies that *xtcointtest* first subtract the cross-sectional averages from the series. When specified, for each time period *xtcointtest* computes the mean of the series across panels and subtracts this mean from the series. [Levin, Lin, and Chu \(2002\)](#) suggest this procedure to mitigate the impact of cross-sectional dependence.

Options for **xtcointtest pedroni**

Main

ar(*panelspecific* | *same*) specifies whether the AR parameter for ADF or Phillips–Perron (PP) regressions is panel specific or the same across panels.

ar(*panelspecific*) specifies that the AR parameter be panel specific in the ADF or PP regressions. The test statistics obtained from using this option are also known as group-mean statistics or between-dimension statistics. This is the default.

ar(*same*) specifies that the AR parameter be the same for all panels in the ADF or PP regressions. The test statistics obtained from using this option are also known as panel cointegration statistics or within-dimension statistics.

trend includes panel-specific linear time trends in the model for the dependent variable on the covariates.

noconstant suppresses the panel-specific means in the model for the dependent variable on the covariates. Specifying **noconstant** imposes the assumption that the series has a mean of zero for all panels. This option may not be specified with **trend**.

lags(*lspec*) specifies the lag structure to use for the ADF regressions performed in computing the test statistic. See the description of **lags**() under [Options for **xtcointtest kao**](#) for additional details.

kernel(*kspec*) specifies the method used to estimate the long-run variance of each panel's series. See the description of **kernel**() under [Options for **xtcointtest kao**](#) for additional details.

demean specifies that **xtcointtest** first subtract the cross-sectional averages from the series. See the description of **demean** under [Options for **xtcointtest kao**](#) for additional details.

Options for **xtcointtest westerlund**

Main

somepanels specifies that the test statistic for panel cointegration be computed using the alternative hypothesis that some of the panels are cointegrated. This statistic is also known as the group-mean variance-ratio (VR) statistic. This option uses a regression in which the AR parameter for Dickey–Fuller (DF) regressions is panel specific. This is the default.

allpanels specifies that the test statistic for panel cointegration be computed using the alternative hypothesis that all the panels are cointegrated, also known as the panel VR statistic. This option also implies that the AR parameter for DF regressions is the same for all panels.

trend includes panel-specific linear time trends in the model for dependent variable on the covariates.

demean specifies that **xtcointtest** first subtract the cross-sectional averages from the series. See the description of **demean** under [Options for **xtcointtest kao**](#) for additional details.

Remarks and examples

[stata.com](http://www.stata.com)

Remarks are presented under the following headings:

[Overview](#)
[Test details](#)
[Kao tests](#)
[Pedroni tests](#)
[Westerlund tests](#)

Overview

A stationary process has a time-invariant mean and a time-invariant variance. By contrast, a nonstationary process has a time-varying mean, a time-varying variance, or both. A nonstationary process may wander arbitrarily over time because its first two moments vary over time.

When the first difference of a nonstationary process is stationary, the process is said to be integrated of order one, denoted $I(1)$. When a linear combination of several $I(1)$ series is stationary, the series are said to be cointegrated (Engle and Granger 1987). We test for cointegration because cointegration implies that the $I(1)$ series are in a long-run equilibrium; they move together, although the group of them can wander arbitrarily.

For example, income and consumption are $I(1)$ series that wander over time. According to economic theory, income determines consumption in the long run. In practice, time-series data on income and consumption typically have periods where the series seem to wander in isolation, which is contrary to the theory. However, when we look at the overall trend, the two series are close to one another, implying a long-run relation. A test of cointegration provides evidence that indeed there is (or is not) a long-run relation between these series even if they tend to deviate temporarily.

`xtcointtest` implements tests of cointegration in panel data, which have many observations on each of many individual units. This type of sample is known as large- N -large- T -panel data. The popular Engle–Granger residual-based test for cointegration has low power when applied to a single time series but has good power when statistics from many individual panels are combined. The Kao tests, the Pedroni tests, and the Westerlund tests implemented in `xtcointtest` combine statistics computed for each individual in the panel, thereby producing a test with higher power. Furthermore, the limiting distribution of the combined test converges to a standard normal distribution after appropriate standardization, whereas tests for cointegration based on a single time series have nonstandard distributions.

All the tests in `xtcointtest` are based on the following panel-data model for the $I(1)$ dependent variable y_{it} , where $i = 1, \dots, N$ denotes the panel (individual) and $t = 1, \dots, T_i$ denotes time:

$$y_{it} = \mathbf{x}'_{it}\boldsymbol{\beta}_i + \mathbf{z}'_{it}\boldsymbol{\gamma}_i + e_{it} \quad (1)$$

For each panel i , each of the covariates in \mathbf{x}_{it} is an $I(1)$ series. All the tests require that the covariates are not cointegrated among themselves. The Pedroni and Westerlund tests allow a maximum of seven covariates in \mathbf{x}_{it} . $\boldsymbol{\beta}_i$ denotes the cointegrating vector, which may vary across panels. $\boldsymbol{\gamma}_i$ is a vector of coefficients on \mathbf{z}_{it} , the deterministic terms that control for panel-specific effects and linear time trends. e_{it} is the error term.

Depending on the options specified with `xtcointtest`, the vector \mathbf{z}_{it} allows for panel-specific means, panel-specific means and panel-specific time trends, or nothing. By default, $\mathbf{z}_{it} = 1$, so the term $\mathbf{z}'_{it}\boldsymbol{\gamma}_i$ represents panel-specific means (fixed effects). If `trend` is specified, $\mathbf{z}'_{it} = (1, t)$ so $\mathbf{z}'_{it}\boldsymbol{\gamma}_i$ represents panel-specific means and panel-specific linear time trends. For tests that allow it, specifying `noconstant` omits the $\mathbf{z}'_{it}\boldsymbol{\gamma}_i$ term.

The tests share a common null hypothesis that y_{it} and \mathbf{x}_{it} are not cointegrated. `xtcointtest` tests for no cointegration by testing that e_t is nonstationary. Rejection of the null hypothesis implies that e_{it} is stationary and that the series y_{it} and \mathbf{x}_{it} are cointegrated. The alternative hypothesis of the Kao tests, the Pedroni tests, and the `allpanels` version of the Westerlund test is that the variables are cointegrated in all panels. The alternative hypothesis of the `somepanels` version of the Westerlund test is that the variables are cointegrated in some of the panels.

All tests allow unbalanced panels and require that N is large enough that the distribution of a sample average of panel-level statistics converges to its population distribution. They also require that each T_i is large enough to run time-series regressions using observations only from that panel. These

tests have nominal coverage only when both T and N are large. The smallest combinations of T and N for which the tests have close to nominal coverage and decent power differs by test and varies with the degree of serial correlation in the residuals. See [Test details](#) for more information. All the tests require that there be no gaps in any panel's series.

Test details

The Kao, Pedroni, and Westerlund tests implement different types of tests for whether e_{it} is nonstationary. The DF t tests, ADF t tests, PP t tests, and their variants that are reported by `xtcointtest kao` and `xtcointtest pedroni` use different regression frameworks to handle serial correlation in e_{it} . The VR tests that are reported by `xtcointtest westerlund` and `xtcointtest pedroni` do not require modeling or accommodating for serial correlation; see [Westerlund \(2005\)](#).

All variants of the DF t test statistics are constructed by fitting the model in (1) using ordinary least squares, obtaining the predicted residuals (\hat{e}_{it}), and then fitting the DF regression model

$$\hat{e}_{it} = \rho \hat{e}_{i,t-1} + \nu_{it} \quad (2)$$

where ρ is the AR parameter and ν_{it} is a stationary error term. The DF t and the unadjusted DF t test whether the coefficient ρ is 1. By contrast, the modified DF t and the unadjusted modified DF t test whether $\rho - 1 = 0$. Nonstationarity under the null hypothesis causes a test of whether $\rho = 1$ to differ from a test of whether $\rho - 1 = 0$; see [Dickey and Fuller \(1979\)](#) and [Kao \(1999\)](#).

The variants of the PP t test statistics are also constructed by fitting the model in (1) using ordinary least squares and obtaining the predicted residuals (\hat{e}_{it}). For the PP t tests, we then fit the DF regression model

$$\hat{e}_{it} = \rho_i \hat{e}_{i,t-1} + \nu_{it} \quad (3)$$

In this case, we have a panel-specific AR parameter ρ_i . The PP t tests whether the ρ_i s are 1, whereas the modified PP t tests whether $\rho_i - 1 = 0$. The PP t test statistic is nonparametrically adjusted for serial correlation in the residuals using the [Newey and West \(1987\)](#) heteroskedasticity- and autocorrelation-consistent (HAC) covariance matrix estimator.

The DF t , the modified DF t , the PP t , the modified PP t , and the modified VR tests are derived by specifying a data-generating process for the dependent variable and the regressors. This specification allows the regressors to be endogenous as well as serially correlated. Therefore, constructing the test statistics requires estimating the contemporaneous and dynamic covariances between the regressors and the dependent variable. The unadjusted DF t and the unadjusted modified DF t assume absence of serial correlation and strictly exogenous covariates and do not require any adjustments in the residuals.

Like the DF and PP tests, the ADF t tests that $\rho = 1$. However, the ADF test uses additional lags of the residuals to control for serial correlation instead of the Newey–West nonparametric adjustments. The ADF regression is

$$\hat{e}_{it} = \rho_i \hat{e}_{i,t-1} + \sum_{j=1}^p \rho_{ij} \Delta \hat{e}_{i,t-j} + \nu_{it}^*$$

where $\Delta \hat{e}_{i,t-j}$ is the j th lag of the first difference of \hat{e}_{it} and $j = 1, \dots, p$ is where p is the number of lag differences.

The VR tests are based on [Phillips and Ouliaris \(1990\)](#) and [Breitung \(2002\)](#), where the test statistic is constructed as a ratio of variances. These tests do not require modeling or accommodating serial correlation; see [Westerlund \(2005\)](#). VR tests also test for no cointegration by testing for the presence

of a unit root in the residuals. However, they do so using the ratio of variances of the predicted residuals. The modified VR test removes estimated conditional variances prior to computing the VR. For further details, see *Methods and formulas*.

These tests get good coverage and power properties by combining panel-level statistics computed from a time-series regression using only the observations in that panel. [Kao \(1999\)](#) finds that his tests have nearly nominal size when $T = 100$ and $N = 300$. [Pedroni \(2004\)](#) finds that his tests have nearly nominal size when $T = 250$ and $N = 60$. [Westerlund \(2005\)](#) limited his simulations to datasets with $T = 150$, and he did not find a combination of T and N in which his tests had nearly nominal size. He said that $T > 150$ should produce better coverage. Each author used a different data-generating process; see [Kao \(1999\)](#), [Pedroni \(2004\)](#), and [Westerlund \(2005\)](#) for details.

□ Technical note

The asymptotic distribution of all the test statistics are obtained using sequential limit theory, denoted as $(T, N) \rightarrow_{\text{seq}} \infty$, in which the time dimension goes to infinity followed by the number of panels going to infinity. See [Phillips and Moon \(2000\)](#) for an introduction to asymptotic theory that depend on both N and T and their relation to nonstationary panels. [Phillips and Moon \(1999\)](#) contains a more technical discussion of “multi-indexed” asymptotic theory. □

Kao tests

The tests derived in [Kao \(1999\)](#) assume a cointegrating vector that is the same across all panels, which restricts $\beta_i = \beta$ in (1). Kao tests estimate panel-specific means and do not allow a time trend, so \mathbf{z} from (1) is always a vector of 1s for Kao tests. This yields the cointegrating relationship

$$y_{it} = \gamma_i + \mathbf{x}'_{it}\beta + e_{it}$$

where γ_i denotes panel-specific means (fixed effects). The null hypothesis of the Kao test is that there is no cointegration among the series. The alternative hypothesis is that the series in all panels are cointegrated with the same cointegrating vector.

`xtcointtest kao` reports the modified DF t , DF t , ADF t , unadjusted modified DF t , and unadjusted DF t statistics. They are constructed using the estimated ρ from DF and ADF regressions; see *Test details*. The test statistics differ in how they formulate the hypothesis and in how they control for serial correlation in e_{it} . See *Test details* for an overview of the differences in the test statistics and see *Kao tests* in *Methods and formulas* for further discussion.

▷ Example 1: Kao tests assuming a constant cointegrating vector

We are interested in the long-run effect of domestic research and development (R&D) and foreign R&D on an economy’s productivity. The fictitious dataset, `xtcoint.dta`, is a balanced panel on 100 countries observed from 1973q3 to 2010q4. It contains quarterly data on the log of productivity (`productivity`), log of domestic R&D capital stock (`rddomestic`), and log of foreign R&D (`rdforeign`).

The cointegrating relationship is specified as

$$\text{productivity}_{it} = \gamma_i + \beta_1 \text{rddomestic}_{it} + \beta_2 \text{rdforeign}_{it} + e_{it}$$

where γ_i is the panel-specific mean and the cointegrating parameters, β_1 and β_2 , are the same across panels. We assume that each series is $I(1)$. A formal test for the presence of a unit root in panel data may be performed using `xtunitroot`. We perform the Kao test of cointegration by typing

```

. use http://www.stata-press.com/data/r15/xtcoint
(Fictitious cointegration data)
. xtcointtest kao productivity rddomestic rdforeign
Kao test for cointegration

```

Ho: No cointegration	Number of panels	=	100
Ha: All panels are cointegrated	Number of periods	=	148
Cointegrating vector: Same			
Panel means:	Included	Kernel:	Bartlett
Time trend:	Not included	Lags:	3.60 (Newey-West)
AR parameter:	Same	Augmented lags:	1

	Statistic	p-value
Modified Dickey-Fuller t	-23.6733	0.0000
Dickey-Fuller t	-15.1293	0.0000
Augmented Dickey-Fuller t	-3.6909	0.0001
Unadjusted modified Dickey-Fuller t	-46.7561	0.0000
Unadjusted Dickey-Fuller t	-20.2521	0.0000

We used a model with panel-specific means and no time trend, as reported in the header. The AR parameter that determines the presence or lack of cointegration is assumed to be the same for all panels and is thus labeled as *Same* in the header.

By default, `xtcointtest kao` uses a Bartlett kernel with [Newey and West \(1994\)](#) automatic lag selection algorithm. In this example, the algorithm chose an average of 3.6 lags across all panels to correct for serial correlation. To choose different kernels and the number of lags, specify the `kernel()` option. The ADF t statistic also includes lagged differences of the dependent variable to control for serial correlation. The number of lags is reported in *Augmented lags*. By default, `xtcointtest kao` uses the first lag. To include more lags, specify the `lags()` option.

The output reports the values of all test statistics with their respective p -values. All test statistics reject the null hypothesis of no cointegration in favor of the alternative hypothesis of the existence of a cointegrating relation among *productivity*, *rddomestic*, and *rdforeign*. The modified DF t , the DF t , and the ADF t test statistics are adjusted for serial correlation using the HAC estimator; see [Methods and formulas](#).

◀

Pedroni tests

The tests derived by [Pedroni \(1999, 2004\)](#) allow for panel-specific cointegrating vectors. This heterogeneity distinguishes Pedroni tests from those derived by Kao. Another difference is that the Pedroni tests allow the AR coefficient (ρ_i) to vary over panels as in (3), while the Kao tests assumed the same AR coefficient. These panel-specific AR coefficients are the default in the Pedroni tests, but the `ar(same)` option restricts the AR coefficients ($\rho_i = \rho$) to be the same over panels.

[Pedroni \(1999, 2004\)](#) refers to the tests based on panel-specific AR parameters as “between-dimension tests” and refers to the tests based on the same AR parameters as “within-dimension tests”.

See [Test details](#) and [Methods and formulas](#) for further discussion of the specific tests.

▷ Example 2: Pedroni cointegration test with panel-specific AR parameter

Continuing with [example 1](#), we perform the Pedroni test of cointegration between productivity, rddomestic, and rdforeign, assuming panel-specific cointegrating vectors and autoregressive parameters. The cointegrating relationship is specified as

$$\text{productivity}_{it} = \gamma_i + \beta_{1i}\text{rddomestic}_{it} + \beta_{2i}\text{rdforeign}_{it} + e_{it}$$

where β_{1i} and β_{2i} represent panel-specific cointegration parameters.

```
. xtcointtest pedroni productivity rddomestic rdforeign
Pedroni test for cointegration
```

Ho: No cointegration	Number of panels	=	100
Ha: All panels are cointegrated	Number of periods	=	149
Cointegrating vector: Panel specific			
Panel means:	Included	Kernel:	Bartlett
Time trend:	Not included	Lags:	4.00 (Newey-West)
AR parameter:	Panel specific	Augmented lags:	1

	Statistic	p-value
Modified Phillips-Perron t	-26.1145	0.0000
Phillips-Perron t	-21.2436	0.0000
Augmented Dickey-Fuller t	-25.3701	0.0000

All the test statistics reject the null hypothesis of no cointegration in favor of the alternative hypothesis that productivity, rddomestic, and rdforeign are cointegrated in all panels with a panel-specific cointegrating vector.

The model underlying the reported statistics includes panel-specific means and panel-specific AR parameters and does not include a time trend. All three statistics used a Bartlett kernel with four lags, as selected by the Newey–West methods, to adjust for serial correlation. The ADF test used a regression with only one additional lag.

◀

▷ Example 3: Pedroni cointegration test with a common AR parameter

The alternative hypothesis in [example 2](#) allows for panel-specific AR parameters. In this example, we use the ar(same) option to specify an alternative hypothesis that assumes the same AR parameter across all panels.

```
. xtcointtest pedroni productivity rddomestic rdforeign, ar(same)
Pedroni test for cointegration
```

Ho: No cointegration	Number of panels	=	100
Ha: All panels are cointegrated	Number of periods	=	149
Cointegrating vector: Panel specific			
Panel means:	Included	Kernel:	Bartlett
Time trend:	Not included	Lags:	4.00 (Newey-West)
AR parameter:	Same	Augmented lags:	1

	Statistic	p-value
Modified variance ratio	14.8852	0.0000
Modified Phillips-Perron t	-23.0042	0.0000
Phillips-Perron t	-17.6735	0.0000
Augmented Dickey-Fuller t	-21.1119	0.0000

All test statistics reject the null hypothesis of no cointegration in favor of the alternative hypothesis of cointegration between `productivity`, `rddomestic`, and `rdforeign`.

The header reports `Same` for the AR parameter, reminding us that we are now using an alternative hypothesis that assumes a constant ρ for all panels.

◀

Westerlund tests

[Westerlund \(2005\)](#) derived a pair of VR test statistics for the null hypothesis of no cointegration. The default test uses a model in which the AR parameter is panel specific and for which the alternative hypothesis is that the series in some of the panels are cointegrated. Specifying the `allpanels` option produces the results for a test in which the alternative hypothesis is that the series in all the panels are cointegrated, and this test uses a model in which the AR parameter is the same over the panels. More specifically, the alternative hypothesis using the `allpanels` option restricts $\rho_i = \rho$ in (3).

See [Test details](#) and [Methods and formulas](#) for further discussion of the specific tests.

► Example 4: Westerlund test with some panels cointegrated under the alternative

Continuing with [example 1](#), we perform the Westerlund test of cointegration between `productivity`, `rddomestic`, and `rdforeign`. The cointegrating relationship is specified as

$$\text{productivity}_{it} = \gamma_i + \beta_{1i}\text{rddomestic}_{it} + \beta_{2i}\text{rdforeign}_{it} + e_{it}$$

where β_{1i} and β_{2i} are panel-specific cointegration parameters. We now test the null hypothesis of no cointegration under the alternative that some of the β_{1i} and β_{2i} produce cointegrated series:

```
. xtointtest westerlund productivity rddomestic rdforeign
```

```
Westerlund test for cointegration
```

Ho: No cointegration	Number of panels	=	100
Ha: Some panels are cointegrated	Number of periods	=	150
Cointegrating vector:	Panel specific		
Panel means:	Included		
Time trend:	Not included		
AR parameter:	Panel specific		

	Statistic	p-value
Variance ratio	-8.0237	0.0000

The VR test statistic rejects the null hypothesis of no cointegration between `productivity`, `rddomestic`, and `rdforeign` in favor of the alternative that at least some panels are cointegrated.

The header tells us that the cointegrating vectors vary by panel, that panel-specific means were included in the model, that no time trend was included in the model, and that the AR parameter varies by panel.

◀

► Example 5: Westerlund test with all panels cointegrated under the alternative

In this example, we use the `allpanels` option to test the null hypothesis of no cointegration under the alternative hypothesis that all panels are cointegrated. This test is based on a model in which the AR parameter is the same over the panels.

```
. xtcointtest westerlund productivity rddomestic rdforeign, allpanels
Westerlund test for cointegration
```

Ho: No cointegration	Number of panels	=	100
Ha: All panels are cointegrated	Number of periods	=	150
Cointegrating vector: Panel specific			
Panel means:	Included		
Time trend:	Not included		
AR parameter:	Same		

	Statistic	p-value
Variance ratio	-5.9709	0.0000

The VR statistic rejects the null hypothesis of no cointegration. This implies all panels are cointegrated.

◀

Stored results

`xtcointtest kao` stores the following in `r()`:

Scalars

<code>r(N)</code>	number of observations
<code>r(N_g)</code>	number of groups
<code>r(N_t)</code>	number of time periods
<code>r(hac_lagm)</code>	average lags used in HAC variance estimator
<code>r(adf_lags)</code>	lags used in ADF regressions

Macros

<code>r(test)</code>	<code>kao</code>
<code>r(hac_kernel)</code>	kernel used in HAC variance estimator
<code>r(hac_method)</code>	HAC lag-selection algorithm
<code>r(adf_method)</code>	ADF regression lag-selection criterion
<code>r(demean)</code>	demean, if the data were demeaned
<code>r(deterministics)</code>	constant

Matrices

<code>r(stats)</code>	Kao test statistics
<code>r(pvals)</code>	<i>p</i> -values

`xtcointtest pedroni` stores the following in `r()`:

Scalars

<code>r(N)</code>	number of observations
<code>r(N_g)</code>	number of groups
<code>r(N_t)</code>	number of time periods
<code>r(hac_lagm)</code>	average lags used in HAC variance estimator
<code>r(adf_lags)</code>	lags used in ADF regressions

Macros

<code>r(test)</code>	<code>pedroni</code>
<code>r(hac_kernel)</code>	kernel used in HAC variance estimator
<code>r(hac_method)</code>	HAC lag-selection algorithm

<code>r(adf_method)</code>	ADF regression lag-selection criterion
<code>r(demean)</code>	<code>demean</code> , if the data were demeaned
<code>r(deterministics)</code>	<code>noconstant</code> , <code>constant</code> , or <code>trend</code>

Matrices

<code>r(stats)</code>	Pedroni test statistics
<code>r(pvals)</code>	<i>p</i> -values

`xtcointtest westerlund` stores the following in `r()`:

Scalars

<code>r(N)</code>	number of observations
<code>r(N_g)</code>	number of groups
<code>r(N_t)</code>	number of time periods
<code>r(stat)</code>	Westerlund test statistic
<code>r(pval)</code>	<i>p</i> -value

Macros

<code>r(test)</code>	<code>westerlund</code>
<code>r(demean)</code>	<code>demean</code> , if the data were demeaned
<code>r(deterministics)</code>	<code>constant</code> or <code>trend</code>

Methods and formulas

Methods and formulas are presented under the following headings:

[Overview](#)
[Kao tests](#)
[Pedroni tests](#)
[Westerlund tests](#)
[Long-run variance](#)

Overview

Consider the panel-data model

$$y_{it} = \mathbf{x}'_{it}\boldsymbol{\beta}_i + \mathbf{z}'_{it}\boldsymbol{\gamma}_i + e_{it} \quad (4)$$

where $i = 1, \dots, N$ denotes the panel and $t = 1, \dots, T_i$ denotes time. For each i , y_{it} is a nonstationary dependent variable for which the first difference is stationary, which is to say that y_{it} is integrated of order 1—denoted $I(1)$ —for each panel. Similarly, \mathbf{x}_{it} is a $k \times 1$ vector of $I(1)$ variables. $\boldsymbol{\beta}_i$ denotes the cointegrating vector that may vary across panels. \mathbf{z}_{it} contains terms to control for panel-specific effects and or panel-specific time trends. $\boldsymbol{\gamma}_i$ denotes the coefficients on the deterministic terms such as panel-specific means and panel-specific linear time trends. e_{it} is an error term.

The vector \mathbf{z}_{it} allows for panel-specific means, panel-specific means and panel-specific time trends, or nothing, depending on the options specified to `xtcointtest`. By default, $\mathbf{z}_{it} = 1$, so the term $\mathbf{z}'_{it}\boldsymbol{\gamma}_i$ represents panel-specific means (fixed effects). If `trend` is specified, then $\mathbf{z}'_{it} = (1, t)$, so $\mathbf{z}'_{it}\boldsymbol{\gamma}_i$ represents panel-specific means and panel-specific linear time trends. For tests that allow it, specifying `noconstant` omits the $\mathbf{z}'_{it}\boldsymbol{\gamma}_i$ term.

The data-generating process for y_{it} and \mathbf{x}_{it} is given by

$$\begin{aligned} y_{it} &= y_{i,t-1} + u_{it} \\ \mathbf{x}_{it} &= \mathbf{x}_{i,t-1} + \boldsymbol{\epsilon}_{it} \end{aligned}$$

Let $\mathbf{w}_{it} = (u_{it}, \epsilon_{it})'$ denote a $(k+1) \times 1$ vector process with zero mean and long-run covariance matrix Ω_i . (A long-run covariance matrix is a covariance matrix that accounts for the serial correlation in the process; see Hall (2005, sec. 3.5) for an introduction.) The long-run matrix can be decomposed as $\Omega_i = \Sigma_i + \Gamma_i' + \Gamma_i$, where Σ_i and Γ_i denote the contemporaneous and autocovariance matrices for a given panel i . The elements of long-run and contemporaneous matrices Ω_i and Σ_i are given by

$$\Omega_i = \begin{bmatrix} \omega_{u,i}^2 & \Omega_{u\epsilon,i} \\ \Omega'_{u\epsilon,i} & \Omega_{\epsilon,i} \end{bmatrix}$$

$$\Sigma_i = \begin{bmatrix} \sigma_{u,i}^2 & \Sigma_{u\epsilon,i} \\ \Sigma'_{u\epsilon,i} & \Sigma_{\epsilon,i} \end{bmatrix}$$

We obtain consistent estimators $\hat{\Omega}_i$ and $\hat{\Sigma}_i$ using Newey and West (1987).

Kao tests

Kao (1999) assumes the same cointegrating vector $\beta_i = \beta$ in (4) so that all panels share a common slope coefficient. This implies a common long-run covariance matrix given by $\Omega = \Sigma + \Gamma' + \Gamma$. The regression model is

$$y_{it} = \gamma_i + \mathbf{x}'_{it}\beta + e_{it}$$

where γ_i denotes panel-specific fixed effects and β is the same cointegrating vector.

Kao (1999) proposes five test statistics. The DF t , the modified DF t , the unadjusted DF t , and the unadjusted modified DF t are based on the DF regression

$$\hat{e}_{it} = \rho \hat{e}_{i,t-1} + \nu_{it}$$

where ρ is the common AR parameter of the estimated residuals.

The test statistics based on DF regressions are

$$\text{DF } t = \frac{t_\rho + \frac{\sqrt{6N} \hat{\sigma}_\nu}{2\hat{\omega}_\nu}}{\sqrt{\frac{\hat{\omega}_\nu^2}{2\hat{\sigma}_\nu^2} + \frac{3\hat{\sigma}_\nu^2}{10\hat{\omega}_\nu^2}}}$$

$$\text{Modified DF } t = \frac{\sqrt{NT}(\hat{\rho} - 1) + \frac{3\sqrt{N} \hat{\sigma}_\nu^2}{\hat{\omega}_\nu^2}}{\sqrt{3 + \frac{36\hat{\sigma}_\nu^4}{5\hat{\omega}_\nu^4}}}$$

where $\hat{\rho}$ is the estimated value of ρ . $\hat{\sigma}_\nu^2$ and $\hat{\omega}_\nu^2$ are scalar terms that are consistent estimates of $\sigma_\nu^2 = \sigma_u^2 - \Sigma'_{u\epsilon} \Sigma_\epsilon \Sigma_{u\epsilon}$ and $\omega_\nu^2 = \omega_u^2 - \Omega'_{u\epsilon} \Omega_\epsilon \Omega_{u\epsilon}$. t_ρ is the t statistic for testing the null hypothesis $H_0: \rho = 1$.

The DF test statistics that assume strict exogeneity and absence of serial correlation are given by

$$\text{Unadjusted DF } t = \sqrt{\frac{5t_\rho}{4}} + \sqrt{\frac{15N}{8}}$$

$$\text{Unadjusted modified DF } t = \frac{\sqrt{NT}(\hat{\rho} - 1) + 3\sqrt{N}}{\sqrt{51/5}}$$

The ADF regression is given by

$$\widehat{e}_{it} = \rho \widehat{e}_{i,t-1} + \sum_{j=1}^p \rho_j \Delta \widehat{e}_{i,t-j} + \nu_{it}^* \quad (5)$$

where p is the number of lagged difference terms.

The test statistic based on ADF regression is

$$\text{ADF } t = \frac{t_{\text{ADF}} + \frac{\sqrt{6N}\widehat{\sigma}_\nu}{2\widehat{\omega}_\nu}}{\sqrt{\frac{\widehat{\omega}_\nu^2}{2\widehat{\sigma}_\nu^2} + \frac{3\widehat{\sigma}_\nu^2}{10\widehat{\omega}_\nu^2}}}$$

where

$$t_{\text{ADF}} = \frac{\widehat{\rho}}{\widehat{\text{SE}}(\widehat{\rho})}$$

is computed from the ADF regression.

The asymptotic distribution of all test statistics converge to $N(0, 1)$.

Pedroni tests

Pedroni (1999) assumes a panel-specific cointegrating vector as in (4), where all panels have individual slope coefficients. The panel cointegration tests are obtained by testing for a unit root in the estimated residuals using the ADF regression in (5) but allowing panel-specific ρ_i instead of ρ or using the PP regressions given in Pedroni (1999).

Pedroni (1999, 2004) derives test statistics based on a model in which the AR parameter either is panel-specific or is the same over the panels. Pedroni (1999, 2004) calls the panel-specific-AR test statistics “group-mean statistics” and the same-AR test statistics “panel cointegration statistics”.

The panel-specific-AR test statistics are

$$\begin{aligned} \text{Modified PP } t &= TN^{-1/2} \sum_{i=1}^N \left(\sum_{t=1}^T \widehat{e}_{i,t-1}^2 \right)^{-1} \sum_{t=1}^T (\widehat{e}_{i,t-1} \Delta \widehat{e}_{i,t} - \widehat{\lambda}_i) \\ \text{PP } t &= N^{-1/2} \sum_{i=1}^N \left(\widehat{\sigma}_i^2 \sum_{t=1}^T \widehat{e}_{i,t-1}^2 \right)^{-1/2} \sum_{t=1}^T (\widehat{e}_{i,t-1} \Delta \widehat{e}_{i,t} - \widehat{\lambda}_i) \\ \text{ADF } t &= N^{-1/2} \sum_{i=1}^N \left(\sum_{t=1}^T \widehat{s}_i^{*2} \widehat{e}_{i,t-1}^2 \right)^{-1/2} \sum_{t=1}^T \widehat{e}_{i,t-1} \Delta \widehat{e}_{i,t} \end{aligned}$$

where \widehat{e}_{it} are the residuals from the panel-data regression model in (4). We calculate

$$\widehat{\lambda}_i = \frac{1}{2}(\widehat{\sigma}_i^2 - \widehat{s}_i^2)$$

where \widehat{s}_i^2 and $\widehat{\sigma}_i^2$ are the individual contemporaneous and long-run variances of the residuals from the DF regression in (3). \widehat{s}_i^{*2} is the individual contemporaneous variance of the residuals from the ADF regression in (5) but with panel-specific ρ_i instead of ρ .

The same-AR test statistics are

$$\begin{aligned} \text{Modified VR} &= T^2 N^{3/2} \left(\sum_{i=1}^N \sum_{t=1}^T \widehat{L}_{11i}^{-2} \widehat{\epsilon}_{i,t-1}^2 \right)^{-1} \\ \text{Modified PP } t &= T\sqrt{N} \left(\sum_{i=1}^N \sum_{t=1}^T \widehat{L}_{11i}^{-2} \widehat{\epsilon}_{i,t-1}^2 \right)^{-1} \sum_{i=1}^N \sum_{t=1}^T \widehat{L}_{11i}^{-2} (\widehat{\epsilon}_{i,t-1} \Delta \widehat{\epsilon}_{i,t} - \widehat{\lambda}_i) \\ \text{PP } t &= \left(\widehat{\sigma}_{N,T}^2 \sum_{i=1}^N \sum_{t=1}^T \widehat{L}_{11i}^{-2} \widehat{\epsilon}_{i,t-1}^2 \right)^{-1/2} \sum_{i=1}^N \sum_{t=1}^T \widehat{L}_{11i}^{-2} (\widehat{\epsilon}_{i,t-1} \Delta \widehat{\epsilon}_{i,t} - \widehat{\lambda}_i) \\ \text{ADF } t &= \left(\widehat{s}_{N,T}^{*2} \sum_{i=1}^N \sum_{t=1}^T \widehat{L}_{11i}^{-2} \widehat{\epsilon}_{i,t-1}^2 \right)^{-1/2} \sum_{i=1}^N \sum_{t=1}^T \widehat{L}_{11i}^{-2} \widehat{\epsilon}_{i,t-1} \Delta \widehat{\epsilon}_{i,t} \end{aligned}$$

where the residuals are as defined above and where

$$\begin{aligned} \widehat{\sigma}_{N,T}^2 &= \frac{1}{N} \sum_{i=1}^N \widehat{L}_{11i}^{-2} \widehat{\sigma}_i^2 \\ \widehat{L}_{11i} &= \widehat{\omega}_{u,i}^2 - \widehat{\Omega}_{u\epsilon,i} \widehat{\Omega}_{\epsilon,i} \widehat{\Omega}_{u\epsilon,i}' \end{aligned}$$

and

$$\widehat{s}_{N,T}^{*2} = \frac{1}{N} \sum_{i=1}^N \widehat{s}_i^{*2}$$

The asymptotic distribution of all test statistics, after appropriate standardization, converges to $N(0, 1)$. The adjustment is given by

$$\frac{\chi - \mu\sqrt{N}}{\sqrt{\nu}}$$

where χ is any of the test statistics given above, and the parameters μ and ν are the mean and variance of the test statistic obtained through simulation. Refer to [Pedroni \(1999\)](#) for details and an algorithm to obtain the predicted residuals. The adjusted statistics are reported in the output.

Westerlund tests

[Westerlund \(2005\)](#) assumes panel-specific cointegrating vectors as in (4), where all panels have individual slope coefficients. The VR test statistics are obtained by testing for a unit root in the predicted residuals using the DF regression in (3).

[Westerlund \(2005\)](#) derives test statistics based on a model in which the AR parameter either is panel-specific or is the same over the panels.

The panel-specific-AR test statistic is used to test the null hypothesis of no cointegration against the alternative hypothesis that some panels are cointegrated. The same-AR test statistic is used to test the null hypothesis of no cointegration against the alternative hypothesis that all the panels are cointegrated.

The panel-specific-AR test statistic is given by

$$\text{VR} = \sum_{i=1}^N \sum_{t=1}^T \widehat{E}_{it}^2 \widehat{R}_i^{-1}$$

The same-AR test statistic is given by

$$\text{VR} = \sum_{i=1}^N \sum_{t=1}^T \widehat{E}_{it}^2 \left(\sum_{i=1}^N \widehat{R}_i \right)^{-1}$$

where $\widehat{E}_{it} = \sum_{j=1}^t \widehat{e}_{ij}$, $\widehat{R}_i = \sum_{t=1}^T \widehat{e}_{it}^2$, and \widehat{e}_{it} are the residuals from the panel-data regression model in (4). The asymptotic distribution of all test statistics, after appropriate standardization, converges to $N(0, 1)$.

Long-run variance

We use the [Newey and West \(1987\)](#) estimator to consistently estimate the long-run variance matrix Ω_i , given by

$$\widehat{\Omega}_i = \frac{1}{T} \sum_{t=1}^T \widehat{\mathbf{w}}_{it} \widehat{\mathbf{w}}'_{it} + \frac{1}{T} \sum_{j=1}^m K(j, m) \sum_{t=j+1}^T (\widehat{\mathbf{w}}_{it} \widehat{\mathbf{w}}'_{it} + \widehat{\mathbf{w}}_{i,t-j} \widehat{\mathbf{w}}'_{it})$$

where m is the maximum number of lags and $K(j, m)$ is the kernel weight function. Define $z = j/(m + 1)$. If `kernel` is `bartlett`, then

$$K(j, m) = \begin{cases} 1 - z & 0 \leq z \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

If `kspec` is `parzen`, then

$$K(j, m) = \begin{cases} 1 - 6z^2 + 6z^3 & 0 \leq z \leq 0.5 \\ 2(1 - z)^3 & 0.5 < z \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

If `kernel` is `quadraticspectral`, then

$$K(j, m) = \begin{cases} 1 & z = 0 \\ 3\{\sin(\theta)/\theta - \cos(\theta)\}/\theta^2 & \text{otherwise} \end{cases}$$

where $\theta = 6\pi z/5$. If we request automatic bandwidth (lag) selection using the Newey–West algorithm, then the method documented in [Methods and formulas](#) of [\[R\] ivregress](#) with $\mathbf{z}_i = \mathbf{h} = 1$ is used.

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Also see

- [XT] [xtunitroot](#) — Panel-data unit-root tests
- [TS] [dfgls](#) — DF-GLS unit-root test
- [TS] [dfuller](#) — Augmented Dickey–Fuller unit-root test
- [TS] [pperron](#) — Phillips–Perron unit-root test